Iraqi Journal of Science, 2023, Vol. 64, No. 2, pp: 809-822
DOI: 10.24996/ijs.2023.64.2.27

ISSN: 0067-2904

On the Estimation of Stress-Strength Model Reliability Parameter of Power Rayleigh Distribution

Abdulrhaman A.J.Ahmed*, Feras Sh. M. Batah
Department of Mathematics, College of Education for pure Science, University Of Anbar, Anbar, Iraq

Received: 26/2/2022 Accepted: 7/6/2022 Published: 28/2/2023

Abstract

The aim of this paper is to estimate a single reliability system (R = P, Z > W) with a strength Z subjected to a stress W in a stress-strength model that follows a power Rayleigh distribution. It proposes, generates and examines eight methods and techniques for estimating distribution parameters and reliability functions. These methods are the maximum likelihood estimation(MLE), the exact moment estimation (EMME), the percentile estimation (PE), the least-squares estimation (LSE), the weighted least squares estimation (WLSE) and three shrinkage estimation methods (sh1) (sh2) (sh3). We also use the mean square error (MSE) Bias and the mean absolute percentage error (MAPE) to compare the estimation methods. Both theoretical comparison, simulation and real data are used. The results in light of this distribution show the advantage of the proposed methods.

Keywords: Power Rayleigh Distribution, Stress-Strength Reliability, Maximum Likelihood, Exact estimators of moments, Percentile, ordinary and weighted Least squares estimators, and the Shrinkage method.

حول تقدير معلمة وموثوقية نموذج المتانة – الإجهاد، لتوزيع *Power Rayleigh*

عبد الرحمن احمد جاسم احمد*, فراس شاكر محمود بطاح
قسم الرياضيات, كلية التربية للعلوم الصرفة, جامعة الانبار , الانبار , العراق

الخلاصة

تقدير نظام موثوقية واحد (R = P (Z> W) مع متانه Z معرضة لإجهاد W في نموذج المتانة والإجهاد الذي يتبع توزيع power Rayleigh هو موضوع هذا البحث. يقوم باقتراح وتوليد وفحص تقنيات ثمان طرق لتقدير معلمات التوزيع ودالة الموثوقية مثل [تقدير الاحتمالية القصوى (MLE) ، والتقدير الدقيق للحظة (EMME) ، والتقدير المئوي (PE) ، وتقدير المربعات الصغرى (LSE) ، وتقدير المربعات الصغرى الموزونة (WLSE) ، وثلاث طرق لتقدير الانكماش (sh3) (sh2)) (sh1) . تم استخدام معياري متوسط الخطأ التربيعي (MSE) ومتوسط النسبة المئوية للخطأ المطلق (MAPE) لمقارنة طرق التقدير اجراءات الدراسة. تم استخدام كل من المقارنة النظرية والمحاكاة وبيانات الحقيقية. وأظهرت النتائج في ظل هذا التوزيع افضلية الطرق المقترحة.

*Email: ferashaker2001@uoanbar.edu.iq

1. Introduction

The Rayleigh distribution is recognized as a lifetime distribution that is effective. Rayleigh was the first one who used it in 1980 in communication theory, physical science, engineering and medical imaging. It is utilized as a model. In aircraft, the authors [6] used this distribution to compute the lifetime of components that are dependent on their age, like resistors, transformers and capacitors. The stress-strength (S.S) model is utilized in a variety of engineering and scientific applications. The Rayleigh distribution with the transformation of the random variable $z = s^{\frac{1}{k}}$ yields the two-parameters power Rayleigh distribution PR(ʒ,k) [2]. The probability density function (P.D.F.) of the PR(ʒ,k), the cumulative function (CDF), the reliability function (R) , the cumulative function(CDF), the reliability function(R) and the Hazard(H) are given as follows:

$$\mathrm{f}(\mathrm{z}; k; ʒ) = \frac{k}{ʒ^2} z^{2k-1} \exp\left(-\frac{z^{2k}}{2ʒ^2}\right); z > 0, k > 0, ʒ > 0, \quad (1)$$

$$\mathrm{F}(\mathrm{z}; k; ʒ) = = 1 - exp\left(-\frac{z^{2k}}{2ʒ^2}\right); z > 0, \mathrm{k} > 0, ʒ > 0 \quad , \quad (2)$$

$$\mathrm{R}(\mathrm{z}; k; ʒ) = 1 - \mathrm{F}(\mathrm{z}; k, ʒ) = exp\left(-\frac{z^{2k}}{2ʒ^2}\right); z > 0, \mathrm{k} > 0, ʒ > 0 \ , \quad (3)$$

$$\mathrm{H}(\mathrm{z}; k; ʒ) = \frac{\mathrm{f}(\mathrm{z}; k, ʒ)}{1 - \mathrm{F}(\mathrm{z}; k, ʒ)} = \frac{k}{ʒ^2} z^{2k-1} \ . \quad (4)$$

Where k is the shape parameter and ʒ is the scale parameter. The rest of the paper is organized as follows: In Section 2, we provide the single reliability system PR(ʒ,k) in the Stress-Strength model. We also derived the eight estimation methods. The numerical studies (simulation and real data) are presented in Sections 3 and 4. Finally, the conclusions and discussions appear in Section 5.

2. Reliability of the Systems for PR Stress-Strength Models

Let Z be the strength and W be the stress random variable independent and each other by PR(ʒ,k_1) and PR(ʒ, k_2) respectively, with two different parameters when $\mathcal{G}=\frac{1}{ʒ^2}$. Then

$$R = \int_0^\infty \int_0^z f(w)f(z)dwdz = 1 - \int_0^\infty \mathcal{G}k1z^{2k1-1}\exp\left(-\frac{\mathcal{G}}{2}z^{2k1}\right)\exp(-\frac{\mathcal{G}}{2}z^{2k2})\,dz$$
$$= 1 - \sum_{r=0}^{\infty} \frac{-1^r\,C}{r!}\,\Gamma\,\frac{k2}{k1}\,r + 1. \quad (5)$$

Where $C = [\frac{\mathcal{G}}{2}(\frac{2}{\mathcal{G}})^{\frac{k2}{k1}}]$.

2.1 Maximum Likelihood Estimator (MLE)[3]

The MLE carefully estimates the parameters of an entire sample power Rayleigh distribution. Let $z_1, z_2, \dots, z_{n1}$ be a random sample for PR(ʒ , k).When ʒ is known and the shape parameter k is unknown then the likelihood function $\mathrm{f}(z_i; k; ʒ)$ in equation (2) is

$$\mathrm{L} = \prod_{\mathrm{i}=1}^{\mathrm{n1}} \mathrm{f}(\mathrm{z_i}; \mathrm{k}; ʒ)$$
$$= \prod_{\mathrm{i}=1}^{\mathrm{n1}} \frac{\mathrm{k}}{ʒ^2} \mathrm{z}^{2\mathrm{k}-1} \exp\left(-\frac{\mathrm{z}^{2\mathrm{k}}}{2ʒ^2}\right)$$

Taking the logarithm of both sides, then k_1

$$\ln l = \text{n}\ln k_1 - 2\text{n}\ln ʒ + (2k_1 - 1)\sum_{i=1}^{n} lnzi - \frac{1}{2ʒ^2}\sum_{i=1}^{n}(zi)^{2Z_1} \quad (6)$$

