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Jordan Permuting 3-Derivations of Prime Rings

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Abstract
The main purpose of this work is to generalize Daif's result by introduceing the concept of Jordan (α,β) permuting 3-derivation on Lie ideal and generalize these result by introducing the concept of generalized Jordan (α,β) permuting 3-derivation
.

Keywords:: Jordan Permuting 3-derivation, Lie ideal, Prime ring,

حول تعميم المشتقات جوردان الثلاثية التبادليه للحلقات الاولية على مثالي لي

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الخلاصة
الهدف الرئيسي من هذا البحث هو تعميم نتائج Daifs وذلك بتقديم مفهوم المشتقات جوردان الثلاثيه التبادليه(α,β) على مثالي لي وتعميم هذه النتائج باعطاء مفهوم تعميم مشتقات جوردان الثلاثيه التبادليه.

1. Introduction

Throughout this paper, Let R be a ring not necessarily with an identity element. A ring R is said to be prime ring if aRb =(0) implies that a=0 or b=0 such that a,b ∈R [1]. A ring R is said to be semiprime ring if aRa =(0) implies that a=0 such that a ∈ R [2].

An additive mapping d from a ring R into R is called a derivation(resp. Jordan derivation) of R if d(xy) = d(x) y + x d(y), for all x , y ∈ R. (rsp.d(x^2) = d(x) x + x d(x)) [3]. Every derivation is Jordan derivation but the converse is not true in general [4].

A ring R is said to be n-torsion-free where n≠0 is an integer number if whenever na=0 with a ∈ R, than a=0 [1].

Additive subgroub U ⊆ R is called a Lie ideal of R if whenever u ∈ U, r ∈ R and [u,r] ∈ U [1]. The concept of a symmetric bi-derivation has been introduced by Maksa in [5] by a bi-derivation we mean a bi-additive map d : R × R → R such that if d(xy, z) = d(x, z)y + xd(y, z), d(x,yz) = d(x, y)z + yd(x, z). Daif prove Jordan derivation is derivation [6]. 2007, Yong–See and Kyoo-Hong introduced the concept of permuting 3-derivation and use concept centerlizing and commuting [7]. In this paper we introduce the concept of Jordan (α,β) permuting 3-derivation to generalizeDaif's result and generalized Jordan (α,β) permuting 3-derivation and prove every (α,β)Jordan permuting 3-derrivation is (α,β)permuting 3-derivation . and generalize the result by introduce the concept generalize (α,β)Jordan permuting 3-derrivation.

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2. Preliminaries

The following lemma is basic to get the main results.

Lemma(2.1) [8]

Let R be a 2-torsion free semiprime ring and let a,b ∈ R. If for all x ∈ R the relation holds, then axb = bxa = 0 is fulfilled for all x ∈ R(axb + bxa = 0 , axb =0 = bxa =0.

3. The main results

we introduce the concept of (α,β) Jordan permuting 3-derivation**.**

Definition (3.1) :

A 3- additive mapping d: R × R × R →R is called (α,β) Jordan permuting 3-derivation if d(x^2 , y ,z) = d (x, y , z) α (x) + β(x) d(x, y ,z) for all x,y,z ∈ R where α and β are endomorphisms of R .

If $\alpha\ and\ \beta$ are the identity mappings then the definition of (α,β) Jordan 3-derivation and the definition Jordan 3-derivation is equal .

Example (3.2):

Let S be a commutative ring and R {($\begin{matrix} a & b \\ 0 & c \end{matrix}$): a , b, c ∈ S } and U={$\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$:,a,b ∈ S } be a Lie ideal of R.Define d: U × U × U →R by d (($\begin{matrix} a & b \\ 0 & 0 \end{matrix}$) , ($\begin{matrix} c & d \\ 0 & 0 \end{matrix}$) , ($\begin{matrix} e & f \\ 0 & 0 \end{matrix}$)) = ($\begin{matrix} ace & 0 \\ 0 & 0 \end{matrix}$) , for all (($\begin{matrix} a & b \\ 0 & 0 \end{matrix}$) , ($\begin{matrix} c & d \\ 0 & 0 \end{matrix}$) , ($\begin{matrix} e & f \\ 0 & 0 \end{matrix}$)) ∈ U .

Let α,β are endomorphism define by α ($\begin{matrix} a & b \\ 0 & c \end{matrix}$) = ($\begin{matrix} a & 0 \\ 0 & 0 \end{matrix}$) and β($\begin{matrix} a & b \\ 0 & c \end{matrix}$) = ($\begin{matrix} 0 & 0 \\ 0 & c \end{matrix}$), for all ($\begin{matrix} a & b \\ 0 & 0 \end{matrix}$) ∈ R

Then d is Jordan (α,β) 3- derivation .

