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Reliability of Stress - Strength and Its Estimation of Exponentiated Q-Exponential Distribution

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Abstract

In this paper, we study a single stress-strength reliability system R = p(ɣ > Ƹ), where Ƹ and ɣ are independently Exponentiated q-Exponential distribution. There are a few traditional estimating approaches that are derived, namely maximum likelihood estimation (MLE) and the Bayes (BE) estimators of R. A wide mainframe simulation is used to compare the performance of the proposed estimators using MATLAB program. A simulation study show that the Bayesian estimator is the best estimator than other estimation method under consideration using two criteria such as the “mean squares error (MSE)” and “mean absolutely error (MAPE)”.

Keywords: Bayesian estimation, Reliability, Stress-strength model, The maximum likelihood (MLE), Exponentiated q-Exponential distribution.

موثوقية الإجهاد – المتانه وتقديرها للتوزيع الأسّي -Q الأسي

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الخلاصة

في هذا البحث ، قمنا بدراسة نظامًا فرديًا لموثوقية الإجهاد – المتانه ، R = p (ɣ> Ƹ)، حيث Ƹ و هما التوزيع الأسي Q بشكل مستقل. هناك عدد قليل من أساليب التقدير التقليدية ، مثل تقدير الاحتمالية لدالة الامكان الاعظم (MLE) وتم اشتقاق تقديرات بايز (BE) لـ R. تم استخدام نظام المحاكاة الحاسب مركزي واسع لمقارنة أداء المقدرين المقترحين باستخدام برنامج MATLAB. أظهرت دراسة المحاكاة أن مقدرات بيز أفضل من المقدرات الاخرى باستخدام معياري "متوسط خطأ المربعات (MSE)" و "معيار الخطأ المطلق (MAPE)".

1. Introduction

In reliability research, the stress-strength model is widely used to characterize the life of a random variable with a strength Ƹ and a stress ɣ. When the stress is applied to the component that exceeds its strength, the component fails, otherwise, we have the case ,if ɣ >Ƹ, then ɣ >Ƹ, the component is work well. The single system R = Pr(ɣ >Ƹ) is a measure of component reliability. It is used by physicists, engineers, geneticists, psychologists, and economists..

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Church and Harris [1] used the term stress-strengths to describe the calculation of R when the strength and stress are evenly distributed. So that several parametric and non-parametric studies have been published. The monograph by Kotz et al. [2]is an excellent resource for explaining the various stress-strength models. The stress-strength model is discussed with some instances in [3] [4] [5].

The Exponentiated q-Exponential distribution (EQED), which was established as an augmentation of the exponentiated generalized family [6]. In [6], the authors explored for stress-strength reliability. The EQED distribution contains the c. d. f. f(ƺ) and p. d. f. F(ƺ) as follows: [7]

$$F_{\alpha}(ƺ, \lambda, M+1) = \left(1 - [1 + M\lambda ƺ]^{\frac{M-1}{M}}\right)^{\alpha}$$

Where ƺ> 0 , α, λ and (M + 1) are all true positive numbers and

$$f_{\alpha}(ƺ, \lambda, M+1) = \alpha(1 - M)\lambda eq(-\lambda ƺ)\left(1 - [1 + M\lambda ƺ]^{\frac{M-1}{M}}\right)^{\alpha-1}$$

Where $eq(-\lambda ƺ) = [1 - M\lambda ƺ]^{\frac{-1}{M}}$.

In this work, we proposed a single stress-strength reliability system in which ƺ and Y are followed Exponentiated q-Exponential distributions. The paper is structured as follows: The maximum likelihood (MLE) and the Bayes (BE) estimation methods of R are presented in Section 2. Simulation study is utilized to compare the performance of these estimations by the MATLAB. In section 3, a large mainframe simulation is utilized to compare the performance of the proposed estimators. Two_criteria for that comparison of these estimators are used,namely the mean squares error (MSE) and mean absolutely error (MAPE) respectively. The result of the analysis reveals that the BE is better than the MLE in section 4. . Finally, the conclusions are given in the last section.

