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Schauder Fixed Point Theorems in Intuitionistic Fuzzy Metric Space

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Abstract

In this paper, we will study a concepts of sectional intuitionistic fuzzy continuous and prove the schauder fixed point theorem in intuitionistic fuzzy metric space as a generalization of fuzzy metric space and prove a nother version of schauder fixed point theorem in intuitionistic fuzzy metric space as a generalization to the other types of fixed point theorems in intuitionistic fuzzy metric space considered by other researchers, as well as, to the usual intuitionistic fuzzy metric space.

Keywords: Fuzzy metric spaces, Intuitionistic fuzzy metric space, fixed point theorem.

مبرهنات شاودر النقطة الصامدة في الفضاء المتري الضبابي الحدسي

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الخلاصة

في هذا البحث ، سنقوم بدراسة المفاهيم الاساسية في استمرارية ضبابية الحدس القطاعي واثبات نظرية شاودر للنقطة الصامدة في الفضاء المتري الضبابي الحدسي ويمكن اعتباره اعمام للفضاء المتري الضبابي ،واثبات الاصصدارات الاخرى في نظرية شاودر للنقطة الصامدة في الفضاء المتري الضبابي الحدسي ويمكن اعتباره اعمام للانواع الاخرى من نظريات النقطة الصامدة في الفضاء المتري الضبابي الحدسي والتي درست من قبل بقية الباحثين بالاضافة الى الفضاء المتري الضبابي الحدسي الاعتيادي.

Introduction

The idea of fuzzy set was first offered in 1965 by Iranian Mathematician Prof. L. A. Zadeh [1].Fuzzy set is characterized by membership function which assigns each object to a grade of membership between zero and one. Following the concept of fuzzy set Kramosil and Michalek [2] introduced the concept of fuzzy metric space. Many authors extend their views. Grorge and Veermanyam [3]. Modified the notion of fuzzy metric spaces with the help of continuous t-norms Grabiec [4].

Atanassove[5] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, park [6] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms. Recently, in 2006, Alaca et al. [7] defined the notion of

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intuitionistic fuzzy metric space by making use of Intuitionistic fuzzy sets, with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [8].In 2006, Turkoglu [9] et al. proved Jungck's common fixed point theorem in the setting of intuitionistic fuzzy metric spaces for commuting mappings [10]. For more details on intuitionistic fuzzy metric space, one can refer to the papers [11-13].

In this paper, we introduce the a definition of sectional intuitionistic fuzzy continuous and prove schauder fixed point theorems in Intuitionistic fuzzy metric space as a generalization of fuzzy metric space and prove another version of schauder fixed point theorem in intuitionistic fuzzy metric space.

Preliminaries

Some basic concepts of fuzzy metric spaces and intuitionistic fuzzy metric spaces are given in this section.

Definition (2.1), [14]:

A binary operation *: $[0, 1]\times [0, 1] \rightarrow [0, 1]$ is a continuous t-norm if * satisfies the following conditions:

1)* is commutative and associative.

2)* is continuous.

3) a * 1=a for all a$\in [0, 1]$

4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, d $\in [0,1]$

Definition (2.2), [15]:

Let X be a nonempty set. A generalized metric (or D-metric) on X is a function D: X*X*X$\rightarrow R^{+}$, that satisfies the following conditions for each x, y, z, a $\in$X:

1) D(x, y, z) $\geq$ 0.

2) D(x, y, z) = 0 if and only if x=y=z.

3) D(x, y, z) = D (p{x, y, z}), where p is the permutation function.

4) D(x, y, z) $\leq$ D(x, y, a) +D (a, z, z).

The pair (X, D) is called the generalized metric or D-metric space.

Definition (2.3), [16]:

A 3 –tuple (X, M_d, *) is said to be M-fuzzy metric space if X is an arbitrary set, * is a continuous T-norm and M is a fuzzy subset of X* X*(0,∞), satisfying the following conditions for all x, y, z$\in$ X and t, s $>$ 0 :

1) M_d(x, y, t) $>$ 0.

2) M_d(x, y, t) =1 if and only if x =y.

3) M_d(x, y, t) = M_d(y, x, t).

4) M_d(x, y, t) *M_d(y, z, s) $\leq$ M_d(x, z, t + s).

