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The Theoretical Solving of Intersection Point of the Horizontal and Vertical Gravity Gradients in Order to Estimate the Depth of Causative Source of Gravity Anomaly

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Abstract
The depth of causative source of gravity is one of the most important parameter of gravity investigation. Present study introduces the theoretical solve of the intersection point of the horizontal and vertical gradients of gravity anomaly. Two constants are obtained to estimate the depth of causative source of gravity anomaly, first one is 1.7807 for spherical body and the second is 2.4142 for the horizontal cylinder body. These constants are tested for estimating the depth of three actual cases and good results are obtained. It is believed that the constants derived on theoretical bases are better than those obtained by empirical experimental studies.

Keywords: Gradient, Gravity, Theoretical Solve, Depth Estimation, Sphere, Horizontal Cylinder

الحل النظري لنقطة تقاطع الانحدار الجذبي الافقي والعمودي لتخمين عمق المصدر المسبب للشذوذ الجذبي

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الخلاصة:
يعد عمق المصدر المسبب للجذبية الارضية احد اهم المعاملات في الاستكشاف الجذبي. تقدم الدراسة الحالية حلا نظريا لنقطة تقاطع الانحدار الافقي والعمودي للشواذ الجذبية. جرى الحصول على ثابتين لتخمين العمق المسبب للشذوذ الجذبي، الاول مقداره 1.7807 للجسم كروي والثاني مقداره 2.4142 للاسطوانه الافقيه. جرى اختبارالثابتين لتخمين العمق لثلاث حالات حقيقة وتم الحصول على نتائج جيدة. لذا يعتقد ان الثابتين اللذين تم الحصول عليهما نظريا هما افضل من تلك التي يتم الحصول عليها من الدراسات التجريبية.

Introduction

Estimation of the causative source depth of gravity anomaly is an important aim for geophysical modeling. Several researchers attempt to prepare methods to estimate the source depth of the gravity anomaly. Some researchers determine the depth from the gravity anomaly data, while others use the

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gravity gradient values. A simple method is developed to determine the depth and shape of a buried structure from the residual gravity data using filters of successive window lengths [1]. The method is applied to theoretical data and known field example and good agreement with the actual ones are obtained. Al-Rawi [2] produced three procedures to determine the depth to the center of the horizontal cylinder from gravity profile. He got accurate results for the considered cases in his study. Hammar and Anzoleaga [3] used the gradient to determine the depth of pinchout stratigraphic traps, they got good results. Miller and Singh [4] identified the location and depth of subsurface bodies depending on the tilt angles that obtained from gravity gradient. The 2D analytical model is applied to find depth of geological body using the gravity gradient values [5]. Veryaskin and Fraser [6] discussed a procedure to model the geological body using combined gravity gradient components.

Two equations to determine the depth of geological bodies depending on gravity gradients are derived by Al-banna and Al-Kasiy [7]. Oruc [8] attempt to estimate the edge and depth of causative sources. His method based on the tilt angle map that obtained from first vertical gradient of gravity anomaly. Beiki et al. [9] introduced a method for interpretation of magnetic and gravity gradient tensor data using the Normalized Source Strength (NSS). The method is demonstrated on collected aeromagnetic and gravity gradient tensor data sets from McFualts Lake region, Canada. An attempt is carried out in this study to solve the intersection point of horizontal and vertical gradient of any gravity anomaly theoretically. This theoretical solve produce a constants which could be used to estimate the depth of subsurface bodied, that approximated to simple geometric bodies (sphere and horizontal cylinder).

Theoretical solve of the intersection point of derivatives

1. Spherical case

The gravity effect of spherical body is represented by the following equation:

$$g_{z} = \frac{4}{3}\pi G\Delta\rho R^3 \frac{Z}{(x^2+Z^2)^{3/2}} \quad [10] \quad \ldots\ldots(1)$$

where

G: gravitational constant ($6.67\text{x}10^{-6}$ cgs)

$\Delta\rho$: density contrast

Z: depth to the center of the sphere.

