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Elastic electron scattering from Te-isotopes in the framework of Skyrme-Hartree-Fock method

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Abstract

The ground state charge, proton and matter densities and their rms radii of some Te-isotopes are studied by means of the Skyrme–Hartree–Fock (SHF) method with the Skyrme parameters namely; SKB, SGI, SKM, SKX, MSK7 and SLy4. Also, the neutron skin thickness, the elastic charge form factor and the binding energy per nucleon are calculated in the same framework. The calculated results have been compared with the available experimental data.

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Keywords: Skyrme–Hartree–Fock (SHF) method, Elastic electron scattering.

الاستطارة الالكترونية المرنة من نظائر التيلوريوم باستخدام طريقة سكيرم- هارتري- فوك

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الخلاصة

تم حساب توزيعات الكثافة الشحنية، البروتونية و الكتلية وانصاف الاقطار المرافقة لها لبعض نظائر التيلوريوم باستخدام طريقة سكيرم- هارتري- فوك مع برامترات مختلفة هي SKB, SGI, SKM, SKX, MSK7, SLy4. كما تم ايضاً حساب السمك النيوتروني، عوامل التشكل و معدل طاقة الربط النووية . لقد تمت مقارنة النتائج النظرية مع نظيراتها من القيم العملية المتاحة .

1. Introduction

The structure of a nucleus can be described in nuclear physics by calculating some of the basic quantities such as nuclear size, various nuclear densities and the associated charge form factor. Among these properties the most important one is the nuclear charge density distribution which gives us much detailed information on the internal structure of nuclei since they are directly related to the wave functions of protons, which are important keys for many calculations in nuclear physics [1]. The Skyrme-Hartree-Fock (SHF) theory has been proved to be very successful for the description of the ground state properties of both stable and unstable nuclei since the implementation of the Skyrme interaction by Vautherin and Brink [2-4]. E. Tel *et al.* [5] have been used the Hartree–Fock method with Skyrme forces to calculate the charge, neutron and proton densities, total binding energy per particle, charge, neutron and proton root-mean-square (rms) radii, neutron skin thickness for the beryllium (Be), copper (Cu) and chromium (Cr) isotopes and compared the obtained results with the experiment. Aytekin *et al.* [6] have been calculated some ground states features of ^{32}S, ^{39}K, ^{40}Ca and ^{48}Ca nuclei by the Hartree–Fock method with the Skyrme SKM* and SLy4 forces using two different code implementations. The obtained results from these code implementations were compared with the

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experimental data. Baldik *et al*. [7] have been used the Skyrme-Hartree-Fock method with SLy4, SLy5, SLy6, and SLy7 force parameters to calculate the neutron and proton densities, proton and neutron root mean square (rms) radii and neutron skin thickness of $^{4-10}$He, $^{6-11}$Li, and $^{7-12}$Be isotopes and compared the evaluated results with experimental data.

In this research, we investigate the ground state features of Te- isotopes using the Skyrme–Hartree–Fock (SHF) method with the Skyrme parameters; SKB [8], SGI [9], SKM [10], SKX [11], MSK7 [12] and SLy4 [13] and compared the obtained results with the available experimental data.

2. Theory

The conventional Skyrme force is the most suitable force has been used to describe the ground states properties of nuclei. This force basically consists of a two-body term and a three-body term [14]:

$$V_{CS} = \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)} \qquad \textbf{(1)}$$

with

$$V_{ij}^{(2)} = t_0\left(1 + x_0 p_\sigma\right)\delta(\vec{r}) + \frac{1}{2}t_1\left[\delta(\vec{r})\vec{k}^2 + \vec{k}'^2\delta(\vec{r})\right] + t_2\vec{k}'.\delta(\vec{r})\vec{k} + iW_0\left(\vec{\sigma}_i - \vec{\sigma}_j\right).\ \vec{k}\times\delta(\vec{r})\vec{k}, \qquad \textbf{(2)}$$

$$V_{ijk}^{(3)} = t_3\,\delta\left(\vec{r}_i - \vec{r}_j\right)\delta\left(\vec{r}_j - \vec{r}_k\right) \qquad \textbf{(3)}$$

