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Fuzzy Survival and Hazard Functions Estimation for Rayleigh distribution

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Abstract

In this article, performing and deriving the probability density function for Rayleigh distribution by using maximum likelihood estimator method and moment estimator method, then crating the crisp survival function and crisp hazard function to find the interval estimation for scale parameter by using a linear trapezoidal membership function. A new proposed procedure used to find the fuzzy numbers for the parameter by utilizing ($\bar{x} \pm s^2$) to find a fuzzy numbers for scale parameter of Rayleigh distribution. applying two algorithms by using ranking functions to make the fuzzy numbers as crisp numbers. Then computed the survival functions and hazard functions by utilizing the real data application.

Keyword: Maximum likelihood estimator, moment estimator, survival function and hazard function, fuzzy number.

تقديردالة البقاء والمخاطرة باستخدام النظرية الضبابية لتوزيع رالي

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الخلاصة

في هذه البحث تم تقديرمعلمة القياس لتوزيع رالي باستخدام عدة طرق منها :- طريقه الأمكان الاعظم ,
maximum likelihood methodطريقه العزومmoment method .
ومن ثم أوجدنا دالة البقاء التقليدية ودالة المخاطرة التقليدية لأيجاد تقدير الفترة لمعلمة القياس لتوزيع رايلي بأستخدام دالة عضوية شبة المنحرف الخطي.
تم أقتراح أسلوب جديد لمعالجة وايجاد الاعداد الضبابية لمعلمة القياس من خلال حساب ($\bar{x} \mp s^2$) وذلك من خلال أستخدام دالة عضوية شبة المنحرف الخطي .
بعد ذلك طبقنا أثنين من الخوارزميات من خلال الدالة الرتبية لتحويل الأعداد التقليدية الى أعداد ضبابية ومن ثم المقارنة بين دالة البقاء ودالة المخاطرة من خلال استخدام تطبيق لبيانات حقيقية .

1. Introduction

One of the most popular functions in statistic is Rayleigh distribution which used in failure and survival times. In (2013) **pak et al.** [1] studied two parameters of weibull distribution when data are fuzzy. In (2014) **pak and Saraj** [2] studied the parameter of exponential distribution in presence of fuzzy data. In (2014) **Shafiq and Viertl** [3] used the maximum likelihood estimator for two parameters of weibull distribution when it's fuzzy data. In (2016) **pak** [4] studied inference for one parameter of lognormal distribution in presence of fuzzy data. In (2016) **Jasim and Hussein** [5] studied the two parameters of weibull distribution by using maximum likelihood method when the data are fuzzy. In (2017) **Shafiq** [6] studied the two parameters of Pareto distribution when it's fuzzy data. In (2017) **Pak** [7] studied statistical inference for the two parameters of Lindley distribution

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when it's fuzzy data. The aim of this article is estimate the parameter of Rayleigh distribution by using maximum likelihood method and moment method then estimate the survival and hazard functions. After that, using interval estimation to find the scale parameter of Rayleigh distribution. Therefore, fuzzify the estimation of scale parameter by using trapezoidal membership depending on ($\bar{x} + s^2$) and ($\bar{x} \quad - s^2$) to fuzzify this parameter, then utilizing the ranking function procedure to transform the fuzzy parameter to crisp parameter. Finally, estimating the fuzzy survival and hazard function and perform comparing between crisp and fuzzy Survival functions by using mean square error to prefer between them.

2. Rayleigh distribution [8]:

The Rayleigh distribution is widely used in probability, Reliability and survival analysis. The Rayleigh distribution is as follows:-

$$f(t;\ B) = \begin{cases} Bt\, e^{-\frac{B}{2}t^2} & 0 \le t < \infty \\ 0 & o.w \end{cases} \quad \text{…. (1)}$$

$\Omega = \{B; B > 0\}$, where B is scale parameter.

