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Semi (1, 2)*-Maximal Soft (1, 2)*-Pre-Open Sets and Semi (1, 2)*-Minimal Soft (1, 2)*-Pre-Closed Sets In Soft Bitopological Spaces

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Abstract
In this paper, we introduce and study new types of soft open sets and soft closed sets in soft bitopological spaces $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$, namely, (1,2)*-maximal soft open sets, (1,2)*-maximal soft (1,2)*-pre-open sets, semi (1,2)*-maximal soft (1,2)*-pre-open sets, (1,2)*-maximal soft closed sets, (1,2)*-maximal soft (1,2)*-pre-closed sets, (1,2)*-minimal soft open sets, (1,2)*-minimal soft (1,2)*-pre-open sets, (1,2)*-minimal soft closed sets, (1,2)*-minimal soft (1,2)*-pre-closed sets, and semi (1,2)*-minimal soft (1,2)*-pre-closed sets. Also, properties and the relation among these concepts have been studied.
Keywords: (1,2)*-maximal soft (1,2)*-pre-open set, (1,2)*-minimal soft (1,2)*-pre-open sets, (1,2)*-maximal soft (1,2)*-pre-closed sets, (1,2)*-minimal soft (1,2)*-pre-closed sets, semi (1,2)*-maximal soft (1,2)*-pre-open sets, semi (1,2)*-minimal soft (1,2)*-pre-closed set.

المجموعات المفتوحة-pre –*(1,2) الميسرة شبة الاكبرية –*(1,2) والمجموعات المغلقة-pre –*(1,2) الميسرة شبة الاصغرية –*(1,2) في الفضاءات التبولوجية الثنائية الميسرة

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الخلاصة
في هذا البحث نحن قدمنا ودرسنا انواع جديدة من المجموعات المفتوحة الميسرة والمجموعات المغلقة الميسرة في الفضاءات التبولوجية الثنائية الميسرة أسميناها بالمجموعات المفتوحة الميسرة الاكبرية –*(1,2) ، المجموعات المفتوحة –pre –*(1,2) الميسرة الاكبرية –*(1,2) ، المجموعات المفتوحة –pre –*(1,2) الميسرة شبة الاكبرية –*(1,2) ، المجموعات المغلقة الميسرة الاكبرية –*(1,2) ، المجموعات المغلقة–pre –*(1,2) الميسرة الاكبرية –*(1,2) ، المجموعات المفتوحة الميسرة الاصغرية –*(1,2) المجموعات المفتوحة –pre –*(1,2) الميسرة الاصغرية –*(1,2) ، المجموعات المغلقة الميسرة الاصغرية –*(1,2) ، المجموعات المغلقة –pre –*(1,2) الميسرة الاصغرية –*(1,2) ، المجموعات المغلقة –pre –*(1,2) الميسرة شبة الاصغرية –*(1,2). كذلك درسنا العلاقة بين هذة المفاهيم و خصائصهم.

Introduction

Nakaoka, F. and Oda, N. in [1-3] introduced the concepts of minimal closed sets, maximal closed sets, minimal open sets and maximal open sets in topological spaces. Also, the notion of soft set theory was introduced by Molodtsov, D. [4], Shabir, M. and Naz, M. [5] introduced and study the concept of

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soft topological spaces. The notion of maximal soft open sets and minimal soft closed sets in soft topological spaces was first introduced by Mahmood, S. I. [6]. Senel, G. and Çagman, N. [7, 8] introduced and investigated soft bitopological spaces and soft (1,2)*-pre-open sets in soft bitopological spaces $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. In this paper we introduce new classes of soft open sets and soft closed sets in soft bitopological spaces $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, namely, (1,2)*-maximal soft open sets, (1,2)*-maximal soft (1,2)*-pre-open sets, semi (1,2)*-maximal soft (1,2)*-pre-open sets, (1,2)*-maximal soft closed sets, (1,2)*-maximal soft (1,2)*-pre-closed sets, (1,2)*-minimal soft open sets, (1,2)*-minimal soft (1,2)*-pre-open sets, (1,2)*-minimal soft closed sets, (1,2)*-minimal soft (1,2)*-pre-closed sets, and semi (1,2)*-minimal soft (1,2)*-pre-closed sets. Also, some of their properties have been studied.

1. Preliminaries

First we recall the following definitions and proposition.

If X is an initial universe set, E is the set of all parameters for X, and $P(X)$ denote the power set of X, then:

Definition (1.1) [4]**:** A soft set over X is a pair (A,P), where A is a function given by $A:P\to P(X)$ and P is a non empty subset of E.

Definition (1.2) [9]**:** If (A,P) is a soft set over X. Then $\tilde{p}=(e,\{p\})$ is called a soft point of (A,P), if $e\in P$ and $p\in A(e)$, and is denoted by $\tilde{p}\,\tilde{\in}\,(A,P)$.

Definition (1.3) [5]**:** If $\tilde{\tau}$ is a collection of soft sets over X. Then $\tilde{\tau}$ is called a soft topology on X if $\tilde{\tau}$ satisfies the following:

i) $\tilde{X}$, $\tilde{\phi}$ belong to $\tilde{\tau}$.

ii) If $(O_1,E),(O_2,E)\,\tilde{\in}\,\tilde{\tau}$, then $(O_1,E)\,\tilde{\bigcap}\,(O_2,E)\,\tilde{\in}\,\tilde{\tau}$.

(iii) If $(O_\alpha,E)\,\tilde{\in}\,\tilde{\tau}$, $\forall\alpha\in\Lambda$, then $\bigcup_{\alpha\in\Lambda}(O_\alpha,E)\,\tilde{\in}\,\tilde{\tau}$.

The triplet $(X,\tilde{\tau},E)$ is called a soft topological space over X. The members of $\tilde{\tau}$ are called soft open sets in $\tilde{X}$.

Definition (1.4) [7]**:** Let X be a non-empty set, and let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two soft topologies over X. Then $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called a soft bitopological space.

Definition (1.5) [7]: A soft subset (A,E) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$-open if $(A,E)=(U_1,E)\,\tilde{\bigcup}\,(U_2,E)$ where $(U_1,E)\,\tilde{\in}\,\tilde{\tau}_1$ and $(U_2,E)\,\tilde{\in}\,\tilde{\tau}_2$. The complement of a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is defined to be soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

Notice that soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets need not necessarily form a soft topology as shown by the following example:

Example (1.6): Let $X=\{a,b,c\}$ and $E=\{e_1,e_2\}$, and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(B,E)\}$ be two soft topologies over X, where $(A,E)=\{(e_1,\{a,b\}),(e_2,\{X\})\}$ and $(B,E)=\{(e_1,\{a,c\}),(e_2,\{X\})\}$. The soft sets in $\{\tilde{X},\tilde{\phi},(A,E),(B,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$. Since $(A,E)\,\tilde{\bigcap}\,(B,E)=\{(e_1,\{a\}),(e_2,\{X\})\}=(C,E)$, but (C,E) is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$. Thus $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is not soft topology over X.

