DOI: 10.24996/ijs.2022.63.11.30

ISSN: 0067-2904

Jordan generalized $\Gamma-(\sigma,\tau)$ −Derivation on Prime Γ-Near Rings

Ali Kareem Kadhim
Department of Mathematics, Basic of education college, Mustansiriyah university, Baghdad, Iraq

Received: 9/1/2022 Accepted: 25/3/2022 Published: 30/11/2022

Abstract

In this paper, we introduce the notion of Jordan generalized$\Gamma-(\sigma,\tau)$ −Derivation on prime $\Gamma-near\ ring$ and then some related concepts are discussed. We also verify that every Jordan generalized $\Gamma-(\sigma,\tau)$ −Derivation is generalized$\Gamma-(\sigma,\tau)$ −Derivation when N is a 2-torsionfree prime $\Gamma-near\ ring$.

Keywords: Γ-near- ring , generalized Γ −Derivation,$(\sigma,\tau)-$ Derivationon$\Gamma-ring$, Jordan derivation on prime $\Gamma-near\ ring$.

اشتقاق –(σ, τ)–Γ جوردان المعمم على الحلقات– Γ المقتربة الأولية

علي كريم كاظم
قسم الرياضيات, كلية التربية الأساسية, الجامعة المستنصرية , بغداد, العراق

الخلاصة

في هذا البحث، تم تقديم مفهوم اشتقاق –(σ, τ)–Γ جوردان المعمم على الحلقات– Γ المقتربة الأولية ومن ثم مناقشة بعض الأمور ذات الصلة بالمفهوم أعلاه. نثبت في هذا البحث ان كل اشتقاق –(σ, τ)–Γ جوردان المعمم على الحلقات– Γ المقتربة الأولية هو اشتقاق –(σ, τ)–Γ المعمم على الحلقات– Γ المقتربة الأولية.

1. Introduction

A $\Gamma-ring$ was initially known in 1964 by Nobusawa [1] to achieve a part of generalizations on the theory of rings. In 1966, a weak version of $\Gamma-ring$was completed by Barnes [2] to add some new generalizations. Thereafter, a number of algebraists[3,4] have studied the structure of $\Gamma-rings$ and obtained various generalizations that are analogous to corresponding parts in ring theory. A $\Gamma-near\ ring$ is a generalization of $\Gamma-\ ring$. This concept was studied by[5, 6, 7, 8].A $\Gamma-near\ ringN$ is said to be prime if $\mathfrak{a}\Gamma N\Gamma\mathfrak{b}=\{0\}$ implies that $\mathfrak{a}=0$ or $\mathfrak{b}=0$ and semiprime if $\mathfrak{a}\Gamma N\Gamma\mathfrak{a}=\{0\}$ implies that $\mathfrak{a}=0$. For any $\mathfrak{a},\mathfrak{b}\in N$, we have $[\mathfrak{a},\mathfrak{b}]\alpha=\mathfrak{a}\alpha\mathfrak{b}-\mathfrak{b}\alpha\mathfrak{a}$. A $\Gamma-prime\ near\ ringN$ is said to be2-torsion free if $2\mathfrak{a}=0$then $\mathfrak{a}=0$. Hvala [9] first introduced the generalized Derivations in rings and obtained some remarkable results in classic rings .Dey, Paul and Rakhimov[10] proved D

*Email: alikkadhim@uomustansiriyah.edu.iq

is a generalized Derivation in semiprime$\Gamma - ring$. Dey and Paul [11]studied some properties that related to generalizedDerivation in prime $\Gamma - ring$. Mamouni and Tamekkante[12] investigate identities with two generalized Derivations in prime rings. Ceven and Ozturk[13] calculated the Jordan generalized$\Gamma -$ Derivations in $\Gamma - rings$ and they proved that every Jordan generalized Derivation on some $\Gamma - rings$ are a generalized Derivation. Wang [14], Bell [15], Majeed and Adhab[16] introduced the Derivation in prime $near\,ring$. (α, β)reverse derivations is defined by Alhaidary and Majeed [17] and studied some results that related to it in prime ring. Ashraf and Shakir [18] studied a new concept $(\sigma, \tau) -$ Derivation on prime $near\,ring$.Sogutcu and Golbasi[19] introduced the comparison between Jordan (σ, τ) $-$Derivation and Jordan Triple (σ, τ) $-$Derivations in semiprime rings. The concept of Γ – Derivation in $\Gamma - near\,ring$ was introduced by [20]. The main purpose of this paper is to define generalized$\Gamma - (\sigma, \tau) -$Derivation and Jordan generalized$\Gamma - (\sigma, \tau) -$ Derivationon $\Gamma - near\,ring$ and study the interrelatedness between them.