The cdf function of Rayleigh distribution is:-

$$F(t) = 1 - e^{-\frac{B}{2}t^2} \quad \text{…. (2)}$$

The survival function S(t) and hazard h(t) function of Rayleigh distribution is:-

$$S(t) = 1 - \mathrm{F(t)}$$

$$S(t) = e^{-\frac{B}{2}t^2} \quad \text{…. (3)}$$

$$h(t) = \frac{f(t)}{s\,(t)}$$

$$h(t) = Bt \quad \text{…. (4)}$$

3. The Maximum likelihood Method [9]:

The maximum likelihood method is the most important technique to estimate the parameter in any probability function. The idea of this method is to attempt to find the value of parameter which made the likelihood function as maximize for any distribution. The Likelihood function for one –parameter of Rayleigh distribution is:

$$L(B;\ t_1, t_2, \dots, t_n) = B^n \prod_{i=1}^{n} t_i \ \ e^{-\sum_{i=1}^{n}\frac{-Bt_i^2}{2}} \quad \text{…. (5)}$$

Taking the logarithm for above likelihood function, we get:-

$$\ln L = n \ln B + \sum_{i=1}^{n} \ln t_i - \sum_{i=1}^{n} \frac{Bt_i^2}{2} \quad \text{…. (6)}$$

Taking the partial derivative for log-likelihood function:-

$$\frac{\partial \ln L}{\partial B} = \frac{n}{B} - \sum_{i=1}^{n} \frac{t_i^2}{2} \quad \text{…. (7)}$$

Equal the above equation to zero as follows:-

$$\frac{\partial \ln L}{\partial B} = \frac{n}{B^{\wedge}} - \sum_{i=1}^{n} \frac{t_i^2}{2} = 0 \quad \text{… (8)}$$

$$B^{\wedge} = \frac{2n}{\sum_{i=1}^{n} t_i^2} \quad \text{…. (9)}$$

4. The Moment Method [9]:

The moment method is the simplest Method for estimate the parameter in pdf. The idea of this method is to find population moments and find the sample moments of the distribution, then equal between them to estimate the parameter of pdf.

$$\acute{m}1 = E(t) = \frac{\sqrt{\pi}}{\sqrt{2\sqrt{B}}} = \frac{1.253}{\sqrt{B}} \quad \text{…. (10)}$$

$$\mathrm{m1} = \frac{\sum_{i=1}^{n} t_i}{n} = \bar{t} \quad \text{…. (11)}$$

$$\bar{t} = \frac{1.253}{\sqrt{B}} \quad \text{…. (12)}$$

$$B^{\wedge} = \frac{(1.253)^2}{\bar{t}^{\,2}} \quad \text{…. (13)}$$

*The interval estimation is as follows:-

$$[B^{\wedge} - t\sqrt{var(B^{\wedge})}\ , B^{\wedge} + t\sqrt{var(B^{\wedge})}\ ] \quad \text{…. (14)}$$

5. Fuzzy sets:

Definition [10]: A crisp set is a special case of a fuzzy set, in which the membership function has only two values, 0 and 1.

Definition [10]: Let x be a nonempty set (universal set).A fuzzy set$\tilde{A}$ in x is characterized by its membership function $\mu_{\tilde{A}}: x \to [0,1]$

$\mu_{\tilde{A}}(x)$ Is the interpreted as a degree of membership of element x in fuzzy set A for each $x \in X$ and denoted for its set by $\tilde{A}$. $\tilde{A} = \{(x, \mu_{\tilde{A}}(x): x \in X\}$

Definition [10]: The fuzzy set $\tilde{A}$ is normal if its core is nonempty equivalently; we can find at least one element $x \in X$ s.t $\mu_{\tilde{A}}(x)$ =1

Definition [11]: The elements of x, such that $\mu_{\tilde{A}}(x) = \frac{1}{2}$ are called crossover points of $\tilde{A}$.

Definition [11]: The crisp set of element that belongs to the fuzzy set $\tilde{A}$ at least to the degree $\propto$ is called the $\propto$ -level set that is:- $A_{\propto} = \{x \in X: \mu_{\tilde{A}} \geq \propto\}$

$A'_{\propto} = \{x \in X: \mu_{\tilde{A}} > \propto\}$ Is called strong $\propto$ - level set or strong $\propto$-cut.

