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Evaluation of Baghdad water quality using Fuzzy logic

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Abstract
Fuzzy measures are considered important tools to solve many environmental problems. Water pollution is one of the environmental problems, which has negatively effect on the health of consumers. In this paper, a mathematical model is proposed to evaluate water quality in the distribution networks of Baghdad city. Fuzzy logic and fuzzy measures have been applied to evaluate water quality with respect to chemical and microbiological contaminants. Our results are evaluate water pollution of some chemical and microbiological contaminants, which are difficult to evaluation through traditional methods.

Keywords: Fuzzy Logic, Fuzzy Integrals, Binary Element Sets, Fuzzy Cognitive Maps, Water Quality, Distribution Networks.

تقييم جودة مياه بغداد باستخدام المنطق الضبابي

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الخلاصة
تعتبر القياسات الضبابية ادوات هامة لحل الكثير من المشاكل البيئية. تلوث المياه هي احدة من المشاكل البيئية ، والذي له تاثير سلبي على صحةالمستهلكين. في هذا البحث، اقترحنا نموذج رياضي لتقييم جودة المياه في شبكات توزيع مدينة بغداد.المنطق الضبابي والقياسات الضبابية تم تطبيقهما لتقييم جودة المياه من حيث الملوثات الكيميائية والمكروبيولوجية. نتائجنا تقييم تلوث المياه لبعض الملوثات الكيميائية والمكروبيولوجية، والتي يصعب تقييمها بالطرق التقليدية.

1. Introduction

Recently, most real-life problems are characterized by uncertainty and complexity due to human environment. Soft computing techniques have the ability to provide an appropriate framework of representing uncertain, imprecise and vague concept. Fuzzy measures are framework of a class of none-additive measures which have ability to represent, manage the uncertain information and imprecise data. Also they have ability to interact with fuzzy integrals such as Choqet integral providing tools of decision making under uncertainty [1]. Choquet [2] in 1954 proposed a functional with respect to a capacity referred now as the Choquet integral. Sugeno [3] in 1974 proposed fuzzy integrals and is now most usually called the Sugeno integral. The Choquet and Sugeno integrals have been applied in the multi-criteria decision making problems (see, [4, 5]).

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On the other hand, fuzzy cognitive map (FCM) is a modeling methodology for simulation and analysis of complex system based on experience and exploiting knowledge. It is a combination of fuzzy logic and Neural Networks [6]. Kosko [7] proposed FCM as popularize of CM as a signed directed graph with nodes and causal relation among nodes taking values in the interval [-1,1].

Some environmental problems, such as pollution of water in distribution networks are complex, uncertainty, and need a wide range of human knowledge and expertise to address them. Diagnosis of pollution in water distribution networks is a difficult task because of many factors that interact with each other's.

Many researchers have contributed to the development strategies to evaluate water quality. Ruey-Tyng Lin, wen- Chen Huang [8] they used fuzzy set to assess water quality rating to investigate the appropriateness of its use in evaluating water quality. In this paper, we applied fuzzy measures with fuzzy cognitive map to evaluate water quality in the distribution networks of Baghdad city according to five chemical and microbiological factors. The structure of this paper is as follows. In section 2, we recall the concept of fuzzy measures and fuzzy integrals based on the bi-element sets. In section 3, we discuss construction and development of fuzzy cognitive map. In section 4, we apply fuzzy measure and its integral based on the bi-element set to evaluate water quality failures in distribution networks of Baghdad city.