5) M_d(x, y, *): $(0, \infty) \rightarrow [0, 1]$ is continuous**.**

Definition (2.4), [15]

A 4 – tuple (X,M_D, *) is called M – fuzzy metric space if X is an arbitrary (nonempty) set, * is M – continuous T-norm and M is a fuzzy subset of X* X* X* (0,∞), satisfying the following conditions for each x, y, z, a $\in$ X and t, s$>$ 0:

1)M_D (x, y, z, t)$>$ 0 .

2) M_D(x, y, z, t)=1 if and only if x = y = z.

3) M_D(x, y, z, t) = M_D (p{x, y, z},t),where p is a permutation function of x, y and z.

4)M_D (x, y, a, t)* M_D(a, z, z, s) $\leq$ M_D (x, y, z, t+s).

5)M_D (x, y, z,*): $(0, \infty) \rightarrow [0, 1]$ is a continuous.

Example (2.1), [17]:

Let X= R and let:

M_D (x, y ,z , t) = $\frac{t}{t+D(x,y,z)}$, t $>$ 0

Where:

D(x, y, z) = max $\{ |x-y| \; |y - z| \; |z - x| \} \forall x, y, z \in X$

Then(X,M_D, *) is M – fuzzy metric space.

Definition (2.5), [14]:

A binary operation◊: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t- conorm if ◊ satisfies the following conditions:

1) ◊ is commutative and associative.

2) ◊is continuous.
3) a◊ 0 = a for all a ∈ [0,1]
4) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for all a, b , c , d ∈ [0,1].

Lemma (2.1), [6]:

If * is a continuous t-norm and continuous t-conorm, then:
1) For every a, b∈ [0, 1], if a >b, there are c, d ∈ [0, 1] such that a* c ≥ b and a ≥ b◊d.
2) If a ∈ [0, 1], there are b, c ∈ [0, 1] such that b*b ≥ a and a ≥ c ◊ c.

Definition (2.6), [7]:

A 5 –tuple (X, M, N, *,◊) is said to be an intuitionistic fuzzy metric space (i. f. m. s) if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t- conorm and M, N are fuzzy sets on X^2×[0, ∞) satisfying the conditions:

1. M (x, y, t) + N(x, y, t) ≤ 1 for all x, y ∈ X and t >0.
2. M(x, y, 0) = 0 for all x, y ∈ X.
3. M (x, y, t) = 1 for all x, y ∈ X and t >0 if and only if x = y;
4. M (x, y, t) = M(y, x, t) for all x, y ∈ X and t >0,
5. M(x, y, t)* M(y, z, s) ≤ M(x, z, t + s) for all x, y, z ∈ X and s, t >0;
6. M(x, y, .) : [0, ∞) → [0, 1] is left continuous, for all x, y ∈ X.
7. $\lim_{x\to\infty} M(x,y,t) = 1$ For all x, y ∈ X and t >0.
8. N(x, y, .) = 1 for all x, y ∈ X.
9. N(x, y, t) = 0 for all x, y ∈ X and t> 0 if and only if x = y.
10. N(x, y, t) = N(y, x, t) for all x, y ∈ X and t > 0.
11. N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) for all x, y, z ∈ X and s, t>0.
12. N(x, y, .) :[0, ∞) → [0, 1] is right continuous for all x, y ∈ X for all x, y ∈ X
13. $\lim_{t\to\infty} N(x,y,t) = 0$

An alternative definition of convergent and Cauchy sequence in intuitionistic fuzzy metric space is given next.

Remark (2.1), [18]:

In intuitionistic fuzzy metric space (X, M, N, *, ◊), M(x, y,*) is non-decreasing and N(x, y, ◊) is non-increasing for all x, y, ∈ X.

Definition (2.7), [7]:

Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space .then

A sequence { X_n} in x is said to be Cauchy sequence if ,for all t > 0 and p >0.

(1) $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 0 \lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and

A sequence $X_n$in X is said to be convergent to a point x ∈ X if, for all t > 0,

(2) $\lim_{n\to\infty} M(x_n, x, t) = 0. \lim_{n\to\infty} M(x_n, x, t)$ and

Definition (2.8), [19]:

A quasi- metric on a set X is a function d: $X^2 \to R^+$ Satisfying the following conditions for every x, y, z∈ X.

(1) d (x, x) = 0
(2) d (x, y) = d(y, x)
(3) d (x, z) ≤ d(x, y) + d(y, z)

Proposition (2.1), [20]:

Let (X, M, N, *, ◊) be the intuitionistic fuzzy metric space, for any r∈ [0, 1], we define d_r: $X^2 \to R^+$ as follows:

$d_r(x, y) = \inf\{t > 0 \mid M(x, y, t) > 1\text{-}r, N(x, y, t) < r)$……………………………………………………(1)

(1) (X, d_r: r ∈ (0, 1] is a generating space of a quasi- metric family.
(2) The topology $T_{(dr)}$ on (X, d_r :r ∈ (0, 1]) coincides with the (M, N)- Topology on (X, M, N, *,◊) (i.e, $d_r$is a compatible symmetric for $T_{(M, N)}$).