2.Jordan generalized$\Gamma -$ (σ, τ) $-$Derivation.

Definition2.1.[8]

Suppose Nbe an additive group (not necessary Abelian) whose elements take the symbol $\mathfrak{a}, \mathfrak{b}, c$ ... and Γ an additive group whose elements take the symbol$\alpha, \beta, \gamma, \ldots$ .Suppose that $\mathfrak{a}\alpha\mathfrak{b}$ is defined to be an element of N for every $\mathfrak{a}, \mathfrak{b}, c \in N$and $\alpha \in \Gamma$. If the product satisfies the following conditions.

1- $(\mathfrak{a} + \mathfrak{b})\alpha c = \mathfrak{a}\alpha c + \mathfrak{b}\alpha c, \mathfrak{a}(\alpha + \beta)\mathfrak{b} = \mathfrak{a}\alpha\mathfrak{b} + \mathfrak{a}\beta\mathfrak{b}, \mathfrak{a}\alpha(\mathfrak{b} + c) = \mathfrak{a}\alpha\mathfrak{b} + \mathfrak{a}\alpha c$

2- $(\mathfrak{a}\alpha\mathfrak{b})\beta c = \mathfrak{a}\alpha\,(\mathfrak{b}\beta c)$,

then$N$is said to b a $\Gamma - near\,ring$.

Definition 2.2.[21]

Suppose$N$is a $\Gamma - near\,ring$ and $\delta$is an additive mapping on ,$\sigma$and τ are endomorphism on N, then δ is called $\Gamma - (\sigma, \tau) -$Derivation if

$$\delta(\mathfrak{a}\ \alpha\ \mathfrak{b}) = \delta(\mathfrak{a})\ \alpha\ \sigma(\mathfrak{b}) + \tau(\mathfrak{a})\alpha\ \delta(\mathfrak{b}) for\ each\ a, b\ \in N\ and\ \alpha \in\ \Gamma.$$

Definition 2.3

Suppose$N$is a$\Gamma - near\,ring$and $f: N \longrightarrow N$isan additive mapping on N, $\sigma$and τ are endomorphism on N, then f is called ageneralized$\Gamma - (\sigma, \tau) -$Derivation associated with $\Gamma - (\sigma, \tau) -$Derivation on N if $f(\mathfrak{a}\ \alpha\ \mathfrak{b}) = f(\mathfrak{a})\ \alpha\ \sigma(\mathfrak{b}) + \tau(\mathfrak{a})\ \alpha\ \delta(\mathfrak{b})$for each$\mathfrak{a}, \mathfrak{b} \in N$ and $\alpha \in \Gamma$**.**

The following is an example of the generalized$\Gamma - (\sigma, \tau) -$derivation.

Example 2.4

Let$R$be a ring. De$f$ine $N = N2\text{x}2(R) = \left\{\begin{bmatrix} \mathfrak{a} & \mathfrak{b} \\ 0 & 0 \end{bmatrix}; \mathfrak{a}, \mathfrak{b} \in R\right\}$ $and\ \Gamma = \left\{\begin{bmatrix} 0 & \mathfrak{b}_1 \\ 0 & 0 \end{bmatrix}; b_1 \in Z\right\}$. Then $N$is a $\Gamma - near\,ring$.

Let $:N \longrightarrow N$ be an additive mapping defined by $\delta(\begin{bmatrix} \mathfrak{a} & \mathfrak{b} \\ 0 & 0 \end{bmatrix}) = \begin{bmatrix} 0 & \mathfrak{b} \\ 0 & 0 \end{bmatrix}$ and f: $N \rightarrow N$be an additive mapping defined by $f(\begin{bmatrix} \mathfrak{a} & \mathfrak{b} \\ 0 & 0 \end{bmatrix}) = \begin{bmatrix} 0 & \mathfrak{b} \\ 0 & 0 \end{bmatrix}$.

