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A Mathematical Model for Iraqi Airways Company about Evaluating Its Objectives and Strategies

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Abstract
In this paper, we introduced a mathematical model for Iraqi Airways Company about evaluating its objectives and strategies. First, we studied Iraqi Airways schedules with different departure cities for each airline path. Then, we applied some fuzzy integrals for determining the best airline path.

Keywords: Fuzzy measures, Sugeno integral, Choquet integral, PAN integral, Lehrer integral, Iraqi Airways strategies.

نموذج رياضي لتقييم الاهداف و الاستراتيجيات المثلى لشركة الخطوط الجوية العراقية

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الخلاصة
في هذا البحث وجدنا الاهداف و الاستراتيجيات المثلى للخطوط الجوية العراقية باستخدام نموذج رياضي مخصص لتقييم عمل شركة الخطوط الجوية العراقية باستخدام مسارات جوية و مدن مغادرة مختلفة.

1-Introduction

Fuzzy integrals with respect to fuzzy measures (generalized measures [1], non-additive measures [2], capacities [3],[4]) are one of the most powerful and flexible functions in the field of aggregation operators .The most well-known fuzzy integrals are Choquet and Sugeno integrals with their generalization (see [5]). These integrals depend on fuzzy measure, which is one of the most important concepts in Mathematics, hence the fuzzy integral with respect to the fuzzy measure have many applications in different fields such as Science, Engineering, Economics…etc.

Many researchers worked in the field of airline transportation; most of them deal with service quality. James J.H. Liou (2007) [6], used fuzzy integral to evaluate the integrated performance of attributes. Chih-Wen Yang (2010) [7], used fuzzy integral to evaluate the effects of service quality on traveling airline choice .Yuan Ho-chen (2011) [8] applied Fuzzy-grey method to deal with domestic airline in flight service quality with uncertainty .Young Dae Ko (2016) [9] provided managerial insights that can be applied in the competitive air transport market using game theory as a tool assuming each airline company to act as a player. Also, Vicence Torra, Yasuo Narakowa (2006) [5] studied the interpretation of Sugeno integral and used it as an aggregation operator with some examples.

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In this paper, we propose a mathematical model of dealing with airlines in Iraq (Iraqi Airways) using many of fuzzy integrals (Sugeno, Shilkrit, Choquet, Pan, and Lehrer integrals) to determine the most suitable town for the passengers from the departure city. The structure of the paper is as follows .In section 2; we give the basic concepts of fuzzy measures and fuzzy integrals with some definitions. Section 3 discusses the mathematical model for passenger objectives and strategies in Iraqi Airways Company. In section 4, we give some study cases with results. Finally, the paper finished with some conclusions.

2-Basic Concepts

In this section, we recall some basic concepts of the fuzzy measures and fuzzy integrals (Sugeno, Shilkrit , Choquet, PAN, and Lehrer) which we are related to our research.

2.1 Fuzzy measures

Fuzzy measures are an extension of the classical measure in the sense that the additivity of the measure is replaced with a weaker condition, the monotonicity. The definition of fuzzy measure is as follows.

Definition 1(fuzzy measure) [1]

Let C be a nonempty set and 2^C is the power set of C. A set function $\mu: 2^C \to [0, \infty]$is a fuzzy measure on (C, Φ), if it satisfies the following axioms:

i- $\mu(\phi) = 0$, and $\mu(C) > 0$.

ii- $A \subseteq B$ implies $\mu(A) \leq \mu(B)$for $A, B \in 2^C$. (monotonicity)

A lot of special fuzzy measures have been presented in the literature. One of them is λ- fuzzy measure defined as follows.

Definition 2 (λ- fuzzy measure) [10]

Let $\lambda \in (-1, \infty)$. A normalized set function g_λ defined on 2^C is called a λ- fuzzy measure on C if for every pair of disjoint subsets A and B of C

$$g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B)$$

2.2 Fuzzy Integrals

There are many kinds of fuzzy integrals with respect to fuzzy measures, in this subsection we shall give the definitions that are related to our work such as Sugeno , Shilkrit, Choquet, Pan, and Lehrer integrals with respect to fuzzy measures.