2. Stress-Strength Reliability

In general, the stress–strength concept is employed in a variety of engineering applications. Let ƺ and ƴ be random variables refer to stress and strength which follow EQE(σ, λ, M + 1) with two distinct parameters, respectively. The formula of the c.d.f. and the p. d. f. of ƺ and ƴ are given by: [6]

$$R = P(ƴ > ƺ) = \int_0^{\infty}\int_0^{ƴ} f(ƺ)f(ƴ)dƺ\, dƴ$$

$$= \int_0^{\infty}\left(\int_0^{ƴ} f(ƺ)dƺ\right)f(ƴ)\, dƴ$$

$$= \int_0^{\infty} F_{ƺ}(ƴ)f(ƴ)dƴ$$

$$= \int_0^{\infty}\left[\begin{matrix}\left(1 - [1 + M\lambda ƴ]^{\frac{M-1}{M}}\right)^{\alpha} \alpha(1 - M)\,\lambda(1 + M\lambda ƴ)^{\frac{-1}{M}} \\ \left[1 - (1 + M\lambda ƴ)^{\frac{M-1}{M}}\right]^{\alpha-1}\end{matrix}\right]dƴ$$

$$R = P(ƴ > ƺ) = \frac{\sigma}{\alpha+\sigma} \quad (1)$$

2.1 The Maximum likelihood estimator (MLE)

Due to its variety of features, the MLE method, which was first presented by R.A. Fisher. It is one of the most essential and popular conventional ways. Good capabilities, such as being impartial or maybe biased, adequate, complete, efficient, consistent, and approximate that determine its capabilities. When the sample size is large, most statisticians prefer this estimate. The purpose of this method is to increase probability as much as feasible [8], [9], [10]. To estimate the parameters of the $EQE(\alpha, \lambda, M+1)$ distribution, the maximum likelihood estimation approach uses a complete sample. Now, when $(M+1)$ is known and σ is unknown parameters, then the likelihood function in equation is as follows:

$$L = \prod_{i=1}^{n} f_\sigma(y_i, \lambda, M+1) = \prod_{i=1}^{n} \sigma(1-M)\lambda(1+M\lambda y_i)^{\frac{-1}{M}}\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]^{\sigma-1}$$

$$= \sigma^n(1-M)^n\lambda^n \prod_{i=1}^{n}(1+M\lambda y_i)^{\frac{-1}{M}} \prod_{i=1}^{n}\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]^{\sigma-1} \qquad (2)$$

$$\ln L = \ln\left(\prod_{i=1}^{n} f_\sigma(y_i, \lambda, M+1)\right) = n\ln\sigma + n\ln(1-M) + n\ln\lambda + \frac{-1}{M}\sum_{i=1}^{n}\ln(1+M\lambda y_i)$$

$$+(\sigma-1)\sum_{i=1}^{n}\ln\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]$$

$$\frac{\partial \ln L}{\partial \sigma} = \frac{n}{\sigma} + \sum_{i=1}^{n}\ln\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]$$

When $\frac{\partial \ln L}{\partial \sigma} = 0$, then

$$\hat{\sigma} = \frac{-n}{\sum_{i=1}^{n}\ln\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]} \qquad (3)$$

Similarly, if $(\xi_1, \dots, \xi_m)$ is a random sample from a stress ξ that is distributed as an $EQE(\alpha, \lambda, M+1)$ distribution with λ and $M+1$ are known but the α shape parameter is unknown, then the likelihood function by the MLE approach is:

$$\hat{\alpha} = \frac{-m}{\sum_{i=1}^{m}\ln\left[1-(1+M\lambda \xi_i)^{\frac{M-1}{M}}\right]} \qquad (4)$$

Where the sample size of y and ξ are n and m, respectively. Equations (3) and (4) are substituted into equation (1) to obtain

$$\hat{R} = \frac{\hat{\sigma}}{\hat{\alpha}+\hat{\sigma}} \qquad (5)$$

$$\hat{R}_{mle} = \frac{\dfrac{n}{\sum_{i=1}^{n}\ln\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]}}{\dfrac{m}{\sum_{i=1}^{m}\ln\left[1-(1+M\lambda \xi_i)^{\frac{M-1}{M}}\right]} + \dfrac{n}{\sum_{i=1}^{n}\ln\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]}} \qquad (6)$$

