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Determination of the Mathematical Model for Plasma Electronic Coefficients of the Earth's Ionosphere

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Abstract

In this research, electron coefficients such as total collision frequency ($\boldsymbol{v_{tot}/N}$), total ionization frequency ($\boldsymbol{v}_{iz}/\boldsymbol{N}$), and Power (P/N) for different gases such as (Ar, He, N_2 and O_2) in Earth's ionosphere have been calculated by applying the Boltzmann equation utilizing BOLSIG +, and it has been discovered that there is a significant impact of reducing the electric field (E/N) on electronic coefficients under which (E/N) increases. In addition, influence of (E/N) on electronic coefficients was studied. Reducing the electric field was chosen in the restricted range (1-100) Td, and the electronic coefficients for gases in the limited range (50-2000) km of the Earth's ionosphere. A positive correlation has been explained between all the plasma electronic coefficients and (E/N) for the four gases. It was found that He gas has higher electronic coefficients than other gases in the Earth's ionosphere because they have lower electron binding energy.

Keywords: Ionosphere; Total collision frequency, Total ionization frequency, Power, Electric field, Boltzmann Equation; Gas Discharge

تحديد النموذج الرياضي لمعاملات البلازما الإلكترونية للغلاف الأيوني للأرض

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3

الخلاصة

في هذا البحث ، تم حساب معاملات الإلكترون مثل تردد التصادم الكلي ($\boldsymbol{v_{tot}/N}$) ، تردد التأين الكلي ($\boldsymbol{v}_{iz}/\ \boldsymbol{N}$) والطاقة (P/N) لغازات مختلفة مثل (Ar, He, N_2 and O_2) في طبقة الأيونوسفير للأرض والتي تم حسابها باستخدام BOLSIG + للتحقق من النتائج التي نحصل عليها من حل معادلة بولتزمان بالإضافة إلى دراسة تأثير المجال الكهربائي المختزل على المعاملات الإلكترونية تم اختيار المجال الكهربائي في المدى المحدود (1–100) Td والمعاملات الألكترونية للغازات في المدى المحدود (50 – 2000) كم للغلاف الأيوني للأرض وتم شرح ارتباط مباشر بين جميع معاملات البلازما الالكترونية و (E/N) للغازات الأربع ,

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لقد وجد ان غاز He لها معاملات الكترونية أعلى من الغازات الأخرى في الغلاف الأيوني للأرض وذلك لامتلاكها طاقة ربط الكتروني واطئة.

1. Introduction

Plasma is a neutral gas composed of neutral and charged particles, such as the upper atmosphere, interstellar matter, and the Earth's ionosphere [1,2]. There are many applications of plasma like solar cells, photodetectors, optoelectronic devices and biomedical device [3-5]. The electron transport coefficients were calculated and rate coefficients were used in fluid models of gas discharges from collision cross-section data by achieving equation of Boltzmann with BOLSIG+ (program) [6]. The effect of reduced electric field on the electronic coefficients was calculated by applying Boltzmann Equation (BE) using BOLSIG + [7]. The transport coefficients for the atmospheres of the planets of our solar system were calculated by BOLSIG +, and the Boltzmann equation was numerically solved [8]. The ionosphere is divided into several layers or zones during the day based on the local time of day, and the concentration of ions varies in these layers depending on the type of interaction with X-ray and EUV or solar radiation from the Sun [9,10]. The Earth's ionosphere is split into three major regions: D (50-90) km, E (90-150) km, and F (150-500) km. For radio waves, D-region acts as both a refracting and an absorbing medium[11]. The F-layer is the highest and most heavily ionized region; the F layer is the most important layer for long-distance communication. During the day, it is frequently divided into sub-layers known as F1 (150-250) km and F2 (250-500) km[12].The F-layers are responsible for reflection of most sky wave propagation of radio waves on the side of the Earth facing [11]. Gas-discharge plasma is a low-temperature plasma caused by an electric current flowing through a gas under the influence of external fields [13]. Ionization in D layer is not very strong. The major ion components are NO^+ and O_2^+, which are ionized from N_2 and O_2. Below 100 km, the dominant atmosphere species are N_2 and O_2, which account for approximately 78 % and 21% of the atmosphere, respectively. The ratio of N_2 to O_2 is approximately 4:1. Above 100 km, the density of O_2 drops sharply and disappears around 200 km. This is explained by O_2 photo dissociation and the lack of turbulent mixing at that altitude. Above 250 km, the density of atomic oxygen (O) surpasses that of nitrogen (N_2) and becomes the dominant species. Changes in O density can also have an impact on plasma composition [11].

