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Calculations of the Quadrupole Moments for Some Nitrogen Isotopes in *p* and *psd* Shell Model Spaces Using Different Effective Charges

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Abstract.

The electric quadrupole moments for some nitrogen isotopes ($^{12,14,15,16,18}N$) are studied by shell model calculations with the proton-neutron formalism. Theoretical calculations performed using the different set of effective charges due to the core polarization effect. The effective charges in the *p*-shell nuclei are found to be slightly different from those in the *sd*-shell nuclei. Most of the results we have obtained are underestimated with the measured data for the isotopes considered in this work.

Keywords: Effective charges and Quadrupole moments of *p* and *psd*-shell isotopes (Z=7)**:** *p* and *psd*- shell model; Core-polarization effects.

حسابات العزوم رباعية القطب لبعض نظائر النيتروجين في *p* –*psd* نموذج فضاء القشرة باستعمال شحنات فعالة مختلفة.

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الخلاصة :

تمت دراسة العزوم رباعية القطب لبعض نظائر النتروجين من خلال حسابات نموذج القشرة وبصيغة برتون نيوترون . الحسابات النظرية التي اجريت باستخدام مجموعة مختلفة من الشحنات الفعالة المختلفة والناشئة عن تأثير استقطاب القلب. وجدت الشحنات الفعالة للنوى في قشرة *p* اختلاف بسيط من تلك نوى في غلاف *sd* . اغلب النتائج التي حصلنا عليها كانت تحت النتائج العملية للنظائر المفترضه في هذا العمل.

1. Introduction

In light and medium mass nuclei, the shell model has been known as one of the most successful models in describing the nuclear structure in both the ground state and the excited states [1]. The study of electric and magnetic moments provides an opportunity for detailed tests in nuclear wave functions. Especially, experimental data for quadrupole moments (Q moments) in nuclei give a measure of the extent to which the nuclear charge distribution deviates from spherical symmetry [2]. The shell model

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is known as it provides reasonable wave functions to describe these observables if appropriate effective operators are used [3].

The effective operators [4] were first introduced to take into account the effect of the larger model space than that adopted in truncated shell model calculations. A well-known example is the effective charge for electric quadrupole (E2) observables. The origin of the E2 effective charge is now well established in p- and sd-shell nuclei as being caused by virtual excitations of particles to E2 giant resonances [4]. The effective charges have been used commonly in the shell model calculations to study Q moments.

In the psd region, several effective interactions have been introduced in shell-model calculations, such as PSDMK [5], WBT [6], WBP [6] and SFO [7]. PSDMK, WBT and WBP interactions are all constructed in $(0-1)\hbar\omega$ model space, which means that $0-1$ nucleons are allowed to be excited from p shell to sd shell.

In present work, shell model calculations are performed with two effective interactions, Cohen-Kurath (CK) interactions in region *p* model space [8, 9] and Millener-Kurath PSDMK interaction in region *psd* model space [5]. Calculations shell model employed the proton neutron formalism to take account of the difference between them in the shell model wave functions [10] and ^{4}He is chosen as the core.

States of mixed configurations the situation differs in the valence shell (*p*-shell model for $N \leq 8$ and *psd*- shell for $N > 8$). Figure-1 indicates how nucleons move via the nucleon–nucleon interaction. The occupancy pattern of nucleons over different orbits is called configuration [11].