The partial derivative of equation (6) with regard to k_1 and the equivalence of the results to zero then we get

$$\hat{k}_{1_{M\,LE}} = \frac{2ʒ^2 n}{\sum_{i=1}^{n}\ (zi)^{2k_{1o}} \ln(zi) - 4ʒ^2 \sum_{i=1}^{n} \ln(zi)} \quad (7)$$

In the same way, let $w_1, w_2, \ldots, w_{n2}$ be a random sample from the stress w which is distributed as power Rayleigh distribution when ʒ is known and shape parameter k_2 unknown then the likelihood function $\text{f}(w_i; k_2; ʒ)$ in equation (2) is done, then the MLE method is presented by

$$\hat{k}_{2_{MLE}} = \frac{2ʒ^2 m}{\sum_{i=1}^{m}(wi)^{2k2\ 0}\ \ln(wi) - 4ʒ^2 \sum_{i=1}^{m} \ln(wi)} \quad (8)$$

Where n and m are the size of Z and W samples, respectively. Now we substitute equations (7) and (8) into equation (5) we obtain.

$$\hat{R}_{MLE} = 1 - \sum_{r=0}^{\infty} \frac{(-1)^r C_{MLE}}{r!}\ \Gamma\left(\frac{\hat{k}_{2_{MLE}}\, r + 1}{\hat{k}_{1_{MLE}}}\right). \quad (9)$$

Where, $C_{MLE} = [\,\frac{\mathcal{G}}{2}\,(\frac{2}{\mathcal{G}})^{\frac{k2MLE}{k1MLE}}\,]$.

2.2 The Exact Estimators of Moments Method (EMME)[11]

We provide the expectation and variance of the power Rayleigh distribution as follows:

$$\text{E(z)} = (2^{\frac{1}{2k}})\ (\frac{1}{ʒ^2})^{\frac{-1}{2k}}\ \Gamma(1 + \frac{1}{2k}) \quad (10)$$

$$\text{Var} = \frac{(2^{\frac{1}{k}})\ (\frac{1}{ʒ^2})^{\frac{-1}{k}}[-k\ \Gamma(1 + \frac{1}{2k})^2 + \Gamma(\frac{1}{k})]}{k} \quad (1\ 1)$$

And then the coefficient of variation is given by

$$cv = \frac{\sqrt{\text{Var}}}{\text{E(z)}} = \frac{\sqrt{\frac{(2^{\frac{1}{k}})\ (\frac{1}{ʒ^2})^{\frac{-1}{k}}[-k\,\Gamma(1+\frac{1}{2k})^2+\Gamma(\frac{1}{k})]}{k}}}{(2^{\frac{1}{2k}})\ (\frac{1}{ʒ^2})^{\frac{-1}{2k}}\ \Gamma(1+\frac{1}{2k})} \quad (12)$$

$$\frac{s}{\bar{y}} = \frac{\sqrt{\frac{(2^{\frac{1}{k}})\ (\frac{1}{ʒ^2})^{\frac{-1}{k}}[-k\ \Gamma(1 + \frac{1}{2k})^2 + \Gamma(\frac{1}{k})]}{k}}}{(2^{\frac{1}{2k}})\ (\frac{1}{ʒ^2})^{\frac{-1}{2k}}\ \Gamma(1 + \frac{1}{2k})}$$

$$\hat{k}_{1_{EMME}} = \frac{(2^{\frac{1}{k_{o1}}})\ (\frac{1}{ʒ^2})^{-\frac{1}{k_{o1}}}\left[-k\ \Gamma\left(1+\frac{1}{k_{o1}}\right)^2+\Gamma\left(\frac{1}{k}\right)\right]\ [\left(\sum_{i=1}^{n}\frac{z\,i}{n}\right)^2]}{\sum_1^n\frac{(z-\bar{z})^2}{(n-1)}\ [(2^{\frac{1}{2k_{o1}}})\ (\frac{1}{ʒ^2})^{-\frac{1}{k_{o1}}}\ \Gamma(1+\frac{1}{2k_{o1}})\ ]^2} \quad (13)$$

where $s^2 = \sum_1^n \frac{(z^i - \bar{z})^2}{(n-1)}$ and $\bar{y} = \sum_{i=1}^{n} \frac{zi}{n}$ and $k_{o2} = k_{o2} = k \mp \in$ and with respect to W, we obtain

$$\hat{k}_{2_{EMME}} = \frac{(2^{\frac{1}{k_{o2}}})\,(\frac{1}{\mathfrak{z}^2})^{-\frac{1}{k_{o2}}}\left[-k\,\Gamma\left(1+\frac{1}{k_{o2}}\right)^2+\Gamma\left(\frac{1}{k}\right)\right]\left[\left(\sum_{i=1}^{n}\frac{wi}{n}\right)^2\right]}{\sum_1^n\frac{(w?-\bar{w})^2}{(n-1)}\left[(2^{\frac{1}{2k_{o2}}}\,(\frac{1}{\mathfrak{z}^2})^{-\frac{1}{k_{o2}}}\,\Gamma(1+\frac{1}{2k_{o2}})\right]^2} \tag{14}$$

From the equation (12) and (13) in (5) we obtain.

$$\hat{R}_{EMME} = 1 - \sum_{r=0}^{\infty}\frac{(-1)^r C_{EMME}}{r!}\,\Gamma\left(\frac{\hat{k}_{2_{EMME}}\,r+1}{\hat{k}_{1_{EMME}}}\right) \tag{15}$$

Where $C_{EMME} = [\frac{\mathcal{G}}{2}(\frac{2}{\mathcal{G}})^{\frac{k2EMME}{k1EMME}}]$.

2.3 The Percentile Estimator (PE)

Let F(z_i,ʒ,k_1) and F(w_j,ʒ,k_2) be two c. d. f. for the random variables of the strength and the stress, respectively.

$$s_1 = \sum_{i=1}^{n}(F(zi)-qi)^2 \tag{16}$$
$$s_1 = \sum_{j=1}^{m}(F(wj)-qj)^2 \tag{17}$$

Where, qi=$\frac{i}{n+1}$, i=1,2,3……..n, and qj= $\frac{j}{m+1}$, j=1,2,3…. represent the expected values of F(zi) and F(wi), respectively [7]. From equation (2) we obtain

$$s_1 = \sum_{i=1}^{n}(F(zi)-qi)^2$$

$$s_1 = \sum_{i=1}^{n}[2k\,1\ln z - \ln(-2\mathfrak{z}^2\ln(1-qi)]^2$$

$$\frac{ds_1}{dk_1} = 2\sum_{i=1}^{n}2\ln z\,[k1\ \ln z - \ln(-2\mathfrak{z}^2\ln(1-qi)]^1$$

$$\hat{k}_{1pE} = \frac{\sum_{i=1}^{n}[\ln(-2\mathfrak{z}^2\ln(1-qi)]}{2\sum_{i=1}^{n}(\ln z)^2} \tag{18}$$

Let $W_1, W_2, W_3 \ \ldots, W_m$ be a random sample of size m from the strength w which is distributed as PR (ʒ, k_2) with unknown shape parameter k_2. We can obtain the following PE estimators:

$$\hat{k}_{2pE} = \frac{\sum_{i=1}^{n}[\ln(-2\mathfrak{z}^2\ln(1-qi)]}{2\sum_{i=1}^{n}(\ln w)^2} \tag{19}$$

We substitute equation (18)and (19) into equation (5) we obtain.

$$\hat{R}_{PE} = 1 - \sum_{r=0}^{\infty}\frac{(-1)^r\,C_{pE}}{r!}\,\Gamma\left(\frac{\hat{k}_{2pE}\,r+1}{\hat{k}_{1pE}}\right). \tag{20}$$

Where, $C_{pE} = [\frac{\mathcal{G}}{2}(\frac{2}{\mathcal{G}})^{\frac{k2pE}{k1pE}}]$.