Definition (1.7) [7]**:** Let (A,E) be a soft subset of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$.Then:

i) The soft $\tilde{\tau}_1\tilde{\tau}_2$-closure of (A,E), denoted by $\tilde{\tau}_1\tilde{\tau}_2 cl(A,E)$ is defined by:

$$\tilde{\tau}_1\tilde{\tau}_2 cl(A,E)=\tilde{\bigcap}\{(F,E):(A,E)\,\tilde{\subseteq}\,(F,E)\ \&\ (F,E)\text{ is soft }\tilde{\tau}_1\tilde{\tau}_2\text{-closed}\}.$$

ii) The soft $\tilde{\tau}_1\tilde{\tau}_2$-interior of (A,E), denoted by $\tilde{\tau}_1\tilde{\tau}_2\,int(A,E)$ is defined by:

$$\tilde{\tau}_1\tilde{\tau}_2\,int(A,E)=\tilde{\bigcup}\{(U,E):(U,E)\,\tilde{\subseteq}\,(A,E)\ \&\ (U,E)\text{ is soft }\tilde{\tau}_1\tilde{\tau}_2\text{-open}\}$$

Proposition (1.8): Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ be a soft bitopological space and (A,E), $(B,E)\,\tilde{\subseteq}\,\tilde{X}$. Then:

i) $\tilde{\tau}_1\tilde{\tau}_2\,int(A,E)\,\tilde{\subseteq}\,(A,E)$ and $(A,E)\,\tilde{\subseteq}\,\tilde{\tau}_1\tilde{\tau}_2 cl(A,E)$.

ii) An arbitrary union of soft $\tilde{\tau}_1\tilde{\tau}_2$ -open sets is soft $\tilde{\tau}_1\tilde{\tau}_2$ -open set.

iii) An arbitrary intersection of soft $\tilde{\tau}_1\tilde{\tau}_2$ -closed sets is soft $\tilde{\tau}_1\tilde{\tau}_2$ -closed set.

iv) $\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(A,E)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$ -open and $\tilde{\tau}_1\tilde{\tau}_2\text{cl}(A,E)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$ -closed.

v) (A,E) is soft $\tilde{\tau}_1\tilde{\tau}_2$ -open iff $(A,E)=\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(A,E)$.

vi) (A,E) is soft $\tilde{\tau}_1\tilde{\tau}_2$ -closed iff $(A,E)=\tilde{\tau}_1\tilde{\tau}_2\text{cl}(A,E)$.

vii) $\tilde{\tau}_1\tilde{\tau}_2\text{cl}(\tilde{\tau}_1\tilde{\tau}_2\text{cl}(A,E))=\tilde{\tau}_1\tilde{\tau}_2\text{cl}(A,E)$ and $\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(A,E))=\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(A,E)$.

viii) $(\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(A,E))^c=\tilde{\tau}_1\tilde{\tau}_2\text{cl}((A,E)^c)$ and $\tilde{\tau}_1\tilde{\tau}_2\,\text{int}((A,E)^c)=(\tilde{\tau}_1\tilde{\tau}_2\text{cl}((A,E))^c$.

ix) If $(A,E)\,\tilde{\subseteq}\,(B,E)$, then $\tilde{\tau}_1\tilde{\tau}_2\text{cl}(A,E)\,\tilde{\subseteq}\,\tilde{\tau}_1\tilde{\tau}_2\text{cl}(B,E)$.

x) If $(A,E)\,\tilde{\subseteq}\,(B,E)$, then $\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(A,E)\,\tilde{\subseteq}\,\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(B,E)$.

Definition (1.9) [8]: A soft subset (A,E) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called soft (1,2)*-pre-open if $(A,E)\,\tilde{\subseteq}\,\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(\tilde{\tau}_1\tilde{\tau}_2\text{cl}(A,E))$. The complement of a soft (1,2)*-pre-open set is defined to be soft (1,2)*-pre-closed. The family of all soft (1,2)*-pre-open (resp.(1,2)*-pre-closed) subsets of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is denoted by (1,2)*-pre-O($\tilde{X}$) (resp. (1,2)*-pre-C($\tilde{X}$)).

Clearly, every soft $\tilde{\tau}_1\tilde{\tau}_2$ -open set is soft (1,2)*-pre-open, but the converse is not true as shown by the following example:

Example (1.10): Let $X=\{a,b,c\}$ and $E=\{e_1,e_2\}$, and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(B,E)\}$ be two soft topologies over X, where $(A,E)=\{(e_1,\{a\}),(e_2,\{X\})\}$ and $(B,E)=\{(e_1,\{a,b\}),(e_2,\{X\})\}$. The soft sets in $\{\tilde{X},\tilde{\phi},(A,E),(B,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$ -open. Thus $(C,E)=\{(e_1,\{a,c\}),(e_2,\{X\})\}$ is a soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but is not soft $\tilde{\tau}_1\tilde{\tau}_2$ -open.

Definition (1.11): Let (A,E) be a soft subset of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. Then:

i) The soft (1,2)*-pre-closure of (A,E), denoted by (1,2)*-pcl(A,E) is the intersection of all soft (1,2)*-pre- closed sets in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ which contains (A,E).

ii) The soft (1,2)*-pre-interior of (A,E), denoted by (1,2)*-pint(A,E) is the union of all soft (1,2)*-pre-open sets in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ which are contained in (A,E).

Proposition (1.12): Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ be a soft bitopological space and (A,E), $(B,E)\,\tilde{\subseteq}\,\tilde{X}$. Then:

i) $\tilde{\tau}_1\tilde{\tau}_2\,\text{int}(A,E)\,\tilde{\subseteq}$ (1,2)*-pint(A,E) $\tilde{\subseteq}\,(A,E)$.

ii) $(A,E)\,\tilde{\subseteq}$ (1,2)*-pcl(A,E) $\tilde{\subseteq}\,\tilde{\tau}_1\tilde{\tau}_2\text{cl}(A,E)$.

iii) An arbitrary union of soft (1,2)*-pre-open sets is soft (1,2)*-pre-open set.

iv) An arbitrary intersection of soft (1,2)*-pre-closed sets is soft (1,2)*-pre-closed set.

v) (1,2)*-pint(A,E) is soft (1,2)*-pre-open and (1,2)*-pcl(A,E) is soft (1,2)*-pre-closed.

vi) (A,E) is soft (1,2)*-pre-open iff (1,2)*-pint$(A,E)=(A,E)$.

vii) (A,E) is soft (1,2)*-pre-closed iff (1,2)*-pcl$(A,E)=(A,E)$.

viii) If $(A,E)\,\tilde{\subseteq}\,(B,E)$, then (1,2)*-pint$(A,E)\,\tilde{\subseteq}$ (1,2)*-pint(B,E).

ix) If $(A,E)\,\tilde{\subseteq}\,(B,E)$, then (1,2)*-pcl$(A,E)\,\tilde{\subseteq}$ (1,2)*-pcl(B,E).

2. (1, 2)*-Maximal Soft (1,2)*-Pre-Open Sets and (1,2)*-Minimal Soft (1,2)*-Pre-Open Sets

In this section we introduce and study new kinds of soft open sets and soft closed sets in soft bitopological spaces called (1,2)*-maximal soft open sets, (1,2)*-minimal soft open sets, (1,2)*-maximal soft closed sets, (1,2)*-minimal soft closed sets, (1,2)*-maximal soft (1,2)*-pre-open sets, (1,2)*-minimal soft (1,2)*-pre-open sets, (1,2)*-maximal soft (1,2)*-pre-closed sets, and (1,2)*-minimal soft (1,2)*-pre-closed sets. Moreover we study the properties and the relation among these concepts.

Definition (2.1): Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ be a soft bitopological space. Then:

i) A proper non-null soft $\tilde{\tau}_1\tilde{\tau}_2$ -open subset (U,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1 ,2)*-maximal soft open if any soft $\tilde{\tau}_1\tilde{\tau}_2$ -open set which contains (U,E) is (U,E) or $\tilde{X}$.

ii) A proper non-null soft $\tilde{\tau}_1\tilde{\tau}_2$-open subset (U,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1, 2)*-minimal soft open if any soft $\tilde{\tau}_1\tilde{\tau}_2$-open set which is contained in (U,E) is $\tilde{\phi}$ or (U,E).

iii) A proper non-null soft $\tilde{\tau}_1\tilde{\tau}_2$-closed subset (K,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1, 2)*-maximal soft closed if any soft $\tilde{\tau}_1\tilde{\tau}_2$-closed set which contains (K,E) is (K,E) or $\tilde{X}$.

iv) A proper non-null soft $\tilde{\tau}_1\tilde{\tau}_2$-closed subset (K,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1,2)*-minimal soft close if any soft $\tilde{\tau}_1\tilde{\tau}_2$-closed set which is contained in (K,E) is $\tilde{\phi}$ or (K,E).