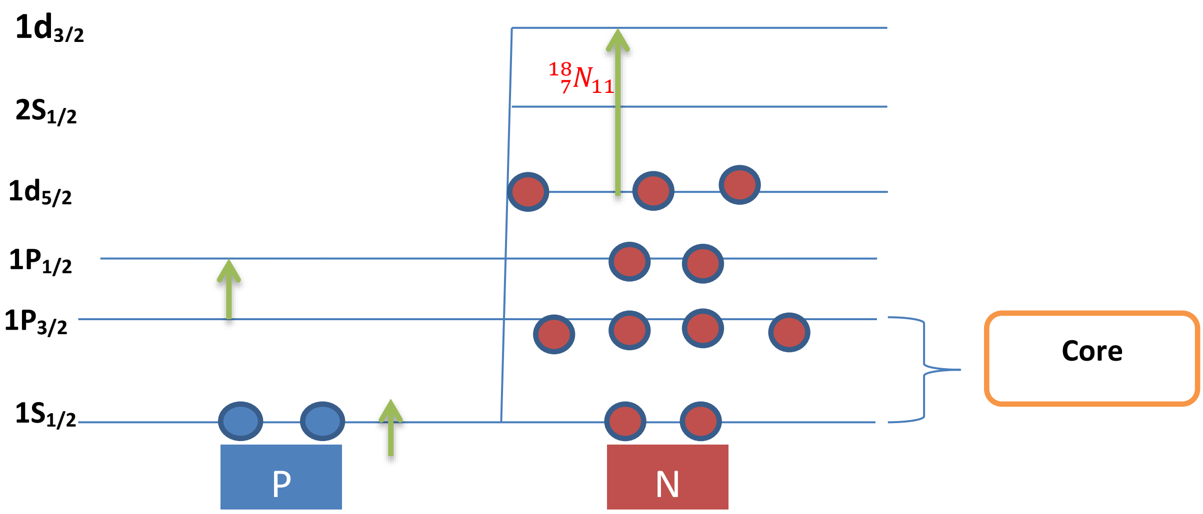

Figure 1-Mixing of different configurations due to the scattering between valence nucleons. A nucleon does not stay in an orbit forever.

Theory

The one-body electric multipole transition operator with multipolarity *J* for a nucleon is given by [12]

$$\hat{O}_{JM}(\vec{r})_k = r_k^J Y_{JM}(\Omega_K) \tag{1}$$

The reduced single-particle matrix element of the transition operator given in eq. (1) in spin space is given by

$$\langle j'||\hat{O}_{Jt_z}||j\rangle = e(t_z)\langle j'||Y_J||j\rangle\langle n'l'|r^J|n\,l\,\rangle \tag{2}$$

where;

$e(t_z) = \frac{1+\tau_z(k)}{2}$ is the electric charge of the *k*-th nucleon, where $t_z = 1/2$ for a proton and $t_z = -1/2$ for a neutron with

$\tau_z(k) = 2t_z(k)$

$\tau_z|p\rangle = |p\rangle$ and $\tau_z|n\rangle = -|n\rangle$

The reduce matrix element of the spherical harmonics part Y_J is given by [12]

$$\langle j'||Y_J||j\rangle = (-1)^{j+1/2}\sqrt{\frac{(2j+1)(2J+1)(2j'+1)}{4\pi}} \times \begin{pmatrix} j' & J & j \\ 1/2 & 0 & -1/2 \end{pmatrix} \times \frac{1}{2}\left[1+(-1)^{l+J+l'}\right] \quad (3)$$

and the radial part of the matrix element of a HO potential is:

$$\langle n'l'|r^J|n\,l\rangle = \int_0^\infty dr r^J r^2\, R_{n'l'}(r) R_{n\,l}(r) \quad (4)$$

Eq. (4) can be written as:

$$\langle n'l'|r^J|n\,l\rangle = \int_0^\infty dr r^\mu\, R_{n'l'}(r) R_{n\,l}(r) \quad (5)$$

where $\mu = J+2$.

The radial integral eq. (5) can solved analytically for a HO radial wave functions as [13].

$$\int_0^\infty dr r^\mu\, R_{n'l'}(r) R_{n\,l}(r) = \frac{2^3}{2^2 2}\sqrt{(n'-1)!\,(n-1)!}\,\sqrt{\Gamma(n'+l'+\tfrac{1}{2})\,\Gamma(n+l+\tfrac{1}{2})} \times \sum_{k'=0}^{n'-1}\sum_{k=0}^{n-1} \frac{(-1)^{k'+k}}{(n'-k'-1)!\,(n-k-1)!\,k'!\,k!\,\Gamma\left(k'+l'+\frac{3}{2}\right)\Gamma\left(k+l+\frac{3}{2}\right)} \times \frac{b^{2m+1}}{b^{3+l'+l+2k'+2k}}\Gamma(m+\tfrac{1}{2}) \quad (6)$$