2.4 The Least Squares Estimator Method (LSE).

By minimizing the sum of square error between the value and its predicted value, least squares technique estimators can be created. The LS approach is frequently used to fit models and solve mathematical and engineering issues, in particular, in linear and non-linear regression [5].

$s = \sum_{i=1}^{n}(\in i)^2$ when $\in i = (wi - \mathfrak{z} - \beta zi)$,

$$s = \sum_{i=1}^{n} (wi - ʒ - \beta zi)^2, \quad (21)$$

$$\sum_{1}^{n} wi = nʒ + \beta \sum_{i=1}^{n} zi, \quad (22)$$

$$\sum_{i}^{n} \mathrm{wizi} = ʒ \sum_{i=1}^{n} \mathrm{zi} - \beta \sum_{i=n}^{n} (\mathrm{zi})^2 \ . \quad (23)$$

Now, we solve the two equations (24) and (25) so we get

$$\hat{\beta}_{LS} = \frac{n \sum_{i=1}^{n} zi\, wi - \sum_{i=1}^{n} zi \sum_{i=1}^{n} wi}{n \sum_{i=1}^{n} (zi)^2 - (\sum_{i=1}^{n} zi)^2} \quad (24)$$

where Zi is the PR distribution's strength random variable with sample size n, and W is the PR distribution's stress random variable with sample size m. A distribution function is derived (CDF) we obtain.

$$1 - \exp(-\frac{zi^{2k}}{2ʒ^2}) = F(zi)$$

$$\ln(zi^{2k}) = ln[-ln(1 - F(zi))] + ln[2ʒ^2]. \quad (25)$$

From the equation (25) we get

$$wi = \ln(zi)\ , a = \frac{\ln(2ʒ^2)}{2k}\ , b = \frac{1}{k}\ , \ zi = \frac{ln[-ln(1- F(zi))]}{2} \quad (26)$$

Now from equation (26), the LS method for the shape parameter k can get the LS estimators of $\hat{k}_{1_{LS}}$ is presented by:

$$\hat{k}_{1_{LS}} = \frac{n\sum_{i=1}^{n}[\frac{ln[-ln(1- F(zi))}{2}]^2 - (\sum_{i=1}^{n} [\frac{ln[-ln(1- F(zi))}{2}])^2}{n \sum_{i=1}^{n}[\frac{\ln[-ln(1- F(zi))}{2}]\ln(zi) - \sum_{i=1}^{n}[\frac{\ln[-ln(1- F(zi))}{2}] \sum_{i=1}^{n}\ln(zi)} \quad i = 1,2,3 \ldots\ldots n \quad (27)$$

And

$$\hat{k}_{2_{LS}} = \frac{m\sum_{i=1}^{m}[\frac{ln[-ln(1- F(wi))}{2}]^2 - (\sum_{i=1}^{m} [\frac{ln[-ln(1- F(wi))}{2}])^2}{m \sum_{i=1}^{m}[\frac{\ln[-ln(1- F(wi))}{2}]\ln(wi) - \sum_{i=1}^{m}[\frac{\ln[-ln(1- F(wi))}{2}] \sum_{i=1}^{n}\ln(wi)} \quad i = 1,2,3 \ldots\ldots m \quad (28)$$

We substitute equations (27)and (28)into equation (5) we obtain.

$$\hat{R}_{LS} = 1 - \sum_{r=0}^{\infty} \frac{(-1)^r C_{LS}}{r!} \Gamma\left(\frac{\hat{k}_{2LS}\, r+1}{\hat{k}_{1LS}}\right). \quad (29)$$

Where $C_{LS} = [\frac{\mathcal{G}}{2} (\frac{2}{\mathcal{G}})^{\frac{k2LS}{k1LS}}]$.

2.5 Weighted least squares (LSE) estimators

The weighted least squares estimators of $k$and k_2 are $\hat{k}_{2WLSE}$ and $\hat{k}_{2WLSE}$, respectively. They can be obtained by minimizing from the equation as follows:

$$s_1 = \sum_{i=1}^{n} WS[(F(zi) - qi)^2]. \quad (30)$$

Where $Ws = \frac{1}{var(F(zi))} = \frac{(n+1)^2\,(n+2)}{(j(n-j+1))}$ and $qi = \frac{i}{n+1}$, i=1,2,3……..n, then we get

$$\hat{k}_{1\,WLSE} = \left[\frac{\sum_{i=1}^{n} ws[\ln(-2ʒ^2 \ln(1- qi)]}{2\sum_{i=1}^{n} ws(\ln z)^2}\right] \quad (31)$$

$$\hat{k}_{2\,WLSE} = \frac{\sum_{i=1}^{n} ws[\ln(-2\mathfrak{z}^2 \ln(1- qi)]}{2\sum_{i=1}^{n} ws(\ln w)^2} \quad (32)$$

We substitute equations (32)and (33)into equation (10) we obtain

$$\hat{R}_{1_{WLSE}} = 1 - \sum_{r=0}^{\infty} \frac{(-1)^r\, C_{WLSE}}{r!}\, \Gamma\left(\frac{\hat{k}_{2WLSE}\, r+1}{\hat{k}_{1WLSE}}\right) \quad (33)$$

Where $C_{WLSE} = [\frac{\mathcal{G}}{2}\left(\frac{2}{\mathcal{G}}\right)^{\frac{k2WLSE}{k1WLSE}}]$.

2.6 The Shrinkage Estimator (Sh) [4].

The shrinkage estimation method can be thought of as a Bayesian strategy that relies on prior knowledge. Thompson had introduced the main arguments for utilizing previous estimating [4]. The parameter was utilized as a starting value k_0 where $[k_0 = k \mp \in], \in = 0.001$ from the past in the shrinkage estimation method, and the normal estimator($\hat{k}_{MLE}$) was employed to them by shrinkage weight factor $\Omega(\hat{k})$, $0 \leq \Omega\,(k) \leq 1$, which can be written as:

$$\hat{k}_{sh} = \Omega\left(\hat{k}\right) k_{MLE} + \left(1 - \Omega\left(\hat{k}\right)\right) k_0 . \quad (34)$$

2.6.1 *The Shrinkage Weight function (Sh1)* [9]

The weight shrinking function will be considered. In this subsection the function form is denoted by $\Omega\,(\hat{k}) = [\frac{\sin n}{n}]$, $0 \leq \Omega\,(\hat{k}) \leq 1$ where $\hat{k}$ is the sample size and n is the number of participants. Taking the forms below as $\Omega\,(\hat{k}_1) = [\frac{\sin n}{n}]$ and $\Omega\,(\hat{k}_1) = [\frac{\sin m}{m}]$, $0 \leq \Omega\left(\hat{k}_1\right) \leq 1$, where n and m refer to the w and z sample sizes as a result, the shrinkage estimator employs the $\hat{k}_1$ and $\hat{k}_2$ shrinkage weight functions, which are specified in equation (34) that will be

$$\hat{k}_{1\,sh1} = (\left[\frac{\sin n}{n}\right] \hat{K}_{1\,MLE} + (1 - |\frac{\sin n}{n}|) k_{1_0}), \quad (35)$$

$$\hat{k}_{2\,sh1} = (\left[\frac{\sin m}{m}\right] \hat{K}_{2\,MLE} + (1 - |\frac{\sin m}{m}|) k_{2_0}). \quad (36)$$

We substitute equations (35)and (36)into equation (5) we obtain

$$\hat{R}_{sh_1} = 1 - \sum_{r=0}^{\infty} \frac{(-1)^r C_{sh1}}{r!}\, \Gamma\left(\frac{\hat{k}_{2\,sh1}\, r+1}{\hat{k}_{1_{sh1}}}\right). \quad (37)$$

Where $C_{sh1} = [\frac{\mathcal{G}}{2}(\frac{2}{\mathcal{G}})^{\frac{k2sh1}{k1sh1}}]$.