Let$\sigma, \tau: N \rightarrow N$ be two endomorphisms which are defined as follows:

$\sigma(\begin{bmatrix} \mathfrak{a} & \mathfrak{b} \\ 0 & 0 \end{bmatrix}) = \begin{bmatrix} \mathfrak{a} & -\mathfrak{b} \\ 0 & 0 \end{bmatrix}, \tau(\begin{bmatrix} \mathfrak{a} & \mathfrak{b} \\ 0 & 0 \end{bmatrix}) = \begin{bmatrix} \mathfrak{a} & 0 \\ 0 & 0 \end{bmatrix}$, then

$$f(\mathcal{X}\alpha\mathcal{Y}) = f(\mathcal{X})\ \alpha\sigma(\mathcal{Y}) + \tau(\mathcal{X})\ \alpha\delta(\mathcal{Y}).$$

Definition 2.5.[21]

Suppose N is a $\Gamma - near\ ring$ and δ is an additive mapping on N, σ and τ are endomorphisms on N, then δ is called a Jordan $\Gamma - (\sigma,\tau) -$ Derivation if $\delta(\mathfrak{a}\ \alpha\ \mathfrak{a}) = \delta(\mathfrak{a})\ \alpha\ \sigma(\mathfrak{a}) + \tau(\mathfrak{a})\ \alpha\ \delta(\mathfrak{a})\ for\ each\ \mathfrak{a} \in N\ and\ \alpha \in \Gamma$.

Definition 2.6

Suppose N is a $\Gamma - near\ ring$ and $f: N \to N$ is an additive mapping on N, σ and τ are endomorphisms on N, then f is called a Jordan generalized $\Gamma - (\sigma,\tau)$ $-$Derivation associated with Jordan $\Gamma - (\sigma,\tau)$ $-$Derivation on N if $f(\mathfrak{a}\ \alpha\ \mathfrak{a}) = f(\mathfrak{a})\ \alpha\ \sigma(\mathfrak{a}) + \tau(\mathfrak{a})\ \alpha\ \delta(\mathfrak{a})$ for all $\mathfrak{a} \in N$ and $\alpha \in \Gamma$.

Remark 2.7

Every generalized $\Gamma - (\sigma,\tau)$ $-$Derivation is Jordan generalized $\Gamma - (\sigma,\tau)$ $-$Derivation.But, the converse is not true in general.

Example 2.8

Assume N is a commutative $\Gamma - near\ ring$ and suppose $a \in N$ such that

$$\sigma(x)\ \Gamma\ \mathfrak{a}\ \Gamma\ \sigma(x) = \tau(x)\ \Gamma\ \mathfrak{a}\ \Gamma\ \tau(x) = \sigma(x)\ \Gamma\ \mathfrak{a}\ \Gamma\ \tau(x) = 0\ for\ each\ x \in N, but$$

$$\sigma(y)\ \Gamma\ \mathfrak{a}\ \Gamma\ \tau(x) \neq 0\ for\ some\ x\ and\ y\ such\ that\ x \neq y.$$

Define a map $f: N \to N$ by $f(x) = \sigma(x)\ \alpha\ \mathfrak{a} + \mathfrak{a}\ \alpha\ \tau(x)$ and $\delta: N \to N$ as follows $\delta(x) = \sigma(x)\ \alpha\ \mathfrak{a}$ where σ, τ are two endomorphism mappings, then f is a Jordan generalized $\Gamma - (\sigma,\tau)$ $-$Derivation, but it does not generalized $\Gamma - (\sigma,\tau)$ $-$Derivation.

Definition 2.9

Suppose N is a $\Gamma - near\ ring$ and δ be an additive mapping on N, σ and τ are endomorphismson N, then f is called a Jordan generalized triple $\Gamma - (\sigma,\tau) -$ Derivation $if\ \delta(\mathfrak{a}\ \alpha\ \mathfrak{b}\ \beta\ c) = \delta(\mathfrak{a})\alpha\ \sigma(\mathfrak{b})\beta\ \sigma(c) + \tau(\mathfrak{a})\alpha\ \delta(\mathfrak{b})\beta\ \sigma(c) +$ $\tau(\mathfrak{a})\alpha\ \tau(\mathfrak{b})\beta\ \delta(c)$,for each $\mathfrak{a}, \mathfrak{b}, c \in N$ and $\alpha, \beta \in \Gamma$.

To investigate the main theorem, The following Lemmas are needed.

Lemma 2.10

Let f be a Jordan generalized $\Gamma - (\sigma,\tau)$ $-$Derivationon $\Gamma - near\ ring\ N$, then for each $\mathfrak{a}, \mathfrak{b} \in N$ and $\alpha \in \Gamma$.

1-$f(\mathfrak{a}\ \alpha\ \mathfrak{b} + \mathfrak{b}\ \alpha\ \mathfrak{a}) = f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + f(\mathfrak{b})\alpha\sigma(\mathfrak{a}) + \tau(\mathfrak{b})\alpha\delta(\mathfrak{a})$.