Lemma (3. 3) :

Let R be a 2-torsion free ring and d : U× U × U → R be (α,β) a Jordan permuting 3-derivation where α and β endomrphsim of R then for all a,b,x ∈ R the following statements ara hold:

i − $d(ab + ba, x, y) = d(a, x, y)\,\alpha(b) + \beta(a)\,d(b, x, y) + d(b, x, y)\,\alpha(a) + \beta(b)\,d(a, x, y)$

ii − $d(aba, x, y) = d(a, x, y)\,\alpha(ba) + \beta(a)\,d(b, x, y)\,\alpha(a) + \beta(ab)\,d(a, x, y)$

ii − $d(abc + cba, x, y) = d(a, x, y)\,\alpha(bc) + \beta(a)\,d(b, x, y)\,\alpha(c) + \beta(a)\,\beta(b)\,d(c, x, y) + d(c, x, y)\,\alpha(ba) + \beta(c)\,d(b, x, y)\,\alpha(a) + \beta(cb)\,d(a, x, y)$.

Proof :

Since d is a Jordan (α,β) -3-derivation , by Definition(3.1)

$d(a^2, x, y) = d(a, x, y)\,\alpha(a) + \beta(a)\,d(a, x, y)$, for all a, x, y ∈ U (1)

Replace a by a+b in equation (1) and by using it , we get

$d((a + b)^2, x, y)) = d(a + b, x, y)\,\alpha(a + b) + \beta(a + b)\,d(a + b, x, y)$

$= d(a, x, y)\,\alpha(a) + d(a, x, y)\,\alpha(b) + d(b, x, y)\,\alpha(a) + d(b, x, y)\,\alpha(b) + \beta(a)\,d(a, x, y) + \beta(b)\,d(a, x, y) + \beta(a)\,d(b, x, y) + \beta(b)\,d(b, x, y)$ for all x, y, z ∈ U. …… ….(2)

On the other hand , since d is 3-additive , we get

$d((a + b)^2, x, y) = d(a^2 + ab + ba + b^2 +, y, z) = d(a^2, x, y) + d(ab + ba, x, y) + d(b^2, x, y)$ for all x, y, z ∈ R .(3)

Compring (2) and (3) and since d is Jordan (α,β) -3- derivation we get

$d(ab + ba, x, y) = d(a, x, y)\,\alpha(b) + \beta(a)\,d(b, x, y) + d(b, x, y)\,\alpha(a) + \beta(b)\,d(a, x, y)$

for all x, y, z ∈ U .

(ii)Consider w = d(a (ab+ ba) + ((ab+ ba) a) ,x,y) and by using (1) we get

w = d(a,x,y) α(ab + ba) + β(a) d((ab + ba),x,y) + d(ab + ba,x,y) α(a) + β(ab + ba) d(a ,x,y) .

Again by (i) , we get

w= d(a,x,y) α(ab)+ d(a,x,y) α(ba) + β(a)(d(a,x,y) α(b) + β(a) d(b,x,y) + d(b,x,y) α(a) + β(b) d(a,x,y)) + (d(a,x,y) α(b) + β(a) d(b,x,y) + d(b,x,y) α(a) + β(b) d(a,x,y)) α(a) + β(ab) d(a,x,y) + β(ba) d(a,x,y) .

= d(a,x,y) α(ab) + d(a,x,y) α(ba) + β(a) d(a,x,y) α(b) + $\beta(a^2)$ d(b,x,y) + β(a) d(b,x,y) α(a) + β(ab) d(a,x,y) + d(a,x,y) α(ba) + β(a) d(b,x,y) α(a) + d(b,x,y) $\alpha(a^2)$ + β(b) d(a,x,y) α(a) + β(ab) d(a,x,y) + β(ba) d(a,x,y) .