2. Fuzzy logic and fuzzy integrals based on bi-element sets

Suppose we have a set of criteria $X = \{1,2,\dots,n\}$. Since the criteria have not always the same importance (weight) and defining a fuzzy measure on a set of criteria can be seen as a way of modeling the interaction phenomena existing among these criteria. Hence, every element i in the set of criteria X has two influences, positive influence represented as (i^+) and negative influence represented as (i^-), and this element is called binary elements (simply bi-element). The set which contains i^+ or i^- for all i. $i = 1,2 \dots, n$ is called binary element set (bi-element set). For this case, we can define the set of all possible combinations of binary elements of n criteria by $(X) = \{\{\Im_1,\dots,\Im_n\} : \forall \Im_i \in \{i^+, i^-\},\ i = \{1,\dots,n\}$, which is equivalent to the power set $P(X)$ in the notion of classical set theory. Due $B(X)$ identified with $\{0,1\}^n$, then the number of elements of $B(X)$ is 2^n. Also the binary alternative of any binary element set $S \in B(X)$ is denoted by $(\Im_1,\dots,\Im_n)$ with $\Im_i = 1\ if\ i^+ \in S$ and $\Im_i = 0\ \ if\ i^- \in S,\ i = 1,\dots,n$. For more details see [9, 10].

Now, we can define fuzzy measure based on bi-element sets as follows :

Definition 3 [9]: A fuzzy measure based on the bi-element set X is a set function $g: B(X) \to [0,1]$ Satisfies the following axioms :

- $g(X^-) = g(\{1^-,\dots,n^-\}) = 0$
- $g(X^+) \coloneqq g(\{1^+,\dots,n^+\}) = 1$ (Boundary condition)
- If $S \subseteq T \subseteq X$, then $g(S) \le g(T),\ \forall S,T \in \mathcal{B}(X)$ (monotonicity)

Where $B(X)$ is the set of all $bi-element\ set$ of X, $g(S)$ represents the weight of importance the set of criteria S. The monotonicity conditions mean that adding new element to coalition does not decrease $g(X)$.

A specific class of fuzzy measure based on $bi-elemet\ set$ which is λ – fuzzy measure based on bi-element sets and denoted by (g_λ).

Definition 4: Let $B(X)$ be the set of bi-element sets X. A set function $g_\lambda: B(X) \to [0,1]$ is called λ – fuzzy measure based on bi-element sets if it satisfies the following axioms :

1. $g_\lambda(X^+) = 1$.
2. If $S \cap T = X^-$, then $g_\lambda(S \cup T) = g_\lambda(S) + g_\lambda(T) + \lambda\, g_\lambda(S)\, \mu_\lambda(T)$ where λ ∈(-1,∞).

The value of λ is obtained through the boundary condition, $g(X^+) = 1$, this gives a polynomial equation (Eq.(1)) with respect to λ:

$$1 + \lambda = \prod_{i=1}^{n}\left(1 + \lambda g_\lambda(i^+)\right) - 1 \qquad (1)$$

Also, the λ – fuzzy measure based on the bi-element sets over the given set $A \subseteq X^+$ is computed using equation -2:

$$g_\lambda(A) = \frac{1}{\lambda}\left[\prod_{x_i \in A}\left(1 + \lambda g_\lambda(i^+)\right) - 1\right] \quad (2)$$

Here, we present an equivalent model of Choquet and Sugeno integrals with respect to fuzzy measure based on the bi-element sets, Assume $\boldsymbol{f} = \{f(x_{1^+}), \dots\dots, f(x_{i^+}), \dots, f(x_{n^+})\}$, $x_{i^+} \in R$ with $\mathrm{i} = \{1, \dots, n\}$ described each alternative in multi-criteria decision making problem and let $X^+ = \{i^+ | x_{i^+} \in R$ for each $i = \{1, \dots\dots, n\}\}$, thus the definition of choquet integral based on bi-element sets of input X as follows:

Definition 5 [9]: Let $g: B(X) \to [0,1]$ be a fuzzy measure based on bi- element set, where $B(X)$ is the set of all bi-element sets. Then the Choquet integral of $\boldsymbol{f}$ with respect to g is given by equation -3

$$ChI_g = \int \boldsymbol{f} dg = \sum_{i=1}^{n}\left[f\left(x_{\sigma(i^+)}\right) - f(x_{\sigma((i-1)^+)})\right]\ g\left(A_{\sigma(i^+)}\right) \quad (3)$$

where $A_{\sigma(i^+)} = \{\dots, \sigma((i-1)^-), \sigma(i^+), \dots, \sigma(n^+)\}$, is bi-element set $\subseteq X^+$, and σ is a permutation on X^+, and $0 \le f\left(x_{\sigma(1^+)}\right) \le \dots \le f\left(x_{\sigma(n^+)}\right) \le 1$ represents the order of $f\left(x_{\sigma(i^+}\right)$ in a set of bi-element X in increasing order, with $f(x_0) = 0$.

