



Examples of α -Skew π -Armendariz Rings

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Abstract

In this paper extensive examples and related counterexamples of the category of α -skew π -Armendariz rings are given. This category of rings regards a new generalization for the concepts of α -skew Armendariz and skew π -Armendariz rings. A ring A is called α -skew π -Armendariz if for any $\varphi(\chi) = \sum_{z=0}^{\lambda} \iota_z \chi^z$ and $\psi(\chi) = \sum_{s=0}^{\mu} \nu_s \chi^s \in A[\chi; \alpha]$ such that $\varphi(\chi)\psi(\chi) \in N(A[\chi; \alpha])$, then $\iota_z \alpha^z(\nu_s) \in N(A)$ for each $0 \leq z \leq \lambda$ and $0 \leq s \leq \mu$. First some general properties of α -skew π -Armendariz rings are studied and then relations between α -skew π -Armendariz rings and other related rings are investigated. Also various examples of non α -skew π -Armendariz rings are established.

Keywords: α -skew π -Armendariz rings, weak α -skew Armendariz, semicommutative ring, weak α -rigid rings.

أمثلة على حلقات تعاكس α -ارمندرايز من النمط π

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الخلاصة

في هذا البحث أُعطيت أمثلة وأمثلة معاكسة، حول حلقات تعاكس α -ارمندرايز من النمط π بشكل واسع. إن هذا الصنف من الحلقات يعتبر تعميم جديد لمفهومي حلقات تعاكس α -ارمندرايز وحلقات تعاكس ارمندرايز من النمط π . الحلقة A تسمى تعاكس α -ارمندرايز من النمط π إذا كان لأي متعددي حدود $\varphi(\chi) = \sum_{z=0}^{\lambda} \iota_z \chi^z$ و $\psi(\chi) = \sum_{s=0}^{\mu} \nu_s \chi^s$ في $A[\chi; \alpha]$ تحقق $\varphi(\chi)\psi(\chi) \in N(A[\chi; \alpha])$ فإن $\iota_z \alpha^z(\nu_s) \in N(A)$ لكل $0 \leq z \leq \lambda$ و $0 \leq s \leq \mu$. أولاً تُرست بعض الخصائص العامة لحلقات تعاكس α -ارمندرايز من النمط π ومن ثم بحثت العلاقات بين حلقات تعاكس α -ارمندرايز من النمط π وحلقات أخرى ذات صلة. أيضاً أنشأت أمثلة على حلقات هي ليست تعاكس α -ارمندرايز من النمط π .

Introduction

Throughout this paper, A will be denote as an associative ring with identity and α is a ring endomorphism of A . The skew polynomial ring whose elements are the polynomials over A will be denote by $A[\chi; \alpha]$, the addition is defined as is customary and the multiplication is defined taking into account the relation $\chi v = \alpha(v)\chi$ for any $v \in A$. The set of all nilpotent elements of A will be denote by $N(A)$. Rege and Chhawchharia [1] named a ring A an Armendariz if for any two polynomials $\varphi(\chi) = \iota_0 + \iota_1 \chi + \dots + \iota_\lambda \chi^\lambda$, $\psi(\chi) = \nu_0 + \nu_1 \chi + \dots + \nu_\mu \chi^\mu \in A[\chi]$ satisfy $\varphi(\chi)\psi(\chi) = 0$, then