The three-body term in Eq. (1) can be replaced by a density-dependent two-body term [3]:

$$V_{ijk}^{(3)} \cong V_{ij}^{(2)} = \frac{1}{6}t_3\,\rho\left(\vec{R}\right)\delta(\vec{r}), \qquad \textbf{(4)}$$

where

$\vec{R} = \left(\vec{r}_i + \vec{r}_j\right)/2$ and $\vec{r} = \left(\vec{r}_i - \vec{r}_j\right)$,

the relative momentum operators $\hat{k} = \left(\nabla_i - \nabla_j\right)/2i$ and $\hat{k}' = -\left(\nabla_i - \nabla_j\right)/2i$ are acting to the right and to the left, respectively [14].

The Skyrme forces, with the three-body term replaced by a density dependent two-body term, are unified in a single form by Ge L.X. et al. [15] as an extended Skyrme force:

$$V_{Skyrme} = \sum_{i<j} V_{ij} = t_0\left(1 + x_0 p_\sigma\right)\delta(\vec{r}) + \frac{t_1}{2}\left(1 + x_1 p_\sigma\right)\left[\delta(\vec{r})\vec{k}^2 + \vec{k}'^2\delta(\vec{r})\right] + t_2\left(1 + x_2 p_\sigma\right)\vec{k}'.\ \delta(\vec{r})\vec{k}$$
$$+ \frac{t_3}{6}\left(1 + x_3 p_\sigma\right)\rho^\alpha\left(\vec{R}\right)\delta(\vec{r}) + iW_0\vec{k}'.\ \delta(\vec{r})\left(\vec{\sigma}_i + \vec{\sigma}_j\right)\times\vec{k}, \qquad \textbf{(5)}$$

where p_σ is the space exchange operator, $\delta(\vec{r})$ is the delta function, $\vec{k}$ is the relative momentum, $\vec{\sigma}$ is the vector of Pauli spin matrices and t_0, t_1, t_2, t_3, W_0, x_0, x_1, x_2, x_3 and α are Skyrme force parameters.

In the SHF method the proton, neutron or charge densities are given by [16]:

$$\rho_g(\vec{r}) = \sum_{\beta\in g} w_\beta \psi_\beta^+(\vec{r})\,\psi_\beta(\vec{r}), \qquad \textbf{(6)}$$

where g is used for proton, neutron and charge densities, ψ_β is the single-particle wave function of the state β and w_β represents the occupation probability of the state β.

The charge form factor, $F(q)$, where q is the momentum transfer, is obtained from the nuclear charge density by the Fourier–Bessel transform [14]:

$$F(q) = 4\pi\int_0^\infty r^2 j_0(qr)\rho_c(r)dr \qquad \textbf{(7)}$$

The root mean square (rms) radii of the proton, neutron and charge distributions can be obtained from equation (6) as follows [14]:

$$r_g = < r_g^2 >^{1/2} = \left[\frac{\int r^2\rho_g(r)dr}{\int \rho_g(r)dr}\right]^{1/2} \qquad g = n, p, c \qquad \textbf{(8)}$$

The neutron skin thickness t, is defined as:

$$t = t_n - t_p \tag{9}$$

3. Results and Discussions

The ground state features of Te-isotopes are carried out by means of the Skyrme-Hartree-Fock (SHF) method with Skyrme parameters; SKB, SGI, SKM, SKX, MSK7 and SLy4 are displayed in Table-1. These nuclear properties include the nuclear binding energy, charge, proton, and matter densities and their rms radii, neutron skin thickness and elastic charge form factors.

Table 1- The values of Skyrme force parameters.