The definition of Sugeno integral based on bi-element sets as follows:

Definition 6: Let $g: B(X) \to [0,1]$ be a fuzzy measure based on bi- element set, where $B(X)$ is the set of all bi-element sets. Then the Sugeno integral of $\boldsymbol{f}$ with respect to g is given by equation - 4

$$SuI_g = \int \boldsymbol{f} dg = \bigvee_{i=1}^{n}\left(f\left(x_{\sigma(i^+)}\right) \bigwedge g\left(A_{\sigma(i^+)}\right)\right) \quad (4)$$

where, $A_{\sigma(i^+)} = \{\dots, \sigma((i-2)^-), \sigma((i-1)^-), \sigma(i^+), \dots, \sigma(n^+)\}$, is bi-element set $\subseteq X^+$, and σ is a permutation on X^+, so that $0 \le f\left(x_{\sigma(1^+)}\right) \le \dots \le f(x_{\sigma(n^+)}) \le 1$, with the condition $x_{\sigma((n+1)^+)} = 0$

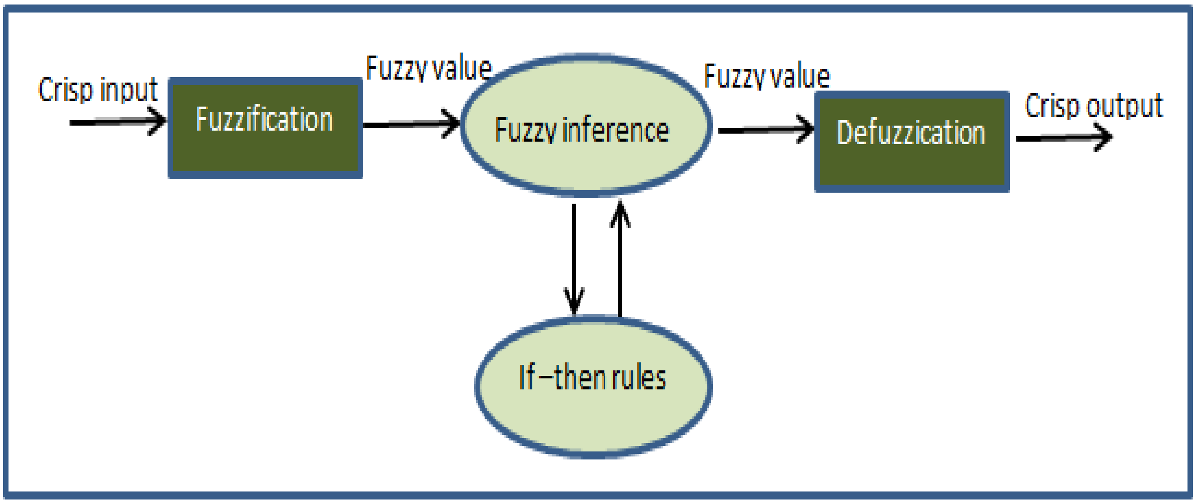
3. Construction and development of fuzzy cognitive map