Definition [11]: The support of a fuzzy set $\tilde{A}$, $S(\tilde{A})$ is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}} > 0$ i.e. supp$(\tilde{A}) = \{x \in X: \mu_{\tilde{A}}(x) > 0\}$

Definition [11]:-The height h (A) of a fuzzy set A is the largest membership grade obtained by any element in that set, formally, $h(A) = sup_{x \in X} A(x)$

-Trapezoidal function [11]:

A fuzzy number $\tilde{A}(a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by:-

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{b-a} & , a \leq x \leq b \\ 1 & , b \leq x \leq c \\ \frac{(d-x)}{(d-c)} & , c \leq x \leq d \\ 0 & , otherewise \end{cases}$$

Figure 1.1- The Trapezoidal Membership Function

-Ranking function]:

The method of ranking function was first introduction by Jain in(1976):Yager in(1981) proposed four indices that may be employed for the purpose of ordering fuzzy quantities in [0,1].

We defined a ranking function $R: F(R) \to R$, which maps each fuzzy number into the real line. Now, suppose that $\tilde{a}$ and $\tilde{b}$ be two trapezoidal fuzzy numbers. Therefore, we define orders on $F(R)$ as following:-

(1) $\tilde{a} \geq \tilde{b}$ if and only if $R(\tilde{a}) \geq R(\tilde{b})$

(2)$\tilde{a} > \tilde{b}$ if and only $R(\tilde{a}) > R(\tilde{b})$

(3)$\tilde{a} = \tilde{b}$ If and only if $R(\tilde{a}) = R(\tilde{b})$ where $\tilde{a}$ and $\tilde{b}$ are in $F(R)$. Also we write

$\tilde{\alpha} \leq \tilde{b}$ If and only if $\tilde{\alpha} \geq \tilde{b}$

Lemma [10]:- let R be any linear ranking function then:-

i- $\tilde{\alpha} \geq \tilde{b}$ iff $-\tilde{b} \geq 0$ iff $-\tilde{b} \geq \tilde{\alpha}$

ii- if $\tilde{\alpha} \geq \tilde{b}$ and $\tilde{c} \geq \tilde{d}$, then $\tilde{\alpha} + \tilde{c} \geq \tilde{d} + \tilde{b}$

-Algorithms of the ranking function:-

(a)The first Yager algorithm:-

Yager (1981) [11] studied the ranking function, $R: F(R) \to R$

Let $\tilde{A} = (a, b, c, d)$ be trapezoidal fuzzy number, and then the following formula is applied to find the ranking function of $\tilde{A}$

Let $\mu^4 = \frac{(x-a)}{b-a}$ by using inverse transformation:-

$\mu^4(b-a) = (x-a)$

$x = \mu^4(b-a) + a = inf\tilde{A}_\mu$

$\mu^2 = \frac{(d-x)}{(d-c)}$ By using inverse transformation

$\mu^2(c-d) = (x-d)$

$x = \mu^2(c-d) + d = sup\tilde{A}_\mu$

$$R(\tilde{A}) = \frac{1}{2}\int_0^1 (\mu^4 b - \mu^4 a + a)\, d\mu + \frac{1}{2}\int_0^1 (\mu^2 c - \mu^2 d + d)\, d\mu$$

$$R(\tilde{A}) = \frac{1}{30}\,[3b + 12a + 5c + 10d] \quad \text{…. (15)}$$

(b)The second Maleki algorithm:-

Maleki (2002) [studied the ranking function, $R: F(R) \to R$

Let $\tilde{A} = (a, b, c, d)$ be trapezoidal fuzzy number, and then the following formula is applied to find the ranking function of $\tilde{A}$

$$R(\tilde{A}) = \frac{1}{2}\int_0^1 (inf\tilde{A}_\mu + sup\tilde{A}_\mu)\, d\mu$$

