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Centralizers on Prime and Semiprime Γ-rings

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Abstract

In this paper, we will generalized some results related to centralizer concept on prime and semiprime Γ-rings of characteristic different from 2 .These results relating to some results concerning left centralizer on Γ-rings.

Keywords: Semiprime Γ-ring , Centralizers .

تمركزات على الحلقات الاولية وشبه اولية من النمط كاما

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الخلاصة

في هذه البحث ، سوف نعمم بعض النتائج المتعلقة بمفهوم التمركز على الحلقات الاولية وشبه الاولية من النمط كاما التي ممثلها لا يساوي 2 هذه النتائج متعلقة مع بعض النتائج للتمركز الايسر على الحلقات من النط كاما.

1. Introduction

Nobusawa in [1] presented the idea of a Γ-ring , the concept of Γ-ring is more general of the Ring Barnes in [2] the definition of the Γ-ring with less conditions . On the basis of these two definitions many researchers in pure mathematics have made working on Γ-ring sense Barnes and Nobusawa see [3-6] , which parallel results in the Ring theory, Barnes in [2] defined it as following : suppose N and Γ be an additive abelaine groups , if there exists a map from N×Γ×N to N, for all a, b, c ∈N and γ, δ ∈Γ satisfying the following conditions :

1. aγb ∈ N .
2. (a+b)γc=aγc+bγc , a(γ+δ)b=aγb+aδb and aγ(b+c)=aγb+aγc
3. (aγb)δc=aγ(bδc) .

Then N is called Γ-ring.

Some preliminaries of Γ-rings was given by S.Kyuno [7] as following : "Let I be a non-zero subset of a Γ-ring N, then I is called a left (right) ideal , if I be an additive subgroup of N and NΓI⊆I (IΓN⊆I), if I be a left and right ideal then I is called an ideal of N . N is called 2-torsion free if 2a=0 obtain a=0 , a∈N . A Γ-ring N is said to be prime if aΓNΓb=(0) with a,b ∈N , obtain a=0 or b=0 and it simeprime if aΓNΓa=(0) with a∈N, obtain a=0 . A Γ-ring N is called commutative if aγb=bγa, for all a,b ∈Γ and γ∈Γ . The subset Z(N)={a∈N| aγb=bγa, for-
all a∈N and γ∈Γ } of a Γ-ring N is called center of N ". An additive mapping T:N→N is called left (right) centralizer if T(aγb)=T(a)γb (T(aγb)=aγT(b)) for all a,b ∈N and γ∈Γ, and T is called Jordan left (right) centralizer if T(aγa)=T(a)γa (T(aγa)=aγT(a)) for all a ∈N and γ∈Γ. If T are both left and right centralizer then T is called centralizer . Also the element (aγb-bγa) ∈ N is called the commutater

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of a and b with respect to γ which is denoted by $[a,b]_\gamma$. In [8] S. Chakraboty and A.C. Paul show that if N is a Γ-ring for all a ,b, c ∈N and γ,δ∈Γ , then

i. $[a+b,c]_\gamma=[a,c]_\gamma+[b,c]_\gamma$
ii. $[a,b+c]_\gamma=[a,b]_\gamma+[a,c]_\gamma$
iii. $[a\delta b,c]_\gamma=a\delta[b,c]_\gamma+[a,c]_\gamma\delta b+a\delta c\gamma b-a\gamma c\delta b$

In this paper we assume that aδcγb=aγcδb which represent by (∗) then from equation (iii) , we get $[a\delta b,c]_\gamma=a\delta[b,c]_\gamma+[a,c]_\gamma\delta b$ .In [9] M.F. Hoque and A.C.Paul proved that if N be a semiprime Γ-ring of characteristic different from 2 with condition(∗) then the Jordan left centralizer is left centralizer on N and they proved if N be a semi-prime Γ-ring of characteristic different from 2 with condition(∗) then the Jordan centralizer is a centralizer on N In this paper we show that if N be a 2-torsion free semi-prime Γ-ring with condition (∗), I be an ideal of N and T:N⟶N be a Jordan left centralizer on I, then N contains a central ideal ideal. and if is a prime Γ-ring of characteristic different from 2 with the same above hypotheses then N is commutative Γ-ring.

2. The Results

To prove the main result , we begin with some lemmas:

Lemma 2.1. [9] Suppose N be a semi-prime Γ-ring, if a,b, ∈N and γ,δ∈Γ, such that aγcδb=0 for all c∈N , then aγb=bγa=0.

Lemma 2.2. [9] Suppose N be a semi-prime Γ-ring and F:N×N→N , bi-additive mapping. If F(a,b)γcδF(a,b)=0 for all a,b,c ∈N and γ,δ∈Γ , then F(a,b)γcδF(u,v)=0 , a,b,c,u,v ∈N.

Lemma 2.3.[9] Suppose N be a semi-prime Γ-ring with condition (∗) and x be a fixed element in N. If $x\delta[a,b]_\gamma=0$, for all a, b ∈N and δ , γ ∈Γ, then N have central ideal I , such that x∈I⊂N.

Theorem 2.4. Suppose N be a 2-torsion free semi-prime Γ-ring with condition (∗),I be an ideal of N and T:N⟶N be a Jordan left centralizer on I , then N contains a central ideal.