Parameter	SKB [8]	SGI [9]	SKM [10]	SKX [11]	MSK7 [12]	SLy4 [13]
t_0 (MeV.fm^3)	-1602.78	-1603.0	-2645.0	-1445.322	-1828.23	-2488.91
t_1 (MeV.fm^5)	570.88	515.9	385.0	246.867	259.40	486.82
t_2 (MeV.fm^5)	-67.70	84.5	-120.0	-131.786	-292.84	-546.39
t_3 (MeV.fm$^{3\alpha}$)	8000	8000	15595	12103.863	13421.7	13777.0
W_0 (MeV.fm^5)	125	115	130	148.637	118.807	123
x_0	-0.165	-0.02	0.09	0.340	0.577	0.834
x_1	0.0	-0.5	0.0	0.580	-0.5	-0.344
x_2	0.0	-1.731	0.0	0.127	0.5	-1.0
x_3	-0.286	0.138	0.0	0.03	0.783	1.354
α	0.333	0.333	0.167	0.5	0.333	0.167

The charge rms radii for Te- isotopes obtained by the Skyrme force parameters used in the present study compared with experimental data [17] are listed in Table- 2 and demonstrated in Figure- 1. According to these results, the charge rms radii of considered isotopes increase from 4.681-4.724 fm for (^{122}Te) to 4.708- 4.751 fm for (^{130}Te) with increasing the neutron number. Also, we can see that the calculated charge rms radii of these isotopes with MSK7 parameter are more close to the experimental data than other parameters.

The calculated results of the proton, neutron and matter rms radii for investigated isotopes using different Skyrme parameters are tabulated in Tables- 3 and 5, respectively. Our analyses show that the values of these rms radii have been increased with the increasing number of neutrons. Besides, we can see from Tables- 2 and 3, that the calculated results of the proton rms radii obtained by all Skyrme parameters are smaller than those of charge rms radii. Furthermore, the neutron skin thickness t values obtained by SGI parameter are shown in Table- 4. It is obvious from this table that the neutron skin thickness t has been increased with increasing the neutron number from 0.120 fm for (^{122}Te) to 0.184 fm for (^{130}Te).

Table 2- The charge rms radii (fm) for Te- isotopes calculated using different parameters along with the experimental data [17].

Nuclei	SKB	SGI	SKM	SKX	MSK7	SLy4	Exp. [17]
^{122}Te	4.724	4.697	4.681	4.694	4.712	4.701	4.7095
^{123}Te	4.727	4.702	4.684	4.698	4.716	4.705	4.7117
^{124}Te	4.731	4.706	4.687	4.702	4.719	4.709	4.7183
^{126}Te	4.737	4.714	4.694	4.711	4.727	4.717	4.7266
^{128}Te	4.744	4.723	4.701	4.720	4.734	4.725	4.7346
^{130}Te	4.751	4.732	4.708	4.736	4.742	4.733	4.7423

Table 3- The proton rms radii (fm) calculated using different parameters.

Nuclei	SKB	SGI	SKM	SKX	MSK7	SLy4
^{122}Te	4.655	4.628	4.608	4.622	4.640	4.629
^{123}Te	4.660	4.634	4.613	4.627	4.645	4.634
^{124}Te	4.664	4.639	4.617	4.632	4.650	4.639
^{126}Te	4.673	4.650	4.626	4.643	4.659	4.649
^{128}Te	4.682	4.661	4.635	4.655	4.669	4.660
^{130}Te	4.691	4.672	4.645	4.676	4.680	4.670

Table 4- The neutron rms radii (fm) calculated using different parameters.

Nuclei	SKB	SGI	SKM	SKX	MSK7	SLy4	*t* (SGI)
^{122}Te	4.737	4.748	4.733	4.719	4.717	4.745	0.120
^{123}Te	4.751	4.762	4.746	4.733	4.729	4.758	0.129
^{124}Te	4.764	4.776	4.759	4.748	4.740	4.772	0.138
^{126}Te	4.790	4.804	4.784	4.776	4.763	4.797	0.154
^{128}Te	4.815	4.830	4.808	4.803	4.785	4.821	0.170
^{130}Te	4.840	4.856	4.832	4.837	4.807	4.845	0.184

Table 5- The matter rms radii (fm) calculated using different parameters.