Example (2.2):

Let (*X, d*) be a metric space. Define *t*-norm *a* * *b* = min {*a, b*} and *t*-co-norm *a* ◊*b* = max {*a, b*} and for all *a, b*∈ Xand *t* > 0. Let us define *M* (*x, y, t*) = *t*/ (*t* + *d*(*x, y*)) and *N* (*x, y, t*) = *d* (*x, y*)/ (*t* +*d*(*x, y*)). Then (*X, M, N*, *, ◊) is an intuitionistic fuzzy metric space.

Schauder Fixed Point Theorem

We give a definition for an intuitionistic schauder fixed point theorem in intuitionistic fuzzy metric (X, M, N,*, ◊).the notion of intuitionistic fuzzy metric spaces was introduced and studied by park. saadati and park, further developed the theory of intuitionistic fuzzy topology(both in metric and normed)spaces. We introduce an intuitionistic fuzzy contraction mapping and prove a fixed point theorem in intuitionistic fuzzy metric space. For the basic notions and concepts, we refer to [6], [21], [22], [23], [24].

Definition (3.1), [25]

Let (X, A) be an intuitionistic fuzzy metric space, is said to be a bounded set in (X, A) if for any r ∈(0, 1) there exist t > 0 such that N (x, y, t) <r and M(x, y, t) > 1-r for all x, y ∈ X.

Definition (3.2), [26]

Let (X, A) be an (I.F.M.S) and B ⊆ X is said to be a closed set in (X, A) if for each r∈ (0, 1) if and only if for any sequence $\{X_n\}$ in B converges to x ∈ X i.e., $\lim_{n\to\infty} M(x_n, x, t) \geq 1-r$ and $\lim_{n\to\infty} N(x_n, x, t) \leq r$ for all t >0 ⇒ x ∈B

Definition (3.3), [25]

Let (X, A) be an intuitionistic fuzzy metric space is said to be compact if every sequence in X contains a convergent subsequence.

In what follows A = (X, N_1, M_1,*, ◊) and B = (Y, N_2, M_2,$\acute{*}$,$\acute{\Diamond}$) will denote two intuitionistic fuzzy metric spaces, where X and Y are metric spaces.

Definition (3.4):

Let A and B be two intuitionistic fuzzy metric space. A mapping T: A → B is said to be sectional intuitionistic fuzzy continuous at $x_0 = (x_{01}, x_{02}, \ldots, x_{0n}) \in X^n$, if ∃ r ∈ (0, 1) such that for each ∈> 0, there exist δ>0 such that, $N_1(x, x_0, \delta) \leq r$ and $M_1(x, x_0, \delta) \geq 1-r$ then $N_2(T(x), T(x_0), \epsilon) \leq r$ and $M_2(T(x), T(x_0), \epsilon) \geq 1-r$ for every $x = (x_1, x_2, \ldots, x_n) \in X^n$.

Example:

Let(X, $d(x, x_0)$) is a metric space. Define

$$N_1(x, x_0, \delta) = \begin{cases} \frac{\delta}{\delta + d(x,x_0)} & , \text{when } \delta>0,\ \delta\in R.\ x, x_0\in X \\ 0 & , when\ \delta \leq 0. \end{cases}$$

$$M_1(x, x_0, \delta) = \begin{cases} \frac{d(x,x_0)}{\delta + d(x,x_0)} & , \text{when } \delta>0,\ \delta\in R.\ x, x_0\in X \\ 1 & , when\ \delta \leq 0. \end{cases}$$

$$N_2(T(x), T(x_0), \epsilon) = \begin{cases} \frac{\epsilon}{\epsilon + d(T(x),T(x_0))} & , \text{when } \epsilon>0,\ \epsilon\in R.\ x, x_0\in X \\ 0 & , when\ \epsilon \leq 0. \end{cases}$$

$$M_2(T(x), T(x_0), \epsilon) = \begin{cases} \frac{d(T(x),T(x_0))}{\epsilon + d(T(x),T(x_0))} & , \text{when } \epsilon>0,\ \epsilon\in R.\ x, x_0\in X \\ 0 & , when\ \epsilon \leq 0. \end{cases}$$

Then A={X, $N_1(x, x_0, \delta)$, $M_1(x, x_0, \delta)$l $x, x_0\in$ X } and B={X, $N_2(x, x_0, \epsilon)$, $M_2(x, x_0, \epsilon)$ l $x, x_0\in$ X}is sectional intuitionistic fuzzy continuous.