2.6.2 *The Constant Shrinkage Estimate (Sh2)* . [9]

In constant shrinkage factor case, it can be assumed that $\Omega\left(\hat{k}\right) = 0.001 \quad 0 \leq \Omega\left(\hat{k}\right) \leq 1$. To get the constant shrinkage estimators, we put into equation (34) $\hat{k}_1$and k_2 as follows:

$$\hat{k}_{1_{sh2}} = (0.001)\, \hat{k}_{1_{MLE}} + (1 - (0.001) k_{1_0} \quad (38)$$

And

$$\hat{k}_{1_{sh2}} = (0.001)\,\hat{k}_{1_{MLE}} + (1 - (0.001)\,k_{1_0} \quad (39)$$

from equations (38) and (39) are substituted into equation (5) we obtain.

$$\hat{R}_{sh2} = 1 - \sum_{r=0}^{\infty} \frac{(-1)^r C_{sh1}}{r!}\,\Gamma\left(\frac{\hat{k}_{2\ sh2}\, r + 1}{\hat{k}_{1_{sh2}}}\right). \quad (40)$$

Where $C_{sh2} = \left[\frac{\mathcal{G}}{2}\left(\frac{2}{\mathcal{G}}\right)^{\frac{k2sh2}{k1sh2}}\right]$.

2.6.3 ***The Shrinkage function (fSh)(sh3)*** [3]

We consider the shrinkage weight factor as a function of the sizes g and h in this situation such that $(\Omega(\hat{k}_1) = e^{-g})$ = and $(\Omega(\hat{k}_2) = e^{-h})$, where$(\Omega\ (\hat{k}) = 0 \leq \Omega(\hat{k}) \leq 1$. Therefore, the shrinkage estimator uses shrinkage function of $\hat{k}_1$and $\hat{k}_2$which is defined in equation (34) as follows:

$$\hat{k}_{1_{sh3}} = (e^{-g})\hat{k}_{1_{MLE}} + \left(1 - (e^{-g})\right)k_{1_0}\ , \quad (41)$$

And

$$\hat{k}_{2_{sh3}} = (e^{-h})\hat{k}_{2_{MLE}} + \left(1 - (e^{-h})\right)k_{2_0}\ . \quad (42)$$

To get the shrinkage function estimator form (40) and (41) into equation (5) as follows:

$$\hat{R}_{sh3} = 1 - \sum_{r=0}^{\infty} \frac{(-1)^r C_{sh3}}{r!}\,\Gamma\left(\frac{\hat{k}_{2_{sh3}}\, r + 1}{\hat{k}_{1_{sh3}}}\right), \quad (43)$$

where $C_{sh3} = \left[\frac{\mathcal{G}}{2}\left(\frac{2}{\mathcal{G}}\right)^{\frac{k2sh3}{k1sh3}}\right]$.

3. Monte Carlo Simulation Study and it's Results

Simulation is a numerical technique of performing experiments on a computer while the Monte Carlo simulation is a computer experiment that involves random sampling of probability distributions. In order to verify the performance of the proposed estimation method that was introduced for estimating the single component reliability system, the Monte Carlo simulation was used. The proposed eight estimation methods are implemented using diverse samples (25, 50, 75, 100). Statistical results for each sample are based on bias, mean absolute percentage error and mean squared error criteria with 1000 replicates. Therefore, the following steps explain the Monte Carlo simulations for each model.

Step1: To find the performance, initialize and generate random samples that follow a continuous uniform distribution defined on the interval (0,1). Z (strength) and W(stress) as $g_1, g_2, \dots, g_{n1}$ and $h_1, h_2, \dots, h_{n2}$, respectively as follows U~ Uniform(0,1).

Step2: : Transform the above uniform random sample to a random samples of power Rayleigh distribution using the cumulative distribution function (cdf) as F(z,ʒ,k)= $[1 - \exp(-\frac{z^{2k}}{2ʒ^2})\ ]$ then $Z_i = ln(1 - \text{ui})^{\frac{-2ʒ^2}{2k}}$, for i =1,2…,n and $Z_j = ln(1 - \text{sj})^{\frac{-2ʒ^2}{2k}}$, for j = 1,2,…, m where (u, and s) are random variables of uniform (0,1).

Step 3: The ʒ is considered the known parameter as the mean of the sample and k is considered as the unknown parameter. The MLE estimators $\hat{k}_{1_{MLE}}$ and $\hat{k}_{2_{MLE}}$ have been calculated respectively from equations (7) and (8).

Step 4: The exact estimators $\hat{k}_{1_{EMME}}$ and $\hat{k}_{2_{EMME}}$ have been calculated from equations (13) and (14), respectively.
Step 5: The percentile estimator, $\hat{k}_{1pE}$ and $\hat{k}_{2pE}$ have been calculated from equations (18) and (19), respectively.
Step 6: The Least Squares Estimator, $\hat{k}_{1_{LS}}$ and $\hat{k}_{2_{LS}}$ have been calculated from equations (27) and (28), respectively.
Step 7: The weighted least squares estimators $\hat{k}_{1\,WLSE}$ and $\hat{k}_{2\,WLSE}$ have been calculated from equations (31) and (32), respectively.
Step 8: The Shrinkage Estimators (Sh1), (Sh2) and (Sh3) have been calculated with ($\hat{k}_{1_{sh1}}$, $\hat{k}_{2_{sh1}}$) ,($\hat{k}_{1_{sh2}}$, $\hat{k}_{2_{sh2}}$) and ($\hat{k}_{1_{sh3}}$, $\hat{k}_{2_{sh3}}$) from equations (35), (36),(38), (39), (41) and(42), respectively.
Step 9: The estimated reliability of stress – strength model of different types of the estimation methods such as $\hat{R}_{MLE}$, $\hat{R}_{EMME}$, $\hat{R}_{PE}$, $\hat{R}_{LS}$, $\hat{R}_{1_{WLSE}}$, $\hat{R}_{sh_1}$, $\hat{R}_{sh2}$ and $\hat{R}_{sh3}$ have been calculated from equations (9), (15), (20), (29), (33), (37), (40) and (43), respectively.
The results in Tables 1, 3 and 5 explain the reliability of estimation ,while the results in Tables 2, 4 and 6 show that the comparison between these methods when the criteria of biased MSE and MAPE are used. However, all estimators depend on the values of the samples size.

4. Real Data analysis [1] [2].

In this part, we have tested all the results above from a real data, as shown below

Table 1: Data set 1 (gauge lengths of 20 mm) [12].