= d(a,x,y) α(ab) + 2 d(a,x,y) α(ba) + β(a) d(a,x,y) α(b) + β (a^2) d(b,x,y) +2 β(a) d(b,x,y) α(a) +2 β(ab) d(a,x,y) + d(b,x,y) $\alpha(a^2)$ + β(b) d(a,x,y) α(a) + β(ba) d(a,x,y) . (4)

On the other hand
w= d ((a^2 b + b a^2) + 2 aba) , x, y)
= d(a^2,x,y) α(b) + $\beta(a^2)$ d(b,x,y) + d(b,x,y) $\alpha(a^2)$ + β(b) d(a^2,x,y) +2d(aba,x,y)
= d(a,x,y) α(ab) + β(a) d(a,x,y) α(b) + $\beta(a^2)$ d(b,x,y) + d(b,x,y) $\alpha(a^2)$
+ β(b) d(a,x,y) α(a) + β(ba) d(a,x,y) + 2 d(aba ,x,y) (5)
By compring equation (4) and equation (5) and since R is 2-torsion free we get
d (aba , x,y) = d(a,x,y) α(ba) + β(a) d(b,x,y) α(a) + β(ab) d(a,x,y) , for all a,b,x,y ∈ R .
(iii) By replacing a+c instead of a in (ii) and using it, we get
d((a +c) b (a +c) ,x,y) =
d((a+c),x,y) α (b(a+c))+ β((a+c) b) d(a+c , x,y) + β (a+c)d(b,x,y) α (a+c) .
= d(a,x,y) α(ba) + d(a,x,y) α(bc) + d(c,x,y) α(ba) + d (c,x,y) α(bc) + β(a b) d(a,x,y) + β(ab) d(c,x,y) + β(cb) d(a,x,y) + β (cb) d(c,x,y) + β(a) d(b,x,y) α(a) + β(c) d(b,x,y) α(c) + β(a) d(b,x,y) α(c) + β(c) d(b,x,y) α(a) … .(6)
On the other hand, since d is 3-derivation and use (ii), we get
d ((a+c) b (a+c),x,y) =
d(aba + cbc + abc + cba ,x,y) = d(a,x,y) α(ba)+ β(a)d(b,x,y) α(a)+ $\beta(ab)$ d(a,x,y) + d(c,x,y) α (bc) + β(c)d(b,x,y) α(c)+ β (cb) d(c,x,y) + d(abc + cba ,x,y) …… .(7)
By compring (6) and (7) we get
d(abc + cba ,x,y) = d(a,x,y)α(bc) + d(c, x, y)α (ba) + β(ab) d(c,x,y) + β(cb) d(a,x,y) + β(a) d(b,x,y)α (c)+ β(c)d(b,x,y) α(a) .

Lemma (3.4):

Let R be a 2- torsion free ring and d: U × U × U → R be a Jordan (α,β) -3- derivation . Then for all elements a, b , x, y , z ∈ U .
(d(ab,y,z) –d(a,y,z) α(b) - β(a) d(b,y,z))α(x) α[a,b] + β [a,b] β(x) ((d(ab,y,z) - d(a,y,z) α(b) – β(a) d(b,y,z)) = 0 .

proof :

Consider w = d(abxba + baxab ,y, z) .Using Lemma (3.3) item (ii) we obtain
w= d(a(bxb)a + b(axa)b, y,z)
w= d(a,y,z) α(bxba) + β(a) d(bxb,y,z) α(a) + β(a(bxb)) d(a,y,z) + d(b,y,z) α(axab) + β(b) d(axa,y,z) α(b) + β(baxa) d(b,y,z)
= d(a,y,z) α(bxba) + β(a) d(b,y,z) α(xba) + β(a) β(b) d(x,y,z) α(b) α(a) + β(a) β(bx) d(b,y,z) α(a) + β(abxb) d(a,y,z) + d(b,y,z) α(axab) + β(b) d(a,y,z) α(xa) α(b) + β(b) β(a) d(x,y,z) α(a) α(b) + β(b) β(ax) d(a,y,z) α(b) + β(baxa) d(b,y,z) + β(a) β(b) d(x,y,z) α(b) α(a)
= d(a,y,z) α(bxba) + β(a) {d(b,y,z) α(xb) + β(bx) d(b,y,z) }α(a) + β(abx) d(a,y,z) +d(b,y,z) α(axab) + β(b) {d(a,y,z) α(xa) + β(ax) d(a,y,z)} α(b) + β(baxa) d(b,y,z)
+ β(b) β(a) d(x,y,z) α(a) α(b). (1)
On other hand , by using Lemma (3.3)
w= d ((ab)x(ba) + (ba)x(ab),y,z)
= d(ab,y,z) α(x) α(ba) + β(ab) d(x,y,z) α(ba) + β(abx) d(ba,y,z) + d(ba,y,z) α(xab) + β(ba) d(x,y,z) α(ab) + β(bax) d(ab,y,z) (2)
Comparing equation (1) and equation (2) we get
0 = (d(ab ,y,z) – d(a,y,z) α(b) – β(a) d(b,y,z)) α(xba)) + β(bax) (d(ab,y,z)
-d(a,y,z) α(b) - β(a) d(b,y,z)) + (d(ba , y,z) –d (b,y,z) α(a) - β(b) d(a,y,z))α(xab)
+ β(abx)(d(ba,y,z) – d(b,y,z) α(a) - β(b) d(a,y,z)) (3)
By Lemma (3.3.3) equation (3) reduce to
0 = (d(ab,y,z) – d(a,y,z) α(b) - β(a) d(b,y,z)) α(xba) + β(bax)(d(ab,y,z)– d(a,y,z) α(b) - β(a) d(b,y,z) – (d(ab , y, z) – d(a,y,z) α(b) - β(b) d(a,y,z) α(xab) – β(abx) d(ab,y,z) – d(a,y,z) α(b) + β(a) d(b,y,z)) ..(4)
That is ,
0=(d(ab,y,z) – d(a,y,z) α(b) - β(a) d(b,y,z)) α(x) α [a,b] + β[a,b] $\beta(x)$ ((d(ab,y,z) – d(a,y,z) α(b) - β(a) d(b,y,z)). for every x, y, z ∈ U .