Remarks (2.2):

i) The family of all (1,2)*-maximal soft open (resp. (1,2)*-maximal soft closed) sets of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is denoted by $(1,2)*M_a$-O($\tilde{X}$) (resp. $(1,2)*M_a$-C($\tilde{X}$)).

ii) The family of all (1,2)*-minimal soft open (resp. (1,2)*-minimal soft closed) sets of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is denoted by $(1,2)*M_i$-O($\tilde{X}$) (resp. $(1,2)*M_i$-C($\tilde{X}$)).

Definition (2.3): Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ be a soft bitopological space. Then:

i) A proper non-null soft (1,2)*-pre-open subset (U,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1,2)*-maximal soft (1, 2)*-pre-open if any soft (1,2)*-pre-open set which contains (U,E) is (U,E) or $\tilde{X}$.

ii) A proper non-null soft (1,2)*-pre-open subset (U,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1,2)*-minimal soft (1, 2)*-pre-open if any soft (1,2)*-pre-open set which is contained in (U,E) is $\tilde{\phi}$ or (U,E).

iii) A proper non-null soft (1,2)*-pre-closed subset (K,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1,2)*-maximal soft (1, 2)*-pre-closed if any soft (1,2)*-pre-closed set which contains (K,E) is (K,E) or $\tilde{X}$.

iv) A proper non-null soft (1,2)*-pre-closed subset (K,E) of $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called (1,2)*-minimal soft (1, 2)*-pre-closed if any soft (1,2)*-pre-closed set which is contained in (K,E) is $\tilde{\phi}$ or (K,E).

Remarks (2.4):

i) The family of all (1,2)*-maximal soft (1,2)*-pre-open (resp. (1,2)*-maximal soft (1,2)*-pre-closed) sets of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is denoted by $(1,2)*M_a$-pre-O($\tilde{X}$) (resp. $(1,2)*M_a$-pre- C($\tilde{X}$)).

ii) The family of all (1,2)*-minimal soft (1,2)*-pre-open (resp. (1,2)*-minimal soft (1,2)*-pre-closed) sets of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is denoted by $(1,2)*M_i$-pre-O($\tilde{X}$) (resp. $(1,2)*M_i$-pre-C($\tilde{X}$)).

Remark (2.5): The concept of (1,2)*-maximal soft (1,2)*-pre-open sets, (1,2)*-minimal soft (1,2)*-pre-open set, (1,2)*-maximal soft (1,2)*-pre-closed sets and (1,2)*-minimal soft (1,2)*-pre-closed sets are independent of each other as in the following example.

Example (2.6): Let $X=\{a,b\}$ and $E=\{e_1,e_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_2,E)\}$ be soft topologies over X, where $(A_1,E)=\{(e_1,\{a\}),(e_2,\{\phi\})\}$ and $(A_2,E)=\{(e_1,\{a\}),(e_2,\{a\})\}$. Thus (1,2)*-pre-O($\tilde{X}$) $=\{\tilde{\phi},\tilde{X},(A_1,E),(A_2,E),(A_3,E),(A_4,E),(A_5,E),(A_6,E),(A_7,E)\}$, where $(A_3,E)=\{(e_1,\{a\}),(e_2,\{b\})\}$, $(A_4,E)=\{(e_1,\{a\}),(e_2,\{X\})\}$, $(A_5,E)=\{(e_1,\{X\}),(e_2,\{\phi\})\}$, $(A_6,E)=\{(e_1,\{X\}),(e_2,\{a\})\}$ and $(A_7,E)=\{(e_1,\{X\}),(e_2,\{b\})\}$ and (1,2)*-pre-C($\tilde{X}$) $=\{\tilde{\phi},\tilde{X},(B_1,E),(B_2,E),(B_3,E),(B_4,E),(B_5,E),(B_6,E),(B_7,E)\}$, where $(B_1,E)=\{(e_1,\{b\}),(e_2,\{X\})\}$, $(B_2,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$, $(B_3,E)=\{(e_1,\{b\}),(e_2,\{a\})\}$, $(B_4,E)=\{(e_1,\{b\}),(e_2,\{\phi\})\}$, $(B_5,E)=\{(e_1,\{\phi\}),(e_2,\{X\})\}$, $(B_6,E)=\{(e_1,\{\phi\}),(e_2,\{b\})\}$ and $(B_7,E)=\{(e_1,\{\phi\}),(e_2,\{a\})\}$. Hence $(1,2)*M_i$-pre-O($\tilde{X}$) $=\{(A_1,E)\}$, $(1,2)*M_a$-pre-O($\tilde{X}$) $=\{(A_4,E),(A_6,E),(A_7,E)\}$, $(1,2)*M_i$-pre-C($\tilde{X}$) $=\{(B_4,E),(B_6,E),(B_7,E)\}$ and $(1,2)*M_a$-pre-C($\tilde{X}$) $=\{(B_1,E)\}$.

Table 1- shows the relation between each of (1,2)*-maximal soft (1,2)*-pre-open sets, (1,2)*-minimal soft (1,2)*-pre-open set, (1,2)*-maximal soft (1,2)*-pre-closed sets and (1,2)*-minimal soft (1,2)*-pre-closed sets are independent.

	(1,2)*-Minimal soft (1,2)*-pre-open set	(1,2)*-Maximal soft (1,2)*-pre-open set	(1,2)*-Minimal soft (1,2)*-pre-closed set	(1,2)*-Maximal soft (1,2)*-pre-closed set
(A_1,E)	Yes	No	No	No
(A_4,E)	No	Yes	No	No
(B_4,E)	No	No	Yes	No
(B_1,E)	No	No	No	Yes

Remarks (2.7):

i) Every (1, 2)*-maximal soft (1,2)*-pre-open set (resp. (1,2)*-maximal soft open set) is soft (1,2)*-pre- open (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-open), but the converse is not true in general. In example (2.6), (A_1,E) is soft (1, 2)*-pre-open (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-open) set, but is not (1,2)*-maximal soft (1,2)*-pre-open (resp. (1,2)*- maximal soft open) set.

ii) Every (1,2)*-minimal soft (1,2)*-pre-open set (resp. (1,2)*-minimal soft open set) is soft (1,2)*-pre- open (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-open), but the converse is not true in general. In example (2.6), (A_2,E) is soft (1,2)*-pre-open (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-open) set, but is not (1,2)*-minimal soft (1,2)*-pre-open (resp. (1,2)*- minimal soft open) set.

iii) Every (1,2)*-maximal soft (1,2)*-pre-closed set (resp. (1,2)*-maximal soft closed set) is soft (1,2)*- pre-closed (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-closed), but the converse is not true in general. In example (2.6), (B_2,E) is soft (1,2)*-pre-closed (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-closed) set, but is not (1,2)*-maximal soft (1,2)*-pre-closed (resp. (1,2)*-maximal soft closed) set.

iv) Every (1,2)*-minimal soft (1,2)*-pre-closed set (resp.(1,2)*-minimal soft closed set) is soft (1,2)*-pre- closed (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-closed) set, but the converse is not true in general. In example (2.6), (B_1,E) is soft (1,2)*-pre-closed (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-closed) set, but is not (1,2)*-minimal soft (1,2)*-pre-closed (resp. (1,2)*-minimal soft closed) set.