2- $f(\mathfrak{a}\ \alpha\ \mathfrak{b}\ \beta\ c) = f(\mathfrak{a})\alpha\ \sigma(\mathfrak{b})\ \beta\ \sigma(c) + \tau(\mathfrak{a})\alpha\ \delta(\mathfrak{b})\ \beta\ \sigma(c) + \tau(\mathfrak{a})\alpha\ \tau(\mathfrak{b})\beta\ \delta(c)$.

Proof

1-$f((\mathfrak{a} + \mathfrak{b})\ \alpha\ (\mathfrak{a} + \mathfrak{b})) = f(\mathfrak{a} + \mathfrak{b})\ \alpha\ \sigma(\mathfrak{a} + \mathfrak{b}) + \tau(\mathfrak{a} + \mathfrak{b})\ \alpha\ \delta(\mathfrak{a} + \mathfrak{b})$

$$= (f(\mathfrak{a}) + f(\mathfrak{b})\ \alpha\ \sigma(\mathfrak{a}) + \sigma(\mathfrak{b})) + (\tau(\mathfrak{a} + \tau(\mathfrak{b})\ \alpha\ \delta(\mathfrak{a}) + \delta(\mathfrak{b}))$$

$$= f(\mathfrak{a})\alpha\ \sigma(\mathfrak{a}) + f(\mathfrak{a})\alpha\ \sigma(\mathfrak{b}) + f(\mathfrak{b})\alpha\ \sigma(\mathfrak{a}) + f(\mathfrak{b})\alpha\ \sigma(\mathfrak{b}) + \tau(\mathfrak{a})\ \alpha\ \delta(\mathfrak{a}) + \tau(\mathfrak{a})\ \alpha\ \delta(\mathfrak{b}) + \tau(\mathfrak{b})\ \alpha\ \delta(\mathfrak{a}) + \tau(\mathfrak{b})\ \alpha\ \delta(\mathfrak{b}) \quad ...(1)$$

on the other hand

$$f((\mathfrak{a} + \mathfrak{b})\alpha(\mathfrak{a} + \mathfrak{b})) = f(\mathfrak{a}\alpha\mathfrak{a} + \mathfrak{a}\alpha\mathfrak{b} + \mathfrak{b}\alpha\mathfrak{a} + \mathfrak{b}\alpha\mathfrak{b})$$

$$= f(\mathfrak{a}\alpha\mathfrak{a}) + f(\mathfrak{b}\alpha\mathfrak{b}) + f(\mathfrak{a}\alpha\mathfrak{b} + \mathfrak{b}\alpha\mathfrak{a})$$

$$= f(\mathfrak{a})\alpha\sigma(\mathfrak{a}) + \tau(\mathfrak{a})\alpha\delta(\mathfrak{a}) + f(\mathfrak{b})\alpha\sigma(\mathfrak{b}) + \tau(\mathfrak{b})\alpha\delta(\mathfrak{b}) + f(\mathfrak{a}\alpha\mathfrak{b} + \mathfrak{b}\alpha\mathfrak{a}) \quad ...(2).$$

From $eq.$ (1)and $eq.$ (2), we get

$$f(\mathfrak{a}\alpha\mathfrak{b} + \mathfrak{b}\alpha\mathfrak{a}) = f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + f(\mathfrak{b})\alpha\sigma(\mathfrak{a}) + \tau(\mathfrak{b})\alpha\delta(\mathfrak{a})$$

2- Replace α for β in Definition (2.9), and we get the required result.

Remark 2.11

Suppose f be a Jordan generalized $\Gamma-(\sigma,\tau)$ −Derivation on $\Gamma - near\ ring\ N$, then we define $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha$ as follows

$\Upsilon(\mathfrak{a},\mathfrak{b})\alpha = f(\mathfrak{a}\alpha\mathfrak{b}) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b})$ foreach $\mathfrak{a},\mathfrak{b} \in N$ and $\alpha \in \Gamma$.

Now, we present the properties of $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha$

Lemma 2.12

Let f be a Jordan generalized $\Gamma-(\sigma,\tau)$ −Derivation on $\Gamma - near\ ring\ N$, then

$1-\ \Upsilon(\mathfrak{a},\mathfrak{b})\alpha = -\Upsilon(\mathfrak{b},\mathfrak{a})\alpha$.

$2-\ \Upsilon(\mathfrak{a}+\mathfrak{b},c)\alpha = \Upsilon(\mathfrak{a},c)\alpha + \Upsilon(\mathfrak{b},c)\alpha$.