Theorem (3.5) :

Let R be a 2-torsion free semiprime ring and d: R× R × R → R be a Jordan (α,α) 3-derivation then d is (α,α) 3-derivation , where α is an automorphsim such that $\alpha^{2=}\alpha$.

Proof :

In Lemma (3.4) take $\alpha = \beta$, then
(d (ab, y,z) – d(a,y,z) α(b) - α(a) d(b,y,z) α(x) α[a, b] + α[a , b] α(x) ((d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z) = 0 , for all a,b,x,z ∈ R
Using Lemma (2.1) to get .
(d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z)) α(x) α[a,b] = 0 (1)
Linearize equation (1) and put b = b + c , then
0 = ((d (a(b+c) ,y,z) – d(a,y,z) α(b + c) - α(a) d(b +c ,y,z) α(x)) α[a, b+c]
0 = (d(ab, y,z) + (d(ac,y,z) – d(a,y,z) α(b) – d(a,y,z) α(c) - α(a) d(b,y,z) - α(a) d(c,y,z)) α(x) (α[a+ b] + [a + c])
By using (1) , the last equation can be reduced to
0 = ((d(ab,y,z) – d(a,y,z) α(b) - α(a)d(b,y,z)) α(x) [a, c] + ((d(ac,y,z) – d(a,y,z) α(c) - α(a) d(c,y,z) α(x) [a,b]. (2)
Hence, by using equation (2), we get
0=((d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z))α(x) α[a,c]) t ((d (ab,y,z) – d(a,y,z) $\alpha(b)$ - α(a) d(b,y,z) α(x) α[a,c]) .
= - ((d(ab,y,z) – d(a,y,z) α(b) – α(a) d(b,y,z)) α(x) α[a,c] t ((d(ac,y,z) – d(a,y,z) α(c) - α(a) d(c,y,z) α(x) α[a,b])
By equation (1) , we get the quation equal zero
- ((d(ab,y,z) – d(a,y,z) α(b) – α(a) d(b,y,z)) {α(x) α[a,c] t d(ac,y,z) – d(a,y,z) α(c) - α(a) d(c,y,z) α(x)} α[a,b]) = 0, for all a,b,c,x,y,z ∈ R.
Since R is semiprime we get
(d(ab,y,z) – d(a,y,z) α(b) – α(a) d(b,y,z) α(x) α[a,c] = 0 , for all a,b,c,x,y,z ∈ U. (3)