Proof:

for all a∈I and γ∈Γ, then T(aγa)=T(a)γa (1)

ifwe replace a by (a+b) in (1), we get for

for all γ∈Γ T(aγb+bγa)=T(a)γb+T(b)γa (2)

in (2) replace b by aγb+bγa and γ by δ , for all b∈I and δ∈Γ , we obtain

T(aδ(aγb+bγa)+(aγb+bγa)δa)

=T(a)δaγb+T(a)δbγa+T(a)δaγb+T(b)γaδa= T(a)δaγb+2T(a)δbγa+ T(b)γaδa (3)

Calculate (3) By deferent way then

T(aδ(aγb+bγa)+(aγb+bγa)δa) =T(aδaγb+bγaδa)+2T(aδbγa) = T(a)δaγb+ T(b)γaδa+2T(aδbγa) (4)

By subtracting Eq.3 from Eq. 4 resulting in

T(aδbγa)=T(a)δbγa (5)

In Eq. 5 replace a by a+c for all c∈I , we obtain

T((a+c)δbγ(a+c))=T((a+c))δbγ(a+c) =(T(a)+T(c)) δbγ(a+c)

=T(a) δbγ(a+c)+T(c)δbγ(a+c)= T(a) δbγa+ T(a) δbγc+ T(c)δbγa+ T(c)δbγc (i)

And we can show that

T((a+c)δbγ(a+c))=T(aδbγa+aδbγc+cδbγa+cδbγc)

T((a+c)δbγ(a+c))=T(a)δbγa+T(aδbγc+cδbγa)+T(c)δbγc (ii)

From (i) and (ii) , we get

T(aδbγc+cδbγa)= T(a)δbγa+ T(c)δbγa (6)

Suppose that J=T(aγbδcαbβa+bγaδcαaβb) , for all a,b,c∈I and γ,δ,α,β∈ Γ , and calculate J by two deferent way as follows:

By using Eq. 5 resulting in

J=T(a)γbδcαbβa+T(b)γaδcαaβb (7)

And by Eq. 6 resulting in

J=T(aγb)δcαbβa+T(bγa)δcαaβb (8)

By subtracting Eq.8 from Eq. 7 resulting in

0=(T(aγb)− T(a)γb)δcαbβa + (T(bγa)− T(b)γa)δcαaβb (9)

Suppose the following bi-additive map F(a,b)= T(aγb)− T(a)γb, and we can show that F(a,b)= −F(b,a). So Eq. 9 become 0= F(a,b) δcαbβa+ F(b,a) δcαaβb and

F(a,b) $\delta c\alpha[a,b]_\beta=0$, using Lemma 2.2. we have F(a,b) $\delta c\alpha[u,v]_\beta=0$ in this equation fix some a,b ∈I and let F= F(a,b) , then $F\delta c\alpha[u,v]_\beta=0$, for all u,v ∈I that mean by lemma 2.1.$F\delta$ $[u,v]_\beta=0$ and by lemma 2.3. we get N have central ideal.

From Theorem 2.4. and using some lemmas in Γ-rings corresponding to lemmas in the Rings Theory we can prove some results.

Lemma 2.5. Suppose N be a semi-prime Γ-ring with condition (∗) and I be a left ideal of N then Z(I)⊆Z(N) .

Proof : if a∈ Z(I) , since I is left ideal then xγa∈I , for all x∈N and γ∈Γ also $0=[a,x\gamma a]_\gamma$, that lead to

$0=[a,x]_\gamma\gamma a$ (1)

By Eq. 1 for all y∈N , then

$0=[a,x]_\gamma\gamma a\delta y$, for all δ∈Γ (2)

In Eq. 1 replace x by xδy we obtain

$0=[a,x]_\gamma\delta y\gamma a$ (3)

From Eq. 2 and Eq. 3 we obtain

$0=[a,x]_\gamma\delta[a,y]_\gamma$ (4)

In Eq.4 replace y by yαx , for all α∈Γ , we get

$0=[a,x]_\gamma\delta y\alpha[a,x]_\gamma$. By the sime-primeness of N , $[a,x]_\gamma=0$.

Lemma 2.6. Suppose N be a semi-prime Γ-ring with condition (∗) and let I be a non-zero left ideal of N . if I be a commutative as a Γ-ring, then I⊆Z(N), if in addition N is a prime Γ-ring, then N must be commutative.

Proof:

By Lemma 2.5. , we get our first desired.

I⊆Z(I)⊆Z(N) (1)

For all x∈N and a∈I , then xΓa⊆I and by Eq. 1, xΓa⊆Z(N) , also for all α∈Γ and y∈N

Then $(0)=[y, x\Gamma a]_\alpha=[y,x]_\alpha\Gamma a$, in general

$[y,x]_\alpha\Gamma I=(0)$ (2)

Since I is left idea and by Eq. 2 , then $[y,x]_\alpha\Gamma N\Gamma I=(0)$, but N is prime Γ-ring and I non-zero ideal that means $[y,x]_\alpha=0$, for all x,y∈N and α∈Γ.

Corollary 2.7. Suppose N be a prime Γ-ring of characteristic different from 2, with condition (∗)**,** I be an ideal of N and T:N⟶N be a Jordan left centralizer on I , then N is commutative.

Proof: by Theorem 2.4. , then N contains a central ideal and by Lemma 2.6. then N is commutative Γ-ring.

Corollary 2.8. Suppose N be a prime Γ-ring of characteristic different from 2, with condition (∗)**,** if T:N⟶N be a left centralizer on N, then T is centralizer on N .

Proof: for all a∈N and γ∈Γ , T(aγa)=T(a)γa , by corollary 2.7. then N is commutative, for all a,b∈N and γ∈Γ ,T(aγb)=T(bγa)=T(b)γa=aγT(b) , that is T also right centralizer.

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