R: Radius of sphere

X : Distance

The horizontal and vertical gravity gradients values of a sphere are expressed as the followings:

$$g_{z,x} = \frac{4}{3}\pi G\Delta\rho R^3 \frac{-3xZ}{(x^2+Z^2)^{5/2}} \quad [11] \quad \ldots\ldots(2)$$

$$g_{z,z} = \frac{4}{3}\pi G\Delta\rho R^3 \frac{2Z^2-x^2}{(x^2+Z^2)^{5/2}} \quad [11] \quad \ldots\ldots(3)$$

The gravity, horizontal and vertical gradients are represented for assumed sphere with radius (1m), density contrast (-2.0 gm/cm) and depth of (5.0m) (Fig. 1). This figure shows the intersection point between the horizontal and vertical gradients curves.

Al-Banna and Al-Kasiy [7] believed that, the distance between the nearest intersection point of the gradients to great gravity value is related to the source depth of the gravity anomaly. They proposed an equation to estimate a value when it is multiply by the distance (x) the depth of the causative source can be obtained.

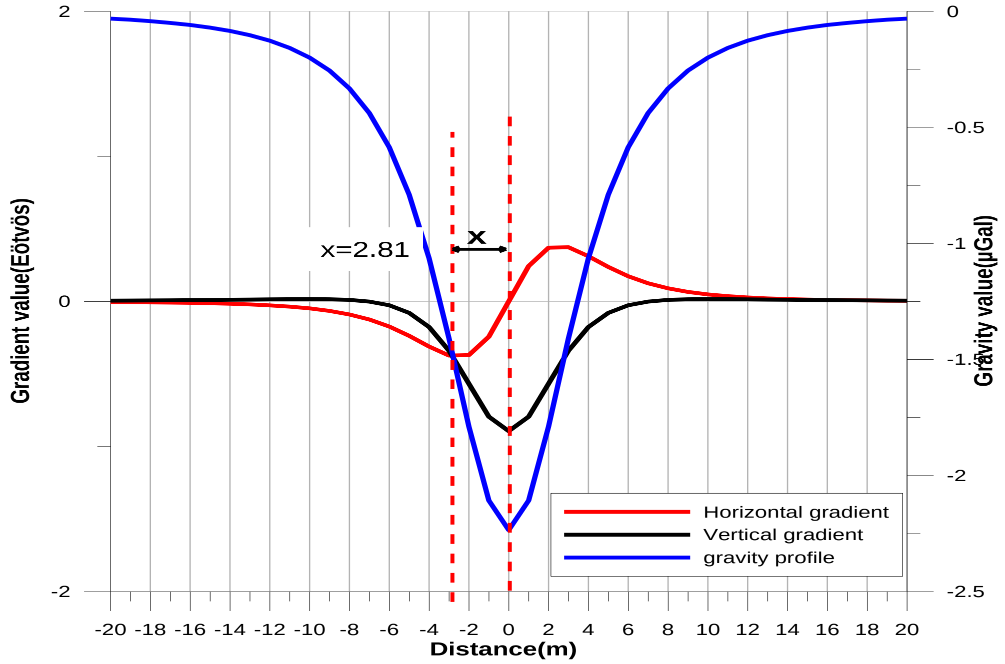

Figure 1- The gravity, horizontal and vertical gradient values of a sphere (r=1m, Δρ=-2gm/cc, Z= 5m.) x is the distance between the center of the gravity anomaly and the intersection point of the horizontal and vertical gradients, x= 2.81m.