Nuclei	SKB	SGI	SKM	SKX	MSK7	SLy4
^{122}Te	4.702	4.697	4.680	4.678	4.684	4.696
^{123}Te	4.712	4.708	4.690	4.689	4.693	4.706
^{124}Te	4.722	4.719	4.700	4.700	4.702	4.717
^{126}Te	4.742	4.741	4.719	4.721	4.720	4.737
^{128}Te	4.762	4.762	4.739	4.743	4.738	4.756
^{130}Te	4.781	4.783	4.758	4.773	4.756	4.776

Figure- 2 shows the calculated charge density distributions for interested isotopes obtained by MSK7 Skyrme parameter along with the experimental data (denoted by filled circle symbols) [18,19]. One can see from this figure that the calculated charge density distributions are in an excellent agreement with experimental data except a slight deviation appearing in the calculated results at the region of small *r*.

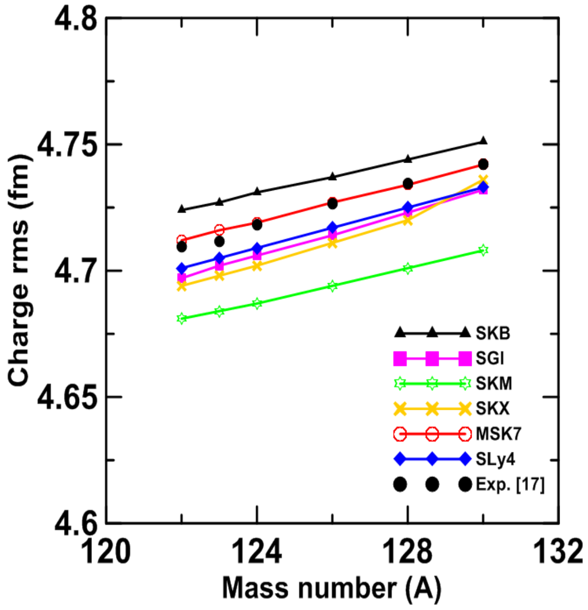

Figure 1- The calculated charge rms radii (fm) for considered isotopes using different parameters along with the experimental data [17].

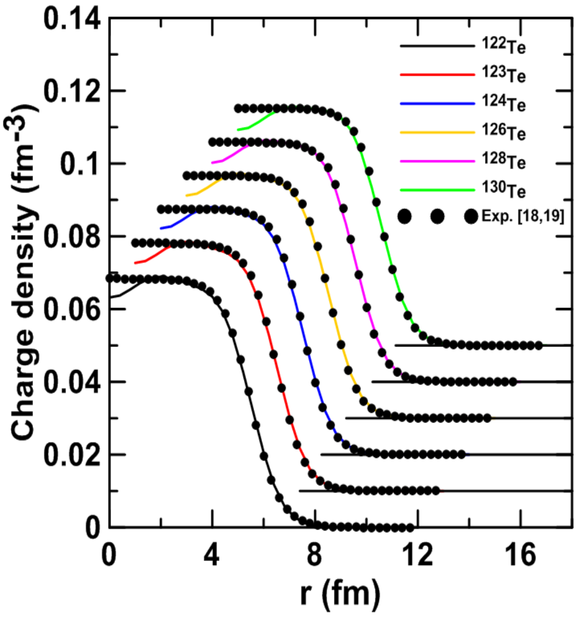

Figure 2- The charge density distributions for interested isotopes calculated by MSK7 Skyrme parameter compared with experimental data [18,19]. The curves and data have been progressively offset by 1 fm and 0.02 in the charge density.

Figures- 3(a) and (b) demonstrate the charge and proton density distributions for Te-isotopes obtained by the MSK7 Skyrme parameter. It is evident from these figures that the calculated values of the charge density at the center (*r*=0) for these isotopes have been decreased with the increasing number of neutrons approximately from (0.062 fm^{-3}) for ^{122}Te to (0.058 fm^{-3}) for ^{130}Te. On the other hand, the proton density decreases approximately from (0.057 fm^{-3}) for ^{122}Te to (0.053 fm^{-3}) for ^{130}Te with the increasing number of neutrons. Moreover, the charge and proton densities for all isotopes have maximum value at *r*=2 fm. All the density distributions have the same shape beyond 4.5 fm and approach smoothly to zero at *r*=8 fm.