Definition 3 (The Sugeno integral) [1]

Let μ be a normalized fuzzy measure on C and f a function on C with range$\{a_1, a_2, a_3, \dots, a_n\}$ where $0 \leq a_1 \leq a_2 \leq \cdots \leq a_n \leq 1$.

The Sugeno integral $\int f(c).\mu(c)$of f w.r.t. μ is defined as

$$SI_\mu(f) = \int f . \mu = \bigvee_{i=1}^{n} (a_i \wedge \mu(\{c \mid f(c) \geq a_i\})) \qquad \ldots(1)$$

Sugeno integral is defined only for functions whose range is included in [*0, 1*], and normalized fuzzy measure.

The Shilkret integral is special type of Sugeno integral has been defined as follows (see [11]).

Definition 4 (The Shilkret integral) [11]

Let μ be a normalized fuzzy measure on C and f a function on C with range$\{a_1, a_2, a_3, \dots, a_n\}$. The Shilkret integral of a measurable function $f: C \to [0, 1]$ is given by

$$Sh_\mu(f) = \underset{a \in [0,1]}{Max} \left(a_i . \mu\left(\{c \mid f(c) \geq a_i\} \right) \right) \qquad \ldots(2)$$

where $0 \leq a_1 \leq a_2 \leq \cdots \leq a_n \leq 1$, $\mu : A \to [0, 1]$ is a fuzzy measure.

The most natural fuzzy integral is Choquet integral, which is an extension of the ordinary integral (Lebesgue's integral). The definition of Choquet integral is as follows.

Definition 5 (The Choquet integral) [1]

Let μ be a fuzzy measure on C and f a function on C with range$\{a_1, a_2, a_3, \dots, a_n\}$, where $0 \leq a_1 \leq a_2 \leq \cdots \leq a_n \leq 1$. The Choquet integral is given by

$$(Ch)\int f \ d\mu = \sum_{i=1}^{n} \left(a_i - a_{i-1}\right).\mu(\{c \mid f(c) \geq a_i\}) \qquad ...(3)$$

where $a_0 = 0$.

The Pan-integral is related to finite partitions of *X*, the definition of Pan-integral is as follows.

Definition 6 (The Pan-integral) [1], [2]

Let μ be a fuzzy measure on C .The PAN integral of a measurable function $f: C\rightarrow [0, \infty]$)

$$(p)\int_A f \ d\mu = Max \left\{ \sum_{i=1}^{n} k_i \,\mu(A_i) \mid \sum_{i=1}^{n} b(k_i, A_i) \leq f \ , (A_i)_{i=1}^{n} \text{is a partition of } C \right\} \qquad ...(4)$$

where $b(k, A)\ (c) = \begin{cases} k & \text{if } c \in A. \\ 0 & \text{elsewhere} . \end{cases}$

and *k* is constant.

The Lehrer (concave) integral recently introduced by Lehrer [12] .The following definition, gives an explicit formula for the Lehrer integral.

Definition7 (The Lehrer integral) [2], [12]

Let μ be a fuzzy measure on C .The Lehrer integral of a measurable function $f: C\rightarrow [0, \infty]$ is

$$(L)\int_A f \ d\mu = Max \left\{ \sum_{i=1}^{n} k_i \,\mu(A_i) \mid n \in N \ , \sum_{i=1}^{n} b(k_i, A_i) \leq f \right\} \qquad ...(5)$$

where $b(k, A)\ (c) = \begin{cases} k & if\ c \in A. \\ 0 & elsewhere . \end{cases}$

and *k* is constant.

The relationships among above fuzzy integrals (Shilkret, Choquet, Pan, and Lehrer) are shown in the following remark (See [2]).