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958
1.966	1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179
2.224	2.240	2.253	2.270	2.272	2.274	2.301	2.301	2.359	2.382
2.382	2.426	2.434	2.435	2.478	2.490	2.511	2.514	2.535	2.554
2.566	2.570	2.586	2.629	2.633	2.642	2.648	2.684	2.697	2.726
2.770	2.773	2.800	2.809	2.818	2.821	2.848	2.880	2.809	2.818
2.821	2.848	2.880	2.954	3.012	3.067	3.084	3.090	3.096	3.128
3.233	3.433	3.585	3.585						

Table 2: Data set 2 (gauge lengths of 10 mm)[12].

1.901	2.132	2.203	2.228	2.257	2.350	2.361	2.396	2.397	2.445
2.454	2.474	2.518	2.522	2.525	2.532	2.575	2.614	2.616	2.618
2.624	2.659	2.675	2.738	2.740	2.856	2.917	2.928	2.937	2.937
2.977	2.996	3.030	3.125	3.139	3.145	3.220	3.223	3.235	3.243
3.264	3.272	3.294	3.332	3.346	3.377	3.408	3.435	3.493	3.501
3.537	3.554	3.562	3.628	3.852	3.871	3.886	3.971	4.024	4.027
4.225	4.395	5.020							

We investigate strength data, which was originally reported by Badar and Priest [1] and represent strength that is measured in the GPA of mono and flooded 1000 - carbon fiber. Single fibers are tested under pressure at gauge lengths of 20 mm (data set 1) and 10 mm (data set 2), with sample sizes n = 74 and m = 63, with $k_1 = 1$, $k_2 = 1.5$, and $ʒ = 1$, respectively. The data is shown in Tables 1 and 2. Several authors have analysed these data sets such as Surles and Padgett [10], Neck and Kondo[8].

Table 3: The Reliability Estimates When R=(0.647505398880016) ʒ=4 k1=2 k2= 3

n,m	$\hat{R}_{Mle}$	$\hat{R}_{Pe}$	$\hat{R}_{Ls}$	$\hat{R}_{wLse}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Sh3}$	$\hat{R}_{Emm}$
25,25	0.917720684078	0.96083582531	0.96082535939	0.96083586907	0.96064672457	0.96083676083	0.96083383455	0.9607623589
25,50	0.919544703319	0.96083613999	0.96081854002	0.96083632328	0.96083497118	0.96083388904	0.96083383455	0.96045326747
25,75	0.918009495434	0.96083603705	0.96079746721	0.96083634074	0.96082517036	0.96083439743	0.96083383455	0.96033296804
25,100	0.918149477144	0.96083426323	0.96079285586	0.96083476891	0.96083588143	0.96083392642	0.96083383455	0.96042327858
50,25	0.916046674749	0.96082591014	0.96080337469	0.96082488437	0.95871044682	0.96083601428	0.96083383455	0.96073807078
50,50	0.945387888656	0.96083232629	0.96083405904	0.96083163747	0.96081733383	0.96083441764	0.96083383455	0.96079696410
50,75	0.928804083745	0.96083614017	0.96083692064	0.96083575987	0.96083139780	0.96083286723	0.96083383455	0.96074764039
50,100	0.921338924766	0.96083702771	0.96083588669	0.96083680101	0.95849497342	0.96083251338	0.96083383455	0.96075433585
75,25	0.941106634068	0.96082624259	0.96080252042	0.96082524385	0.96082930016	0.96083313388	0.96083383455	0.96078266202
75,50	0.930173244438	0.96083313388	0.96083035317	0.96083249501	0.96079925872	0.96083401715	0.96083383455	0.96082090541
75,75	0.937924111656	0.96083230005	0.96083311512	0.96083163423	0.96083523230	0.96083384311	0.96083383455	0.96081448423
75,100	0.918949913392	0.96083642713	0.96083703403	0.96083609235	0.92960657310	0.95685903202	0.96083383455	0.96081224392
100,25	0.911217436553	0.96081881283	0.96078887846	0.96081760486	0.96083047457	0.96083369482	0.96083383455	0.96065390633
100,50	0.930827605862	0.96082760586	0.96081448007	0.96082669872	0.96077086295	0.96083640609	0.96083383455	0.96081913452
100,75	0.925500349613	0.96083298500	0.96083385103	0.96083235239	0.96083575714	0.96083428650	0.96083383455	0.96083416872
10,100	0.930870290645	0.96083642775	0.96083714781	0.96083608271	0.96083435916	0.96083391704	0.96083383455	0.96083484499

Table 2: The Bias, MSE and MAPE of The Simulated Estimates When R=(0.647505398880016) ʒ=4 k1=2 ,k2=3

n,m		$\hat{R}_{Mle}$	$\hat{R}_{Pe}$	$\hat{R}_{Ls}$	$\hat{R}_{wLse}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Sh3}$	$\hat{R}_{Emm}$	Best
25 50	**Bias** **Mse** **Mape**	0.27203930443 0.07404382938 0.42013441881	0.31333074111 0.0981761535 4 0.48390444566	0.31331314114 0.09816512480 0.48387726448	0.31333092440 0.09817626840 0.48390472874	0.31332957230 0.09817542088 0.48390264057	0.31332849016 0.09817474274 0.48390096932	0.31332843567 0.09817470860 0.48390088517	0.31294786859 0.09793641135 0.48331314168	***MLE***
25 100	**BiasMse** **Mape**	0.27064407826 0.07325154214 0.41797964732	0.31332886435 0.09817497724 0.48390154722	0.31328745698 0.09814903082 0.48383759816	0.31332937003 0.09817529413 0.48390232818	0.31333048255 0.09817599129 0.48390404634	0.31332852754 0.09817476617 0.48390102705	0.31332843567 0.09817470860 0.48390088517	0.31291787970 0.09791764210 0.48326682719	***MLE***
50 25	**Bias** **Mse** **Mape**	0.26854127586 0.07221379874 0.41473210313	0.31332051126 0.09816974279 0.48388864680	0.31329797581 0.09815562178 0.48385384331	0.31331948549 0.09816910001 0.48388706262	0.31120504794 0.09685075956 0.48062154923	0.31333061540 0.09817607455 0.48390425153	0.31332843567 0.09817470860 0.48390088517	0.31323267190 0.09811827777 0.48375298869	***MLE***
50 100	**Bias** **Mse** **Mape**	0.27383352588 0.07506142048 0.42290539408	0.31333162883 0.09817670966 0.48390581665	0.31333048781 0.09817599473 0.48390405447	0.31333140213 0.09817656760 0.48390546655	0.31098957454 0.09671854516 0.48028877455	0.31332711450 0.09817388073 0.48389884477	0.31332843567 0.09817470860 0.48390088517	0.31324893697 0.09812489774 0.48377810827	***MLE***