Figures- 4(a) and (b) illustrate the evaluated results of matter density distributions for Te- isotopes obtained by SHF method with MSK7 parameter. For comparison the experimental matter densities of these isotopes [18,19] are also shown in Figure- 4(a). The experimental data of selected isotopes are very well reproduced by the evaluated results. Inspection of Figure- 4(b) reveals that the matter density distributions of these isotopes generally show the similar behavior where all distributions are constant in the range *r* =0-3 fm and they decreases drastically to zero after *r* =3 fm.

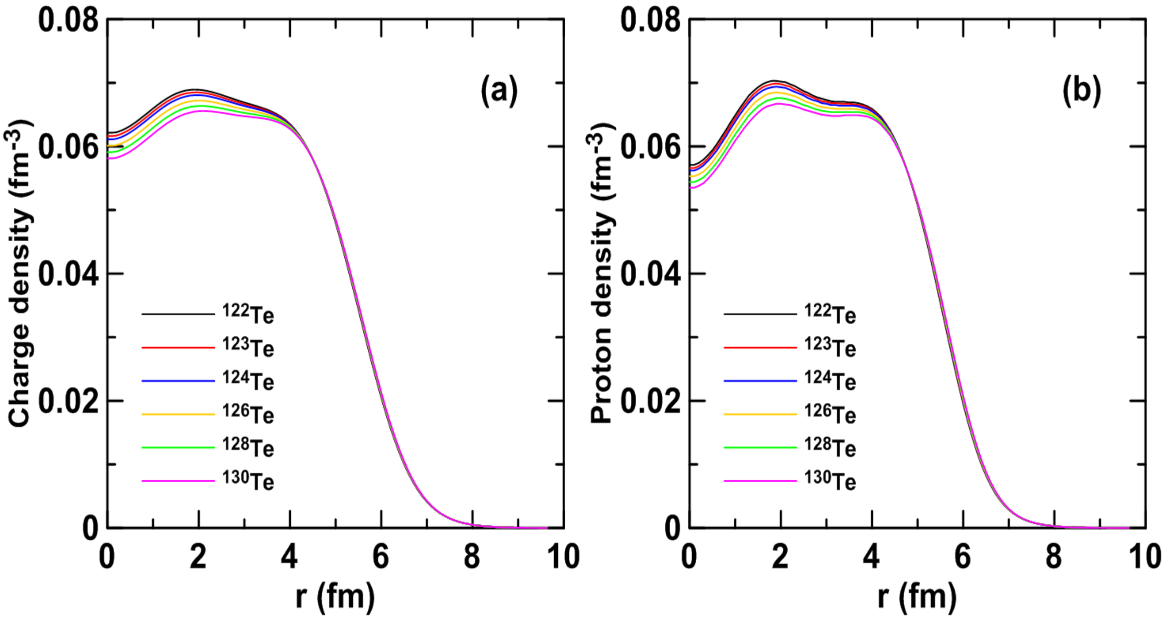

Figure 3- The density distributions of the selected isotopes obtained by SKM7 parameter; (a) charge and (b) proton.