2.2 Bayes estimation

In this section, we use Gamma prior loss function to estimate the distribution parameters by Bayesian estimation method as follows

$$\prod_{i=1}^{r} f_\alpha(\xi_i, \lambda, M+1) = \alpha^r(1-M)^r\lambda^r\prod_{i=1}^{r}(1+M\lambda\xi_i)^{\frac{-1}{M}}\prod_{i=1}^{r}\left[1-(1+M\lambda\xi_i)^{\frac{M-1}{M}}\right]^{\alpha-1}$$

$$\prod_{i=1}^{r} f_\sigma(y_i, \lambda, M+1) = \sigma^r(1-M)^r\lambda^r\prod_{i=1}^{r}(1+M\lambda y_i)^{\frac{-1}{M}}\prod_{i=1}^{r}\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]^{\alpha-1}$$

$$L(\alpha, \lambda, M+1 \backslash \xi) = \alpha^r(1-M)^r\lambda^r\prod_{i=1}^{r}(1+M\lambda\xi_i)^{\frac{-1}{M}}\prod_{i=1}^{r}\left[1-(1+M\lambda\xi_i)^{\frac{M-1}{M}}\right]^{\alpha-1} \qquad (7)$$

$$L(\alpha,\lambda,M+1\backslash y) = \sigma^{r}(1-M)^{r}\lambda^{r}\prod_{i=1}^{r}(1+M\lambda y_i)^{\frac{-1}{M}}\prod_{i=1}^{r}\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]^{\sigma-1} \quad (8)$$

2.2.1 Bayesian estimation based on Gamma prior

The most frequently used parameter prior distribution (α,σ) is the Gamma distribution with hyper-parameters a, b_1 and b_2 with pdf is given by: [11]

$$g(\alpha) = \frac{b_1^a}{\Gamma a}\alpha^{a-1}e^{-b_1\alpha}, \alpha > 0;\ b_1, a > 0\,, \quad (9)$$

$$g(\sigma) = \frac{b_2^a}{\Gamma a}\sigma^{a-1}e^{-b_2\sigma},\ \sigma > 0;\ b_2, a > 0, \quad (10)$$

then the joint p. d. f. is $g(\alpha,\sigma) = \frac{b_1^a}{\Gamma a}\frac{b_2^a}{\Gamma a}\alpha^{a-1}\sigma^{a-1}e^{-b_1\alpha}e^{-b_2\sigma}$ and the likelihood function is

$$L\left(\alpha,\sigma \,\middle|\, \underline{\xi}, y\right) = \alpha^{r}\sigma^{r}(1-M)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1+M\lambda\xi_i)^{\frac{-1}{M}}\prod_{i=1}^{r}\left[1-(1+M\lambda\xi_i)^{\frac{M-1}{M}}\right]^{\alpha-1}$$
$$\prod_{i=1}^{r}(1+M\lambda y_i)^{\frac{-1}{M}}\prod_{i=1}^{r}\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]^{\sigma-1}$$

The posterior function is as follows :

$$P\left(\alpha,\sigma \,\middle|\, \underline{\xi}, \underline{y}\right) = \frac{L\left(\alpha,\sigma \,\middle|\, \underline{\xi}, \underline{y}\right)g(\alpha,\sigma)}{\int_0^\infty\int_0^\infty L\left(\alpha,\sigma \,\middle|\, \underline{\xi}, \underline{y}\right)g(\alpha,\sigma)d\alpha d\sigma}$$

Using Equations (7),(8) ,(9) and (10), it will be:

$$\int_0^\infty\int_0^\infty L\left(\alpha,\sigma \,\middle|\, \underline{\xi}, y\right)g(\alpha,\sigma)d\alpha d\sigma$$
$$= \frac{b_1^a}{\Gamma a}\frac{b_2^a}{\Gamma a}(1-M)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1+M\lambda\xi_i)^{\frac{-1}{M}}\prod_{i=1}^{r}(1+M\lambda y_i)^{\frac{-1}{M}}$$
$$\int_0^\infty\int_0^\infty \alpha^{r+a-1}\sigma^{r+a-1}\prod_{i=1}^{r}\left[1-(1+M\lambda\xi_i)^{\frac{M-1}{M}}\right]^{\alpha-1}$$
$$\prod_{i=1}^{r}\left[1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right]^{\sigma-1} e^{-b_1\alpha}e^{-b_2\sigma}\,d\alpha d\sigma$$
$$= \frac{b_1^a}{\Gamma a}\frac{b_2^a}{\Gamma a}(1-M)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1+M\lambda\xi_i)^{\frac{-1}{M}}\prod_{i=1}^{r}(1+M\lambda y_i)^{\frac{-1}{M}}$$
$$e^{-\sum_{i=1}^{r} z_i}\int_0^\infty \alpha^{r+a-1}\,e^{-(b_1-\sum_{i=1}^{r} z_i)\alpha}d\alpha$$
$$e^{-\sum_{i=1}^{r} w_i}\int_0^\infty \sigma^{r+a-1}\,e^{-(b_2-\sum_{i=1}^{r} w_i)\sigma}d\sigma$$
$$= \frac{b_1^a}{\Gamma a}\frac{b_2^a}{\Gamma a}(1-M)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1+M\lambda\xi_i)^{\frac{-1}{M}}\prod_{i=1}^{r}(1+M\lambda y_i)^{\frac{-1}{M}}$$
$$\frac{(r+a-1)!\,e^{-\sum_{i=1}^{r} z_i}}{(b_1-\sum_{i=1}^{r} z_i)^{r+a}} * \frac{(r+a-1)!\,e^{-\sum_{i=1}^{r} w_i}}{(b_2-\sum_{i=1}^{r} w_i)^{r+a}}$$

$$P\left(\alpha,\sigma \,\middle|\, \underline{\xi}, \underline{y}\right) = \frac{L\left(\alpha,\sigma \,\middle|\, \underline{\xi}, \underline{y}\right)g(\alpha,\sigma)}{\int_0^\infty\int_0^\infty L\left(\alpha,\sigma \,\middle|\, \underline{\xi}, \underline{y}\right)g(\alpha,\sigma)d\alpha d\sigma}$$

$$= \frac{\alpha^{r+a-1}\sigma^{r+a-1}e^{-(b_1-\sum_{i=1}^{r} z_i)\alpha}e^{-(b_2-\sum_{i=1}^{r} w_i)\sigma}}{\frac{(r+a-1)!}{(b_1-\sum_{i=1}^{r} z_i)^{r+a}} * \frac{(r+a-1)!}{(b_2-\sum_{i=1}^{r} w_i)^{r+a}}}$$

$$\hat{\alpha} = E(\alpha) = \int_0^\infty \int_0^\infty \frac{\alpha^{r+a}\sigma^{r+a-1}e^{-(b_1-\sum_{i=1}^{r} z_i)\alpha}e^{-(b_2-\sum_{i=1}^{r} w_i)\sigma}}{\frac{(r+a-2)!}{(b_1-\sum_{i=1}^{r} z_i)^{r+a-1}} * \frac{(r+a-2)!}{(b_2-\sum_{i=1}^{r} w_i)^{r+a-1}}} d\alpha d\sigma$$

$$\hat{\alpha} = \frac{(r+a)}{(b_1-\sum_{i=1}^{r} z_i)} \tag{11}$$

Similarly, we get

$$\hat{\sigma} = \frac{(r+a)}{(b_2-\sum_{i=1}^{r} w_i)} \tag{12}$$

Where

$$z = \ln\left(1 - (1 + M\lambda \varsigma_i)^{\frac{M-1}{M}}\right) \tag{13}$$

$$w = \ln\left(1 - (1 + M\lambda y_i)^{\frac{M-1}{M}}\right) \tag{14}$$

Equations (11), (12), (13) and (4) are substituted into equation (1) to obtain

$$\hat{R}_{Bayes} = \frac{\frac{(r+a)}{\left(b_2-\sum_{i=1}^{r}\ln\left(1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right)\right)}}{\frac{(r+a)}{\left(b_1-\sum_{i=1}^{r}\ln\left(1-(1+M\lambda \varsigma_i)^{\frac{M-1}{M}}\right)\right)} + \frac{(r+a)}{\left(b_2-\sum_{i=1}^{r}\ln\left(1-(1+M\lambda y_i)^{\frac{M-1}{M}}\right)\right)}}. \tag{15}$$