$3-\ \Upsilon(\mathfrak{a},\mathfrak{b}+c)\alpha = \Upsilon(\mathfrak{a},\mathfrak{b})\alpha + \Upsilon(\mathfrak{a},c)\alpha$.

$4-\ \Upsilon(\mathfrak{a},\mathfrak{b})\alpha+\beta = \Upsilon(\mathfrak{a},\mathfrak{b})\alpha + \Upsilon(\mathfrak{a},\mathfrak{b})\beta$.

Proof

1- From Lemma 2.10,

$$f(\mathfrak{a}\alpha\mathfrak{b} + \mathfrak{b}\alpha\mathfrak{a}) = f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + f(\mathfrak{b})\alpha\sigma(\mathfrak{a}) + \tau(\mathfrak{b})\alpha\delta(\mathfrak{a})$$
$$f(\mathfrak{a}\alpha\mathfrak{b}) + f(\mathfrak{b}\alpha\mathfrak{a}) = f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + f(\mathfrak{b})\alpha\sigma(\mathfrak{a}) + \tau(\mathfrak{b})\alpha\delta(\mathfrak{a})$$
$$f(\mathfrak{a}\alpha\mathfrak{b}) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) = -f(\mathfrak{b}\alpha\mathfrak{a}) + f(\mathfrak{b})\alpha\sigma(\mathfrak{a}) + \tau(\mathfrak{b})\alpha\delta(\mathfrak{a})$$
$$f(\mathfrak{a}\alpha\mathfrak{b}) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) = -(f(\mathfrak{b}\alpha\mathfrak{a}) - f(\mathfrak{b})\alpha\sigma(\mathfrak{a}) - \tau(\mathfrak{b})\alpha\delta(\mathfrak{a}))$$
$$\Upsilon(\mathfrak{a},\mathfrak{b})\alpha = -\Upsilon(\mathfrak{b},\mathfrak{a})\alpha.$$

2- $\Upsilon(\mathfrak{a}+\mathfrak{b},c)\alpha = f((\mathfrak{a}+\mathfrak{b})\alpha c) - f(\mathfrak{a}+\mathfrak{b})\alpha\sigma(c) - \tau(\mathfrak{a}+\mathfrak{b})\alpha\delta(c)$

$$= f(\mathfrak{a}\alpha c + \mathfrak{b}\alpha c) - (f(\mathfrak{a}) + f(\mathfrak{b}))\alpha\sigma(c) - (\tau(\mathfrak{a}) - \tau(\mathfrak{b}))\alpha\delta(c)$$
$$= f(\mathfrak{a}\alpha c) + f(\mathfrak{b}\alpha c) - f(\mathfrak{a})\alpha\sigma(c) - f(\mathfrak{b})\alpha\sigma(c) - (\tau(\mathfrak{a})\alpha\delta(c) - \tau(\mathfrak{b})\alpha\delta(c)$$
$$= f(\mathfrak{a}\alpha c) - f(\mathfrak{a})\alpha\sigma(c) - (\tau(\mathfrak{a})\alpha\delta(c) + f(\mathfrak{b}\alpha c) - f(\mathfrak{b})\alpha\sigma(c) - \tau(\mathfrak{b})\alpha\delta(c)$$
$$= \Upsilon(\mathfrak{a},c)\alpha + \Upsilon(\mathfrak{b},c)\alpha.$$

3- $\Upsilon(\mathfrak{a},\mathfrak{b}+c)\alpha = f(\mathfrak{a}\alpha(\mathfrak{b}+c)) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}+c) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}+c)$

$$= f(\mathfrak{a}\alpha\mathfrak{b} + \mathfrak{a}\alpha c)) - f(\mathfrak{a})\alpha(\sigma(\mathfrak{b}) + \sigma(c) - \tau(\mathfrak{a})\alpha(\delta(\mathfrak{b}) + \delta(c)$$
$$= f(\mathfrak{a}\alpha\mathfrak{b} + \mathfrak{a}\alpha c) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) - f(\mathfrak{a})\alpha\sigma(c) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) - \tau(\mathfrak{a})\alpha\delta(c)$$
$$= f(\mathfrak{a}\alpha\mathfrak{b}) + f(\mathfrak{a}\alpha c) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) - f(\mathfrak{a})\alpha\sigma(c) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) - \tau(\mathfrak{a})\alpha\delta(c)$$
$$= f(\mathfrak{a}\alpha\mathfrak{b}) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + f(\mathfrak{a}\alpha c) - f(\mathfrak{a})\alpha\sigma(c) - \tau(\mathfrak{a})\alpha\delta(c)$$
$$= \Upsilon(\mathfrak{a},\mathfrak{b})\alpha + \Upsilon(\mathfrak{a},c)\alpha.$$