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$\iota_z v_s = 0$ for each $0 \leq z \leq \lambda$, $0 \leq s \leq \mu$ and for some positive integers λ and μ . This naming was used in [1] because Armendariz showed at first in [2] that a reduced ring (a ring is said to be reduced if it has no nonzero nilpotent elements) satisfies this condition permanently. Several articles have been published on the category of Armendariz rings. For more information on Armendariz rings see [1-3]. Many generalizations of the notion of Armendariz rings arise in several trends; one of them is the concept weak Armendariz rings introduced by Liu and Zhao [4]. A ring A is called weak Armendariz if whenever any two polynomials $\varphi(\chi) = \iota_0 + \iota_1\chi + \dots + \iota_\lambda\chi^\lambda$, $\psi(\chi) = v_0 + v_1\chi + \dots + v_\mu\chi^\mu \in A[\chi]$ satisfy $\varphi(\chi)\psi(\chi) = 0$, then $\iota_z v_s \in N(A)$ for each z and s . The Armendariz property extended to skew polynomial rings in [5] as the following: For an endomorphism α of a ring A , A is said to be α -skew Armendariz ring if for $\varphi(\chi) = \sum_{z=0}^{\lambda} \iota_z \chi^z$, $\psi(\chi) = \sum_{s=0}^{\mu} v_s \chi^s \in A[\chi; \alpha]$, such that $\varphi(\chi)\psi(\chi) = 0$ implies $\iota_z \alpha^z(v_s) = 0$ for all $0 \leq z \leq \lambda$, $0 \leq s \leq \mu$. As a generalization of the α -skew Armendariz rings, Ouyang in [6] introduced the notion of weak α -skew Armendariz rings. A ring A is said to be weak α -skew Armendariz if any two polynomials $\varphi(\chi) = \iota_0 + \iota_1\chi + \dots + \iota_\lambda\chi^\lambda$, $\psi(\chi) = v_0 + v_1\chi + \dots + v_\mu\chi^\mu$ in $A[\chi; \alpha]$ such that $\varphi(\chi)\psi(\chi) = 0$ implies $\iota_z \alpha^z(v_s) \in N(A)$ for all z, s . According to Hong et. al. and Krempa [7, 8], an endomorphism α of a ring A is said to be rigid if $\iota\alpha(\iota) = 0$ implies $\iota = 0$ for $\iota \in A$. A ring A is said to be an α -rigid if there exists a rigid endomorphism α of A . Note that any rigid endomorphism of a ring is a monomorphism, and α -rigid rings are reduced rings by [9]. Following [5], A is an α -rigid ring if and only if $A[\chi; \alpha]$ is reduced; also every α -rigid ring is an α -skew Armendariz ring, while the converse does not hold. Cohn in [10] introduced the concept of reversible rings. A ring A is said to be reversible if $\iota v = 0$ implies $v \iota = 0$ for $\iota, v \in A$. Reduced rings are clearly reversible [11] and reversible rings are semicommutative [12] where A is said to be semicommutative if for all $\iota, v \in A$, $\iota v = 0$ implies $\iota A v = 0$, but the converse is not true in general [13].

The aim of this work is to give a comprehensive classification for the relationships of a new generalization for the notions of α -skew Armendariz and skew π -Armendariz rings with some other rings in the family of Armendariz. This new generalization is called α -skew π -Armendariz rings. In section 2 we give some examples of this class of rings and present counterexamples to disprove some implications. We show that every weak α -skew Armendariz ring is α -skew π -Armendariz and that the concepts of Armendariz rings and α -skew π -Armendariz rings are different. Also we prove that if A is α -skew π -Armendariz ring such that the polynomial ring $A[\chi; \alpha]$ is 2-primal semiprime, then A is weak α -skew Armendariz ring. We devote section 3 to study some general results using the fact that every α -skew Armendariz ring is α -skew π -Armendariz given in section 2.

It is worth to mention that there are many generalizations of the concept of Armendariz rings to different rings and modules such as central Armendariz rings and Armendariz modules [14, 15] are studied. Also, certainly there are many related concepts with the family of Armendariz rings most notably π -McCoy rings [16].

Examples of α -skew π -Armendariz and Non α -skew π -Armendariz Rings

Motivating by [4, 17, 18], in this section we give a new concept, named, α -skew π -Armendariz rings and concentrate to give various affirmative examples and counterexamples in order to illustrate relations between nonequivalent and deferent concepts.

Definition 2.1: Let α be an endomorphism of a ring A . A ring A is called α -skew π -Armendariz if two polynomials $\varphi(\chi) = \sum_{z=0}^{\lambda} \iota_z \chi^z$ and $\psi(\chi) = \sum_{s=0}^{\mu} v_s \chi^s \in A[\chi; \alpha]$ such that $\varphi(\chi)\psi(\chi) \in N(A[\chi; \alpha])$ then $\iota_z \alpha^z(v_s) \in N(A)$ for each $0 \leq z \leq \lambda$ and $0 \leq s \leq \mu$.