Remark (3.1):

If (X, M) is a fuzzy metric space and K ⊂ X, then $\bar{k}$ is closed in (X, d_r(x, y)) and $\overline{k^r} \subset \bar{k}$∀ r ∈ (0, 1), where $\overline{k^r}$ denotes the closure of K in (X, d_r(x, y)).

Theorem (3.1)

Let K be a non- empty convex, intuitionistic fuzzy compact subset of an intuitionistic fuzzy metric space satisfying (proposition (2.1)) and T: K→K be sectional intuitionistic fuzzy continuous. Then T has a fixed point.

Proof

Since satisfies (proposition (2.1)) then (X, d_r(x, y)) is a metric space. As K is an intuitionistic fuzzy compact subset of X, K is a compact subset of (X, d_r(x, y)) by definition of (3.3) for each r∈ (0, 1).

Where $d_r(x, y)$ denotes the r-metric space of X. First suppose that X is intuitionistic fuzzy bounded then for each r∈ (0, 1) t > 0 such that

N(x, y, t) ≤ r and M(x, y, t) ≥ 1-r for all x, y ∈ X...…………………………………………. (2)

Now from the definition of bounded we have

$[d_r(x, y)]$= inf {t > 0: N(x, y, t) < r and M (x, y, t) > 1-r} r∈ (0, 1)…….by (1)

From (2) we have $d_r(x, y) \le t$ for all x, y ∈ X ⇒ X is bounded with respect to $d_r(x, y)$

Conversely, suppose that X is bounded with respect to $d_r(x, y)$, 0 < r < 1.

Then for each r∈ (0, 1) there exist t such that

$d_r(x, y) \le t$ for all x, y ∈X that is, $d_r(x, y) \le t < t+1$ for all x, y ∈ X.

N(x, y, t + 1) <r

M(x, y, t + 1)> 1-r for all x, y ∈ X ⇒ X is I .f. b.

Second there exist subsequence $\{x_{n_k}\}$ and x in B (both depending on r_0) such that $\lim_{n\to\infty} d_{r_0}(x_{n_k}, x) = 0$. Then for a given ∊> 0, there exist a positive integer N (∊) such that

$d_{r_0}(x_{n_k}, x) < \epsilon \Rightarrow N(x_{n_k}, x, \epsilon) < r_0$ for all n ≥ N (∊)

$\lim_{n\to\infty} N(x_{n_k}, x, \epsilon) < r_0$ for all t > 0,

Since ∊ is arbitrary and by condition (1) of definition (2.6) it follows that $\lim_{n\to\infty} M(x_{n_k}, x, \epsilon) > 1 - r_0 \Rightarrow x \in B$

⇒B is closed with respect to $d_{r_0}(x_{n_k}, x)$ since $0 < r_0 < 1$ is arbitrary, it follows that B is closed with respect to $d_r(x_{n_k}, x)$, 0 < r <1.

Converse follows by condition (1), (6) and (12) of definition (2.6) for a given ∊> 0 with $r_0 - \epsilon > 0$ and for a t > 0 ∃ a positive integer K(∊, t) such that $N(x_{n_k}, x, t) \le r_0 - \epsilon$ and $M(x_{n_k}, x, t) \ge 1 - r_0 + \epsilon$ ∀ k ≥ K(∊, t) $\Rightarrow d_{r_0-\epsilon}(x_{n_k}, x) \le t$, ∀ k ≥ K(∊, t) $\Rightarrow \lim_{k\to\infty} d_{r_0-\epsilon}(x_{n_k}, x) = 0$

⇒B is compact with respect to $d_{r_0-\epsilon}(x_{n_k}, x)$ for since $r_0 \in (0, 1)$ and ∊> 0 are arbitrary it follows that B is compact with respect to $d_r(x, y)$ for each r ∈ (0, 1).

A gain since T: K→K is sectional intuitionistic fuzzy continuous ∃ $r_0 \in (0, 1)$ such that T: K→K is continuous with respect to (x, $d_{r_0}(x, y)$) it follows A = (x, N_1, M_1, *, ◊) and B = (y, N_2, M_2, $\acute{*}$, $\hat{\lozenge}$)

Will denote two intuitionistic fuzzy metric spaces, where x and y are metric spaces.