Alinearization of equation (3) on a give such that a = a + h
0= (d(a+ h) b,y,z) – d(a+ h,y,z) α(b) – α(a+ h) d(b,y,z) α(x)) α[a+ h,c]
=((d(ab,y,z) + d(hb,y,z) – d(a,y,z) α(b) – d(h,y,z) α(b) – α(a) d(b,y,z) – α(h) d(b,y,z)) α(x) (α[a,c] + α[h,c])
By using (3) the last equation can be written as
(d(ab,y,z) - d(a,y,z) α(b)- α(a)d(b,y,z) - α(a)d(b,y,z)) α(x)) α[h,c] +(d(hb,y,z) - d(h,y,z) α(b) - α(h)d(b,y,z)) α(x) α[a,c] = 0 for all a,b,c,x,y,z,h ∈ R. ……. .(4)
Multiply equation (4) by t(d(ab,y,z) - α(a)d(b,y,z) - d(a,y,z) α(b)) frome right to get
(d(ab,y,z) - d(a,y,z) α(b)- α(a)d(b,y,z)) α(x) α[h,c]) t (d(ab,y,z) - d(a,y,z) α(b)- α(a)d(b,y,z) α(x) α[h,c] = 0
Since α is automorphsime and R is semiprime , we get
(d(ab,y,z) - d(a,y,z) α(b)- α(a)d(b,y,z)) α(x)α[h,c] = 0 for all a,b,c,x,y,z ∈ R …(5)
In particular,
[d(ab,y,z) – α(a) d(b,y,z) – d(a,y,z) α(b) ,c] x [d(ab,y,z) – α(a) d(b,y,z) – d(a,y,z) α(b) ,c]
= (d(ab,y,z) - α(a)d(b,y,z) – d(a,y,z) α(b)) cx [d(ab,y,z) – α(a) d(b,y,z) – d(a,y,z) α(b) ,c] – c (d(ab,y,z) - α(a)d(b,y,z) – d(a,y,z) α(b)) x [d(ab,y,z) – α(a) d(b,y,z) – d(a,y,z) α(b) ,c] =0 (by equation (1)).
Since R is semiprime, then
[d(ab,y,z) – α(a) d(b,y,z) – d(a,y,z) α(b) ,R] = 0 , for all a,b,c,y,z ∈ R.
Hence by Lemma (2.1)
(d(ab,y,z) – α(a) d(b,y,z) – d(a,y,z) α(b)) ∈ Z(R) …… ..(6)
Notice that
$2(d(ab,y,z) - d(a,y,z)\alpha(b) - \alpha(a)d(b,y,z))^2$ = ((d(ab,y,z) – d(a,y,z) α(b)- α(a)d(b,y,z)) { d(ab,y,z) – d(a,y,z) α(b)- α(a)d(b,y,z) – (d(ba,y,z) – d(b,y,z) α(a)- α(b)d(a,y,z)} . (7)
= (d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z)) d ([a,b],y,z] (by equation (6) and equation (5)) (8)
Since α [a,b] (d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z)) + (d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z)) α [a,b] = 0 .
By Lemma (3.3)
0= d([a,b],y,z) α (d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z)) ,y,z)) + α([a,b] d(d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z),y,z) + d((ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z) y,z) α ([a,b])+ α(d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z)) d([a,b] ,y,z) .= 2 d([a,b],y,z) α (d(ab,y,z) – d(a,y,z) α(b) -

α(a) d(b,y,z) ,y,z)) + α ([a,b],y,z) d (d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z) ,y,z)) + d (d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z) ,y,z) α ([a,b],y,z) .

= 4 $\alpha(\mathrm{d(ab,y,z)} - \mathrm{d(a,y,z)}\,\alpha(\mathrm{b}) - \alpha(\mathrm{a})\,\mathrm{d(b,y,z)}\,,\mathrm{y,z}))^{2}$+ α([a,b]) α(d(a,y,z) α(b) - α(a) d(b,y,z) ,y,z)) + α(d (d(ab,y,z) – d(a,y,z) α(b) - α(a) d(b,y,z) ,y,z)) α [a,b]

Multiply the last equation by α (d(ab, y, z) −d(a,y,z) α(b) - α(a) d(b,y,z) ,y,z) from right= 4 $\alpha(\mathrm{d(ab,y,z)} - \mathrm{d(a,y,z)}\,\alpha(\mathrm{b}) - \alpha(\mathrm{a})\,\mathrm{d(b,y,z)}\,,\mathrm{y,z}))^{3}$

Since R 2-torsion free semiprime ring, we get

$\mathrm{d(ab,y,z)} - \mathrm{d(a,y,z)}\,\alpha(\mathrm{b}) - \alpha(\mathrm{a})\,\mathrm{d(b,y,z)}\,,\mathrm{y,z}) = 0$, for all a,b, y,z ∈ R.

This is, d is (α,α) permuting 3 − derivation.

4. Generlazied (α,β) Jordan permuting 3-derivation

We generalize the results in the above.

Definition (4.1) :

A 3- additive mapping F: U × U × U →R is called generalized (α,β) Jordan permuting 3-derivation if there exist a Jordan(α,β) permuting 3-derivation d: U × U × U →R such that F(x^2 , y ,z) = F (x, y , z) α (x) + β(x) d(x, y ,z) for all x,y,z ∈ U.

Example (4.2) :

Let R be a ring of all 2× 2 matrices over a commutative ring S. Let U = { $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}$: a , b ∈ S } be a Lie ideal of R ={ $\begin{pmatrix}a & b\\ 0 & c\end{pmatrix}$: a , b,c ∈ S } .