75 75	**Bias**	0.29041871277	0.31332690117	0.31332771624	0.31332623535	0.31332983342	0.31332844423	0.31332843567	0.31330908535	***MLE***
	Mse	0.08436677322	0.09817374709	0.09817425780	0.09817332986	0.09817558451	0.09817471396	0.09817470860	0.09816259478	
	Mape	0.44851936876	0.48389851531	0.48389977410	0.48389748703	0.48390304383	0.48390089840	0.48390088517	0.48387100076	
75 100	**Bias**	0.27144451451	0.31333102825	0.31333163515	0.31333069347	0.28210117422	0.30935363314	0.31332843567	0.31330684504	***MLE***
	Mse	0.07375931859	0.09817633327	0.09817671361	0.09817612347	0.07967099742	0.09571542797	0.09817470860	0.09816117976	
	Mape	0.41921583199	0.48390488912	0.48390582642	0.48390437209	0.43567385648	0.47776224520	0.48390088517	0.48386754084	
100 25	**Bias**	0.26371203767	0.31331341395	0.31328347958	0.31331220598	0.31332507569	0.31332829594	0.31332843567	0.31314850745	***MLE***
	Mse	0.07105030972	0.09816529887	0.09814654549	0.09816454200	0.09817260320	0.09817462104	0.09817470860	0.09806202803	
	Mape	0.40774032020	0.48387768579	0.48383145549	0.48387582022	0.48389569605	0.48390066938	0.48390088517	0.48362300607	
100 100	**Bias**	0.28336489176	0.31333102887	0.31333174893	0.31333068383	0.31332896028	0.31332851816	0.31332843567	0.31332944611	***MLE***
	Mse	0.08034074442	0.09817633366	0.09817678490	0.09817611744	0.09817503735	0.09817476029	0.09817470860	0.09817534208	
	Mape	0.43762552753	0.48390489009	0.48390600213	0.48390435721	0.48390169537	0.48390101257	0.48390088517	0.48390244569	

Table 3: The Reliability Estimates R=(0.6250000) δ=0.5 k1=1 ,k2=1

n,m	$\hat{R}_{Mle}$	$\hat{R}_{Pe}$	$\hat{R}_{Ls}$	$\hat{R}_{wLse}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Sh3}$	$\hat{R}_{Emm}$
25,25	0.9632751894580	0.9605164535011	0.9615425244952	0.9604477642596	0.9583288650746	0.9583324565609	0.9583333333333	0.9640214859800
25,50	0.9646982110320	0.9613107743832	0.9615969315392	0.9612445280815	0.9583402322564	0.9583345706952	0.9583333333333	0.9648837286890
25,75	0.9666492799387	0.9619744023797	0.9626144842401	0.9619101993239	0.9583526503535	0.9583369071819	0.9583333333333	0.9680370499740
25,100	0.9671321977184	0.9611393751909	0.9633961899733	0.9610725713544	0.9583500224911	0.9583363625229	0.9583333333333	0.9668335823662
50,25	0.9574365304512	0.9584189508555	0.9570702630065	0.9583435968856	0.9583142383575	0.9583297300119	0.9583333333333	0.9576792817742
50,50	0.9614959801882	0.9594511895441	0.9588112259377	0.9593792056409	0.9583354021257	0.9583337137725	0.9583333333333	0.9624472391021
50,75	0.9612704892412	0.9602700496275	0.9597546171747	0.9602006201297	0.9583353625892	0.9583336868983	0.9583333333333	0.9611700220272
50,100	0.9633530287616	0.9608041459570	0.9600595590516	0.9607363758166	0.9583497429544	0.9583363986787	0.9583333333333	0.9639870699068
75,25	0.9557160869807	0.9595962902239	0.9547813981797	0.9595247571426	0.9583089610309	0.9583287656505	0.9583333333333	0.9554538325627
75,50	0.9582302685781	0.9584149134096	0.9592008464563	0.9583395704954	0.9583235582060	0.9583314804438	0.9583333333333	0.9587981646990
75,75	0.9604317041351	0.9593418280276	0.9585569969980	0.9592694834859	0.9583358081079	0.9583338043226	0.9583333333333	0.9607759759284
75,100	0.9611982760728	0.9601936713512	0.9588827982951	0.9601239430632	0.9583380415409	0.9583341951027	0.9583333333333	0.9611708732532
100,25	0.9533823522239	0.9578104955934	0.9561222720878	0.9577332995649	0.9583004561246	0.9583272129750	0.9583333333333	0.9536226660497
100,50	0.9569480158910	0.9584764243041	0.9577448768214	0.9584012983144	0.9583184847121	0.9583305549217	0.9583333333333	0.9572521883736
100,75	0.9586508041842	0.9585155231017	0.9585730266676	0.9584405970599	0.9583286043162	0.9583324301113	0.9583333333333	0.9586641113119
100,100	0.9600726673937	0.9597260967698	0.9581070291697	0.9596549794831	0.9583364311377	0.9583339399040	0.9583333333333	0.9600072836912

Table 4: The Bias, MSE and MAPE of The Simulated Estimates when R=(0.6250000) ʒ=0.5 k1=1 ,k2=1

n,m		$\hat{R}_{Mle}$	$\hat{R}_{Pe}$	$\hat{R}_{Ls}$	$\hat{R}_{wLse}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Sh3}$	$\hat{R}_{Emm}$	Best
25 50	Bias	0.33969821103	0.33631077438	0.33659693153	0.33624452808	0.33334023225	0.33333457069	0.33333333333	0.33988372868	***Sh3***
	Mse	0.11539951950	0.11310748071	0.11329958668	0.11306294236	0.11111571060	0.11111193602	0.11111111111	0.11552366240	
	Mape	0.54351713765	0.53809723901	0.53855509046	0.53799124493	0.53334437161	0.53333531311	0.53333333333	0.54381396590	
25 100	Bias	0.34213219771	0.33613937519	0.33839618997	0.33607257135	0.33335002249	0.33333636252	0.33333333333	0.34183358236	***Sh3***
	Mse	0.11705980650	0.11299462457	0.11451286543	0.11294975195	0.11112223755	0.11111313058	0.11111111111	0.11685142058	
	Mape	0.54741151634	0.53782300030	0.54143390395	0.53771611416	0.53336003598	0.53333818003	0.53333333333	0.54693373178	
50 25	Bias	0.33649598018	0.33445118954	0.33381122593	0.33437920564	0.33333540212	0.33333371377	0.33333333333	0.33744723910	***Sh3***
	Mse	0.11323081718	0.11185831665	0.11143018760	0.11181017638	0.11111249035	0.11111136473	0.11111111111	0.11387289796	
	Mape	0.53839356830	0.53512190327	0.53409796150	0.53500672902	0.53333664340	0.53333394203	0.53333333333	0.53991558256	
50 100	Bias	0.33835302876	0.33580414595	0.33505955905	0.33573637581	0.33334974295	0.33333639867	0.33333333333	0.33898706990	***Sh3***
	Mse	0.11448355553	0.11276481921	0.11226564632	0.11271931116	0.11112205119	0.11111315468	0.11111111111	0.11491324494	
	Mape	0.54136484601	0.53728663353	0.53609529448	0.53717820130	0.53335958872	0.53333823788	0.53333333333	0.54237931185	
75 75	Bias	0.33543170413	0.33434182802	0.33355699699	0.33426948348	0.33333580810	0.33333380432	0.33333333333	0.33577597592	***Sh3***
	Mse	0.11251535484	0.11178635054	0.11126054313	0.11173799283	0.11111276098	0.11111142510	0.11111111111	0.11274590750	
	Mape	0.53669072661	0.53494692484	0.53369119519	0.53483117357	0.53333729297	0.53333408691	0.53333333333	0.53724156148	
75 100	Bias	0.33619827607	0.33519367135	0.33388279829	0.33512394306	0.33333804154	0.33333419510	0.33333333333	0.33617087325	***Sh3***
	Mse	0.11303283377	0.11236031747	0.11147850878	0.11231361763	0.11111424997	0.11111168562	0.11111111111	0.11301409913	
	Mape	0.53791724171	0.53630987416	0.53421247727	0.53619830890	0.53334086646	0.53333471216	0.53333333333	0.53787339720	
100 25	Bias	0.32838235222	0.33281049559	0.33112227208	0.33273329956	0.33330045612	0.33332721297	0.33333333333	0.32862266604	***MLE***
	Mse	0.10784110344	0.11076484333	0.10964404045	0.11071347823	0.11108919424	0.11110703091	0.11111111111	0.10799730755	
	Mape	0.52541176355	0.53249679294	0.52979563534	0.53237327930	0.53328072979	0.53332354076	0.53333333333	0.52579626567	
100 100	Bias	0.33365080418	0.33351552310	0.33357302666	0.33344059705	0.33332860431	0.33333243011	0.33333333333	0.33366411131	***Sh3***
	Mse	0.11132463748	0.11123295884	0.11127111399	0.11118298876	0.11110795848	0.11111050896	0.11111111111	0.11133449545	
	Mape	0.53384128669	0.53362483696	0.53371684266	0.53350495529	0.53332576690	0.53333188817	0.53333333333	0.53386257809	