Fuzzy cognitive map (FCM) is a qualitative soft computing method that represents domain knowledge on a map by nodes and directed connection weights between them to represent factors and relationships between factors respectively, with taking into account the degree of uncertainty that characterize these relationships to the real world using fuzzy logic [11]. causal relations between concepts of FCM can be represented by $n \times n$ influence matrix $\mathbf{W} = [w_{ij}]$. There are three possible types of the causal relationships between concepts of FCM, these are: $w_{ij} > 0$, which means a decrease (or increase) in the value of concept f_i will cause a decrease (or increase) in the value of concept f_j or $w_{ij} < 0$, which means decrease (or increase) in the value of concept f_i will cause an increase (or decrease) in the value of concept f_j. There is no relation between concept f_i and concept f_j, when $w_{ij} = 0$. Mathematically, the value $\boldsymbol{f_i}$ for each node f_i is computed by equation -5

$$\boldsymbol{f_i}(t) = \mathcal{T}\Big(\sum_{\substack{j=1 \\ j \neq i}}^{n} \boldsymbol{f_j}(t-1)\, w_{ji}\Big) \quad (5)$$

where, $\boldsymbol{f_i}(t)$ is the value of concept f_i at simulation step t, $\boldsymbol{f}_j(t-1)$ is the value of concept f_j at simulation step t-1, w_{ji} is the weight of causal relation between concept f_j and concept f_i, and $\mathcal{T}$ is the sigmoid function $\mathcal{T}(x) = \frac{1}{1+e^{-\alpha x}}$ which is used to transform the result in to the fuzzy interval [0,1]. where $\alpha > 0$ is a parameter that determines its steepness [12].

The construct and design of fuzzy cognitive map is based on the experts. Experts identify the number and kind of concepts which consist of fuzzy cognitive map and the relationships between concepts. The methodology for developing fuzzy cognitive map is based on fuzzy logic. At first, Fuzzification model is used to transform the input real values into membership of the term linguistic variable grade to fuzzy set and they use if- then rule to justify the cause and effect relation among concepts and infer a linguistic weight for each interconnection, then Fuzzy inference engine is used to combine the fuzzy if-then rule in the fuzzy value. Finally, defuzzification model is used to convert the fuzzy values output by the inference procedure into a crisp number that can be used in the real world. Figure- 1 presents the stages of fuzzy expert systems, for more details see [13-16].



`**Figure 1-** The stage of fuzzy expert [13] [system[

4. Application of fuzzy measures with FCM to evaluate water quality

In this section, we apply fuzzy measures and fuzzy integrals based on bi-element sets with the help of fuzzy cognitive map to evaluate water quality in the distribution networks of Bagdad city.

4.1 FCM for water quality failures

There are many factors that causes of water pollution in the distribution networks of Baghdad city. In our application, we have identified five key concepts (factors) to model water quality deterioration in the distribution networks of Baghdad city which are considered as chemical and microbiological contaminants ((CH-M)WQF). These factors are: Arsenic ($f_{1(AS)}$), Dichloromethane ($f_{2(CH2CL2)}$), chlorine ($f_{3(CL)}$), Total Coli forms ($f_{4(TC)}$), and Fecal Coli forms ($f_{5(FC)}$). Figure-2 gives the proposed model FCM for describing water quality failures in the distribution networks of Baghdad city, whose nodes were assigned with concepts f_1, f_2, f_3, f_4 and f_5 , whereas edges with linguistic weights. The corresponding **W** matrix is given by the formula (1)