Define F :U ×U × U → R by

F $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix},\begin{pmatrix}0 & c\\ 0 & d\end{pmatrix},\begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}$ = $\begin{pmatrix}0 & 0\\ 0 & bdf\end{pmatrix}$ for all $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix},\begin{pmatrix}0 & c\\ 0 & d\end{pmatrix},\begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}$ ∈ U . Also α, β: R→R are defined by $\alpha\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}$= $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}$, for all $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}$ ∈ U and β $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}$ = $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$, for all $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}$ ∈ U.

Then F is gereralized Jordan (α,β) permuting 3- derivation ,since there exists (α,β) a permuting 3-derivation d wich is defined as Example(3.2).

Lemma (4.3) :

Let R be a 2-torsion free ring and F : U × U × U → R be a generalized Jordan (α,β) permuting 3-derivation then for all a,b,x ∈ U the following statements areholds:

i- F(ab + ba , x, y) = F(a,x,y) α(b) + β(a) d(b,x,y) + F(b,x,y) α(a) + β(b) d(a,x,y)

ii- F(aba , x, y) = F(a, x, y) α(ba) + β(a) d(b,x,y) α(a) + β(ab) d(a,x,y)

iii- F(abc + cba ,x,y) = F(a, x, y) α(bc) + F(c,x,y)α(ba) + β(ab) d(c,x,y) + β(cb) d(a,x,y)+β(a) d(b,x,y) α(c) + α(c) d(b,x,y) α(a).

Proof :

i. Since F is a generlaized Jordan (α,β) permuting 3-derivation , Then

F(a^2, x, y) = F(a, x,y) α(a) + β(a) d(a, x,y), for all a, x, y ∈ U. …… .(1)

Replace a by a+b in equation (1) and by using it , we get

F ($(a+b)^2$, x, y) = F(a^2+ ab +ba + b^2, x, y) = F(a^2, x, y) + F(ab +ba, x, y) + F(b^2, x, y) for all x, y, z ∈ U. …… ….(2)

On the other hand , since d is 3-additive , we get

F($(a+b)^2$,x,y)) = F(a+b ,x,y) α(a+b) + β(a+b) d(a+b,x,y)

= F(a, x,y) α(a) + F(a, x,y) α(b) + F(b, x,y) α(a) + F(b, x,y) α(b) + β(a) d(a,x,y)+ β(b) d(a,x,y) + β(a) d(b,x,y) + β(b) d(b,x,y) for all x, y, z ∈ U. …… … .(3)

Compring (2) and (3) and since F is generalized Jordan (α,β) permuting 3- derivation we get

F(ab + ba ,x, y) = F(a,x,y) α(b) + β(a) d(b,x,y) + F(b,x,y) α(a) + β(b) d(a,x,y)

for all x, y, z ∈ U .

ii. Consider w =4 F(a (ab+ ba) + ((ab+ ba) a) ,x,y) and by using (i) we get

w = 4(F(a,x,y) α(ab + ba) + β(a) d((ab + ba),x,y) + F(ab + ba,x,y) α(a) + β(ab + ba) d(a ,x,y) .

Again by (i) , we get

w=4(F(a,x,y) α(ab)+ F(a,x,y) α(ba) + β(a)(d(a,x,y) α(b) + β(a) d(b,x,y) + d(b,x,y) α(a) + β(b) d(a,x,y)) + (F(a,x,y) α(b) + β(a) d(b,x,y) + F(b,x,y) α(a) + β(b) d(a,x,y)) α(a) + β(ab) d(a,x,y) + β(ba) d(a,x,y)) .= 4(F(a,x,y) α(ab) + F(a,x,y) α(ba) + β(a) d(a,x,y) α(b) + $\beta(a^2)$ d(b,x,y) + β(a) d(b,x,y) α(a) + β(ab) d(a,x,y) + F(a,x,y) α(ba) + β(a) d(b,x,y) α(a) + F(b,x,y) $\alpha(a^2)$ + β(b) d(a,x,y) α(a) + β(ab) d(a,x,y) + β(ba) d(a,x,y) . (4)

On the other hand

w= 4 F ((a^2 b + b a^2) + 2 aba) , x, y) = 4(F(a^2,x,y) α(b) + $\beta(a^2)$ d(b,x,y) + F(b,x,y) $\alpha(a^2)$ + β(b) d(a^2,x,y) +2F(aba,x,y))= 4(F(a,x,y) α(ab) + β(a) d(a,x,y) α(b) + $\beta(a^2)$ d(b,x,y) + F(b,x,y) $\alpha(a^2)$ + β(b) d(a,x,y) α(a) + β(ba) d(a,x,y) + 2 F(aba ,x,y)) …… .(5)