v) The concept of (1,2)*-maximal soft open (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-open) sets and (1,2)*-maximal soft (1,2)*-pre-open sets are in general independent. In example (2.6), (A_4,E) is a (1,2)*-maximal soft (1,2)*-pre- open set which is not (1,2)*-maximal soft open (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-open) and (A_2,E) is a (1,2)*-maximal soft open (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-open) set which is not (1,2)*-maximal soft (1,2)*-pre-open.

vi) The concept of (1,2)*-minimal soft closed (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-closed) sets and (1,2)*-minimal soft (1,2)*- pre-closed sets are in general independent. In example (2.6), (B_4,E) is a (1,2)*-minimal soft (1,2)*- pre-closed set which is not (1,2)*-minimal soft closed (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-closed), and (B_2,E) is a (1,2)*- minimal soft closed (resp. soft $\tilde{\tau}_1\tilde{\tau}_2$-closed) set which is not (1,2)*-minimal soft (1,2)*-pre-closed.

vii) The intersection of two (1,2)*-maximal soft (1,2)*-pre-open sets need not to be (1,2)*-maximal soft (1,2)*-pre-open set. In example (2.6), (A_4,E) and (A_6,E) are (1,2)*-maximal soft (1,2)*-pre-open sets, but $(A_4,E)\,\tilde{\cap}\,(A_6,E)=(A_2,E)$ which is not (1,2)*-maximal soft (1,2)*-pre-open.

viii) The union of two (1,2)*-minimal soft (1,2)*-pre-closed sets need not to be (1,2)*-minimal soft (1,2)* -pre-closed set. In example (2.6), (B_4,E) and (B_6,E) are (1,2)*-minimal soft (1,2)*-pre-closed sets, but $(B_4, E)\,\tilde{\bigcup}\,(B_6, E) = (B_2, E)$ which is not (1,2)*-minimal soft (1,2)*-pre-closed.

Proposition (2.8): Any soft $\tilde{\tau}_1\tilde{\tau}_2$-open and (1,2)*-maximal soft (1,2)*-pre-open set in a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a (1,2)*-maximal soft open set.

Proof: Let (O, E) be any soft $\tilde{\tau}_1\tilde{\tau}_2$-open and (1,2)*-maximal soft (1,2)*-pre-open set in $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$. To prove that (O, E) is a (1,2)*-maximal soft open set. Suppose that (O, E) is not (1,2)*-maximal soft open, then there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set $(U, E) \neq (O, E)$ such that $(O, E)\,\tilde{\subset}\,(U, E) \neq \tilde{X}$. But every soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is soft (1,2)*-pre-open, this implies that (U, E) is a soft (1,2)*-pre-open set such that $(O, E) \neq (U, E)$ and $(O, E)\,\tilde{\subset}\,(U, E) \neq \tilde{X}$, which is a contradiction. Hence (O, E) is a (1,2)*-maximal soft open set.

Proposition (2.9):

i) Every (1, 2)*-minimal soft (1,2)*-pre-open set in a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a (1,2)*-minimal soft open set.

ii) Every (1, 2)*-minimal soft (1,2)*-pre-open set in a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft $\tilde{\tau}_1\tilde{\tau}_2$ - open set.

Proof:i) Let (A, E) be a (1,2)*-minimal soft (1,2)*-pre-open set in $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$. To prove that (A, E) is (1, 2)*-minimal soft open. If not, then there is a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (U, E) in $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ such that $\tilde{\phi} \neq (U, E)\,\tilde{\subset}\,(A, E)$. Since every soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is soft (1,2)*-pre-open, hence (U, E) is a soft (1,2)*-pre-open set such that $(U, E)\,\tilde{\subset}\,(A, E)$, this is a contradiction, since (A, E) is a (1,2)*-minimal soft (1,2)*-pre-open set. Thus (A, E) is a (1, 2)*-minimal soft open set.

ii) It is obvious.

Remark (2.10): The converse of proposition ((2.9),(ii)) may not be true in general. In example (2.6), (A_2, E) is a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$, but is not (1,2)*-minimal soft (1,2)*-pre-open.

Proposition (2.11): Let (A, E) be a soft subset of a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$. Then the following duality principle holds:

i) (A, E) is a (1,2)*-minimal soft (1,2)*-pre-closed set if and only if $(A, E)^c$ is a (1,2)*-maximal soft (1, 2)*-pre-open set.

ii) (A, E) is a (1,2)*-maximal soft (1,2)*-pre-closed set if and only if $(A, E)^c$ is a (1,2)*-minimal soft (1, 2)*-pre-open set.

Proof:i) $\Rightarrow$ Let (A, E) be a (1,2)*-minimal soft (1,2)*-pre-closed set and suppose that $(A, E)^c$ is not a (1,2)*-maximal soft (1,2)*-pre-open set. Then there exists a soft (1,2)*-pre-open set $(B, E) \neq (A, E)^c$ such that $(A, E)^c\,\tilde{\subset}\,(B, E) \neq \tilde{X}$. That is $\tilde{\phi} \neq (B, E)^c\,\tilde{\subset}\,(A, E)$ and $(B, E)^c$ is a soft (1,2)*-pre-closed set. This is a contradiction to the fact that (A, E) is a (1,2)*-minimal soft (1,2)*-pre-closed set. Hence $(A, E)^c$ is a (1,2)*-maximal soft (1,2)*-pre-open set. **Conversely,** let $(A, E)^c$ be a (1,2)*-maximal soft (1,2)*-pre-open set and suppose that (A, E) is not a (1,2)*-minimal soft (1,2)*-pre-closed set. Then there exists a non-null soft (1,2)*-pre-closed set (B, E) such that $(B, E)\,\tilde{\subset}\,(A, E)$. That is $(A, E)^c\,\tilde{\subset}\,(B, E)^c$ and $(B, E)^c$ is a soft (1,2)*-pre-open set. This is a contradiction to the fact that $(A, E)^c$ is a (1, 2)*-maximal soft (1,2)*-pre-open set. Hence (A, E) is a (1,2)*-minimal soft (1,2)*-pre-closed set.

ii) Similar to (i).

Theorem (2.12): The following statements are true for any soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$.

i) If (V, E) is a (1, 2)*-maximal soft (1,2)*-pre-open set and (W, E) is a soft (1,2)*-pre-open set. Then either $(V, E)\,\tilde{\bigcup}\,(W, E) = \tilde{X}$ or $(W, E)\,\tilde{\subset}\,(V, E)$.

ii) If (V,E) and (W,E) are (1,2)*-maximal soft (1,2)*-pre-open sets. Then either $(V,E)\tilde{\bigcup}(W,E)=\tilde{X}$ or $(V,E)=(W,E)$.

iii) If (K,E) is a (1, 2)*-minimal soft (1,2)*-pre-closed set and (H,E) is a soft (1,2)*-pre-closed set. Then either $(K,E)\tilde{\bigcap}(H,E)=\tilde{\phi}$ or $(K,E)\tilde{\subseteq}(H,E)$.

iv) If (K,E) and (H,E) are (1,2)*-minimal soft (1,2)*-pre-closed sets. Then either $(K,E)\tilde{\bigcap}(H,E)=\tilde{\phi}$ or $(K,E)=(H,E)$.