Remark 1[2]:

The relationships among Shilkret(Sh),Choquet(Ch),and Lehrer(L) is $\mathrm{Sh} \leq \mathrm{Ch} \leq \mathrm{L}$

and the relationship between Shilkret(Sh) and Pan(P) is $\mathrm{Sh} \leq \mathrm{PAN}$

3-A Mathematical model for passenger strategies in Iraqi Airways Company

Consider a passenger intends to travel via Iraqi Airways from city c_1 then as transit city c_2 and at last city c_3 with several alternative places as departure cities .To construct a mathematical model, we need to consider the first effective factor: the degree of importance of the passenger reaching these cities which represent their importance (e.g.; economically, tourism…etc) to the passenger, these degrees are expressed by the fuzzy measures. There is another effective factor in the mathematical model which is the degree of accessibility of such cities from particular departure location (i.e.; the distances between departure cities and the first arrival city). The combination of both degrees(i.e.; the degree of importance and the degree of accessibility) of traveling from a certain particular departure city will be expressed by a function $f: C \rightarrow [0, \infty]$. The passenger at a departure cityc_4, and the most accessible city is c_1, then the transit city is c_2, finally c_3,therefore $f(c_1) \geq f(c_2) \geq f(c_3)$.The values for the measure are expressed using the same terms as the degree of importance , hence $f(c)$ is comparable with $\mu(A)$, where *A* is a subset of *C* . Therefore, from the values of μ and *f*, we can define a certain relation $\mu_f(c_i) = \mu(\{c|f(c) \geq f(c_i)\})$. This expression stands for the degree of importance of reaching c_i and all those cities that are at least as accessible as c_i , hence if we reach c_i , it is possible to reach as well all those cities with a greater degree of possibility than c_i (i.e.; all those cities in the set $\{c|f(c) \geq f(c_i)\}$), therefore, $\mu_f(c_i)$ is the degree of importance the passenger achieves when decides to reach c_i .

These degrees are expressed with the fuzzy measure μ(A) for all $A \subseteq C$. This fuzzy measure is monotonic increasing and is bounded; the boundary conditions mean that no reach to cities implies no any importance. For each city c_i , both degrees of accessibility $f(c_i)$ and importance $\mu_f(c_i)$ must be considered for possible strategies. In this paper, we have studied the possible strategies of Iraqi's Airways using fuzzy measures and fuzzy integrals to achieve the best performance including Iraqi's airports, and the airways paths in Iraq and other countries.

To achieve this target, we apply fuzzy integrals: Sugeno, Shilkret, Choquet, PAN and Lehrer for combining both importance and reliability of some study cases of Iraqi's airways as shown in the next section.

4-Case studies

In this section, we deal with some case studies, which have been taken (according to year 2016) from Iraqi Airways schedules. These cases are:
case1 (Baghdad-Basrah-Dubai), and arbitrary departure city Sulaimaniya or Mosul.
case2 (Baghdad-Erbil-Dubai), and arbitrary departure city Najaf or Basrah.
case3 (Erbil-Sulaimaniya-Dubai), and arbitrary departure city Mosul or Tikrit.
case4 (Erbil-Amman), and arbitrary departure city Mosul or Sulaimaniya.
case5 (Basrah-Amman). and arbitrary departure city Najaf or Nasriyah.

Here, we discuss the cases 1 and 2 in details and other cases are shown in Table-1.

Case Study 1: From Sulaimaniya as a departure city to Baghdad, then as transit city Basrah and at last Dubai city.(Baghdad→Basrah→Dubai**)**
c_1: Baghdad, c_2: Basrah, c_3: Dubai, the departure city is Sulaimaniya

Fuzzy measures (importance factor) for reaching cities $\{c_1, c_2, c_3\}$ with their values are shown in the following table.

Table 1- Fuzzy measures for reaching cities$\{c_1,c_2,c_3\}$

set	$\{c_1\}$	$\{c_2\}$	$\{c_3\}$	$\{c_1,c_2\}$	$\{c_1,c_3\}$	$\{c_2,c_3\}$	$\{c_1,c_2, c_3\}$
μ	0.7	0.4	0.5	0.86	0.9	0.73	1

Note that, we obtained $\{c_1,c_2\},\{c_1,c_3\},\{c_2,c_3\},\{c_1,c_2, c_3\}$ by using λ-fuzzy measures, where $\lambda = -0.84$.

Accessibility degrees from Sulaimaniya city are shown in the following table.

Table 2-Accessibility degrees from Sulaimaniya city

set	c_1	c_2	c_3
f	0.9	0.8	0.3

Importance degrees for each city are shown in the following table.