where;

$$m = \frac{1}{2}(\mu + l' + l + 2k' + 2k)$$

$$\frac{b^{2m+1}}{b^{3+l'+l+2k'+2k}} = b^{\mu-2}$$

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$$

The reduced electric matrix element between the initial and final nuclear states is [12]

$$M(EJ) = \left\langle J_f \right|\left| \sum_k e(k)\hat{O}_J(\vec{r})_k \right|\left| J_i \right\rangle \quad (7)$$

where $e(k)$ is the electric charge for the k-th nucleon. Since $e(k) = 0$ for neutron, there should appear no direct contribution from neutrons; however, this point requires further attention: The addition of a valence neutron will induce polarization of the core into configurations outside the adopted model space. Such core polarization effect is included through perturbation theory which gives effective charges for the proton and neutron. The reduced electric matrix element can be written in terms of the proton and neutron contributions

$$M(EJ) = \sum_{t_z} e(t_z)\,\langle J_f||\hat{O}_J(\vec{r},t_z)||J_i\rangle \quad (8)$$

where $\langle J_f||\hat{O}_J(\vec{r},t_z)||J_i\rangle$ is the electric matrix element which is expressed as the sum of the products of the one-body density matrix (*OBDM*) times the single-particle matrix elements,

$$\langle J_f||\hat{O}_J(\vec{r},t_z)||J_i\rangle = \sum_{jj'} OBDM(J_i,J_f,J,t_z,j,j')\,\langle j'||\hat{O}_J(\vec{r},t_z)||j\rangle \quad (9)$$

with j and j' label single-particle states for the shell model space.

The role of the core and the truncated space can be taken into consideration through a microscopic theory, which combines shell model wave functions and configurations with higher energy as first order perturbation to describe *EJ* excitation: these are called *CP* effects. The reduced matrix elements of the transition operator is expressed as a sum of the model space (*MS*) contribution and the *CP* contribution, as follows:

$$M(EJ) = \sum_{t_z}\left[e(t_z)\,\langle J_f||\hat{O}_J(\vec{r},t_z)||J_i\rangle_{MS} + e\langle J_f||\Delta\hat{O}_J(\vec{r},t_z)||J_i\rangle_{CP}\right] \quad (10)$$

Similarly, the *CP* electric matrix element is expressed as the sum of the products of the *OBDM* times the single-particle matrix elements [14],

$$\langle J_f||\Delta\hat{O}_J(\vec{r},t_z)||J_i\rangle = \sum_{jj'} OBDM(J_i,J_f,J,t_z,j,j')\,\langle j'||\Delta\hat{O}_J(\vec{r},t_z)||j\rangle \quad (11)$$

The single- particle matrix element of the *CP* term is

$$\langle j'||\Delta\hat{O}_J||j\rangle = \left\langle j' \right|\left| \hat{O}_J \frac{\hat{Q}}{E_i - H_\circ} V_{res} \right|\left| j \right\rangle + \left\langle j' \right|\left| V_{res} \frac{\hat{Q}}{E_i - H_\circ} \hat{O}_J \right|\left| j \right\rangle \quad (12)$$

where the operator $\hat{Q}$ is the projection operator onto the space outside the model space. The single particle *CP* terms given in eq. (10) are written as [12]

$$\langle j' || \Delta\hat{O}_J || j \rangle = \sum_{j_1 j_2 \lambda} \frac{(-1)^{j+j_2+\lambda}}{\varepsilon_j - \varepsilon_{j'} - \varepsilon_{j_1} + \varepsilon_{j_2}} (2\lambda + 1) \begin{Bmatrix} j' & j & J \\ j_2 & j_1 & \lambda \end{Bmatrix}$$
$$\times \sqrt{(1+\delta_{j_1 j'})(1+\delta_{j_2 j})} \, \langle j' j_1 | V_{res} | j j_2 \rangle_\lambda \langle j_2 | \hat{O}_J | j_1 \rangle \quad (13)$$

+terms with j_1 and j_2 exchanged with an overall minus sign, where the index j_1 runs over particle states and j_2 over hole states and ε is the single-particle energy. For the residual two-body interaction V_{res}, the two-body M3Y interaction of Bertsch *et al*. [15] is adopted.