By compring equation (4) and equation (5) and since R is 2-torsion free we get

F (aba , x,y) = F(a,x,y) α(ba) + β(a) d(b,x,y) α(a) + β(ab) d(a,x,y) , for all a,b,x,y ∈ U .

iii. By replacing a+c instead of a in (ii) and using it, we get

F((a +c) b (a +c) ,x,y) =

F((a+c),x,y) α (b(a+c))+ β((a+c) b) d(a+c , x,y) + β (a+c)d(b,x,y) α (a+c) .

= F(a,x,y) α(ba) + F(a,x,y) α(bc) + F(c,x,y) α(ba) + F (c,x,y) α(bc) + β(a b) d(a,x,y) + β(ab) d(c,x,y) + β(cb) d(a,x,y) + β (cb) d(c,x,y) + β(a) d(b,x,y) α(a) + β(c) d(b,x,y) α(c) + β(a) d(b,x,y) α(c) + β(c) d(b,x,y) α(a) … … .(6)

On the other hand,

F ((a+c) b (a+c),x,y) = F(aba + cbc + abc + cba ,x,y) (7)

By compring (3) and (4) we get

F(abc + cba ,x,y) = F(a,x,y)α(bc) + F(c,x,y)α (ba) + β(ab) d(c,x,y) + β(cb) d(a,x,y) + β(a) d(b,x,y)α (c)+ β(c)d(b,x,y) α(a) .

Lemma (4.4):

Let R be a 2- torsion free ring and F: U × U × U → R be a generalized Jordan (α,β) permuting 3-derivation . Then for all elements a, b , x, y , z ∈ U .

(F(ab,y,z) –F(a,y,z) α(b) - β(a) d(b,y,z) α(x) α[a,b] + β [a,b] β(x) ((d(ab,y,z) - d(a,y,z) α(b) – β(a) d(b,y,z)) = 0 .

proof :

Consider w = = 2^4F(abxba + baxab ,y, z) .Using Lemma (4. 3) we obtain

w=2^4 F(a(bxb)a + b(axa)b, y,z)

w= 2^4(F(a,y,z) α(bxba) + β(a(bxb)) d(a,y,z)+ β(a) d(bxb,y,z) α(a) + d (b,y,z) α(axab) + β(baxa) d(b,y,z) + β(b) d(axa,y,z) α(b)

=2^4(F(a,y,z) α(bxba) + β(a) d(b,y,z) α(xba) + β(a) β(b) d(x,y,z) α(b) α(a) + β(a) β(bx) d(b,y,z) α(a) + β(abxb) d(a,y,z) + F(b,y,z) α(axab) + β(b) d(a,y,z) α(xa) α(b) + β(b) β(a) d(x,y,z) α(a) α(b) + β(b) β(ax) d(a,y,z) α(b) + β(baxa) d(b,y,z).

= 2^4(F(a,y,z) α(bxba) + β(a) {d(b,y,z) α(xb) + β(bx) d(b,y,z) }α(a) + β(abx) d(a,y,z) +F(b,y,z) α(axab) + β(b) {d(a,y,z) α(xa) + β(ax) d(a,y,z)} α(b) + β(baxa) d(b,y,z) +β(ab) d(x,y,z) α(ba) + β(b) β(a) d(x,y,z) α(a) α(b) . ……… … …(1)

On other hand , by using Lemma (4. 3)

w=2^4 F ((ab)x(ba) + (ba)x(ab),y,z)

= 2^4F(ab,y,z) α(x) α(ba) +β(ab) d(x,y,z) α(ba) + β(abx) d(ba,y,z) + F(ba,y,z) α(xab) + β(ba) d(x,y,z) α(ab) + β(bax) d(ab,y,z) + α(ba) d(x,y,z) α(ab) (2)

Comparing equation (1) and equation (2) , and since R is 2-torsion free we get

0 = (F(ab ,y,z) – F(a,y,z) α(b) – β(a) d(b,y,z)) α(xba)) + β(bax) (d(ab,y,z) -d(a,y,z) α(b) - β(a) d(b,y,z)) + (F(ba , y,z) –F (b,y,z) α(a) - β(b) d(a,y,z))α(xab) + β(abx)(d(ba,y,z) – d(b,y,z) α(a) - β(b) d(a,y,z)) … (3)