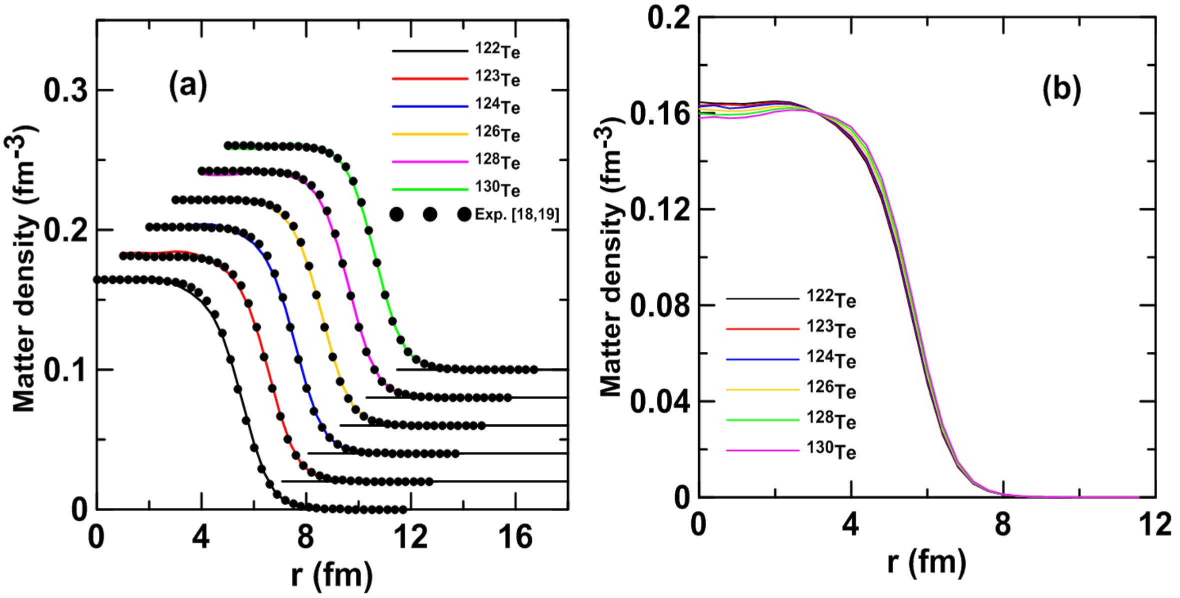

Figure 4-The calculated matter density distributions for Te-isotopes using MSK7 parameter. In Figure- 4 (a) the curves and data have been progressively offset by 1 fm and 0.02 in the matter density. The filled circle symbols are the experimental data of Refs. [18,19].

The calculated binding energy per nucleon obtained by different parameters for the isotopes under study are given in Table- 6 and shown in Figure- 5 compared with experimental data [20]. Inspection to the Figure- 5 gives an indication that the calculated result obtained using MSK7 parameter is better describing the experimental data than the other results.

Table 6- The binding energy per nucleon (MeV) for the isotopes under study calculated using different parameters along with experimental data [20].

nuclei	SKB	SGI	SKM	SKX	MSK7	SLy4	Exp. [20]
^{122}Te	8.509	8.518	8.503	8.414	8.438	8.443	8.478
^{123}Te	8.516	8.517	8.504	8.412	8.442	8.436	8.465
^{124}Te	8.524	8.518	8.505	8.409	8.446	8.429	8.473
^{126}Te	8.543	8.521	8.508	8.403	8.452	8.417	8.463
^{128}Te	8.567	8.528	8.512	8.395	8.457	8.406	8.448
^{130}Te	8.595	8.539	8.516	8.390	8.461	8.398	8.430

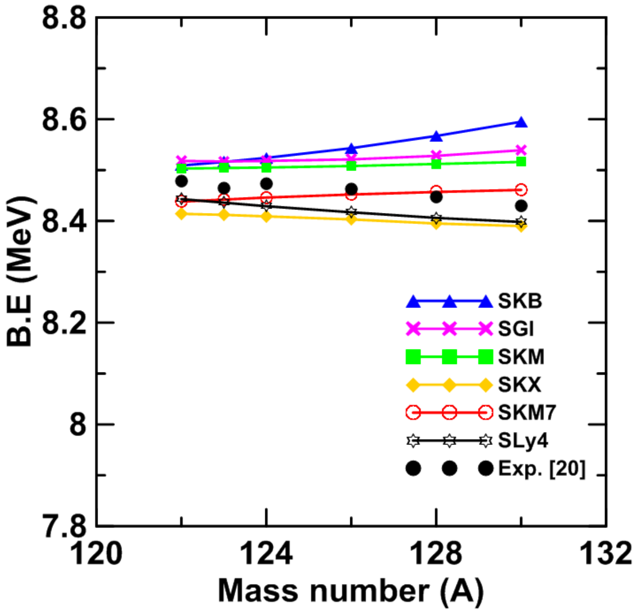

Figure 5- The binding energy per nucleon for isotopes under study obtained by different Skyrme parameters along with experimental results [20].