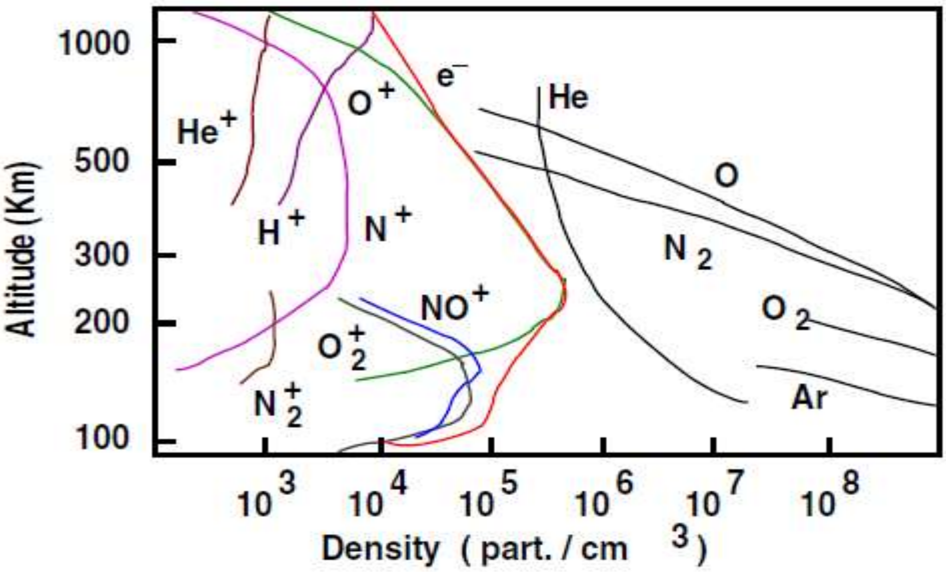


Figure 1: The altitude dependent chemical composition of the Earth's ionosphere[4].

At altitudes above 1000 km, as shown in Figure 1, H^+ dominates the ion composition. Near the peak (200km-600km) of e^+, which is the electron density, the ions are almost entirely O^+. This corresponds directly to the high density of atomic O at the same altitude. NO^+ and O_2^+ become the most important components and dominate the ionosphere below 200 km [11]. The general objective of this paper is to study the properties of the plasma (electronic coefficients) and describe the effect of the reduced electric field (E/ N) on it of the Earth's ionosphere for (Ar, He, N_2 and O_2) gases by estimating a mathematical model obtained using the "Origin" program.

2. Calculation and Methods:

2.1 Boltzmann Equation (BE)

Boltzmann equation describes a kinetics of gases and plasma[10]. This numerical method is used for providing the ratio of density distribution between one energy state (termed by N_2) to the next lower energy state (termed by N_1), so it is presented by [7].

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\, e^{(-U/KT)} \tag{1}$$

Where g_1 and g_2 denote a range of the two energy states, U the energy needed to excite particles, k the Boltzmann constant, and T the kinetic temperature.

2.2 Transport coefficients

It is the process by which charged particles in a gas are transported. BOLSIG+ math expressions are used to calculate the output parameters, and they help to illustrate some additional brief explanations and to recognize mathematical symbols. It is worth noting that all transport coefficients were multiplied or divided by the BOLSIG + output with N (the total gas density). This coefficient reduced the independent of N. All units are with SI units [15,16].

- **Total collision frequency $(\boldsymbol{\upsilon_{tot}/N})$:**

Summarizes the collision frequency on all neutral electron collisions, without Coulomb collisions [13].

$$(\upsilon_{tot}/\,N) \;=\; \gamma \int_0^\infty \sum_k x_k\, \sigma_k \varepsilon\, f_0 d\varepsilon \tag{2}$$

Where: σ_k in m^2: Cross section of electron-neutral collision process *k*. This is the total cross section for inelastic processes and the momentum-transfer cross section for elastic processes.

x_k : Fractional particle number density of target gas species of collision.

$\boldsymbol{\varepsilon}$: Electron energy in eV.

f_0: electron energy probability function" (EEPF), $\boldsymbol{\gamma}$: constant coefficient, = $(2e/m_e)^{1/2}$.