It is clear that every Armendariz, weak Armendariz and π -Armendariz ring is I_A -skew π -Armendariz ring, where I_A is the identity endomorphism of A .

Next we show that the concept of α -skew π -Armendariz rings represents a generalization of the concept α -skew Armendariz rings.

Proposition 2.2: Every α -skew Armendariz ring is α -skew π -Armendariz ring.

Proof: Assume that A is an α -skew Armendariz ring, then $\varphi(\chi)\psi(\chi) = 0$ implies $\iota_z \alpha^z(v_s) = 0$, where $\varphi(\chi), \psi(\chi) \in A[\chi; \alpha]$. Suppose that $\varphi(\chi)\psi(\chi) \in N(A[\chi; \alpha])$, we have to prove that $\iota_z \alpha^z(v_s) \in N(A)$. Since A is α -skew Armendariz, then $\iota_z \alpha^z(v_s) = 0 \in N(A)$, therefore A is α -skew π -Armendariz ring.

Note that Hong et al. in [5] proved that every Armendariz ring is an I_A -skew Armendariz, where I_A is an identity endomorphism of A . Hence if A is reduced ring, then A is I_A -skew Armendariz. Consequently, we have that every reduced ring is I_A -skew π -Armendariz ring.

It is known that every Armendariz, weak Armendariz [4]. Now we have the following result:

Proposition 2.3: Every weak α -skew Armendariz is α -skew π -Armendariz ring.

Proof: Assume that A is weak α -skew Armendariz ring, so for each $\varphi(\chi) = \sum_{z=0}^{\lambda} \iota_z \chi^z$, $\psi(\chi) = \sum_{s=0}^{\mu} \nu_s \chi^s \in A[\chi; \alpha]$ such that $\varphi(\chi)\psi(\chi) = 0$ then $\iota_z \alpha^z(\nu_s) \in N(A)$. Now suppose that $\varphi(\chi)\psi(\chi) \in N(A[\chi; \alpha])$, thus we have to prove that $\iota_z \alpha^z(\nu_s) \in N(A)$ for each z, s which is already satisfied because A is weak α -skew Armendariz ring. Therefore A is α -skew π -Armendariz ring.

Proposition 2.4: If A is α -skew π -Armendariz ring such that the polynomial ring $A[\chi; \alpha]$ is 2-primal semiprime, then A is weak α -skew Armendariz ring.

Proof: Assume that $\varphi(\chi)\psi(\chi) = 0$, we claim $\iota_z \alpha^z(\nu_s) \in N(A)$. Since A is α -skew π -Armendariz ring, then $\varphi(\chi)\psi(\chi) \in N(A[\chi; \alpha])$ and since $A[\chi; \alpha]$ 2-primal semiprime we have $\varphi(\chi)\psi(\chi) = 0$, so A is weak α -skew Armendariz ring

We mentioned before that every Armendariz ring is an I_A -skew Armendariz. However, in the following examples we show that the concepts of Armendariz rings and α -skew π -Armendariz rings are different. Let α be an endomorphism of a ring A and $M_k(A)$ be the $k \times k$ full matrix ring over A . Let $\bar{\alpha}: M_k(R) \rightarrow M_k(A)$ defined by $\bar{\alpha}((\iota_{zs})) = (\alpha(\iota_{zs}))$ where $0 \leq z \leq \lambda$ and $0 \leq s \leq \mu$ for some positive integers λ, μ , then $\bar{\alpha}$ is an endomorphism of $M_k(A)$.

Example 2.5: Let A be a reduced ring,

$$\omega_4 = \left\{ \begin{pmatrix} \iota & \iota_{12} & \iota_{13} & \iota_{14} \\ 0 & \iota & \iota_{23} & \iota_{24} \\ 0 & 0 & \iota & \iota_{34} \\ 0 & 0 & 0 & \iota \end{pmatrix} \mid \iota, \iota_{zs} \in A, 1 \leq z \leq 3, 2 \leq s \leq 4 \right\}.$$

The ring ω_4 is weak $\bar{\alpha}$ -skew Armendariz [19, Example 2.6], hence ω_4 is $\bar{\alpha}$ -skew π -Armendariz by Proposition 2.3, but S_4 is not Armendariz [3].