4- $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha+\beta = f(\mathfrak{a}(\alpha+\beta)\mathfrak{b}) - f(\mathfrak{a})(\alpha+\beta)\sigma(\mathfrak{b}) - \tau(\mathfrak{a})(\alpha+\beta)\delta(\mathfrak{b})$

$$= f(\mathfrak{a}\alpha\mathfrak{b} + \mathfrak{a}\beta\mathfrak{b}) - (f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + f(\mathfrak{a})\beta\sigma(\mathfrak{b})) - (\tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + \tau(\mathfrak{a})\beta\delta(\mathfrak{b}))$$
$$= f(\mathfrak{a}\alpha\mathfrak{b}) + f(\mathfrak{a}\beta\mathfrak{b}) - (f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + f(\mathfrak{a})\beta\sigma(\mathfrak{b})) - (\tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + \tau(\mathfrak{a})\beta\delta(\mathfrak{b}))$$
$$= f(\mathfrak{a}\alpha\mathfrak{b}) - f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) - \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}) + f(\mathfrak{a}\beta\mathfrak{b}) - f(\mathfrak{a})\beta\sigma(\mathfrak{b}) - \tau(\mathfrak{a})\beta\delta(\mathfrak{b})$$
$$= \Upsilon(\mathfrak{a},\mathfrak{b})\alpha + \Upsilon(\mathfrak{a},\mathfrak{b})\beta.$$

Remark 2.13

If N is a $\Gamma - near\ ring$, then f is a generalized $\Gamma-(\sigma,\tau)$ −Derivation on N if and only if $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha = 0$

Lemma 2.14

Suppose f be a Jordangeneralized $\Gamma-(\sigma,\tau)-$ Derivationof a 2-torsion free prime $\Gamma - near\ ring\ N$ and suppose that $\mathfrak{a},\mathfrak{b} \in N$ and $\alpha,\beta \in \Gamma$.

Then $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a},\mathfrak{b}]\alpha + [\mathfrak{a},\mathfrak{b}]\alpha\,\beta\,m\,\beta\,\Upsilon(\mathfrak{a},\mathfrak{b})\alpha = 0$.

Proof

Initially, we compute the value of $f(\mathfrak{a}\alpha(\mathfrak{b}\beta m\beta\mathfrak{b})\alpha\mathfrak{a}) + \mathfrak{b}\alpha(\mathfrak{a}\beta m\beta\mathfrak{a})\alpha\mathfrak{b})$

Applying Lemma3.3 (iii)[22], then we get $f((\mathfrak{a}\alpha\mathfrak{b})\beta m\beta(\mathfrak{b}\alpha\mathfrak{a}) + (\mathfrak{b}\alpha\mathfrak{a})\beta m\beta(\mathfrak{a}\alpha\mathfrak{b}))$.Now, by applying Lemma 3.3 (iv) [22] and Lemma 3.8(i) [22] with some simple calculations, we get the proof.

Lemma 2.15.[22]

Let$N$be a 2-torsion free semiprime$\Gamma - near\ ring$ and suppose that$\mathfrak{a}, \mathfrak{b} \in N$. If $\mathfrak{a}\Gamma m\Gamma\mathfrak{b} + \mathfrak{b}\Gamma m\Gamma\mathfrak{a} = 0$ for each$\in N$, then $\mathfrak{a}\Gamma m\Gamma\mathfrak{b} = \mathfrak{b}\Gamma m\Gamma\mathfrak{a} = 0$.

Lemma 2.16

If N is a 2-torsion free semiprime$\Gamma - near\ ring$, then for each $\mathfrak{a}, \mathfrak{b} \in N$ and $\alpha, \beta \in \Gamma$, $\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha = [\mathfrak{a}, \mathfrak{b}]\alpha\,\beta\,m\,\beta\,\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha = 0$.

Proof

By Lemma (2.14), $\Upsilon(\mathfrak{a}, b)\alpha\,\beta\,m\,\beta[\mathfrak{a}, b]\alpha + [\mathfrak{a}, b]\alpha\,\beta\,m\,\beta\,\Upsilon(\mathfrak{a}, b)\alpha = 0$.