- **Total ionization frequency $(\boldsymbol{\upsilon_{iz}/N})$:**

The total ionization frequency is the number of electrons generated per unit time[17].

$$(\upsilon_{iz}/\,N) \;=\; \gamma \int_0^\infty \sum_{k=iZ} x_k\, \sigma_k \varepsilon\, f_0 d\varepsilon \tag{3}$$

- **Power $(\mathbf{P/N})$:**

The energy absorbed by electrons from the electric field per unit time is calculated using different expressions depending on the field configuration, as shown below. DC electric field with temporal growth (PT) or without growth [17]:

$$(P/N) = (\mu N)(E/N)^2 \quad (4)$$

Where: μ in (1/m/V/s): Mobility of electron, E: Electric field in (Td), N: Number of particles

3. Results and Discussion

3.1 Influence of reduced electric field on total collision frequency

The change in total collision frequency as (E/N) changes is displayed in Figure (2). The values of the total collision frequency gradually increase as (E/N) increases due to electrons gaining energy as a result of elastic collisions. Hence, the total collision frequency is very high at (1-20) Td for all gases in the Earth's ionosphere, but the total collision frequency increases slowly from (20-100) Td due to inelastic collision, whereas the increase corresponds to a slight increase in the total collision frequency of (20-45) Td. The value (45-100) Td also shows a slight increase for (Ar and N_2), but it is very low in total collision frequency for (O_2 and He). In general, Ar has the highest increase due to the high effect of elastic collisions, while other gases have the lowest increase due to the low effect of elastic collisions. The reason for this is that if an internal or external disturbance occurs in the plasma, causing particle (electrons and ions) to be displaced from their equilibrium position due to the accumulation of electrical charges, it quickly returns to its normal state [18].

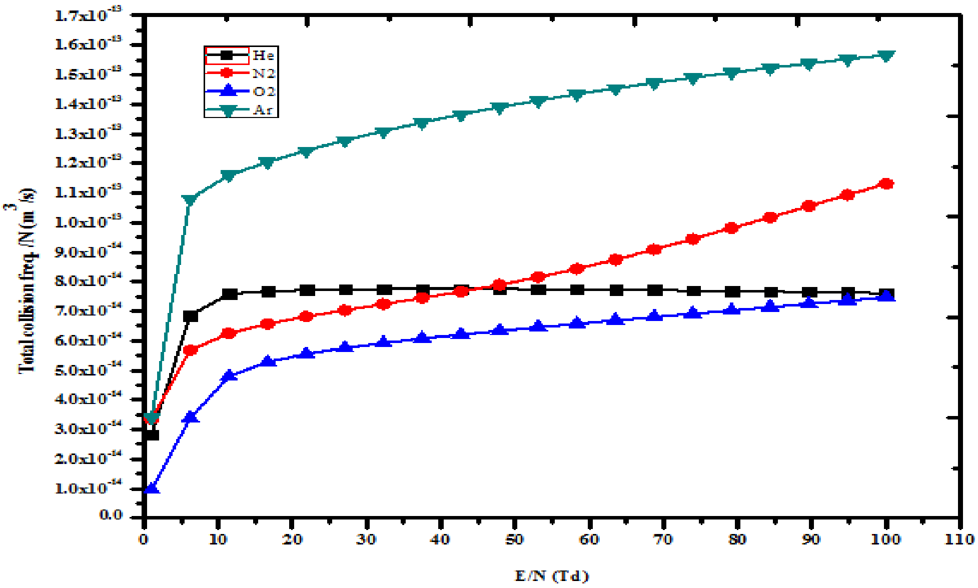


Figure 2: Total collision frequency as the function of (E/N) for different gases (He, N_2, O_2 and Ar) in the Earth's ionosphere.

3.2 Total collision frequency modeling

Figure 3 shows a high fitting/matching ratio between original data (using Bolsig+) and our simulation model (using Orgin) which determine the general behavior of the total collision frequency ($\upsilon_{tot}/$ N)for Ar gas in the Earth's ionosphere with increasing electric field (E/N). This relationship can be expressed by

$$(\boldsymbol{\upsilon}_{\mathbf{tot}}/\ \mathbf{N}) = \boldsymbol{A}\left|\left(\frac{\mathrm{E}}{\mathrm{N}}\right) - \mathrm{T}\right| \mathrm{i} + y_0 \quad (\mathbf{5})$$

Where: E/N= (1-100) Td, (A, T, i and y0): are the constants of equation (5) and the highest value that matches between the original data and the simulation model (Adj.R-Square) for Ar gas shown in the following table:

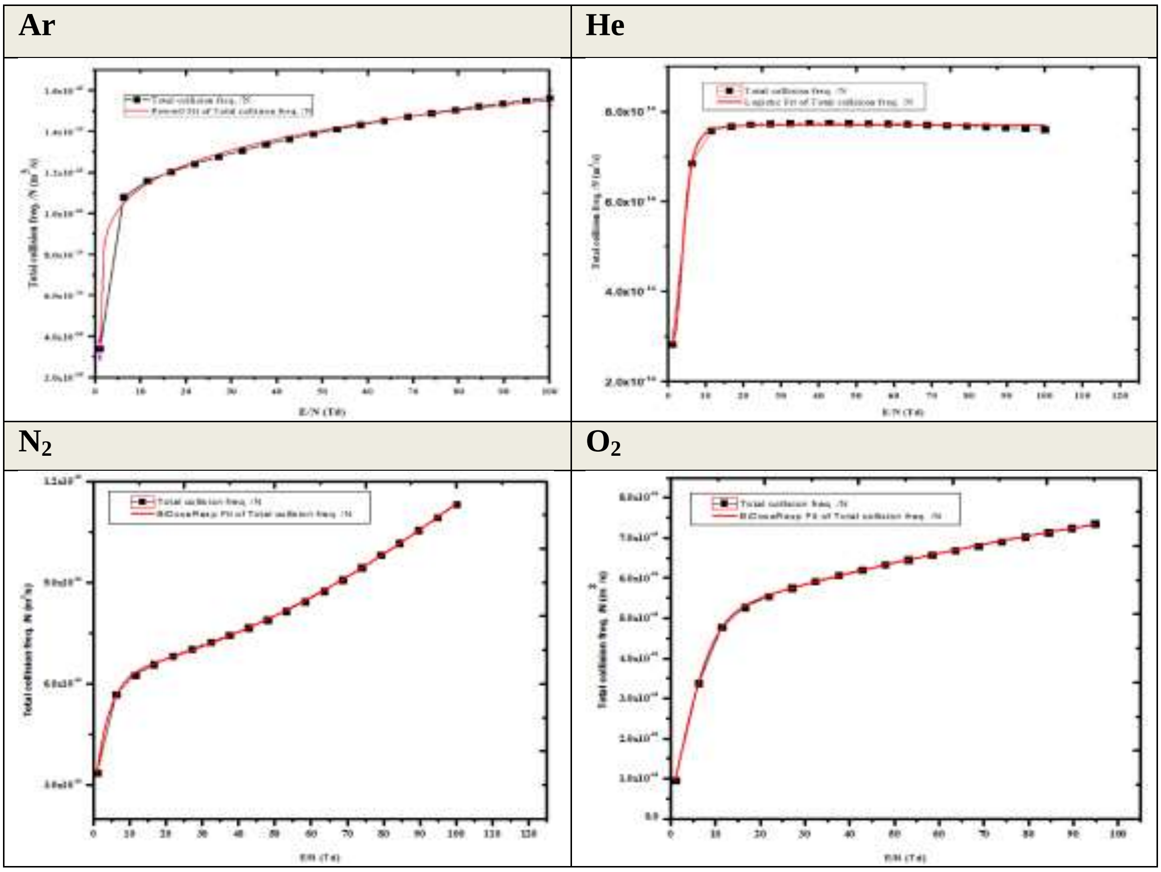


Figure 3: Total collision frequency estimates/simulations for Ar, He, N_2, and O_2 gases

Table 1: Demonstrates the relationship between constants (A, T, i and y_0) and ($\boldsymbol{\upsilon_{tot/}}$ $\boldsymbol{N}$), as represented by equation (5).

$\boldsymbol{\upsilon_{tot}/N}$ (m^3/s) for Ar			
Adj.R-Square	99.782 %		
A	5.2×10^{-14}	I	0.182
T	I	y_0	3.4×10^{-14}

Figure 3 shows a high fitting/matching ratio between original data (using Bolsig+) and our simulation model (using Orgin) which determine the general behavior of the total collision frequency (υ_{tot}/ N) for He gas in the Earth's ionosphere with increasing electric field (E/N). This relationship can be expressed by

$$(\upsilon_{tot}/\,N) = Q_2 + (Q_1 - Q_2)/(1 + (\frac{E/N}{U_0})^a) \qquad (6)$$

(Q_1, Q_2, a and U_0): are the constants of equation (6) and the highest value that matches between the original data and the simulation model (Adj.R-Square) for He gas shown in the following table:

Table 2: Demonstrates the relationship between constants (Q_1, Q_2, U_0 and a) and ($\boldsymbol{\upsilon_{tot/}N}$), as represented by equation (6).