Example 2.6: Let \mathbb{Z}_2 be the ring of integers modulo 2 and suppose that $A = \mathbb{Z}_2 \oplus \mathbb{Z}_2$ with the habitual addition and multiplication. Then A is a commutative reduced ring [19], hence R is Armendariz [1]. Now let $\alpha: A \rightarrow A$ defined by $\alpha((\iota, \nu)) = (\nu, \iota)$, so A is not α -skew π -Armendariz for if

$$\varphi(\chi) = (1,0) - (1,0)\chi, \psi(\chi) = (0,1) + (1,0)\chi \in A[\chi; \alpha]$$

we have

$$\begin{aligned} \varphi(\chi)\psi(\chi) &= (1,0)(0,1) + (1,0)(1,0)\chi - (1,0)\chi(0,1) - (1,0)\chi(1,0)\chi \\ &= (0,0) + (1,0)\chi - (1,0)\alpha(0,1)\chi - (1,0)\alpha(1,0)\chi^2 \\ &= (0,0) + (1,0)\chi - (1,0)(1,0)\chi - (1,0)(0,1)\chi^2 \\ &= (0,0) + (1,0)\chi - (1,0)\chi - (0,0)\chi^2 \\ &= (0,0) \in N(A[\chi; \alpha]) \end{aligned}$$

but

$$(1,0)\alpha((0,1)) = (1,0) \notin N(A)$$

Therefore A is not be α -skew π -Armendariz.

According to Hong et. al. and Krempa [7, 8], an endomorphism α of a ring A is called rigid if $\iota\alpha(\iota) = 0$ implies $\iota = 0$ for $\iota \in A$. And a ring A is said to be an α -rigid if there is a rigid endomorphism α of A . Hong in [5] proved that every α -rigid ring is α -skew Armendariz. Consequently, every α -rigid rings is α -skew π -Armendariz. The following example shows that α -skew π -Armendariz rings may not be α -rigid:

Example 2.7: Suppose that

$$A = \left\{ \begin{pmatrix} \iota & \rho \\ 0 & \iota \end{pmatrix} \mid \iota \in \mathbb{Z}, \rho \in \mathbb{Q} \right\},$$

be a ring where \mathbb{Z} and \mathbb{Q} are the set of all integers and all rational numbers, respectively, then A is a commutative ring. Let $\alpha: A \rightarrow A$ be an automorphism defined by

$$\alpha\left(\begin{pmatrix} \iota & \rho \\ 0 & \iota \end{pmatrix}\right) = \begin{pmatrix} \iota & \rho/2 \\ 0 & \iota \end{pmatrix}.$$

Since every α -skew Armendariz ring is α -skew π -Armendariz as we mentioned, then A is α -skew π -Armendariz (because A is α -skew Armendariz [5]). But A is not α -rigid, for if

$$\begin{pmatrix} 0 & \rho \\ 0 & 0 \end{pmatrix} \alpha\left(\begin{pmatrix} 0 & \rho \\ 0 & 0 \end{pmatrix}\right) = 0$$

while

$$\begin{pmatrix} 0 & \rho \\ 0 & 0 \end{pmatrix} \neq 0 \quad \text{if } \rho \neq 0.$$

Recall that a ring A is weak α -rigid when $\iota\alpha(\iota) \in N(A)$ if and only if $\iota \in N(A)$ for $\iota \in A$ and an endomorphism α of R [6]. Next we show that weak α -rigid rings may not imply α -skew π -Armendariz rings.