By Lemma(2.15), we get $\Upsilon(\mathfrak{a}, b)\alpha\,\beta\,m\ \beta[\mathfrak{a}, b]\alpha = [\mathfrak{a}, b]\alpha\,\beta\,m\,\beta\,\Upsilon(\mathfrak{a}, b)\alpha = 0$.

Lemma 2.17

Suppose$f$is a Jordan generalized$\Gamma - (\sigma, \tau) -$ Derivation of a 2-torsionfree prime $\Gamma - near\ ringN$and suppose that $\mathfrak{a}, b \in N$and $\alpha, \beta \in \Gamma$. Then $\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\beta[c, d]\alpha = 0$.

Proof

By Lemma 2.16, $\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha = 0 \ldots (1)$

Replace $\mathfrak{a} + c$ for $a$in (1), we get

$\Upsilon(\mathfrak{a} + c, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a} + c, \mathfrak{b}]\alpha = \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha + \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha + \Upsilon(c, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha + \Upsilon(c, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha =$

$\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha + \Upsilon(c, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha = 0$, which implies that

$$-\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha = \Upsilon(c, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha$$

Now, we have, $\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\ \beta[c, \mathfrak{b}]\alpha\,\gamma\,m\,\gamma\,\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha = 0$, so

$$-\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha\,\gamma\,m\,\gamma\,\Upsilon(c, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha = 0$$

By primness, we get

$$\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha = 0 \ldots (2)$$

Replace $\mathfrak{b} + d$ for b in (1)

$$\begin{aligned}\Upsilon(\mathfrak{a}, \mathfrak{b} + d)&\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b} + d]\alpha \\ &= \Upsilon(\mathfrak{a}, d)\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha + \Upsilon(\mathfrak{a}, d)\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha \\ &\quad + \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha + \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha \\ &= \Upsilon(\mathfrak{a}, d)\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha + \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha = 0\end{aligned}$$

$$\Upsilon(\mathfrak{a}, d)\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha = -\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha$$

We have, $\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha\,\gamma m\gamma\,\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha = 0$, so

$$-\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha\,\gamma\,m\,\gamma\,\Upsilon(\mathfrak{a}, d)\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha = 0$$

By primness, we get

$$\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha = 0 \ \ldots \ (3).$$

Now, replace $\mathfrak{a} + c$for $\mathfrak{a}$ and $\mathfrak{b} + d$ for $\mathfrak{b}$ in (1)

$$\begin{aligned}\Upsilon(\mathfrak{a}, \mathfrak{b})&\alpha\,\beta\,m\,\beta[\mathfrak{a} + c, \mathfrak{b} + d]\alpha \\ &= \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, \mathfrak{b}]\alpha + \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, \mathfrak{b}]\alpha + \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[\mathfrak{a}, d]\alpha \\ &\quad + \Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, d]\alpha\end{aligned}$$

From(1), (2) and(3) , we get $\Upsilon(\mathfrak{a}, \mathfrak{b})\alpha\,\beta\,m\,\beta[c, d]\alpha = 0$.

Theorem 2.18

Suppose that N is a commutative2-torsionfree prime $\Gamma - near\, ring$, then every Jordangeneralized$\Gamma - (\sigma,\tau) -$ Derivation is a generalized$\Gamma - (\sigma,\tau) -$Derivation.

Proof

Since$N$ is prime$\Gamma - near\, ring$and by Lemma 2.17, we get either $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha = 0or[c\,,d]\alpha = 0$. If $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha \neq 0$, then $[c,d]\alpha = 0$ and N is commutative Γ – $near - ring$,

$f(2\mathfrak{a}\alpha\mathfrak{b}\,) = 2\,(\,f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + \tau(\mathfrak{a})\alpha\delta(\mathfrak{b}))$, since N is 2 – torsion free,

then we obtain the required result.

$f(\mathfrak{a}\alpha\mathfrak{b}) = f(\mathfrak{a})\alpha\sigma(\mathfrak{b}) + \tau(\mathfrak{a})\alpha\delta(\mathfrak{b})$,implies that f is a generalized$\Gamma - (\sigma,\tau) - Derivation$.

If $\Upsilon(\mathfrak{a},\mathfrak{b})\alpha = 0$, then by Remark 2.13, we get f is generalized$\Gamma - (\sigma,\tau) -$Derivation.

Conclusion

In this paper, the notion of Jordan generalized $\Gamma - (\sigma,\tau) -$Derivation on prime $\Gamma - near\, ring$ is introduced. We also discuss some related concepts. Also, every Jordan generalized $\Gamma -\ (\sigma,\tau) -$Derivation is generalized $\Gamma -\ (\sigma,\tau)\ -$Derivation when N is a 2-torsion free prime $\Gamma - near\, ring$
is verified. Some of the important properties are also given.