Proof:

i) If $(V,E)\tilde{\bigcup}(W,E)=\tilde{X}$, then the proof is complete. If $(V,E)\tilde{\bigcup}(W,E)\neq\tilde{X}$, then we have to prove that $(W,E)\tilde{\subseteq}(V,E)$. Now, $(V,E)\tilde{\bigcup}(W,E)\neq\tilde{X}$ means $(V,E)\tilde{\subseteq}(V,E)\tilde{\bigcup}(W,E)$. Since (V,E) is a (1,2)*-maximal soft (1,2)*-pre-open set, then by definition ((2.3),(i)) we have $(V,E)\tilde{\bigcup}(W,E)=\tilde{X}$ or $(V,E)\tilde{\bigcup}(W,E)=(V,E)$, but $(V,E)\tilde{\bigcup}(W,E)\neq\tilde{X}$, then $(V,E)\tilde{\bigcup}(W,E)=(V,E)$ which implies $(W,E)\tilde{\subseteq}(V,E)$.

ii) If $(V,E)\tilde{\bigcup}(W,E)=\tilde{X}$, then the proof is complete. If $(V,E)\tilde{\bigcup}(W,E)\neq\tilde{X}$, then we have to prove that $(V,E)=(W,E)$. Now, $(V,E)\tilde{\bigcup}(W,E)\neq\tilde{X}$ means $(V,E)\tilde{\subseteq}(V,E)\tilde{\bigcup}(W,E)$ and $(W,E)\tilde{\subseteq}(V,E)\tilde{\bigcup}(W,E)$. Since $(V,E)\tilde{\subseteq}(V,E)\tilde{\bigcup}(W,E)$ and (V,E) is (1,2)*-maximal soft (1,2)*-pre-open, then by definition ((2.3),(i)) we have $(V,E)\tilde{\bigcup}(W,E)=\tilde{X}$ or $(V,E)\tilde{\bigcup}(W,E)=(V,E)$, but $(V,E)\tilde{\bigcup}(W,E)\neq\tilde{X}$, therefore $(V,E)\tilde{\bigcup}(W,E)=(V,E)$ which implies $(W,E)\tilde{\subseteq}(V,E)$. Similarly, if $(W,E)\tilde{\subseteq}(V,E)\tilde{\bigcup}(W,E)$ we obtain $(V,E)\tilde{\subseteq}(W,E)$. Therefore $(V,E)=(W,E)$.

iii) Since (K,E) is a (1,2)*-minimal soft (1,2)*-pre-closed set, then by proposition ((2.11),(i)) $(K,E)^c$ is a (1,2)*-maximal soft (1,2)*-pre-open set. Also, since (H,E) is a soft (1,2)*-pre-closed set, then $(H,E)^c$ is a soft (1,2)*-pre-open set. Hence by (i) $(K,E)^c\tilde{\bigcup}(H,E)^c=\tilde{X}$ or $(H,E)^c\tilde{\subseteq}(K,E)^c$. Therefore

$$(K,E)\tilde{\bigcap}(H,E)=\tilde{\phi} \text{ or } (K,E)\tilde{\subseteq}(H,E).$$

iv) Since (K,E) and (H,E) are (1,2)*-minimal soft (1,2)*-pre-closed sets, then by proposition ((2.11),(i)) $(K,E)^c$ and $(H,E)^c$ are (1,2)*-maximal soft (1,2)*-pre-open sets. Hence by (ii) $(K,E)^c\tilde{\bigcup}(H,E)^c=\tilde{X}$ or $(H,E)^c=(K,E)^c$. Therefore $(K,E)\tilde{\bigcap}(H,E)=\tilde{\phi}$ or $(K,E)=(H,E)$.

Theorem (2.13): Let (U,E) be a soft subset of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. Then

i) If (U,E) is a (1,2)*-maximal soft (1,2)*-pre-open set and $\tilde{x}\tilde{\in}(U,E)^c$, then $(U,E)^c\tilde{\subseteq}(V,E)$ for any soft (1,2)*-pre-open set (V,E) containing $\tilde{x}$.

ii) If (U,E) is a (1,2)*-maximal soft (1,2)*-pre-open set. Then either of the following (1) and (2) holds:

1) For each $\tilde{x}\tilde{\in}(U,E)^c$ and each soft (1,2)*-pre-open set (V,E) containing $\tilde{x}$, $(V,E)=\tilde{X}$.

2) There exists a soft (1,2)*-pre-open set (V,E) such that $(U,E)^c\tilde{\subseteq}(V,E)$ and $(V,E)\tilde{\subseteq}\tilde{X}$.

iii) If (U,E) is a (1,2)*-maximal soft (1,2)*-pre-open set. Then either of the following (1) and (2) holds:

1) For each $\tilde{x}\tilde{\in}(U,E)^c$ and each soft (1,2)*-pre-open set (V,E) containing $\tilde{x}$, we have $(U,E)^c \tilde{\subset} (V,E)$

2) There exists a soft (1,2)*-pre-open set (V,E) such that $(U,E)^c = (V,E) \neq \tilde{X}$.

Proof:

i) Since $\tilde{x}\tilde{\in}(U,E)^c$, we have $(V,E)\tilde{\not\subset}(U,E)$ for any soft (1,2)*-pre-open set (V,E) containing $\tilde{x}$. Hence by theorem ((2.12),(i)) we get $(U,E)\tilde{\bigcup}(V,E) = \tilde{X}$, therefore $(U,E)^c \tilde{\bigcap}(V,E)^c = \tilde{\phi}$, thus $(U,E)^c \tilde{\subset}(V,E)$.

ii) If (1) does not hold, then there exists $\tilde{x}\tilde{\in}(U,E)^c$ and a soft (1,2)*-pre-open set (V,E) containing $\tilde{x}$ such that $(V,E)\tilde{\subset}\tilde{X}$. By (i) we have $(U,E)^c \tilde{\subset}(V,E)$.

iii) If (2) does not hold, then, by (i), we have $(U,E)^c \tilde{\subset}(V,E)$ for each $\tilde{x}\tilde{\in}(U,E)^c$ and each soft (1,2)*- pre-open set (V,E) containing $\tilde{x}$.

Theorem (2.14): Let (U,E), (V,E) and (W,E) be (1,2)*-maximal soft (1,2)*-pre-open subsets of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $(U,E)\neq(V,E)$. If $(U,E)\tilde{\bigcap}(V,E)\tilde{\subseteq}(W,E)$, then either $(U,E)=(W,E)$ or $(V,E)=(W,E)$.

Proof: Suppose that $(U,E)\tilde{\bigcap}(V,E)\tilde{\subseteq}(W,E)$. If $(U,E)=(W,E)$, then the proof is complete. If $(U,E)\neq(W,E)$, then we have to prove $(V,E)=(W,E)$.

$$
\begin{aligned}
&(V,E)\tilde{\bigcap}(W,E) = (V,E)\tilde{\bigcap}[(W,E)\tilde{\bigcap}\tilde{X}]\\
&=(V,E)\tilde{\bigcap}[(W,E)\tilde{\bigcap}((U,E)\tilde{\bigcup}(V,E))]\\
&=(V,E)\tilde{\bigcap}[((W,E)\tilde{\bigcap}(U,E))\tilde{\bigcup}((W,E)\tilde{\bigcap}(V,E))]\\
&=[(V,E)\tilde{\bigcap}(W,E)\tilde{\bigcap}(U,E)]\tilde{\bigcup}[(V,E)\tilde{\bigcap}(W,E)\tilde{\bigcap}(V,E)]\\
&=[(U,E)\tilde{\bigcap}(V,E)]\tilde{\bigcup}[(W,E)\tilde{\bigcap}(V,E)] \quad (\text{since } (U,E)\tilde{\bigcap}(V,E)\tilde{\subseteq}(W,E))\\
&=[(U,E)\tilde{\bigcup}(W,E)]\tilde{\bigcap}(V,E)\\
&=\tilde{X}\tilde{\bigcap}(V,E)=(V,E) \quad (\text{since } (U,E)\tilde{\bigcup}(W,E)=\tilde{X})
\end{aligned}
$$

This implies $(V,E)\tilde{\subseteq}(W,E)$, but (V,E) is a (1,2)*-maximal soft (1,2)*-pre-open set, therefore $(V,E)=(W,E)$.