$\boldsymbol{\upsilon_{tot}/N}$ (m^3/s) for He			
Adj.R-Square	99.816 %		
Q_1	2.79×10^{-14}	U_0	4.063
Q_2	7.709×10^{-14}	a	3.657

Figure 3 shows a high fitting/matching ratio between original data (using Bolsig+) and our simulation model (using Orgin) which determine the general behavior of the total collision frequency($\upsilon_{tot}/$ N) for (N_2 and O_2) gases in the Earth's ionosphere with increasing electric field (E/N). This relationship can be expressed by

$$(\boldsymbol{\upsilon_{tot}/N}) = S_1 + (S_2 - S_1)\left[\frac{r}{1+10^{\left(\log b_1-\left(\frac{E}{N}\right)\right)L_1}} + \frac{1-r}{1+10^{\left(\log b_2-\left(\frac{E}{N}\right)\right)L_2}}\right] \quad (7)$$

(S_1, S_2, r, log b_1, log b_2, L_1 and L_2): are the constants of equation (7) and the highest value that

matches between the original data and the simulation model (Adj.R-Square) for (N_2 and O_2) gases shown in the following table:

Table 3: Demonstrates the relationship between constants (S_1, S_2, $\log b_1$, $\log b_2$, r, L_1 and L_2) in the Earth's ionosphere, as represented by equation (7).

	$\boldsymbol{\upsilon_{tot}/N}$ (m^3/s) for N_2	$\boldsymbol{\upsilon_{tot}/N}$ (m^3/s) for O_2
Adj.R-Square	99.97 %	99.985 %
S_1	-2.2×10^{-12}	$- 2.8\times10^{-12}$
S_2	2.01×10^{-13}	9.8×10^{-14}
$\log b_1$	-14.929	2.11
$\log b_2$	LOGx02	-559.69
R	0.936	0.02309
L_1	0.11	0.117
L_2	0.009	0.0031

3.3 Influence of reduced electric field on total ionization frequency

Figure (4) shows the change in total ionization frequency as a function to (E/N). The calculations of total ionization frequency gradually increase as the reduced electric field increases due to an increase in the cross section caused by the electrons gaining energy and inelastic collisions caused by the particle's loss of kinetic energy. As a result, in a reduced electric field (25-100), the total ionization frequency in all gases increases dramatically. Because the electron binding energy of He gas is lower than that of other gases (Ar, N_2, and O_2), the total ionization frequency of He gas is higher. As a result, the other gases require a larger electric field to accelerate electrons and increase cross section in order to increase their total ionization.