Example 2.8: Let A be a ring and $M_2(A)$ the 2 by 2 matrix ring over A . Let

$$\omega = \left\{ \begin{pmatrix} \iota & \nu \\ 0 & o \end{pmatrix} \mid \iota, \nu, o \in M_2(A) \right\}.$$

Under usual matrix addition and multiplication, ω is a ring. The endomorphism $\alpha: \omega \rightarrow \omega$ is defined by

$$\alpha\left(\begin{pmatrix} \iota & \nu \\ 0 & o \end{pmatrix}\right) = \begin{pmatrix} \iota & -\nu \\ 0 & o \end{pmatrix}$$

for any $\begin{pmatrix} \iota & \nu \\ 0 & o \end{pmatrix} \in \omega$, then ω is weak α -rigid ring [6]. Let $\varphi(\chi), \psi(\chi) \in \omega[\chi; \alpha]$ such that

$$\begin{aligned} \varphi(\chi) &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} + \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \end{pmatrix} \chi, \\ \psi(\chi) &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \end{pmatrix} + \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} \chi. \end{aligned}$$

Now

$$\begin{aligned} \varphi(\chi)\psi(\chi) &= \left(\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} + \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \end{pmatrix} \chi \right) \left(\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \right. \\ &\quad \left. + \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \chi \right) \\ &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} + \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \chi + \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \chi^2 \\ &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}. \end{aligned}$$

Thus $\varphi(\chi)\psi(\chi) = 0 \in N(\omega[\chi; \alpha])$, but for any positive integer μ we have

$$\begin{aligned}
 & \left(\begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & 0) \end{pmatrix} \alpha \begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & 1) \end{pmatrix} \right)^\mu \\
 &= \begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & -1) \end{pmatrix}^\mu \\
 &= \begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & -1) \end{pmatrix} \cdots \begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & -1) \end{pmatrix} \quad (\mu \text{ times}) \\
 &= \begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & 1) \end{pmatrix} \notin N(\omega)
 \end{aligned}$$

which means that

$$\begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & 0) \end{pmatrix} \alpha \begin{pmatrix} (0 & 0) & (0 & 0) \\ (0 & 0) & (0 & 1) \end{pmatrix}$$

is not nilpotent in ω , hence ω is not α -skew π -Armendariz.

The following two examples show that the notions of α -skew π -Armendariz rings and reversible rings are different:

Example 2.9: Consider the ring of integer module 2, \mathbb{Z}_2 and let $A = \mathbb{Z}_2 \oplus \mathbb{Z}_2$. It is clear that A is a commutative reduced ring [19], hence A is reversible [11]. Now define an endomorphism $\alpha: A \rightarrow A$ by $\alpha((\iota, \nu)) = (\nu, \iota)$, so A is not α -skew π -Armendariz ring as shown in Example 2.5.

Example 2.10: Let A be an α -rigid ring. Then

$$\omega = \left\{ \begin{pmatrix} \iota & \nu & o \\ 0 & \iota & b \\ 0 & 0 & \iota \end{pmatrix} \mid \iota, \nu, o, b \in A \right\}$$

is $\bar{\alpha}$ -skew Armendariz ring [5], then is $\bar{\alpha}$ -skew π -Armendariz.

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \neq 0$$

and

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0.$$

So that ω is not reversible ring [13].

Hong et. al. in [20] introduced the notion of α -Armendariz. A ring A is called α -Armendariz if for any endomorphism α of A and for $\varphi(\chi) = \sum_{z=0}^{\lambda} \iota_z \chi^z$ and $\psi(\chi) = \sum_{s=0}^{\mu} \nu_s \chi^s$ in $A[\chi, \alpha]$ such that $\varphi(\chi)\psi(\chi) = 0$, then $\iota_z \nu_s = 0$ for all $0 \leq z \leq \lambda$, $0 \leq s \leq \mu$. In addition, it was proved that if A is an α -Armendariz ring, then A is α -skew Armendariz. Consequently, and since every α -skew Armendariz ring is α -skew π -Amendariz, then every α -Armendariz ring is α -skew π -Amendariz. The following example shows that the converse of the latest statement is not hold in general.