Theorem (2.15): Let (U,E), (V,E) and (W,E) be (1,2)*-maximal soft (1,2)*-pre-open subsets of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ which are different from each other. Then $(U,E)\tilde{\bigcap}(V,E)\tilde{\not\subset}(U,E)\tilde{\bigcap}(W,E)$.

Proof: Let $(U,E)\tilde{\bigcap}(V,E)\tilde{\subseteq}(U,E)\tilde{\bigcap}(W,E)$. Then $[(U,E)\tilde{\bigcap}(V,E)]\tilde{\bigcup}[(W,E)\tilde{\bigcap}(V,E)]\tilde{\subseteq}$ $[(U,E)\tilde{\bigcap}(W,E)]\tilde{\bigcup}[(W,E)\tilde{\bigcap}(V,E)]$. Hence $[(U,E)\tilde{\bigcup}(W,E)]\tilde{\bigcap}(V,E)\tilde{\subseteq}(W,E)\tilde{\bigcap}[(U,E)\tilde{\bigcup}(V,E)]$. But by theorem ((2.12),(ii)) we get $(U,E)\tilde{\bigcup}(W,E) = (U,E)\tilde{\bigcup}(V,E)=\tilde{X}$. Therefore $\tilde{X}\tilde{\bigcap}(V,E)\tilde{\subseteq}$ $(W,E)\tilde{\bigcap}\tilde{X}$ which implies $(V,E)\tilde{\subseteq}(W,E)$. From the definition of (1,2)*-maximal soft (1,2)*-pre-open set it follows that $(V,E)=(W,E)$. Contradiction to the fact that (U,E), (V,E) and (W,E) are different from each other. Hence $(U,E)\tilde{\bigcap}(V,E)\tilde{\not\subset}(U,E)\tilde{\bigcap}(W,E)$.

Theorem (2.16): Let (U,E) and (V,E) be (1,2)*-maximal soft (1,2)*-pre-open sets in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ and $(O,E)\tilde{\subseteq}\tilde{X}$ such that $(V,E)\tilde{\subseteq}(O,E)\tilde{\subseteq}$ (1,2)*-pcl(V,E), if $(U,E)\tilde{\bigcap}(V,E)=\tilde{\phi}$, then $(O,E)\tilde{\bigcap}(U,E)=\tilde{\phi}$.

Proof: Since $(U,E)\tilde{\bigcap}(V,E)=\tilde{\phi}$, it follows that $(V,E)\tilde{\subseteq}(U,E)^c$, therefore (1,2)*-pcl$(V,E)\tilde{\subseteq}$

(1,2)*-pcl($(U,E)^c$). Since $(U,E)^c$ is a (1,2)*-minimal soft (1,2)*-pre-closed set and every (1,2)*-minimal soft (1,2)*-pre-closed set is soft (1,2)*-pre-closed, then (1,2)*-pcl($(U,E)^c$) $= (U,E)^c$. But $(V,E) \tilde{\subseteq} (O,E) \tilde{\subseteq}$ (1,2)*-p cl(V,E). Therefore $(O,E) \tilde{\subseteq}$ (1,2)*-p cl$(V,E) \tilde{\subseteq} (U,E)^c$. Thus $(O,E) \tilde{\subseteq} (U,E)^c$ which means $(O,E) \tilde{\cap} (U,E) = \tilde{\phi}$.

Proposition (2.17): Let (U,E) be a (1,2)*-maximal soft (1,2)*-pre-open set and (V,E) be a proper soft subset of $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ with $(U,E) \tilde{\subseteq} (V,E)$. Then (1,2)*-pint$(V,E) = (U,E)$.

Proof: If $(V,E) = (U,E)$, then (1,2)*-pint$(V,E) =$ (1,2)*-int$(U,E) = (U,E)$. Otherwise, $(V,E) \neq (U,E)$, and hence $(U,E) \tilde{\subset} (V,E)$. It follows that $(U,E) \tilde{\subset}$ (1,2)*-pint(V,E). Since (U,E) is a (1,2)*-maximal soft (1,2)*-pre-open set, we have (1,2)*-pint$(V,E) \tilde{\subset} (U,E)$. Thus (1,2)*-pint$(V,E) = (U,E)$.

Theorem (2.18): If (K,E) is a (1,2)*-minimal soft (1,2)*-pre-closed set in $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ and $\tilde{x} \tilde{\in} (K,E)$, then $(K,E) \tilde{\subset} (H,E)$ for any soft (1,2)*-pre-closed set (H,E) containing $\tilde{x}$.

Proof: Let (K,E) be a (1,2)*-minimal soft (1,2)*-pre-closed set such that $\tilde{x} \tilde{\in} (K,E)$ and (H,E) be a soft (1,2)*-pre-closed set containing $\tilde{x}$. If $(K,E) \tilde{\not\subset} (H,E)$, then $(K,E) \tilde{\cap} (H,E) \tilde{\subset} (K,E)$ and $(K,E) \tilde{\cap} (H,E) \neq \tilde{\phi}$. Since (K,E) is a (1,2)*-minimal soft (1,2)*-pre-closed set, then by definition ((2.3),(iv)), $(K,E) \tilde{\cap} (H,E) = (K,E)$ which contradicts the relation $(K,E) \tilde{\cap} (H,E) \tilde{\subset} (K,E)$. Therefore $(K,E) \tilde{\subset} (H,E)$.

Theorem (2.19): Let (K,E) and $\{(K_\alpha, E)\}_{\alpha\in\Lambda}$ be (1,2)*-minimal soft (1,2)*-pre-closed sets. Then:

i) If $(K,E) \tilde{\subset} \bigcup_{\alpha\in\Lambda} (K_\alpha, E)$, then there exists $\alpha_0 \in \Lambda$ such that $(K,E) = (K_{\alpha_0}, E)$.

ii) If $(K,E) \neq (K_\alpha, E)$ for each $\alpha \in \Lambda$, then $(\bigcup_{\alpha\in\Lambda} (K_\alpha, E)) \tilde{\cap} (K,E) = \tilde{\phi}$.

Proof:

i) Since $(K,E) \tilde{\subset} \bigcup_{\alpha\in\Lambda} (K_\alpha, E)$, we get $(K,E) = (K,E) \tilde{\cap} \bigcup_{\alpha\in\Lambda} (K_\alpha, E) = \bigcup_{\alpha\in\Lambda} ((K,E) \tilde{\cap} (K_\alpha, E))$. If $(K,E) \neq (K_\alpha, E)$ for each $\alpha \in \Lambda$, then by theorem ((2.12),(iv)), $(K,E) \tilde{\cap} (K_\alpha, E) = \tilde{\phi}$ for each $\alpha \in \Lambda$, hence we have $(K,E) = \bigcup_{\alpha\in\Lambda} ((K_\alpha, E) \tilde{\cap} (K,E)) = \tilde{\phi}$. This contradicts our assumption that (K,E) is a (1,2)*-minimal soft (1,2)*-pre-closed set. Thus there exists $\alpha_0 \in \Lambda$ such that $(K,E) = (K_{\alpha_0}, E)$.

ii) If $(K,E) \neq (K_\alpha, E)$ for each $\alpha \in \Lambda$, then by theorem ((2.12),(iv)), $(K,E) \tilde{\cap} (K_\alpha, E) = \tilde{\phi}$ for each $\alpha \in \Lambda$,
therefore $(\bigcup_{\alpha\in\Lambda} (K_\alpha, E)) \tilde{\cap} (K,E) = \bigcup_{\alpha\in\Lambda} ((K_\alpha, E) \tilde{\cap} (K,E)) = \tilde{\phi}$.