Acknowledgements

The author would like to thank Mustansiriyah university- Baghdad – Iraq for its support in the present work.

References

[1] N., Nobusawa, On a Generalization of the Ring Theory, *Osaka J. Math.*,1: 81-89, 1964.
[2] W.E., Barnes, On the Gamma-Rings of Nobusawa, *Pacific J. Math..*,18: 411 – 422, 1966.
[3] L., Luh, On the Theory of Simple Gamma-Rings, *Michigan Math. J.*, 16: 65 – 75, 1969.
[4] S., Kyuno, On Prime Gamma-Ring, *pacific J. Math..*, 75: 185 – 190, 1975.
[5] B., Satyanarayana, Contribution to Near- Ring Theory, *Doctoral Thesis*, NagarjunaUniversity, India, 1984.
[6] E.F., Adhab, Near ring with Generalized Right n-Derivations, *Iraqi journal of Science*, 62(7): 2334 – 2342, 2021.
[7] B., Satyanarayana, A Note on Γ- Near Rings, *Indian J. of Math*., 41: 427 – 433, 1999.
[8] M.R., Khan, and M.M., Hasnain, Γ-Near Rings with Generalized Γ-Derivation, *NoVI SAD J. Math.*,44 (2): 19 – 28, 2014.
[9] B., Hvala, Generalized Derivations in Rings, *Communications in Algebra*, **26**(4): 1147 – 1166, 1998.
[10] K.K., Dey, A.C., Paul, and I.S., Rakhimov, Generalized Derivations in Semiprime *G*amma - Rings, *International Journal of Mathematics and Mathematical Sciences*. Volume **2012**, Article ID 270132, 14 pages, doi:10.1155/270132, 2012.
[11] K.K., Dey, and A.C., Paul, Generalized Derivations of Prime Gamma-Rings, *GANIT J. Bangladesh Math. Soc.*, (ISSN 1606-3694) 33: 33 -39, 2013.
[12] A., Mamouni, and M., Tamekkante, Commutativity of Prime Rings Involving Generalized Derivations, *Palestine Journal of Mathematics*, 10(2): 407 – 413, 2021
[13] Y., Ceven, and M., Ali Ozturk, On Jordan Generalized Derivations Gamma-Rings, *Hacettepe Journal of Mathematics and Statistics*,33: 11-14, 2004.
[14] X. K., Wang, Derivations in Prime Near – Ring, *Proc. Amer. Math. Soc.*,121: 361-366, 1994.
[15] H.E., Bell, and G., Mason, On Derivations in Near – Rings, Near-Rings and Near-Fields. *G. Betsch, ed.*) North-Holland, Amsterdam, 31 – 35, 1987.
[16] A.H., Majeed, and E.F., .Adhab, On Two Sided α-n- Derivation in Prime Near- Rings, *Iraqi Journal of Science*, 56(3): 2674 – 2681, 2015.

[17] Z.S., Alhaidary, and A.H., Majeed, Commutativity Results for Multiplication (Generalized) (α, β) Reverse Derivations on Prime Rings, *Iraqi Journal of Science*, 62(9) : 3102-3113, 2021.
[18] M., Ashraf, and A., Shakir, (σ, τ) –Derivations on Prime Near Rings. *Arch Math. (Brno)*, 40: 281-286, 2004.
[19] E.K., Sogutcu, and O., Golbasi, Comparison of Jordan (σ,τ) $-$Derivation and Jordan Triple $(\boldsymbol{\sigma}, \boldsymbol{\tau})$ - Derivations in Semiprime Rings. *Adiyaman university, Journal of Science*, 10(1): 264 – 272, 2020.
[20] Y., Jun, H., Kim, and Y., Cho, On Gamma - Derivations in Gamma-Near Rings, *Soochow J. Math.*29: 275 – 282, 2003.
[21] A.K., Kadhim, Jordan $\Gamma - (\sigma,\tau)$ $-$Derivation on Prime Γ-Near-Ring, *AIP conference proceedings (ISSN: 0094-243x), sixth national scientific/third international conference* , (Accepted for publication), 2021.
[22] S., Chakraborty. and A.C., Paul, On Jordan k-Derivation of 2- Torsion Free Prime Γ_N-Rings,*International Mathematical Forum*, **2**(57): 2823 – 2829, 2007.