3. The simulation study

Simulation is a way of employing computer models to replicate or simulate real-world occurrences. We regularly encounter processes that are difficult to interpret in real life; as a result, it is desirable to depict these processes in a style that is comparable to real-life images using certain models. [8]

Many phases of application of techniques for measuring the system reliability of single systems were used in simulation tests. In this paper, the estimation methods are established on a variety of six samples (15, 20, 25, 30, 40, and 90). Statistical data for each sample is based on mean absolute error (MAPE) and mean squared error (MSE) with 1000 repetitions are used as follows

$MSE = \frac{1}{L}\sum_{i=1}^{n}\left(\hat{R}_i - R\right)^2$ and $MAPE = \frac{1}{L}\sum_{i=1}^{n}\frac{|\hat{R}_i - R|}{|R|}$

For this purpose, the Monte Carlo simulation stages are as follows:

Stage 1: Create random samples as $u_1, \dots, u_n$ and $w_1, \dots, w_m$ that follow a continuous uniform distribution that is well-defined on the interval (0, 1).

Stage 2: We substituted $F(\varsigma)$ and $F(y)$, to assemble discrete values for the two random variables, respectively, by the inverse p.d.f. we obtain that

$$F(\varsigma) = \left(1 - [1 + M\lambda\varsigma]^{\frac{M-1}{M}}\right)^{\alpha} = U_i,\ \forall i = 1, \dots, n$$

and

$$F(y) = \left(1 - [1 + M\lambda y]^{\frac{M-1}{M}}\right)^{\sigma} = W_j,\ \forall j = 1, \dots, m$$

convert the previously mentioned random uniform samples to samples that are random having the followed EQED.

$$\varepsilon_i = \frac{\left(1-U_i^{\frac{1}{\alpha}}\right)^{\frac{M-1}{M}}-1}{M\lambda} \quad ,\forall i = 1, \dots, n \text{ and } y_j = \frac{\left(1-w_j^{\frac{1}{\sigma}}\right)^{\frac{M-1}{M}}-1}{M\lambda} \quad ,\forall j = 1, \dots, m$$

Stage3: Using the formula (1) to compute R.

Stage4: Using formulae (6), and (15), determine R of the MLE, and Bayes, respectively.

Stage5: Using two parameters (α, σ) formulae (3), (4), (11) and (12), compute the MLE, and Bayes estimators.

Stage6: Compute MSE and MAPE criteria are based on replication of (L=1000) and (n=15, m = 15) represented the smallest sample size, (n = 30, m = 30) for moderate, and (n = 90, m = 90) for large sample sizes in three experiments with varied parameter values.

4. Conclusions:

Tables 1-3 summarize the simulation study's findings, which show how system reliability estimations increase as sample sizes change. Using the Mean Square Error (MSE) and Mean Absolute Error (MAPE) criterion, MSE decreases while n remains fixed and m changes. We concluded that the Bayes estimators (BE) for R are better than the MLE estimators, and that their performance is superior for all small, medium, and large sample sizes.

Table 1: Summary of the MSE and MAPE values of reliability estimators of first experiment

α=0.2, σ=0.5, R= 0.7143												
λ=0.4, M =4, a=4, b1=0.4, b2=0.8												
n	**m**	**r**	$\hat{\alpha}_{mle}$	$\hat{\sigma}_{mle}$	$\hat{\alpha}_{bayes}$	$\hat{\sigma}_{bayes}$	$\hat{R}_{mle}$	$\hat{R}_{Bayes}$	**MSE** $\hat{R}_{mle}$	**MAPE** $\hat{R}_{mle}$	**MSE** $\hat{R}_{Bayes}$	**MAPE** $\hat{R}_{Bayes}$
15	15	5	0.4876	1.4592	0.2915	0.8222	0.7400	0.7297	0.0080	0.1012	0.0075	0.0965
15	20	10	0.4965	1.4149	0.4587	0.9270	0.7346	0.6661	0.0068	0.0933	0.0105	0.1105
20	25	15	0.4847	1.4267	0.4512	1.0212	0.7410	0.6903	0.0058	0.0844	0.0069	0.0898
25	25	20	0.4727	1.4178	0.4478	1.2896	0.7452	0.7378	0.0051	0.0786	0.0056	0.0796
25	30	22	0.4743	1.4029	0.4933	1.1772	0.7432	0.7018	0.0056	0.0813	0.0050	0.0755
30	40	25	0.4726	1.3750	0.4493	0.9757	0.7422	0.6839	0.0042	0.0707	0.0064	0.0834
40	50	30	0.4580	1.3692	0.3889	0.9162	0.7480	0.7019	0.0047	0.0704	0.0051	0.0699
90	90	30	0.4449	1.3544	0.1678	0.4786	0.7538	0.7412	0.0049	0.0639	0.0040	0.0552