3. Semi (1,2)*-Maximal Soft (1,2)*-Pre-Open Sets and Semi (1,2)*-Minimal Soft (1,2)*-Pre-Closed Sets

In this section we introduce and study new concepts in soft bitopological spaces, namely, semi (1,2)*-maximal soft (1,2)*-pre-open sets and semi (1,2)*-minimal soft (1,2)*-pre-closed sets. Also, we study the properties of these soft sets and its relationship with the previous concepts.

Definition (3.1): A soft subset (A,E) of a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called semi (1,2)*-maximal soft (1,2)*-pre-open if there exists a (1,2)*-maximal soft (1,2)*-pre-open set (N,E) in $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ such that $(N,E) \tilde{\subseteq} (A,E) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(N,E)$. The family of all semi (1,2)*-maximal soft (1,2)*-pre-open sets in $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is denoted by $S(1,2)*M_a$ pre-O($\tilde{X}$).

Remark (3.2): Every (1,2)*-maximal soft (1,2)*-pre-open set is semi (1,2)*-maximal soft (1,2)*-pre-open, but the converse is not true as shown by the following example:

Example (3.3): Let $X=\{a,b\}$ and $E=\{e_1,e_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_2,E)\}$ be soft topologies over X, where $(A_1,E)=\{(e_1,\{a\}),(e_2,\{\phi\})\}$ and $(A_2,E)=\{(e_1,\{a\}),(e_2,\{a\})\}$. The soft sets in $\{\tilde{X},\tilde{\phi},(A_1,E),(A_2,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Then $\tilde{X}$ is a semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, since $\exists$ a (1,2)*-maximal soft (1,2)*-pre-open set $(N,E)=\{(e_1,\{a\}),(e_2,\{X\})\}$ in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $(N,E)\tilde{\subseteq}\tilde{X}\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(N,E)$, but $\tilde{X}$ is not (1,2)*-maximal soft (1,2)*-pre-open set.

Remark (3.4): Semi (1,2)*-maximal soft (1,2)*-pre-open sets and soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets are in general independent as shown by the following examples:

Examples (3.5): Let $X=\{a,b\}$ and $E=\{e_1,e_2\}$, and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_2,E)\}$ be soft topologies over X, where $(A_1,E)=\{(e_1,\{a\}),(e_2,\{a\})\}$ and $(A_2,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$. The soft sets in $\{\tilde{X},\tilde{\phi},(A_1,E),(A_2,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Since $(N,E)=\{(e_1,\{a\}),(e_2,\{X\})\}$ is a (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, thus (N,E) is a semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but (N,E) is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. Also, (A_1,E) is a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but is not semi (1,2)*-maximal soft (1,2)*-pre-open.

Remark (3.6): In example (3.5) $(A_1,E)=\{(e_1,\{a\}),(e_2,\{a\})\}$ is soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but is not semi (1,2)*-maximal soft (1,2)*-pre-open set.

The following diagram shows the relationships between semi (1,2)*-maximal soft (1,2)*-pre-open sets and some types of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets:

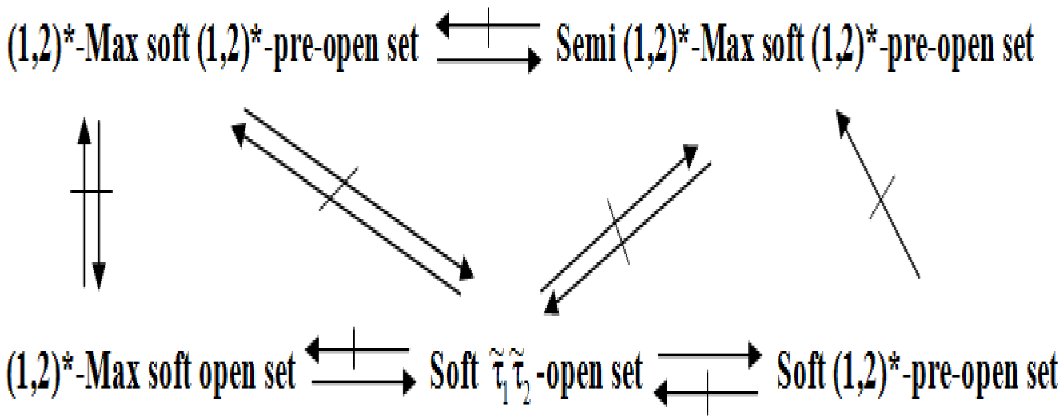

Figure 1- the relationships between semi (1, 2)*-maximal soft (1,2)*-pre-open sets and some types of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets.

Theorem (3.7): If (A,E) is a semi (1,2)*-maximal soft (1,2)*-pre-open set in a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ and $(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(A,E)$, then (B,E) is also semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$.

Proof: Since (A,E) is a semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, then by definition (3.1), there exists a (1,2)*-maximal soft (1,2)*-pre-open set (N,E) in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $(N,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(N,E)$. Since $(A,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(N,E)$ it follows from proposition ((1.8),(vii)) $\tilde{\tau}_1\tilde{\tau}_2(A,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2cl(N,E))=\tilde{\tau}_1\tilde{\tau}_2cl(N,E)$. But from hypothesis $(B,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(A,E)$ it follows that $(N,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(N,E)$. Hence by definition (3.1), (B,E) is a semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$.

Remark (3.8): The intersection of two semi (1,2)*-maximal soft (1,2)*-pre-open sets need not to be semi (1,2)*-maximal soft (1,2)*-pre-open as shown by the following example:

Examples (3.9): Let $X=\{a,b\}$ and $E=\{e_1,e_2\}$, and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_2,E)\}$ be two soft topologies over X, where $(A_1,E)=\{(e_1,\{a\}),(e_2,\{X\})\}$ and $(A_2,E)=\{(e_1,\{b\}),(e_2,\{\phi\})\}$. Also,

$S(1,2)*M_a$-pre-O($\tilde{X}$) $=\{\tilde{X},(N_1,E),(N_2,E),(N_3,E),(N_4,E)\}$ where $(N_1,E)=\{(e_1,\{a\}),(e_2,\{X\})\}$, $(N_2,E)=\{(e_1,\{b\}),(e_2,\{X\})\}$, $(N_3,E)=\{(e_1,\{X\}),(e_2,\{a\})\}$, $(N_4,E)=\{(e_1,\{X\}),(e_2,\{b\})\}$. Then (N_1,E) and (N_3,E) are semi (1,2)*-maximal soft (1,2)*-pre-open sets, but $(N_1,E)\tilde{\cap}(N_3,E)=(N,E)$ $=\{(e_1,\{a\}),(e_2,\{a\})\}$ which is not semi (1,2)*-maximal soft (1,2)*-pre-open.

Definition (3.10): A soft subset (A,E) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called semi (1,2)*-minimal soft (1,2)*-pre-closed set if $(A,E)^c$ is semi (1,2)*-maximal soft (1,2)*-pre-open. The family of all semi (1, 2)*-minimal soft (1,2)*-pre-closed sets in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is denoted by $S(1,2)*M_i$-pre-C($\tilde{X}$).

Remark (3.11): Every (1, 2)*-minimal soft (1,2)*-pre-closed set is semi (1,2)*-minimal soft (1,2)*-pre-closed, but the converse is not true in general. In example (3.3), $\tilde{\phi}$ is semi (1,2)*-minimal soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, since $(\tilde{\phi})^c=\tilde{X}$ is semi (1,2)*-maximal soft (1,2)*-pre-open set, but $\tilde{\phi}$ is not (1,2)*-minimal soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$.