By Lemma (4.3.3) equation (3) reduce to

0 = (F(ab,y,z) – F(a,y,z) α(b) - β(a) d(b,y,z)) α(xba) + β(bax)(d(ab,y,z)– d(a,y,z) α(b) - β(a) d(b,y,z) – ((F(ab , y, z) – F(a,y,z) α(b) - β(a) d(b,y,z)) α(xab) – β(abx) d(ab,y,z) – d(a,y,z) α(b) + β(a) d(b,y,z)). (4)

That is ,

(F(ab,y,z) – F(a,y,z) α(b) - β(a) d(b,y,z)) α(x) α [a,b] + β[a,b] $\beta(x)$ ((d(ab,y,z) – d(a,y,z) α(b) - β(a) d(b,y,z)) =0 . every x, y, z ∈ U .

Theorem (4. 5) :

Let R be a 2-torsion free semiprime ring and F: R× R× R → R be a generalized (α, α)Jordan permuting 3-derivation then (F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) α (x [d,c])=0 , for all a,b,c,d,x,y,z ∈ R .

Proof :

By Lemma (4. 4), we get

(F (ab, y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) $\alpha(x$[a, b]) + $\alpha($ [a , b] $x)$ ((d(ab,y,z) – d(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) = 0 and implies this implies that , for all a,b,x,z ∈ R.

Since d is (α, α) Jordan permuting 3-derivation and Lemma (2.1), we get $\alpha[a,b]$x) (d(ab,y,z) – d(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z))= 0

(F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) $\alpha(x$ [a,b]) = 0 (1)

Linearize equation (1) on b with regard to b = b + c for all a,b,x,y,z ∈ R .

0 = ((F (a(b+c) ,y,z) – F(a,y,z) α (b + c) - $\alpha(a)$ d(b +c ,y,z) $\alpha(x$ [a, b+c])

0 = (F(ab, y,z) + (F(ac,y,z) – F(a,y,z) $\alpha(b)$ – F(a,y,z) $\alpha(c)$ - $\alpha(a)$ d(b,y,z) - $\alpha(a)$ d(c,y,z)) α(x [a+ b] + [a + c]))

By using (1) , the last equation can be reduced to

0 = ((F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z))α (x [a, c]) + ((F(ac,y,z) – F(a,y,z) $\alpha(c)$ - $\alpha(a)$ d(c,y,z))$\alpha(x$ [a,b]) for all a,b,c,y,z ∈ U. … …(2)

By using (2) and (1) the following can be reduce to

((F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) $\alpha(x$[a,c])) t ((F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) $\alpha(x$[a,c])) .

= - ((F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) $\alpha(x$[a,c])) t ((F(ac,y,z) – F(a,y,z) $\alpha(c)$ - $\alpha(a)$ d(c,y,z) $\alpha(x$[a,b])) .

= - ((F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) {$\alpha(x$ [a,c])t F(ac,y,z) – F(a,y,z) $\alpha(c)$ - $\alpha(a)$ d(c,y,z) α x}α [a,b]) = 0 , for all a,b,c,x,y,z ∈ R. (by equation (1))..

Since R is semiprime we get

(F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z)) ($\alpha(x)$ [a,c] = 0 , for all a,b,c,x,y,z ∈ R. (3)

A linearization of equation (3) on a give such that a = a + d

0= (F((a+ h) b,y,z) – F(a+ h,y,z) $\alpha($ b)– (α) (a+ h) d(b,y,z)) α(x [a+ d,c])=((F(ab,y,z) + F(hb,y,z) – F(a,y,z) $\alpha($ b)– F(h,y,z) $\alpha(b)$– $\alpha(a)$ d(b,y,z) – α(x) d(b,y,z)) α (x([a,c] + [d,c])) = ((F(ab,y,z) – F(h,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) α (x [h,c]) + F(hb,y,z) – $\alpha(h)$ d(b,y,z) - F(h,y,z) α (b)) [a,c].

The last equation lead us to

(F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) α (x [h,c]t (F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) α (x [h,c]= - (F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z) α (x [h,c])t (F(hb,y,z) – $\alpha($ $h)$ d(b,y,z) - F(h,y,z) α (b)) α (x [a,c]).

Hence ,(F(ab,y,z) – F(a,y,z) $\alpha(b)$ - $\alpha(a)$ d(b,y,z))α (x [h,c])=0 .

It is immedaitly from Theorem (4. 5) we get, the following corollary

Corollary (4.6)

Let R be a 2-torsion free Prime ring and F: R× R × R → R be a generalized (α, α)Jordan permuting 3-derivation then either F is generalized (α, α) permuting 3-derivation or R is commutative.

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