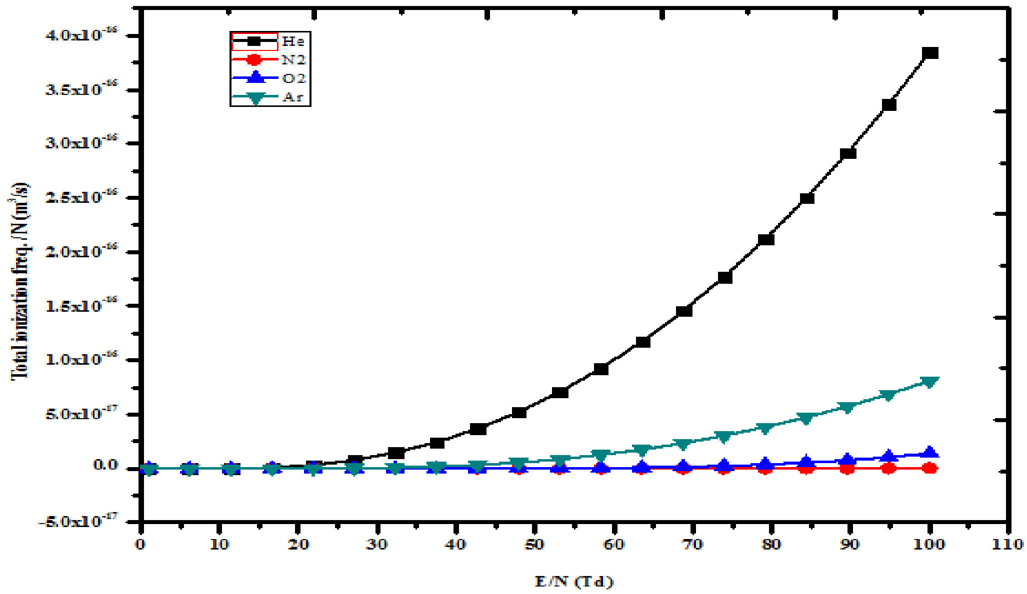


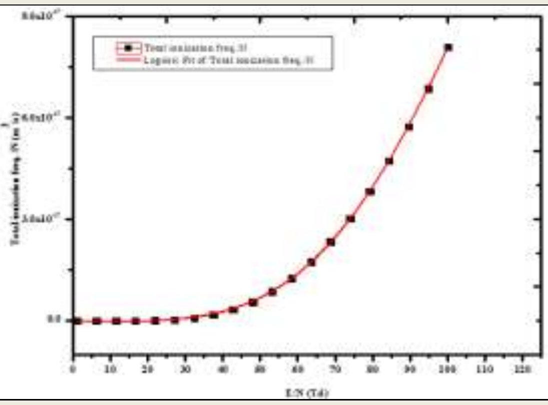
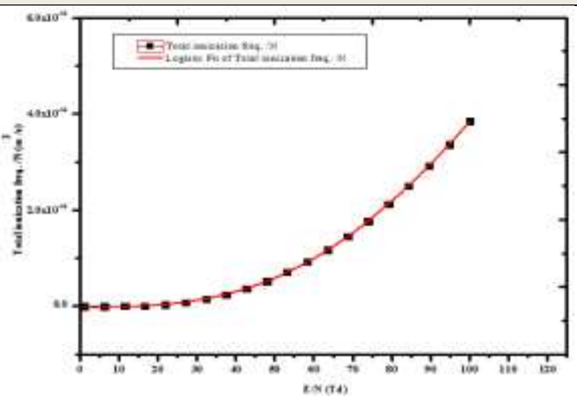
Figure 4: Total ionization frequency /N as the function of (E/N) for different gases (He, N_2, O_2 and Ar) in the Earth's ionosphere.

3.4 Total ionization frequency modeling

Figure 5 shows a high fitting/matching ratio between original data (using Bolsig+) and our simulation model (using Orgin) which determine the general behavior of the total ionization frequency (υ_{iz}/N) for four gases (Ar, He, N_2 and O_2) in the Earth's ionosphere with increasing electric field (E/N). This relationship can be expressed by

$$(\upsilon_{iz}/N) = C_2 + (C_1 - C_2)/(1 + (\frac{E/N}{V_0})^b) \quad (8)$$

(C_1, C_2, b and V_0): are the constants of equation (8) and the highest value that matches between the original data and the simulation model (Adj.R-Square) for (Ar, He, N_2 and O_2) gases shown in the following table:

Ar	**He**
	
N_2	**O_2**

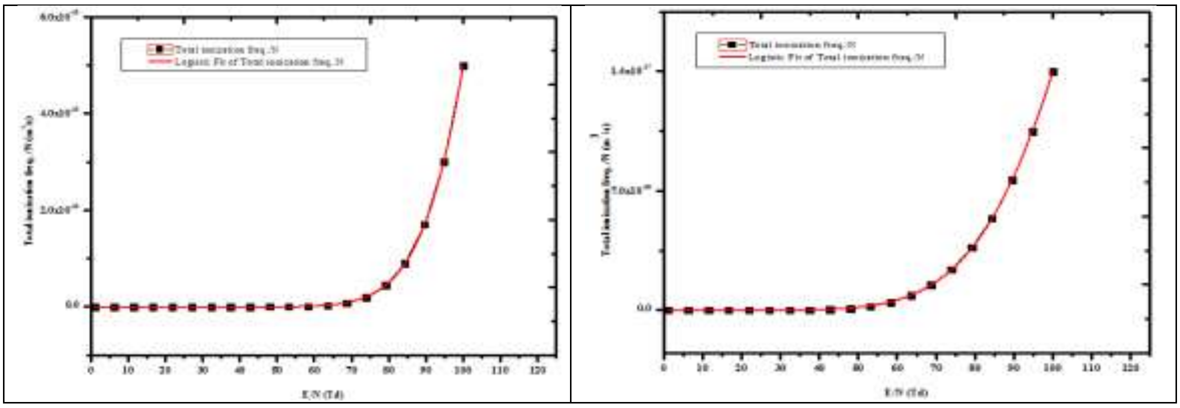


Figure 5: Total ionization frequency /N estimates/simulations for Ar, He, N_2, and O_2 gases

Table 4: Demonstrates the relationship between constants (C_1, C_2, V_0 and b) in the Earth's ionosphere, as represented by equation (8).