If and only if T: (X, $d_r(x, y)^1$)→(y, $d_r(x, y)^2$)is continuous for some r ∈ (0, 1).

First suppose that T: A→B is sectional intuitionistic fuzzy continuous. Thus ∀ y∈ X^n, ∃ $r_0 \in (0, 1)$ such that for each ∊>0, ∃δ> 0, $N_1(x, y, \delta) \le r_0$ and $M_1(x, y, \delta) \ge 1-r_0 \Rightarrow N_2(T(x), T(y), \epsilon) \le r_0$ and $M_2(T(x), T(y), \epsilon) \ge 1-r_0$ ∀ x ∈ X^n.

Choose $\boldsymbol{\eta}_0$ such that $\delta_1 = \delta - \boldsymbol{\eta}_0 > 0$, let

$[d(x,y)]^1_{r_0} \le \delta - \boldsymbol{\eta}_0 = \delta_1$ then

$[d(x,y)]^1_{r_0} \le \delta - \boldsymbol{\eta}_0 = \delta$

$N_1(x, y, \delta) \le r_0$ and $M_1(x, y, \delta) \ge 1-r_0$

$N_2(T(x), T(y), \epsilon) \le r_0$ and $M_2(T(x), T(y), \epsilon) \ge 1-r_0$

$[d(T(x), T(y))]^2_{r_0} \le \epsilon$

Thus T is continuous with respect to $[d(x,y)]^1_{r_0}$ and $[d(x,y)]^2_{r_0}$ Next let T be continuous with respect to $[d(x,y)]^1_{r_0}$ and $[d(x,y)]^2_{r_0}$.thus ∀ y ∈ X^n and ∊> 0 ∃δ> 0 such that

$[d(x,y)]^1_{r_0} \le \delta \Rightarrow [d(T(x), T(y))]^2_{r_0} \le \frac{\epsilon}{2}$

Let $N_1(x, y, \delta) \le r_0$ and $M_1(x, y, \delta) \ge 1-r_0$

Then $[d(x,y)]^1_{r_0} \le \delta$

Hence it follows that K is a non-empty convex and compact subset of the metric space (X, $d_{r_0}(x, y)$) and T: k →K is a continuous mapping. So by Schauder fixed point theorem [27] (let A be a closed convex subset of a banach space and a assume there exists a continuous map T sending A to a countablycompact subset T(A) of A then T has fixed points) it follows that T has a fixed point.

We also give another version of schauder fixed point theorem in intuitionistic fuzzy metric space.

Theorem (3.2):

Let K be a non-empty, I-f- closed, convex subset of an intuitionistic fuzzy metric space satisfying (proposition (2.1)) and let T: K→K be sectional intuitionistic fuzzy continuous with $\bar{T}$(K) being intuitionistic fuzzy compact. Then T has a fixed point in K.

Proof: Since satisfies ((proposition (2.1)), then from theorem (3.1) that (X, d_r(x, y)) is a metric space. Again since K is i-f-closed, K is closed with respect to d_r(x, y) for each r∈ (0, 1). Now T: K→ K is sectional intuitionistic fuzzy continuous.

Thus ∃ r_0 (say) ∈ (0, 1) such that T: K → K is continuous with respect to d_{r_0}(x, y)).

Also since $\overline{\mathrm{T}}$(K) is intuitionistic fuzzy compact by theorem (3.1) $\overline{\mathrm{T}}$(K) is compact with respect to d_r(x, y) for each r ∈ (0, 1).

In particular $\overline{\mathrm{T}}$(K) is compact an (X,d_{r_0}(x, y)).

Also from Remark (3.2) $\overline{\mathrm{T}}$(K) is closed in (X, $d(x,y)_{r_0}$) and $\overline{\mathrm{T}}(k)^{r_0} \subset$ $\overline{\mathrm{T}}$(K) Where $\overline{\mathrm{T}}(k)^{r_0}$ is the closure of T (K) in (X, $d(x,y)_{r_0}$). So$\overline{\mathrm{T}}(k)^{r_0}$ is compact in (X, $d(x,y)_{r_0}$).

Thus K is a non-empty closed, convex subset of a fuzzy metric space (X,$d(x,y)_r$) and T: K → K is continuous with $\overline{\mathrm{T}}(k)^{r_0}$ compact.

Therefore by Schauder fixed point theorem [27] it follows that T has a fixed point.

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