The electric matrix element can be represented in terms of only the model space matrix elements by assigning effective charges ($e^{eff}(t_z)$) to the protons and neutrons which are active in the model space,

$$M(EJ) = \sum_{t_z} e^{eff}(t_z) \langle J_f || \hat{O}_2(\vec{r}, t_z) || J_i \rangle_{MS} \quad (14)$$

They formulated an expression for the effective charges to explicitly include neutron excess via [16]

$$e^{eff}(t_z) = e(t_z) + \delta e(t_z), \delta e(-t_z) = Z/A - 0.32(N-Z)/A - 2t_z[0.32 - 0.3(N-Z)/A] . \quad (15)$$

The electric quadrupole moment in a state $|J = 2\ M = 0 >$ for $J_i = J_f$ is [12]:

$$Q(J=2) = \begin{pmatrix} J_i & J & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{16\pi}{5}} M(EJ) \quad (16)$$

Results and discussion

The conventional approach to supplying this added ingredient to shell model wave functions is to redefine the properties of the valence nucleons from those exhibited by actual nucleons in free space to model effective values [17]. Effective charges are introduced to take into account effects of model-space truncation. The quadrupole moment gives a useful measure of how the core is polarized especially if the valence nucleons are neutrons which do not directly participate to the electric quadrupole moment.

Shell model calculations are performed with NuShellX [18] with the p model space for neutron number N ≤ 8, which covered the orbits $1s_{1/2}$, $1p_{1/2}$ and $1p_{3/2}$ 1d3/5 and psd model space for N > 8 to find the one body matrix elements (OBME) with harmonic oscillator parameter and the radial wave functions for the single-particle matrix elements are calculated with the HO potential. The size parameters *b* is calculated for each nucleus with mass number *A* as:

$$b = \sqrt{\frac{\hbar}{M_P \omega}} \ , \quad \hbar\omega = 45A^{-1/3} - 25A^{-2/3} \qquad Mp = \text{mass of proton [17].}$$

Shell-model in the *p* shell is used for neutron number $N \leq 8$. For neutron with $N \geq 8$, *psd* shell is used with *p* shell full protons and (*N*-8) neutrons. Our calculations used two kinds intractions in region *p* model space (CK) interaction [8, 9] and used in region the *psd* model space PSDMK interaction [5]. Effective charges are needed because of the polarization of the core which is not included in the model space [19, 20]. One set of effective charges, conventional effective charges *ep* = 1.3 and *en* = 0.5, is suitable for *sd*-shell nuclei [1], which means that both valence protons and neutrons are excited in the *sd*-shell. For valence protons and/or neutrons locate in *p* shell in neutron-rich nuclei, this set of effective charges becomes invalid [21, 22]. Two set of effective charges, Bohr-Mottelson (B-M) [16] are calculated according to equation (15) and tabulated in Table- 1. Three sets of effective charges, standard effective charges (ST) ep = 1.36 and en = 0.45 [19].

The quadrupole moments are calculated for nitrogen isotopes with mass number *A* = 12, 14, 15, 16, 18 and with neutron number *N* = 5, 7, 8, 9, 11, respectively. Figure-2 and Table-1 shows the quadrupole moments in N isotopes with three sets of effective charges, to calculate quadrupole moments and comparison with experimental values are taken from Refs. [23].

The calculated quadrupole moments using these effective charges are underestimated with the experimental data of Ref. [23], are shown in Figure-2 as a function of neutron number *N*.

Conclusions

The results that obtained from this work have been deduced by using the shell model calculations. The main conclusions that may be drawn from the calculations are briefly summarized as:

1- The *p* model space for neutron number $N \leq 8$, which covered the orbits $1s_{1/2}$, $1p_{1/2}$ and $1p_{3/2}$ and *psd* model space for $N > 8$.
2- Two effective interactions, Cohen-Kurath (CK) interactions in region p model space and illener-Kurath PSDMK interaction in region psd model space.
3- In our calculations employed different effective charges.
4- Calculations of *Q* moments with standard effective charges and conventional effective charges

are better than of B-M effective charges. In Figure-2, it may be seen clearly that this set of effective charges is closer to the experimental values.