Table 2: Summary of the MSE and MAPE values of reliability estimators of second experiment

α=0.3, σ=0.9, R= 0.7500												
λ=0.4, **M**=4, a=4, b1=2.1, b2=1.5												
n	**m**	**r**	$\hat{\alpha}_{mle}$	$\hat{\sigma}_{mle}$	$\hat{\alpha}_{bayes}$	$\hat{\sigma}_{bayes}$	$\hat{R}_{mle}$	$\hat{R}_{Bayes}$	**MSE** $\hat{R}_{mle}$	**MAPE** $\hat{R}_{mle}$	**MSE** $\hat{R}_{Bayes}$	**MAPE** $\hat{R}_{Bayes}$
15	15	5	0.7991	3.2749	0.4205	1.4078	0.7889	0.7637	0.0076	0.0964	0.0048	0.0760
15	20	10	0.7940	3.0996	0.6634	1.7139	0.7880	0.7168	0.0063	0.0889	0.0065	0.0818
20	25	15	0.7690	3.0488	0.6792	1.9522	0.7903	0.7371	0.0059	0.0865	0.0043	0.0683
25	25	20	0.7733	2.9937	0.6824	2.4547	0.7876	0.7780	0.0048	0.0778	0.0034	0.0643
25	30	22	0.7653	2.9536	0.7444	2.2561	0.7883	0.7473	0.0046	0.0753	0.0035	0.0631
30	40	25	0.7544	2.9966	0.6945	1.9175	0.7941	0.7316	0.0045	0.0746	0.0032	0.0582
40	50	30	0.7465	2.9495	0.6057	1.8280	0.7945	0.7481	0.0039	0.0709	0.0022	0.0494
90	90	30	0.7350	2.8800	0.2719	0.9900	0.7948	0.7828	0.0029	0.0630	0.0019	0.0497

Table 3: The MSE and MAPE values of reliability estimators of third experiment

α=2 , σ=3.7, R= 0.6491												
λ=0.8, **M**=4, a=4, b1=0.4, b2=0.8												
n	**m**	**r**	$\hat{\alpha}_{mle}$	$\hat{\sigma}_{mle}$	$\hat{\alpha}_{bayes}$	$\hat{\sigma}_{bayes}$	R_{mle}	R_{bayes}	**MSE** (R_{mle})	**MAPE** (R_{mle})	**MSE** (R_{Bayes})	**MAPE** (R_{Bayes})
15	15	5	10.3598	28.1370	4.7532	6.4731	0.7141	0.5823	0.0180	0.1714	0.0125	0.1377
15	20	10	10.3129	26.8308	7.3619	8.7441	0.7096	0.5500	0.0156	0.1607	0.0180	0.1703
20	25	15	10.0489	25.7933	7.7867	10.3350	0.7091	0.5742	0.0135	0.1499	0.0121	0.1349
25	25	20	9.8387	25.3868	7.8906	13.3117	0.7109	0.6289	0.0125	0.1428	0.0063	0.0966
25	30	22	9.5584	25.7105	8.6668	12.8820	0.7197	0.5997	0.0130	0.1487	0.0083	0.1113
30	40	25	9.4969	24.9027	8.2330	11.7399	0.7181	0.5899	0.0110	0.1366	0.0086	0.1161
40	50	30	9.3531	24.3734	7.2476	11.8560	0.7181	0.6210	0.0093	0.1269	0.0050	0.0858
90	90	30	9.0322	23.8660	3.2602	7.0730	0.7224	0.6833	0.0076	0.1182	0.0031	0.0704

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