Example 2.11: Consider the ring

$$\omega_4 = \left\{ \begin{pmatrix} \iota & \iota_{12} & \iota_{13} & \iota_{14} \\ 0 & \iota & \iota_{23} & \iota_{24} \\ 0 & 0 & \iota & \iota_{34} \\ 0 & 0 & 0 & \iota \end{pmatrix} \mid \iota, \iota_{zs} \in A, 1 \leq z \leq 3, 2 \leq s \leq 4 \right\},$$

where A is an α -rigid ring. Extending the endomorphism α of A to the endomorphism $\bar{\alpha}: \omega_4 \rightarrow \omega_4$ by $\bar{\alpha}((\iota_{zs})) = (\alpha(\iota_{zs}))$. The ring ω_4 is not $\bar{\alpha}$ -Armendariz because if ω_4 is $\bar{\alpha}$ -Armendariz ring, then ω_4 is $\bar{\alpha}$ -skew Armendariz but ω_4 is not $\bar{\alpha}$ -skew Armendariz [5]. Now, since A is α -rigid, then A is α -skew π -Armendariz (as we mentioned before) and hence ω_4 is $\bar{\alpha}$ -skew π -Armendariz.

Some General Results of α -skew π -Armendariz Rings

In this section we investigate some general relationships between the concept of α -skew π -Armendariz rings and other rings of the family of Armendariz rings depending on previous proved statements.

Theorem 3.1: Let A be a nil-semicommutative and α -condition ring. If $A[\chi; \alpha]$ is weak Armendariz, then A is α -skew π -Armendariz.

Proof: Suppose that $A[\chi; \alpha]$ is weak Armendariz ring and $\varphi(\chi)\psi(\chi) = 0$, where $\varphi(\chi) = \sum_{z=0}^{\lambda} \iota_z \chi^z$ and $\psi(\chi) = \sum_{s=0}^{\mu} \nu_s \chi^s \in A[\chi; \alpha]$. Then $\theta(\gamma)\vartheta(\gamma) = 0$, for $\theta(\gamma) = \iota_0 + (\iota_1 \chi)\gamma + \dots + (\iota_\lambda \chi^\lambda)\gamma^\lambda, \vartheta(\gamma) = \nu_0 + (\nu_1 \chi)\gamma + \dots + (\nu_\mu \chi^\mu)\gamma^\mu \in A[\chi; \alpha][\gamma]$. Since $A[\chi; \alpha]$ is weak Armendariz, $\iota_z \chi^z \nu_s \chi^s \in N(A[\chi; \alpha])$, for each z, s which implies that $\iota_z \alpha^z(\nu_s) \in N(R)$ for each z, s . Therefore A is α -skew π -Armendariz.

Corollary 3.2: If A is a nil-semicommutative ring, then A is weak Armendariz ring if and only if $A[x]$ is weak Armendariz.

The following theorem appears in [6]:

Theorem 3.4: If A is a weak α -rigid ring such that $N(A)$ be an ideal in A , then A is a weak α -skew Armendariz ring.

Using the above Theorem we get the following result:

Corollary 3.5: If A is a weak α -rigid ring such that $N(A)$ is an ideal in A , then A is α -skew π -Armendariz ring.

Proof: Since every weak α -skew Armendariz is α -skew π -Armendariz ring Proposition 2.3, then A is α -skew π -Armendariz ring.

The following two examples show that the notions of α -skew π -Armendariz rings and semicommutative rings are different:

Example 3.6: Let A be a reduced ring,

$$UTM_4 = \left\{ \left(\begin{array}{cccc} \iota & \iota_{12} & \iota_{13} & \iota_{14} \\ 0 & \iota & \iota_{23} & \iota_{24} \\ 0 & 0 & \iota & \iota_{34} \\ 0 & 0 & 0 & \iota \end{array} \right) \middle| \iota, \iota_{zs} \in A \right\}.$$

Since, UTM_4 is $\bar{\alpha}$ -skew π -Armendariz, but UTM_4 is not semicommutative for $\mu \geq 4$ [13].

$$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 0.$$

But we have

$$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \neq 0.$$

And it is proved similarly that UTM_μ is not semicommutative for $\mu \geq 5$.