Table 3-Importance degrees for c_1, c_2, and c_3

set	c_1	c_2	c_3
μ_f	0.7	0.86	1

To apply Sugeno integral, we shall use a permutation on $f(c_i)$. That is
$f(c_3) \le f(c_2) \le f(c_1)$ $(0.3 \le 0.8 \le 0.9)$.

To find μ_f, we use the formula $\mu_f(\{c_i\}) = \mu(\{c \mid f(c) \ge f(c_i)\})$:
$\mu_f(\{c_3\}) = \mu(\{c \mid f(c) \ge f(c_3)\})=1$
$\mu_f(\{c_2\}) = \mu(\{c \mid f(c) \ge f(c_2)\})= \mu\{c_1,c_2\}=0.86$
$\mu_f(\{c_1\}) = \mu(\{c \mid f(c) \ge f(c_1)\})= \mu\{c_1\}=0.7$

Now, we apply Sugeno integral (equation (1)) with $a_i = f(c_i)$, for this case we obtained

$$SI_\mu(\mathrm{f}) = Max\left\{\min\{f(c_1), \mu_f(\{c_1\})\}, \min\{f(c_2), \mu_f(\{c_2\})\}, \min\{f(c_3), \mu_f(\{c_3\})\}\right\}$$
$$= Max\{\min\{0.9, 0.7\}, \min\{0.8, 0.86\}, \min\{0.3, 1\}\}$$
$$= Max\{0.7, 0.8, 0.3\} = 0.8$$

By using Shilkrit integral (equation (2)), we obtain

$$Sh_\mu(f) = Max\{f(c_1).\mu f(\{c1\}), f(c_2).\mu f(\{c2\}), f(c_3).\mu f(\{c3\}\}.$$
$$= Max\{(0.9)(0.7), (0.8)(0.86), (0.3)(1)\} = 0.688$$

If we apply Choquet integral (equation (3)) with $a_i = f(c_i)$, we obtain

$$(Ch)\int f d\mu = \sum_{i=1}^{n} (a_i - a_{i-1}).\mu(\{c|f(c) \ge a_i\})$$
$$(Ch)\int f d\mu = (0.3-0)(1) + (0.8-0.3)(0.86) + (0.9-0.8)(0.7)$$
$$=0.3+0.43+0.07$$

=0.8

Also we can apply Pan–integral using equation (4), we get

$$(p)\int_A fd\,\mu = Max\left\{\begin{array}{l} f\left(c_2\right).\mu\left(\left\{c_1,c_2\right\}\right)+f\left(c_3\right).\mu\left(\left\{c_3\right\}\right), f\left(c_3\right).\mu\left(\left\{c_1,c_3\right\}\right)+f\left(c_2\right).\mu\left(\left\{c_2\right\}\right), \\ f\left(c_3\right).\mu\left(\left\{c_2,c_3\right\}\right)+f\left(c_1\right).\mu\left(\left\{c_1\right\}\right), f(c_3).\mu(\{C\}), \\ f\left(c_1\right).\mu\left(\left\{c_1\right\}\right)+f\left(c_2\right).\mu\left(\left\{c_2\right\}\right)+f\left(c_3\right).\mu\left(\left\{c_3\right\}\right) \end{array}\right\}$$

$$=Max\left\{0.838, 0.59, 0.849, 0.3, 1.1\right\}$$

$$=1.1$$

Finally, we can apply Lehrer–integral using equation (5), we obtain

$$(L)\int_A fd\,\mu = Max\left\{\begin{array}{l} f\left(c_2\right).\mu\left(\left\{c_1,c_2\right\}\right)+f\left(c_3\right).\mu\left(\left\{c_3\right\}\right)+(f\left(c_1\right)-f\left(c_2\right)).\mu\left(\left\{c_1\right\}\right), \\ f\left(c_3\right).\mu\left(\left\{c_1,c_3\right\}\right)+f\left(c_2\right).\mu\left(\left\{c_2\right\}\right)+(f\left(c_1\right)-f\left(c_3\right)).\mu\left(\left\{c_1\right\}\right), \\ f\left(c_3\right).\mu\left(\left\{c_2,c_3\right\}\right)+f\left(c_1\right).\mu\left(\left\{c_1\right\}\right)+(f\left(c_2\right)-f\left(c_3\right)).\mu\left(\left\{c_2\right\}\right), \\ f(c_3).\mu(\{C\})+\left(f(c_2)-f(c_3)\right).\mu(\{c_2\})+\left(f(c_1)-f(c_3)\right).\mu(\{c_1\}), \\ (f(c_1).\mu\left(\left\{c_1\right\}\right)+f(c_2).\mu\left(\left\{c_2\right\}\right)+f\left(c_3\right).\mu\left(\left\{c_3\right\}\right) \end{array}\right\}$$