Table 5: The Reliability Estimates R=(0.894906414692539) ʒ=0.2 k1=2 ,k2=1

n,m	$\hat{R}_{Mle}$	$\hat{R}_{Pe}$	$\hat{R}_{Ls}$	$\hat{R}_{wLse}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Sh3}$	$\hat{R}_{Emm}$
25,25	0.995552795838	0.9883077145136	0.9889153857995	0.9882940050072	0.9888622101518	0.9883737684996	0.9883297644256	0.9895886474004
25,50	0.995161107102	0.9883131449841	0.9889266864959	0.9882994480485	0.9883287330780	0.9883272225134	0.9883297644253	0.9896609287063
25,75	0.995206866100	0.9883332530672	0.9891184718485	0.9883196038887	0.9883476438438	0.9883324988370	0.9883297644253	0.9902685621166
25,100	0.995424534526	0.9882589410227	0.9892674624585	0.9882451152399	0.9888133985055	0.9883643499761	0.9883297644255	0.9900249757944
50,25	0.995627808493	0.9883610126927	0.9880961798202	0.9883474288662	0.9916033850203	0.9912478921415	0.9883297644190	0.9881684919299
50,50	0.992788709889	0.9883441934368	0.9884099852333	0.9883305707320	0.9883426787081	0.9883321707703	0.9883297644252	0.9891496497521
50,75	0.993621886346	0.9883549892837	0.9885829547934	0.9883413921102	0.9883437319029	0.9883323236577	0.9883297644253	0.9889418277385
50,100	0.994830827558	0.9883958262920	0.9886393003267	0.9883823258159	0.9883497692491	0.9883330785770	0.9883297644253	0.9894620048330
75,25	0.989751769039	0.9884538667678	0.9878814862312	0.9884405033004	0.9883252112238	0.9883289236429	0.9883297644252	0.9881141710484
75,50	0.989239722553	0.9883814183605	0.9882461121098	0.9883678836658	0.9883222915791	0.9883283438730	0.9883297644252	0.9886645180631
75,75	0.991464407062	0.9883569355648	0.9883978808086	0.9883433429741	0.9886986980440	0.9883410390771	0.9883297644251	0.9887316709344
75,100	0.990529473903	0.9883901849304	0.9884336715450	0.9883766712179	0.9883320942235	0.9883301847721	0.9883297644252	0.9889316200804
100,25	0.998305637639	0.9884606359864	0.9877506845374	0.9884472884759	0.9888356498616	0.9884286412439	0.9883297644252	0.9878448403923
100,50	0.987631815012	0.9883600967839	0.9882759502156	0.9883465118032	0.9883193516564	0.9883278020013	0.9883297644252	0.9885582370178
100,75	0.989617561255	0.9883480354993	0.9883545592219	0.9883344218842	0.9883284659870	0.9883295281064	0.9883297644252	0.9885483482704
100,100	0.989736156632	0.9883550784098	0.9883212703091	0.9883414815463	0.9883301439429	0.9883298316777	0.9883297644252	0.9886006299337

Table 6: The Bias, MSE and MAPE of The Simulated Estimates When R=(0.894906414692539) ʒ=0.2 k1=2 ,k2=1

n, m		$\hat{R}_{Mle}$	$\hat{R}_{Pe}$	$\hat{R}_{Ls}$	$\hat{R}_{wLse}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Sh3}$	$\hat{R}_{Emm}$	Best
25 50	**Bias**	0.10025469240	0.09340673029	0.09402027180	0.09339303335	0.09342231838	0.09342080782	0.09342334973	0.09475451401	***WLS***
	Mse	0.01005233603	0.00872483097	0.00883987318	0.00872227245	0.00872773119	0.00872744737	0.00872792227	0.00897856745	
	Mape	0.11202813027	0.10437597580	0.10506156874	0.10436067037	0.10439339449	0.10439170653	0.10439454695	0.10588203689	
25 100	**Bias**	0.10051811983	0.09335252633	0.09436104776	0.09333870054	0.09390698381	0.09345793528	0.09342334973	0.09511856110	***WLS***
	Mse	0.01010653239	0.00871470849	0.00890404183	0.00871212740	0.00881866359	0.00873438631	0.00872792227	0.00904759468	
	Mape	0.11232249337	0.10431540639	0.10544236382	0.10429995697	0.10493497674	0.10443319407	0.10439454695	0.10628883595	
50 25	**Bias**	0.09788229519	0.09343777874	0.09350357054	0.09342415603	0.09343626401	0.09342575607	0.09342334973	0.09424323505	***Sh3***
	Mse	0.00958329516	0.00873062155	0.00874292612	0.00872807600	0.00873033599	0.00872837191	0.00872792227	0.00888186756	
	Mape	0.10937712993	0.10441067044	0.10448418852	0.10439544795	0.10440897783	0.10439723589	0.10439454695	0.10531071574	
50 100	**Bias**	0.09992441286	0.09348941159	0.09373288563	0.09347591112	0.09344335455	0.09342666388	0.09342334973	0.09455559014	***Sh3***
	Mse	0.00998584502	0.00874027351	0.00878587748	0.00873774940	0.00873166058	0.00872854152	0.00872792227	0.00894080047	
	Mape	0.11165906426	0.10446836681	0.10474043329	0.10445328091	0.10441690105	0.10439825030	0.10439454695	0.10565975233	