In order to solve the intersection point of gradients theoretically the horizontal gradient ($g_{z,x}$) is divided by the vertical gradient then we get the following:

$$\frac{gz,x}{gz,z} = \frac{-3xZ}{2Z^2-x^2} \quad \ldots\ldots(4)$$

at the intersection point

$$\frac{gz,x}{gz,z} = 1 \quad \ldots\ldots(5)$$

$$\frac{-3xZ}{2Z^2-x^2} = 1 \quad \ldots\ldots(6)$$

then

$2z^2-x^2= -3xz$ ……(7)

or

$2z^2+3xz-x^2=0$ ……(8)

The solution of the above equation for the variable z (depth) gives

$Z= \frac{1}{4}(-\sqrt{17}\,x- 3x)$ ……(9)

or

$Z= \frac{1}{4}(\sqrt{17}\,x- 3x)$ ……(10)

And simplification gives

$Z= -1.7807\ x$ ……(11)

where x is the distance (with negligence negative sign) for the closest intersection point of horizontal and vertical gradients from the great value of certain gravity anomaly due to sphere.

And

$Z= 0.2807x$ ……(12)

where x is the distance of the farthest intersection point of horizontal and vertical gradients from the great value for the gravity anomaly.

The value (1.7807) considers here as the theoretical constant which can be used to estimate the depth (z) of the spherical causative source.

2. The horizontal cylinder case

The gravity effect of the horizontal cylinder represents by the following equation:

$$g_z = 2\pi G\Delta\rho R^2 \frac{Z}{(x^2+Z^2)} \quad [10] \quad \ldots\ldots(13)$$

where G, $\Delta\rho, z, x$ as previously defined for the sphere case, and R = radius of the horizontal cylinder.

The horizontal cylinder and vertical gravity gradients of gravity anomaly of the the horizontal cylinder are

$g_{z,x} = 2\pi G\Delta\rho R^2 \frac{-2xZ}{(x^2+Z^2)^2}$ [11]……………………………………………………………….……..(14)

and the vertical gradient is

$g_{z,z} = 2\pi G\Delta\rho R^2 \frac{Z^2-x^2}{(x^2+Z^2)^2}$ [11]…………………………………………………………….....(15)

The gravity, horizontal and vertical gradients of the horizontal cylinder are represented in Figure-2 The radius of the cylinder used is (1m) and the density contrast is (-2.0 gm/cc) at depth of (5m).

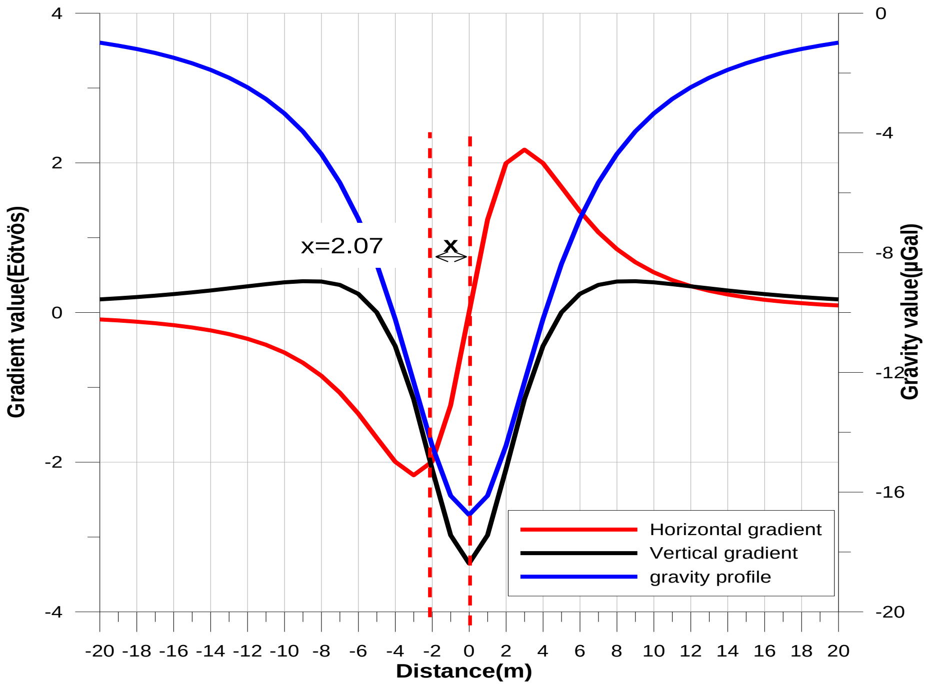

Figure 2- The gravity, horizontal and vertical values of a horizontal cylinder (r=1, Δρ=-2gm/cc, Z= 5m.) and x is the distance between the center of the gravity anomaly and the intersection point of the horizontal and vertical gradients curves, x= 2.07m.