5- Most of the results we have obtained are much underestimated with this set of effective charges.

Table 1- Quadrupole moments in units efm2 of calculated with HO potential for Nitrogen (N) isotopes (Z=7).

A, N $_7$N	J_i^π	$b(fm)$	e_p	e_n	$Q_{theo.}$ B-M , ST	$Q_{theo.}$ ep=1.3,en= 0.5	$Q_{exp.}$
12,5	1^+	1.67	1.27	1.01	1.315, 0.72	0.768	+0.98±0.09
14, 7	1^+	1.70	1.18	0.82	1.56, 1.41	1.40	+2.0±0.03
15,8	$5/2^+$	1.71	1.15	0.75	-2.63,-1.58	-1.76	
16, 9	2^-	1.727	1.12	0.68	-3.243, -2.14	-2.38	(-)1.8±0.02
18, 11	1^-	1.751	1.06	0.57	1.86, 1.47	1.63	+2.7±0.04

Experimental quadrupole moments are taken from Stone [23]. Quadrupole moments calculated with effective charges of B-M model [16], standard effective charges for proton and neutron 1.36, 0.45 [19], respectively and convention effective charges proton and neutron1.3, 0.5 [1], respectively are presented.

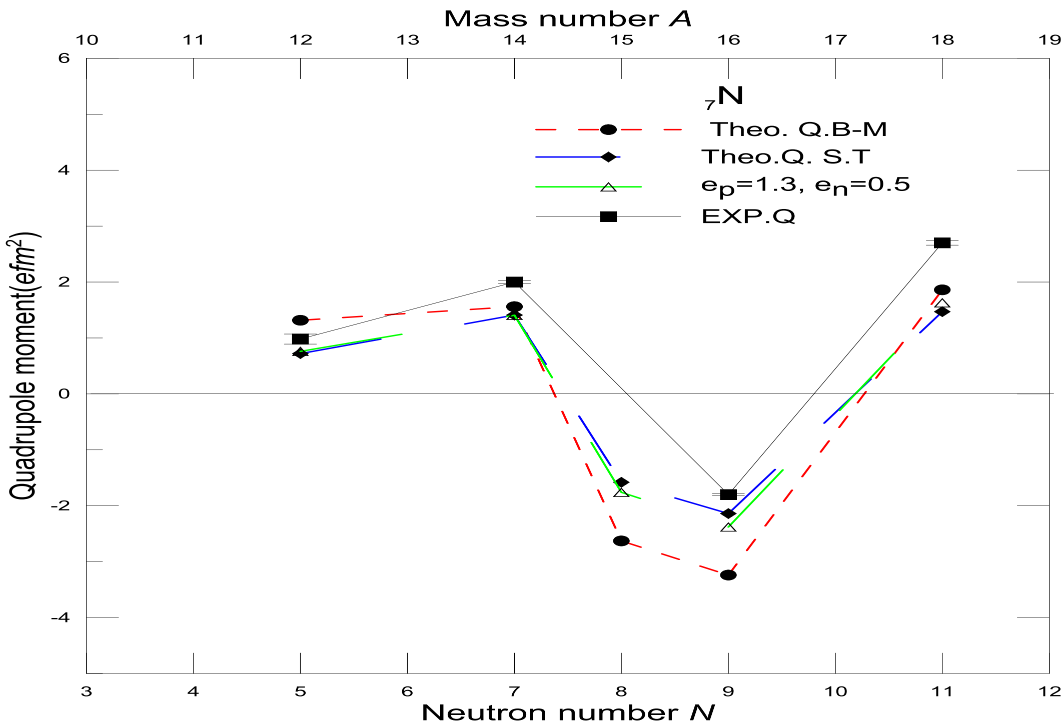

Figure 2- Experimental [23] and theoretical quadrupole moments with versus Neutron and mass number for N isotopes.

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