$\mu = \frac{(x-a)}{b-a}$ By using inverse transformation:-

$\mu\,(b-a) = (x-a)$

$x = \mu\,(b-a) + a = inf\tilde{A}_\mu$

$\mu = \frac{(d-x)}{(d-c)}$ By using inverse transformation

$\mu\,(c-d) = (x-d)$

$x = \mu\,(c-d) + d = sup\tilde{A}_\mu$

$$R(\tilde{A}) = \frac{1}{2}\int_0^1 (\mu\,(b-a) + a + \mu\,(c-d) + d)\, d\mu$$

$$R(\tilde{A}) = \frac{1}{4}[\,b + a + c + d] \quad \text{…. (16)}$$

-Mean Time To Failure (MTTF):-

MTTF=$\int_0^\infty s(t)\, dt$, MTTF=$\int_0^\infty e^{-\frac{B}{2}t^2}\, dt$

Let u=$\frac{B}{2}t^2$, $dt = \frac{1}{\sqrt{B}\sqrt{2u}}$

MTTF=$\int_0^\infty e^{-u}\frac{1}{\sqrt{B}\sqrt{2}}\, u^{-\frac{1}{2}}du$

$$\text{MTTF}=\frac{\sqrt{\pi}}{\sqrt{B}\sqrt{2}} \quad \text{…. (17)}$$

- Mean Squared Error can be calculated by:-

$$\text{MSE } [S^{\wedge}(t_i)] = \sum_{i=1}^{n}\frac{[S^{\wedge}(t_i) - S(t_i)]^2}{n} \quad \text{…. (18)}$$

Where: - $S^{\wedge}(t_i)$ Is estimated survival function, $S(t_i)$ Is empirical survival which:- $S(t_i) = \frac{i-0.5}{n}$

-Real data application

In this article, choosing on real data for lung cancer disease because it is widespread and deadly in Iraq Depending data for the lung cancer disease from Radiation and Nuclear Medicine Hospital in Iraq The time of study in this article determined from 1-1-2017 until 31-12-2017 .The number of patients in this time is (68): twenty patients are dead and forty eight patients remained alive, that means the data became complete data are (20) patients where:

T=[15,22,26,30,35,42,44,58,60,65,66,71,73,75,80,86,91,104,121,190]

(a)Maximum likelihood method:-

***** The value of $B^{\wedge}$ from equation (9) is:-$B^{\wedge} = 0.00032$

***** f (t), s (t) and h (t) in equations (1), (3) and (4) then we obtaining the following result in Table-1

T	f(t)	S(t)	h(t)
15	0.0047	0.9639	0.0049
22	0.0066	0.9240	0.0072
26	0.0076	0.8954	0.0085
30	0.0085	0.8632	0.0098
35	0.0094	0.8186	0.0114
42	0.0103	0.7496	0.0137
44	0.0105	0.7288	0.0144
58	0.0109	0.5771	0.0190
60	0.0109	0.5553	0.0196
65	0.0107	0.5014	0.0212
66	0.0106	0.4907	0.0216
71	0.0102	0.4388	0.0232
73	0.0100	0.4186	0.0239
75	0.0098	0.3988	0.0245
80	0.0092	0.3514	0.0261
86	0.0084	0.2986	0.0281
91	0.0077	0.2584	0.0297
104	0.0058	0.1708	0.0340
121	0.0036	0.0914	0.0395
190	0.0002	0.0027	0.0621

-By applying the equation (17) is: MTTF=70.0446

-By applying the equation (18) is: MSE $[S^{\wedge}(t_i)]$ =0.3205

***** To find the interval estimation applying the equation (14) as follows:-

$B^{\wedge} = [0.00017, 0.00046] = [a, d]$

***** Then applying $(\bar{x} \; -s^2) = b$ and $(\bar{x} \; +s^2) = c$, therefore the trapezoidal became as follow:- $B^{\wedge} = [0.00017, 0.00030, 0.00031, 0.00046]$