Remark (3.12): Semi (1,2)*-minimal soft (1,2)*-pre-closed sets and soft $\tilde{\tau}_1\tilde{\tau}_2$-closed sets are in general independent. In example (3.5), $(M,E)=\{(e_1,\{b\}),(e_2,\{\phi\})\}$ is a semi (1,2)*-minimal soft (1,2)*- pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, since $(M,E)^c=(N,E)$ is semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but (M,E) is not soft $\tilde{\tau}_1\tilde{\tau}_2$-closed in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. Also, $(A_1,E)=\{(e_1,\{a\}),(e_2,\{a\})\}$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but is not semi (1,2)*-minimal soft (1,2)*-pre-closed.

Remark (3.13): In example (3.5), $(A_2,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$ is soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but is not semi (1,2)*-minimal soft (1,2)*-pre-closed.

The following diagram shows the relationships between semi (1,2)*-minimal soft (1,2)*-pre-closed sets and some types of soft $\tilde{\tau}_1\tilde{\tau}_2$-closed sets:

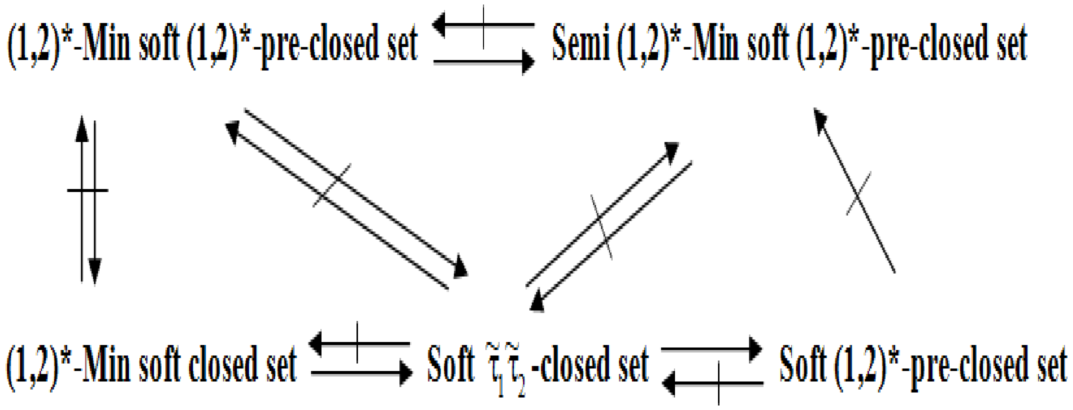

Figure 2- the relationships between semi (1,2)*-minimal soft (1,2)*-pre-closed sets and some types of soft $\tilde{\tau}_1\tilde{\tau}_2$-closed sets.

Theorem (3.14): A soft subset (B,E) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is semi (1,2)*-minimal soft (1,2)*-pre-closed if and only if there exists a (1,2)*-minimal soft (1,2)*-pre-closed set (M,E) in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $\tilde{\tau}_1\tilde{\tau}_2\,int(M,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(M,E)$.

Proof: Suppose that (B,E) is a semi (1,2)*-minimal soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, then by definition (3.10), $(B,E)^c$ is semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. Hence by definition (3.1), there exists a (1,2)*-maximal soft (1,2)*-pre-open set (N,E) in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $(N,E)\tilde{\subseteq}(B,E)^c\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(N,E)$ which implies that $(\tilde{\tau}_1\tilde{\tau}_2cl(N,E))^c\tilde{\subseteq}(B,E)\tilde{\subseteq}(N,E)^c$. By proposition ((1.8),(viii)), $(\tilde{\tau}_1\tilde{\tau}_2cl(N,E))^c=\tilde{\tau}_1\tilde{\tau}_2\,int((N,E)^c)$, therefore $\tilde{\tau}_1\tilde{\tau}_2\,int((N,E)^c)\tilde{\subseteq}(B,E)\tilde{\subseteq}(N,E)^c$. Put $(N,E)^c=(M,E)$, hence (M,E) is a (1,2)*-minimal soft (1,2)* -pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $\tilde{\tau}_1\tilde{\tau}_2\,int(M,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(M,E)$. **Conversely,** suppose that there exists a (1,2)*-minimal soft (1,2)*-pre-closed set (M,E) in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $\tilde{\tau}_1\tilde{\tau}_2\,int(M,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(M,E)$. Hence $(M,E)^c\tilde{\subseteq}(B,E)^c\tilde{\subseteq}(\tilde{\tau}_1\tilde{\tau}_2\,int(M,E))^c$. By proposition ((1.8), (viii)), $(\tilde{\tau}_1\tilde{\tau}_2cl((M,E)^c)=(\tilde{\tau}_1\tilde{\tau}_2\,int(M,E))^c$, therefore there exists a (1,2)*-maximal soft (1,2)*-pre-open set $(M,E)^c$ in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $(M,E)^c\tilde{\subseteq}(B,E)^c\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl((M,E)^c)$. Thus by definition (3.1), $(B,E)^c$ is a semi (1,2)*-maximal soft (1,2)*-pre-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. Hence by definition (3.10), (B,E) is a semi (1,2)*-minimal soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$.

Theorem (3.15): If (M,E) is a semi (1,2)*-minimal soft (1,2)*-pre-closed set in a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ and $\tilde{\tau}_1\tilde{\tau}_2\,int(M,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(M,E)$, then (B,E) is also semi (1,2)*-minimal soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$.

Proof: Since (M,E) is a semi (1,2)*-minimal soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, then by theorem (3.14) there exists a (1,2)*-minimal soft (1,2)*-pre-closed set (N,E) in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $\tilde{\tau}_1\tilde{\tau}_2\,int(N,E)\tilde{\subseteq}(M,E)\tilde{\subseteq}(N,E)$. Now, $\tilde{\tau}_1\tilde{\tau}_2\,int(N,E)\tilde{\subseteq}(M,E)$ which implies $\tilde{\tau}_1\tilde{\tau}_2\,int(N,E)=\tilde{\tau}_1\tilde{\tau}_2\,int(\tilde{\tau}_1\tilde{\tau}_2\,int(N,E))\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\,int(M,E)$. Since $\tilde{\tau}_1\tilde{\tau}_2\,int(M,E)\tilde{\subseteq}(B,E)$, then $\tilde{\tau}_1\tilde{\tau}_2\,int(N,E)\tilde{\subseteq}(B,E)$. Hence $\tilde{\tau}_1\tilde{\tau}_2\,int(N,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\,int(M,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(M,E)\tilde{\subseteq}(N,E)$. It follows that $\tilde{\tau}_1\tilde{\tau}_2\,int(N,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(N,E)$. Thus there exists a (1,2)*-minimal soft (1,2)*-pre-closed set (N,E) in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ such that $\tilde{\tau}_1\tilde{\tau}_2\,int(N,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(N,E)$. Therefore (B,E) is a semi (1,2)*-minimal soft (1,2)*-pre-closed set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$.

Remark (3.16): The union of two semi (1,2)*-minimal soft (1,2)*-pre-closed sets need not to be semi (1,2)*-minimal soft (1,2)*-pre-closed. In example (3.9), $S(1,2)*M_i$ -pre-C($\tilde{X}$) $=\{\tilde{\phi},(M_1,E),(M_2,E),(M_3,E),(M_4,E)\}$, where $(M_1,E)=\{(e_1,\{b\}),(e_2,\{\phi\})\}$, $(M_2,E)=\{(e_1,\{a\}),(e_2,\{\phi\})\}$, $(M_3,E)=\{(e_1,\{\phi\}),(e_2,\{b\})\}$, $(M_4,E)=\{(e_1,\{\phi\}),(e_2,\{a\})\}$. Then (M_1,E) and (M_3,E) are semi (1,2)*-minimal soft (1,2)*-pre-closed sets, but $(M_1,E)\tilde{\bigcup}(M_2,E)=(M,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$ which is not semi (1,2)*-minimal soft (1,2)*-pre-closed set.

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