The intersection point of the horizontal and vertical gradients is exhibited in Figure-2 as discussed in spherical case.

The intersection point is used to solve theoretically the relation expressed by equation 14 and 15 by dividing the horizontal gravity gradient to the vertical gradient and the following obtained

$\frac{gz,x}{gz,z} = \frac{-2xZ}{Z^2-x^2}$ ……………………………………………………………………………...(16)

At the intersection point is

$\frac{gz,x}{gz,z}$ =1 ………………………………………………………………………………...…(17)

then,

$z^2-x^2= -2xz$ ………………...……………………………………………………………….(18)

$z^2 +2xz-x^2 =0$ ……………………………………………………………………..…...…(19)

the solutions of the equation for the variable z are

$z= -\sqrt{2}\ x- x$ ……………………………………………………………………………...…(20)

and

z= -2.4142 x ………………………………………………………………………..……..(21)

where x is the distance (with negligence negative sign) for the closest intersection point between the horizontal and vertical gradients from the great value of the certain gravity anomaly, due to horizontal cylinder

z= $\sqrt{2}$ x- x ……………………………………………………………………………..…… ..…(22)
z= 0.4142x ……………………………………………………………………………………. (23)

where x is the distance for the furthest intersection point of horizontal and vertical gradients from the great value of the gravity anomaly.

The value of 2.4142 considered as the theoretical solve to determine the depth of the horizontal cylinder from the relation of the intersection point of the gradient for the horizontal cylinder. Therefore this constant can be considered to obtain the depth of the causative source of the horizontal cylindrical.

Testing the preference of the constants

In order to test the preference of the derived constants, several cases from the world which were previously discussed by Al-Banna and Al-Kasiy [7] are tested Figure-3. The actual published depth was compared with that obtained by using the derived constants and the results of comparison are summarized in table-1. The parameters in the table show that the depth obtained by using the current derived constants give results range between 90%—96% of the actual depth of the considered cases.

Table 1- summarizes the depths and references of the considered area. (x) is the distance from the great gravity value of the anomaly to the intersection point

Considered area	**Profile length**	**Approximation of the causative source to simple body**	**Distance (x) Current study**	**The actual depth**	**Depth of source using the constant of present study of the actual case**	**Remark and reference**
West Senegal West Africa Louga area	25km	Horizontal cylinder	1.65km	4.15km	3.98km 96%	[12] Depth to the basement N-S profile (Fig. 3,A)
Bakaro Coalfield, India	13km	Horizontal cylinder	1.87km	≈5.0km	4.5 km 90%	[13] Depth to basement (figure 3, B)
Rawa Cave, West Iraq	42m	Sphere	1.95m	3.6m	3.4m 96%	[14] Depth to the center of the cave E-W profile, (Fig. 3, C)