$$=Max\left\{0.908, 1.01, 1.049, 0.92, 1.1\right\}$$

$$=1.1$$

Now if we take the same path with different departure city.Say, Mosul as a departure city to Baghdad then as transit city Basrah and at last Dubai city.

(Baghdad→Basrah→Dubai)

c_1: Baghdad, c_2: Basrah, c_3: Dubai, the departure city is Mosul

Accessibility degrees from Mosul city are shown in the following table.

Table 4- Accessibility degrees from Mosul city

set	c_1	c_2	c_3
g	0.8	0.7	0.1

Importance degrees for each city are shown in the following table.

Table 5- Importance degrees for each city

set	c_1	c_2	c_3
μ_g	0.7	0.86	1

To apply Sugeno integral, we shall use a permutation on $g(c_i)$. That is

$g(c_3) \leq g(c_2) \leq g(c_1)$ $(0.1 \leq 0.7 \leq 0.8)$.

To find μ_g, we use the same previous method,

$\mu_g(\{c_3\}) = 1$

$\mu_g(\{c_2\}) = \mu(\{c_1,c_2\}) = 0.86$

$\mu_g(\{c_1\}) = \mu(\{c_1\}) = 0.7$

First, we apply Sugeno integral (equation (1)) with $a_i = g(c_i)$, for this case

$SI_\mu(g) = Max\,\{0.7, 0.7, 0.1\} = 0.7$

By using Shilkrit integral (equation (2)), we obtain

$Sh_\mu(f) = Max\,\{0.56, 0.602, 0.1\} = 0.602$

If we apply Choquet integral (equation (3)) with $a_i = g(c_i)$, we get

$(Ch)\int gd\mu = 0.1+0.516+0.07 = 0.686$

Also we can apply Pan-integral (equation (4)), we obtain

$$(p)\int_A gd\,\mu = Max\left\{0.652, 0.37, 0.633, 0.1, 0.89\right\} = 0.89$$

Finally, we can apply Lehrer integral (equation (5)), we obtain

$$(L)\int_A gd\mu = Max\{0.722, 0.86, 0.873, 0.83, 0.89\} = 0.89$$

Case Study 2: As in case 1, we can calculate the fuzzy integrals(Sugeno, Shilkret ,Choquet, ,PAN, Lehrer) for case 2 as follows.

From Najaf as a departure city to Baghdad then as transit city Erbil and at last Dubai city. (Baghdad→Erbil→Dubai)

c_1: Baghdad, c_2: Erbil, c_3: Dubai;the departure city is Najaf

Fuzzy measures (importance factor) for reaching cities $\{c_1, c_2, c_3\}$ with their values are shown in the following table.

Table 6- Fuzzy measures (importance factor) for reaching cities $\{c_1, c_2, c_3\}$ with their values

set	$\{c_1\}$	$\{c_2\}$	$\{c_3\}$	$\{c_1,c_2\}$	$\{c_1,c_3\}$	$\{c_2,c_3\}$	$\{c_1,c_2, c_3\}$
μ	0.5	0.3	0.7	0.68	0.92	0.83	1

We obtained the same previous by using λ-fuzzy measure, where $\lambda = -0.8$.

Accessibility degrees from Najaf city are shown in the following table.

Table 7- Accessibility degrees from Najaf city

set	c_1	c_2	c_3
f	0.8	0.3	0.1

Importance degrees for each city are shown in the following table.

Table 8- Importance degrees for each city

set	c_1	c_2	c_3
μ_f	0.5	0.68	1

To apply Sugeno integral, we shall use a permutation on $f(c_i)$. That is

$f(c_3) \le f(c_2) \le f(c_1)$ $(0.1 \le 0.3 \le 0.8)$.