75 75	Bias	0.09655799237	0.09345052087	0.09349146611	0.09343692828	0.09379228335	0.09343462438	0.09342334973	0.09382525624	***Sh3***
	Mse	0.00932905372	0.00873300445	0.00874067128	0.00873046419	0.00879751862	0.00873002952	0.00872792227	0.00880319259	
	Mape	0.10789730723	0.10442490894	0.10447066260	0.10440972010	0.10480680640	0.10440714565	0.10439454695	0.10484365147	
75 100	Bias	0.09562305921	0.09348377023	0.09352725685	0.09347025652	0.09342567953	0.09342377007	0.09342334973	0.09402520538	***Sh3***
	Mse	0.00914419091	0.00873921684	0.00874735130	0.00873669040	0.00872835759	0.00872800081	0.00872792227	0.00884079503	
	Mape	0.10685257993	0.10446206296	0.10451065644	0.10444696226	0.10439715035	0.10439501666	0.10439454695	0.10506708170	
100 25	Bias	0.09272540032	0.09345368209	0.09336953552	0.09344009711	0.09341293696	0.09342138730	0.09342334973	0.09365182232	***MLE***
	Mse	0.00859834678	0.00873359302	0.00871788047	0.00873105408	0.00872597680	0.00872755560	0.00872792227	0.00877074403	
	Mape	0.10361463366	0.10442844140	0.10433441306	0.10441326107	0.10438291136	0.10439235407	0.10439454695	0.10464985029	
100 100	Bias	0.09482974194	0.09344866371	0.09341485561	0.09343506685	0.09342372925	0.09342341698	0.09342334973	0.09369421524	***LS***
	Mse	0.00899288582	0.00873265500	0.00872633987	0.00873011397	0.00872799319	0.00872793484	0.00872792227	0.00877861748	
	Mape	0.10596609922	0.10442283369	0.10438505533	0.10440764008	0.10439497104	0.10439462210	0.10439454695	0.10469722163	

Table 7: The Bias, MSEs And MAPEs For The All Estimation Methods Using Real Data.

n, m		$\hat{R}_{Mle}$	$\hat{R}_{Pe}$	$\hat{R}_{Ls}$	$\hat{R}_{wLse}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Sh3}$	$\hat{R}_{Emm}$	Best
R=(0.2950107977) ʒ=1 k1=1 , k2=1.5										
74 63	R^	0.97865323321	0.96544789001	0.71853985278	0.96541957459	0.92033172446	0.92164793315	0.92161309999	0.96973830153	***LS***
	Bias	0.68364243545	0.67043709225	0.42352905502	0.67040877683	0.62532092670	0.62663713539	0.62660230223	0.67472750377	
	Mse	0.46736697955	0.44948589467	0.17937686045	0.44944792805	0.39102626137	0.39267409946	0.39263044516	0.45525720434	
	Mape	2.31734716370	2.27258492688	1.43563916386	2.27248894590	2.11965437011	2.12411593120	2.12399785701	2.28712816241	
R=(0.731308969808190) ʒ=1 k1=1.5 , k2=1										
74 63	R^	0.90707411349	0.96544789001	0.97400905331	0.96541957459	0.96996764470	0.97015014141	0.97015124827	0.94032492227	***MLE***
	Bias	0.17576514368	0.23413892020	0.24270008350	0.23411060478	0.23865867489	0.23884117160	0.23884227846	0.20901595246	
	Mse	0.03089338573	0.05482103395	0.05890333053	0.05480777527	0.05695796310	0.05704510525	0.05704563398	0.04368766838	
	Mape	0.24034320778	0.32016415752	0.33187078721	0.32012543869	0.32634452023	0.32659406825	0.32659558178	0.28581073266	
R=(0.625000) ʒ=1 k1=1 , k2=1										
74 63	R^	0.97445223707	0.96544789001	0.95588596723	0.96541957459	0.95796054182	0.95833900885	0.95833333333	0.96095223809	***LS***
	Bias	0.34945223707	0.34044789001	0.33088596723	0.34041957459	0.33296054182	0.33339000885	0.33333333333	0.33595223809	
	Mse	0.12211686599	0.11590476581	0.10948552331	0.11588548676	0.11086272241	0.11111489482	0.11111111111	0.11286390627	
	Mape	0.55912357932	0.54471662402	0.52941754758	0.54467131934	0.53273686691	0.53334241416	0.53333333333	0.53752358094	

5.Conclusion

In Tables (1-6) for the random data, we conclude the following:
1- The value of MSE decreases with increasing sample size (n, m) for all the factors under study.
2- When the value of k decreases, the value of the estimated reliability decreases.
3 - When k1 < k2, the MLE method is the best possible.
4 - When k1 = k2, the (Sh3) method is the best possible
5- When k1 > k2, the estimation methods alternate with each other according to the sample size, but the best methods are (Sh3, MLE, LE, WLS, LS), respectively.
In Table (7) the practical example, we conclude the following:
1 - When k1 > k2, MLE is the best possible method.
2 - When k1 < k2 the (LS) method is the best possible.
3 - When k1=k2, the LS method is the best it can be.
In general, the (LS) method is the best possible in the applied example.
In general, the Monte Carlo simulations are used to compare reliability estimates for small samples, with the MLE approach under k1 and k2 estimators providing the best results, and using three criteria (Bias, Mse, Mape).

6.Acknowledgements

We thank all the people and staff of Mathematical Department/ Education collage for pure Science/ Anbar University who helped us most in carrying out our research work.

References

[1] Bader, M. and Priest, A. "Statistical Aspects of Fibre and Bundle Strength in Hybrid Composites," *Progress in Science and Engineering of Composites*, *ICCM-IV, Tokyo*, pp. 1129-1136, 1982.
[2] Bhat, A.A.and Ahmad, S. P., "A New Generalization of Rayleigh Distribution: Properties and Applications," *Pakistan Journal of Statistics*, vol. 36, no. 3, pp. 225-250, 2020.
[3] Dey, S., Dey, T. and Kundu, D., "Two-parameter Rayleigh distribution: Different methods of estimation," *American Journal of Mathematical and Management Sciences*, vol. 33, pp. 55-74, 2014.
[4] Haddad, E., and Batah, F., "On Estimating Reliability of a Stress–Strength Model in Case of Rayleigh Pareto Distribution," *Iraqi Journal of Science*, vol. 62, no. 12, pp.4847-4858, 2021.
[5] Hassan, A. and Basheikh,H., "Estimation of Reliability in Multicomponent Stress–strength Model Following Exponentiated Pareto Distribution," *The Egyptian Statistical Journal, Institute Of Statistical Studies & Research, Cairo University*, vol. 56, no. 2, pp. 82-95, 2012.
[6] Iriarte, Y. A., Gómez, H. W., Varela, H. and Bolfarine, H., "Slashed Rayleigh distribution," *Revista Colombiana de Estadística*, vol. 38, no. 1, pp. 31-44, 2015.
[7] Kao , J., "computer methods for estimating Weibull parameters in reliability studies,*" transaction of IRE- Reliability and Quality control*, vol. 13, pp. 15- 22, 1958.
[8] Kundu, D. and Gupta, R., "Estimation of P(Y < X) for the generalized exponential distribution," *Metrika*, vol. 61, no. 3, pp 291–308, 2005.
[9] Salman ,A and Hamad ,A. "estimating the shape parameter for power function distribution through shrinkage technique,*" International Journal of science and research* , vol. 78, no. 96, pp.1316-1319, 2015.
[10] Surles, J., and Padgett, W., " Inference for P(Y < X) in the Burr type X model," *Journal of Applied Statistical Sciences*, vol. 7, pp. 225–238, 1998.
[11] Abid , salah." The fréchet stress-strength model"International Journal of Applied Mathematical Research," *International Journal of Applied Mathematical Research*, vol. 3, no. 3, pp. 207-213,2014.
[12] Knndu, D. and Raqab,M . " *Estimation of R=P(Y<X) for three-parameter Weibull distribution," Published by Elsevier, Statistics and Probaibilty Letters*, vol. 79, no. 17, pp.1839-1846, 2009.