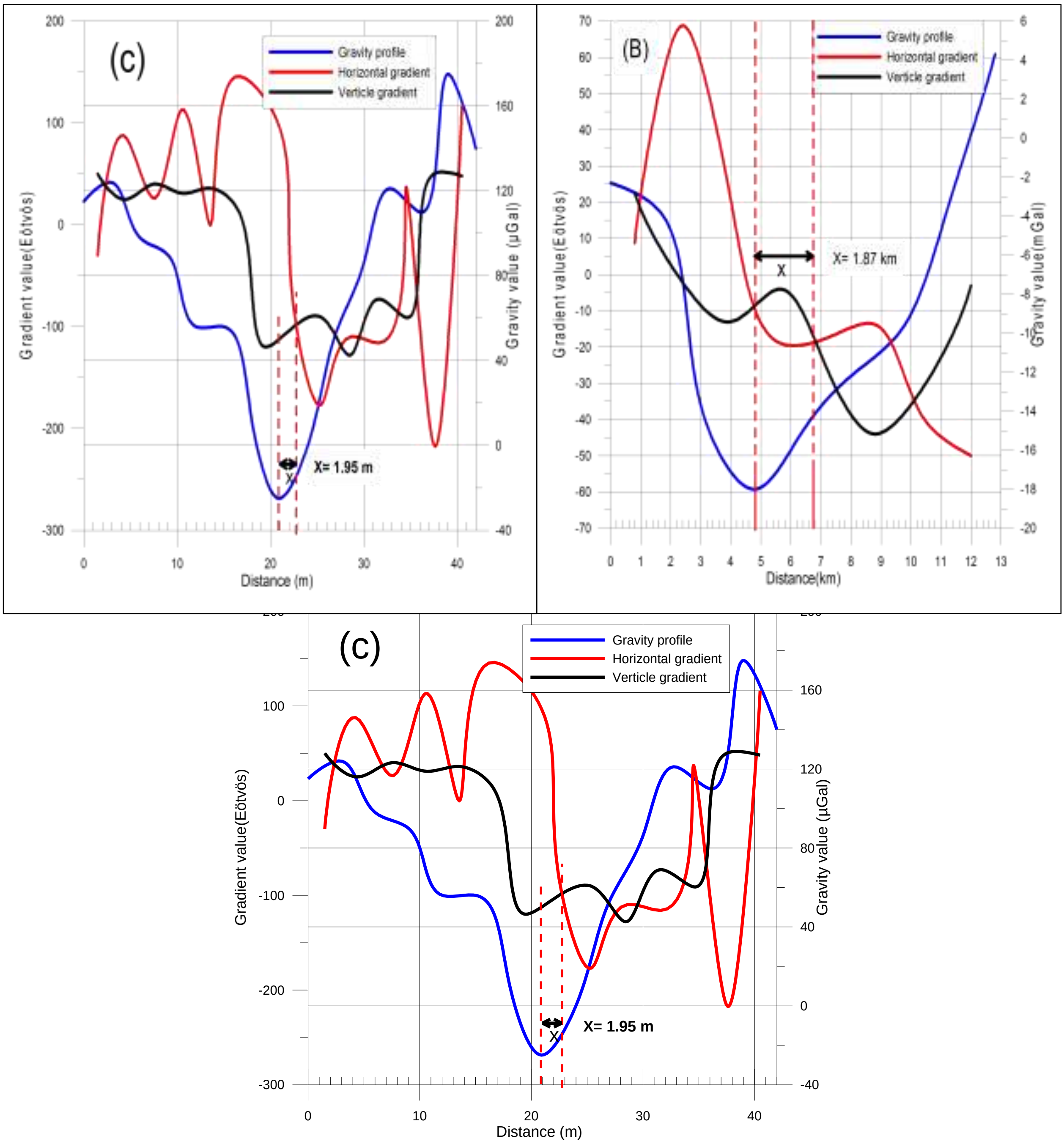

Figure 3- The gravity, horizontal and vertical gradients curves with the great distance (x) between the great gravity value and the intersection point of the three considered studies. (A) profile over Louga area anomaly, West Senegal, West Africa [12]. (B) profile across the central part of Bakaro coal field, North Karanapura, India [13] (C) profile over a known cavity at Rawa area, Western Iraq [14].

Conclusions

The intersection point of the horizontal and the vertical gradients is solved theoretically that is used for depth estimation of causative source of gravity anomaly. Theoretical solve are deduced for simple geometric bodies (sphere and horizontal cylinder). Two constants are obtained and are considered for depth estimation for spherical and horizontal cylinder bodies. The causative source depth of gravity anomaly is determined by multiply the constant by the distance (x) between the great gravity value and the nearest intersection point of the horizontal and vertical gradients. The obtained constants are tested for actual depth of three cases; they give good and reliable depth value in comparison with the previous methods published by various authors. It is believed that the constants derived on theoretical bases are better than those obtained by empirical experimental studies.

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