First applying Sugeno Integral (equation (1)), $SI_\mu(f) = 0.5$

By using Shilkret integral (equation (2)), $Sh_\mu(f) = 0.4$

If we apply Choquet integral (equation (3)), $(Ch)\int f\, d\mu = 0.486$

Also we can apply Pan integral (equation (4)), $(p)\int_A fd\mu = 0.56$.

Finally,we can apply Lehrer integral (equation (5)), $(L)\int_A fd\mu = 0.56$.

Now if we take the same path with different departure city. Say Basrah as a departure city to Baghdad then as transit city Erbil and at last Dubai city.

c1: Baghdad, c_2: Erbil, c_3: Dubai; the departure city is Basrah.

Accessibility degrees from Basrah city are shown in the following table.

Table 9- Accessibility degrees from Basrah city

set	c_1	c_2	c_3
g	0.5	0.4	0.2

Importance degrees for each city are shown in the following table.

Table 10- Importance degrees for each city

set	c_1	c_2	c_3
μ_g	0.5	0.68	1

To apply Sugeno integral, we shall use a permutation on $g(c_i)$. That is

$g(c_3) \le g(c_2) \le g(c_1)$ $(0.2 \le 0.4 \le 0.5)$.

First we apply Sugeno Integral, $SI_\mu(g) = 0.5$

By using Shilkret integral, $Sh_\mu(f) = 0.272$

If we apply Choquet integral, $(Ch)\int g\, d\mu = 0.386$

Also we can apply Pan-integral, $(p)\int_A gd\mu = 0.51.$

Finally, we can apply Lehrer integral to get $(L)\int_A gd\mu = 0.51.$

The cases (case1, case2, case3, case4, case5) in Table-11 represent the usual airlines paths used by the Iraqi's Airways [13] for different cases with the values of each path using different fuzzy integrals.

Table 11- Case studies

cases	The path	Departure city	Sugeno integral $SI_\mu(f)$	Shilkrit Integral $Sh_\mu(f)$	Choquet integral (C)$\int f d\mu$	Pan integral (p) $\int f d\mu$	Lehrer Integral (L)$\int f d\mu$
1	Baghdad →Basrah→Dubai	Sulaimania	0.8	0.688	0.8	1.1	1.1
		Mosul	0.7	0.602	0.686	0.89	0.89
2	Baghdad→Erbil→Dubai	Najaf	0.5	0.4	0.486	0.56	0.56
		Basrah	0.5	0.272	0.386	0.51	0.51
3	Erbil→Sulaimaniya→Dubai	Mosul	0.616	0.45	0.646	0.83	0.83
		Tikrit	0.616	0.431	0. 552	0.74	0.74
4	Erbil→Amman	Mosul	0.6	0.54	0.62	0.7	0.7
		Sulaimania	0.6	0.42	0.54	0.66	0.66
5	Basrah→ Amman	Nasriyah	0.7	0.56	0.65	0.83	0.83
		Najaf	0.5	0.4	0.47	0.71	0.71

From the Table- 11, we deduce the following results.

In cases 2, 3, and 4, same results were obtained when we used Sugeno integral choosing two arbitrary departure cities for the same airline path , while in cases 1, 5, Sulaimaniya and Nasriyah represent the best departure cities for the suggested paths. The same conclusion in case of Shilkret integral.

When we used Choquet integral, which concentrate on the degree of accessibility, a certain difference have been appeared among the values of each path, which can be considered normal since the distances are different from departure cities with respect to destinations, this can be noticed in all cases. The same for Pan and Lehrer integrals. Our results are coincides with Remark1 for all study cases as shown in Table-11.

5-Conclusions

In this paper, we have proposed a mathematical model for Iraqi Airways schedules about evaluating its objectives and strategies.Fuzzy integrals (Sugeno, Shilkret, Choquet, Pan,Lehrer) have been applied for different cases of Iraqi's Airways with optional departure cities to determine the most economical path. The results were found to be conforming to the real results on the realistic. This model can be used to investigate new future plans, considering the best airlines paths, also the capability of fuzzy integrals to deal with Airways problems concerning choosing the departures and transit cities.

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