Figure -6 exhibits the calculated elastic charge form factors for Te-isotopes obtained with MSK7 parameter compared with the experimental data (denoted by filled circle symbols) [18,19]. The calculated form factors are quite consistent with experimental results throughout all range of momentum transfer (q) as well as a good agreement can be noticed for the observed four diffraction minima (at q~0.8 fm^{-1}, q~1.35fm^{-1}, q~1.9 fm^{-1} and q~2.5 fm^{-1}, respectively) for these isotopes.

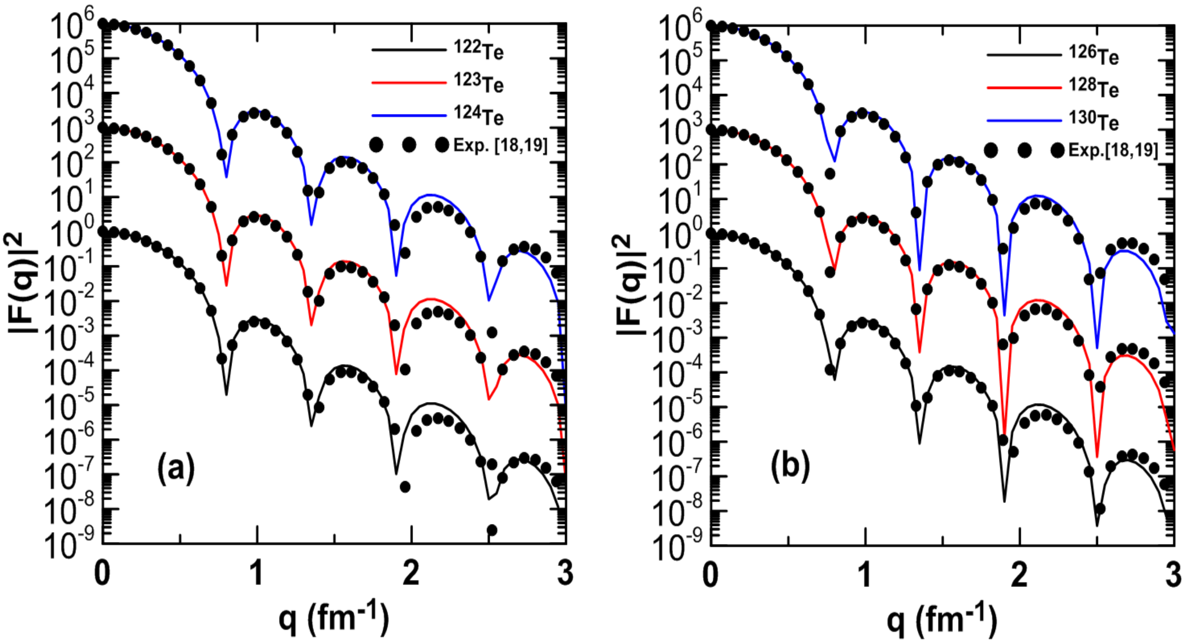

Figure 6- Elastic charge form factors for investigated isotopes calculated by MSK7 parameter. **(a)** ^{122}Te, ^{123}Te ($\times 10^3$), ^{124}Te ($\times 10^6$), **(b)** ^{126}Te, ^{128}Te ($\times 10^3$) and ^{130}Te ($\times 10^6$).

4. Conclusions

Several conclusions can be drawn from the present study:

1- In the description of the rms charge density radii of investigated isotopes, one found that the MSK7 parameter give well agreement with the experimental data.

2- The results for binding energies of considered isotopes calculated by MSK7 parameter are more close to the experimental data than the calculated results with the other selected parameters.

3- Generally, good agreement has also been found in extensive comparisons of the measured charge and matter density distributions with the calculated results using the MSK7 Skyrme parameter.

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