(a)- applying the first Ranking function by using equation (15) which as follow:-$B^{\wedge} = 0.00030$

Finding the f(t), s(t), h(t) and tabulating in Table-2

T	f(t)	S(t)	h(t)
15	0.0044	0.9668	0.0045
22	0.0061	0.9300	0.0066
26	0.0070	0.9036	0.0078
30	0.0079	0.8737	0.0090
35	0.0087	0.8321	0.0105
42	0.0097	0.7675	0.0126
44	0.0099	0.7480	0.0132
58	0.0105	0.6037	0.0174
60	0.0105	0.5827	0.0180
65	0.0103	0.5306	0.0195
66	0.0103	0.5203	0.0198
71	0.0100	0.4695	0.0213
73	0.0098	0.4496	0.0219
75	0.0097	0.4301	0.0225
80	0.0092	0.3829	0.0240

86	0.0085	0.3298	0.0258
91	0.0079	0.2888	0.0273
104	0.0062	0.1974	0.0312
121	0.0040	0.1112	0.0363
190	0.0003	0.0044	0.0570

-By applying the equation (17) is: MTTF=72.3418
- By applying the equation (18) is: MSE $[S^{\wedge}(t_i)]$ =0.3175
***(b)** applying the second Ranking function method by using equation (16) which as follow:- $B^{\wedge} = 0.00031$
Finding the f(t), s (t), h(t) and tabulating in Table-3

T	f(t)	S(t)	h(t)
15	0.0045	0.9657	0.0046
22	0.0063	0.9277	0.0068
26	0.0073	0.9005	0.0081
30	0.0081	0.8698	0.0093
35	0.0090	0.8271	0.0109
42	0.0099	0.7608	0.0130
44	0.0101	0.7408	0.0136
58	0.0107	0.5937	0.0180
60	0.0106	0.5724	0.0186
65	0.0105	0.5195	0.0202
66	0.0104	0.5091	0.0205
71	0.0101	0.4578	0.0220
73	0.0099	0.4378	0.0226
75	0.0097	0.4182	0.0233
80	0.0092	0.3708	0.0248
86	0.0085	0.3178	0.0267
91	0.0078	0.2771	0.0282
104	0.0060	0.1870	0.0322
121	0.0039	0.1034	0.0375
190	0.0002	0.0037	0.0589

-By applying the equation (17) is: MTTF=71.1654
-By applying the equation (18) is: MSE $[S^{\wedge}(t_i)]$ =0.3190
(b)-Moment method:-
***** The value of $B^{\wedge}$ from equation (13) is:- $B^{\wedge} = 0.00034$
***** f (t), s (t), h (t) in equation (1), (3), (4) then tabulating in Table-4

T	f(t)	S(t)	h(t)
15	0.0049	0.9622	0.0051
22	0.0069	0.9204	0.0075
26	0.0079	0.8907	0.0089
30	0.0088	0.8571	0.0103
35	0.0097	0.8107	0.0120
42	0.0106	0.7392	0.0144
44	0.0108	0.7178	0.0151
58	0.0112	0.5620	0.0199
60	0.0111	0.5398	0.0206
65	0.0108	0.4850	0.0223

66	0.0107	0.4742	0.0226
71	0.0103	0.4217	0.0243
73	0.0100	0.4014	0.0250
75	0.0098	0.3816	0.0257
80	0.0092	0.3342	0.0274
86	0.0083	0.2817	0.0295
91	0.0075	0.2421	0.0312
104	0.0056	0.1568	0.0356
121	0.0034	0.0815	0.0414
190	0.0001	0.0021	0.0651

-By applying the equation (17) is: MTTF=67.9533

- By applying the equation (18) is: MSE $[S^{\wedge}(t_i)]$ =0.3235

***** To find the interval estimation applying the equation (14) as follows:-

$$B^{\wedge} = [0.00018, 0.00049] = [a, d\ ]$$

***** Then applying ($\bar{x} \ -s^2) = b$ and ($\bar{x} \ +s^2) = c$, therefore the trapezoidal became as follow:- $B^{\wedge} = [0.00018, 0.00032, 0.00033, 0.00049]$