	($\boldsymbol{\upsilon}_{iz}$/ $\boldsymbol{N}$) for Ar	($\boldsymbol{\upsilon}_{iz}$/ $\boldsymbol{N}$) for He	($\boldsymbol{\upsilon}_{iz}$/ $\boldsymbol{N}$) for N_2	($\boldsymbol{\upsilon}$iz/ $\boldsymbol{N}$) for O_2
Adj.R-Square	99.991 %	99.995 %	99.997 %	99.995 %
C_1	-2.35×10^{-19}	-1.3×10^{-18}	-3.07×10^{-22}	-1.91×10^{-20}
C_2	3.01×10^{-16}	2.14×10^{-15}	2.53×10^{-18}	6.39×10^{-17}
V_0	129.17	168.24	113.43	121.51
B	3.93	2.91	11.16	6.54

3.5 Influence of reduced electric field on power

The relationship between power and reduced electric field is depicted in Figure (6). The power increases slightly in the reduced electric field (1-25), but strongly in the reduced electric field (25-100) due to an increase in the potential difference, which causes an increase in acceleration of electrons, so the power increases.

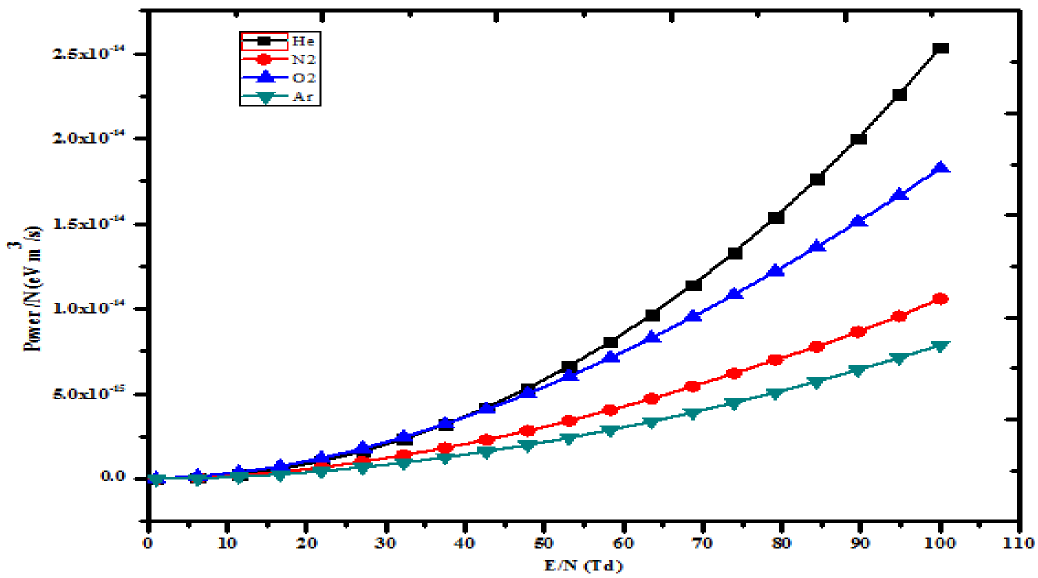


Figure 6: Power/N as the function of (E/N) for different gases (He, N_2, O_2 and Ar) in the Earth's ionosphere

3.6 Power modeling

Figure 7 shows a high fitting/matching ratio between original data (using Bolsig+) and our simulation model (using Orgin) which determine the general behavior of the power (P/N) for (Ar, He, N_2 and O_2) gases in the Earth's ionosphere with increasing electric field (E/N). This relationship can be expressed by

$$(P/N) = I_2 + (I_1 - I_2)/(1 + (\frac{E/N}{K_0})^f) \quad (9)$$

(I_1, I_2, f and K_0): are the constants of equation (9) and the highest value that matches between the original data and the simulation model (Adj.R-Square) for (Ar, He, N_2 and O_2) gases shown in the following table:

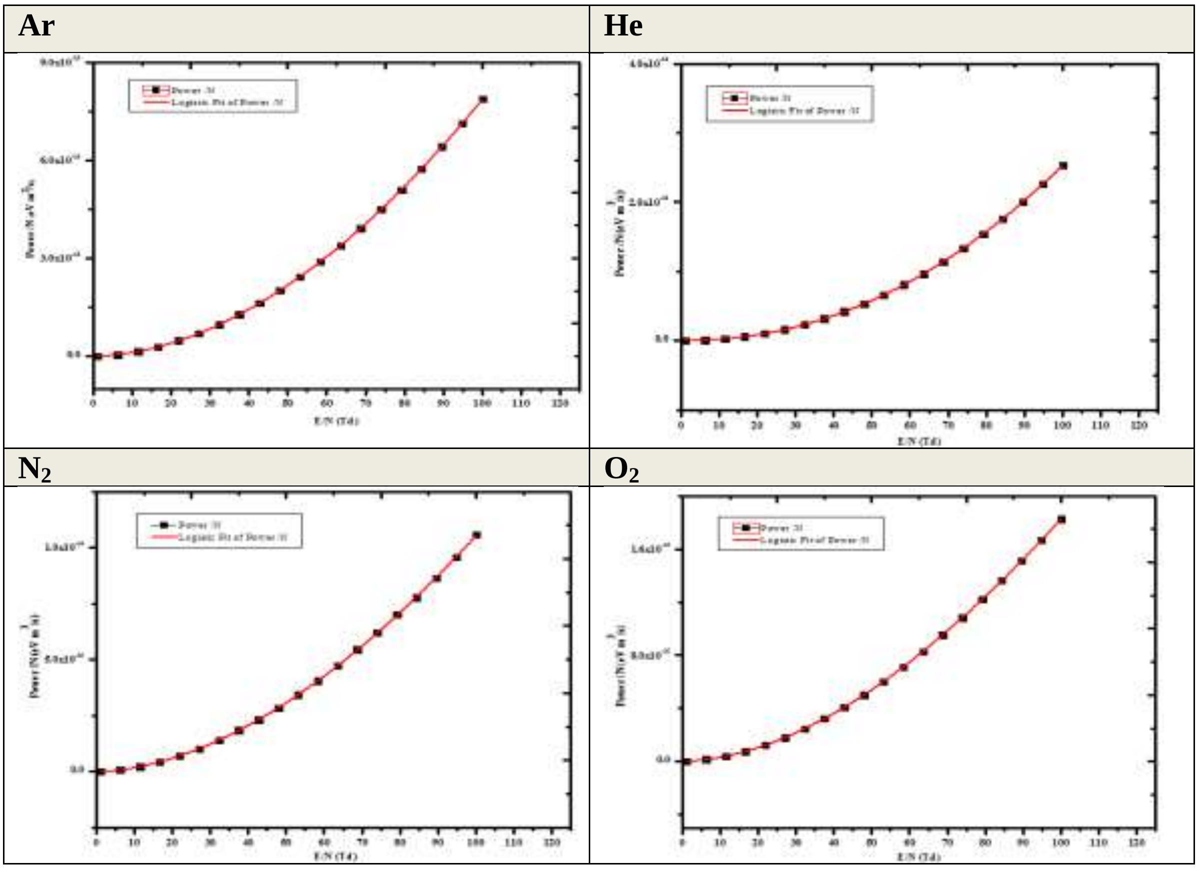


Figure 7: Power/N data estimates/simulations for Ar, He, N_2, and O_2 gases

Table 5: Demonstrates the relationship between constants (I_1, I_2, k_0 and f) and (P/N) in the Earth's ionosphere, as represented by equation (9).

	(P/N) for Ar	(P/N) for He	(P/N) for N_2	(P/N) for O_2
Adj.R-Square	99.999 %	99.999 %	99.996 %	99.999 %
I_1	3.91×10^{-18}	4.85×10^{-17}	1.65×10^{-18}	2.9×10^{-17}
I_2	1.6×10^{-11}	7.65×10^{-11}	4.4×10^{-13}	2.63×10^{-13}
K_0	5991.48	4324.67	784.51	413.18
F	1.86	2.12	1.8	1.83

4. Conclusion

The percent work and the values of plasma electronic coefficients obtained with the BOLSIG+ program have a strong correlation. This study demonstrated that a change in the reduced electric field has a significant impact on the plasma electronic coefficients (Total collision frequency, Total ionization frequency, and Power) for gases (Ar, He, N_2, and O_2) in the Earth's ionosphere. It is clear that the He gas is more affected by a change in (E/N) than other gases in all electronic coefficients. For all gases, there is a positive relationship between (E/N) and the electronic transport coefficients ((υ_{tot}/N) ,(υ_{iz}/N) ,and (P/N)). The relationship between ((υ_{iz}/N), (P/N)) and (E/N) at (1-25) Td has been determined to be linear. Logistic functions outperform other functions in terms of matching.

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