Example 3.7: Let A_1 and A_2 be two reduced rings such that $A = A_1 \oplus A_2$, then A is a semicommutative ring [21]. Define an endomorphism $\alpha: A \rightarrow A$ by $\alpha((\iota, \nu)) = (\nu, \iota)$ such that

$$\varphi(\chi) = (0,1) - (0,1)x, \psi(\chi) = (1,0) + (0,1)x \in A[\chi; \alpha]$$

hence

$$\varphi(\chi)\psi(\chi) = ((0,1)(1,0) + (0,1)(0,1)\chi - (0,1)\chi(1,0) - (0,1)\chi(0,1)\chi)$$

$$\begin{aligned}
&= ((0,0) + (0,1)\chi - (0,1)(0,1)\chi - (0,1)(1,0)\chi^2) \\
&= ((0,0) + (0,1)\chi - (0,1)x - (0,0)\chi^2) \\
&= 0 \in N(A[\chi; \alpha]),
\end{aligned}$$

but

$$(0,1)(0,1) = (0,1) \notin N(A).$$

Therefore A is not α -skew π -Armendariz.

The following Theorem appears in [6]:

Theorem 3.8: If A is a weak α -rigid semicommutative ring, then $A[\chi]$ is a weak α -skew Armendariz ring.

Using the above Theorem we get:

Corollary 3.9: If A is a weak α -rigid semicommutative ring, then $A[\chi]$ is an α -skew π -Armendariz ring.

Proof: Since every weak α -skew Armendariz is α -skew π -Armendariz ring Proposition 2.3, then A is α -skew π -Armendariz ring.

The following Corollary appears in [6]:

Corollary 3.10: Let A be a weak α -rigid ring. If A is a semicommutative ring, then $A[\chi]/\langle\chi^2\rangle$ is a weak α -skew Armendariz ring, for a principal ideal $\langle\chi^2\rangle$ of $A[\chi]$.

Using Corollary 3.10 we get:

Corollary 3.11: Let A be a weak α -rigid ring. If A is a semicommutative ring, then $A[\chi]/\langle\chi^2\rangle$ is an α -skew π -Armendariz ring, for a principal ideal $\langle\chi^2\rangle$ of $A[\chi]$.

Proof: Since every weak α -skew Armendariz is α -skew π -Armendariz ring Proposition 2.3, then A is α -skew π -Armendariz ring.

The following Theorem appears in [6]:

Theorem 3.12: Let A be a weak α -rigid ring such that A is a nil-semicommutative ring, then $A[x]$ is a weak α -skew Armendariz ring.

Using the above Theorem we get:

Corollary 3.13: Let A be a weak α -rigid ring such that A is a nil-semicommutative ring, then $A[\chi]$ is an α -skew π -Armendariz ring.

Proof: Since every weak α -skew Armendariz is α -skew π -Armendariz ring Proposition 2.3, then A is α -skew π -Armendariz ring.

The following Corollary appears in [6]:

Corollary 3.14: Let A be a weak α -rigid ring such that A is a nil-semicommutative ring, then $A[\chi]/\langle\chi^2\rangle$ is a weak α -skew Armendariz ring, for a principal ideal $\langle\chi^2\rangle$ of $A[\chi]$.

Using Corollary 3.14 we get:

Corollary 3.15: Let A be a weak α -rigid ring such that A is a nil-semicommutative ring, then $A[\chi]/\langle\chi^2\rangle$ is an α -skew π -Armendariz ring, for a principal ideal $\langle\chi^2\rangle$ of $A[\chi]$.

Proof: Since every weak α -skew Armendariz is α -skew π -Armendariz ring Proposition 2.3, then A is α -skew π -Armendariz ring.

The following Theorem appears in [22]:

Theorem 3.16: Let α be an endomorphism of A . If R is semicommutative ring and satisfies α -condition, then A is α -skew weakly Armendariz.

Since the concept of weakly Armendariz rings given in [22] is equivalent to the concept of weak Armendariz rings, then by using Theorem 3.13 we get the following result:

Corollary 3.17: Let α be an endomorphism of A . If A is semicommutative ring and satisfies α -condition, then A is α -skew π -Armendariz.

Proof: Since every weak α -skew Armendariz is α -skew π -Armendariz ring Proposition 2.3, then A is α -skew π -Armendariz ring.

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