(a)- applying the first Ranking function by using equation (15) which as follow:-

$$B^{\wedge} = 0.00032$$

Finding the f(t), s(t), h(t) and tabulating in following Table-5

t	f(t)	S(t)	h(t)
15	0.0046	0.9646	0.0048
22	0.0065	0.9255	0.0070
26	0.0075	0.8975	0.0083
30	0.0083	0.8659	0.0096
35	0.0092	0.8220	0.0112
42	0.0101	0.7541	0.0134
44	0.0103	0.7336	0.0141
58	0.0108	0.5838	0.0186
60	0.0108	0.5621	0.0192
65	0.0106	0.5086	0.0208
66	0.0105	0.4981	0.0211
71	0.0101	0.4464	0.0227
73	0.0100	0.4263	0.0234
75	0.0098	0.4066	0.0240
80	0.0092	0.3592	0.0256
86	0.0084	0.3062	0.0275
91	0.0077	0.2658	0.0291
104	0.0059	0.1772	0.0333
121	0.0037	0.0961	0.0387
190	0.0002	0.0031	0.0608

-By applying the equation (17) is: MTTF=70.0446

- By applying the equation (18) is: MSE $[S^{\wedge}(t_i)]$ =0.3205

*** (b)** applying the second Ranking function method by using equation (16) which as follow:- $B^{\wedge} = 0.00033$

Finding the f(t), s(t), h(t) and tabulating in Table-6

t	f(t)	S(t)	h(t)
15	0.0048	0.9636	0.0049
22	0.0067	0.9232	0.0073

26	0.0077	0.8945	0.0086
30	0.0085	0.8620	0.0099
35	0.0094	0.8170	0.0116
42	0.0104	0.7475	0.0139
44	0.0105	0.7266	0.0145
58	0.0110	0.5740	0.0191
60	0.0109	0.5521	0.0198
65	0.0107	0.4980	0.0215
66	0.0106	0.4874	0.0218
71	0.0102	0.4353	0.0234
73	0.0100	0.4151	0.0241
75	0.0098	0.3953	0.0248
80	0.0092	0.3478	0.0264
86	0.0084	0.2951	0.0284
91	0.0077	0.2550	0.0300
104	0.0058	0.1679	0.0343
121	0.0036	0.0893	0.0399
190	0.0002	0.0026	0.0627

-By applying the equation (17) is: MTTF=68.9752

-By applying the equation (18) is: MSE $[S^{\wedge}(t_i)]$ =0.3220

6- Comparison algorithms

To compare between crisp numbers (MLE, ME) and two algorithm (MLE, ME) which are fuzzy numbers depending on mean squares error and mean time to failure.

If the estimate value of parameter in crisp or in two algorithms has best estimation when own the minimum mean squares error.

If the estimate value of parameter in crisp or in two algorithms has best estimation when own the maximum mean time to failure.

Table 7

Algorithm	MTTF	MSE
Crisp $\begin{cases} MLE \\ ME \end{cases}$	$\begin{cases} 70.0446 \\ 67.9533 \end{cases}$	$\begin{cases} 0.3205 \\ 0.3235 \end{cases}$
First Algorithm $\begin{cases} MLE \\ ME \end{cases}$	$\begin{cases} 72.3418 \\ 70.0446 \end{cases}$	$\begin{cases} 0.3175 \\ 0.3205 \end{cases}$
second Algorithm $\begin{cases} MLE \\ ME \end{cases}$	$\begin{cases} 71.1654 \\ 68.9752 \end{cases}$	$\begin{cases} 0.3190 \\ 0.3220 \end{cases}$

- Noting that from above table that minimum mean squares error is first Yager algorithm which presents maximum likelihood method but the high mean squares error is crisp of moment method. Showing that the high mean time to failure is the first Yager algorithm which presents maximum likelihood method but the minimum mean time to failure is crisp of moment method.

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