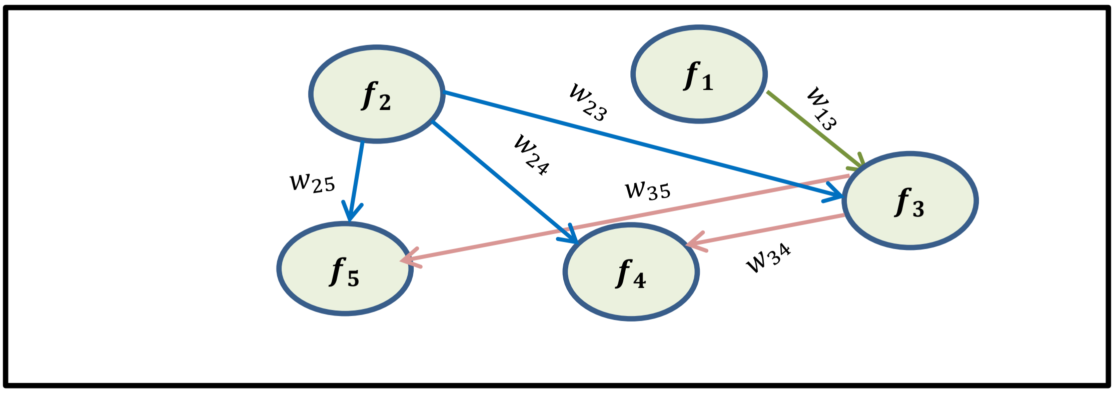


Figure 2- FCM for evaluate water quality in the distribution networks

$$\boldsymbol{W} = \begin{bmatrix} 0 & 0 & w_{13} & 0 & 0 \\ 0 & 0 & w_{23} & w_{24} & w_{25} \\ 0 & 0 & 0 & w_{34} & w_{35} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (1)$$

The initial values of state factor are chosen by experts. Suppose the values of the initial state factors of the proposed FCM model are f(0)=[0.02 0.04 0.6 0 0] at time step 0.

To calculate the activation level of factors that contribute to water quality failures fuzzy expert systems are need to be considered. First, we need to calculate the values of the initial weights matrix which correspond to the initial state factors. Assuming there are seven linguistic variables are used to express the relationship among concepts of FCM. The corresponding memberships functions that describe these linguistic variables are shown in Figure-4 and they are: μ_{NL}, μ_{NM}, μ_{NS}, μ_{Z}, μ_{PS}, μ_{PM}, μ_{PL}

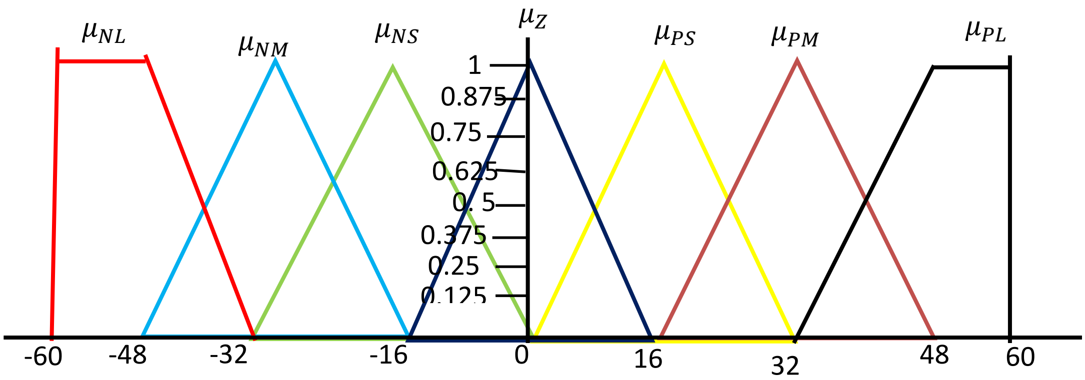


Figure 3- The seven membership functions corresponding to each of one of the seven linguistic variables

The fuzzy rule-base of water quality failures in the distribution networks presented in Table- 1**,** for more details see [16].

Table 1- Fuzzy rule-base of water quality failures

Linguistic variables	NL (-3)	NM (-2)	NS (-1)	Z (0)	PS(1)	PM (2)	PL (3)
NL (-3)	NL	NL	NL	NL	NM	NS	Z
NM (-2)	NL	NL	NL	NM	NS	Z	PS
NS (-1)	NL	NL	NM	NS	Z	PS	PM
Z (0)	NL	NM	NS	Z	PS	PM	PL
PS(1)	NM	NS	Z	PS	PM	PL	PL
PM (2)	NS	Z	PS	NM	PL	PL	PL
PL (3)	Z	PS	PM	PL	PL	PL	PL

Here, the numbers -3,-2,-1,0,1,2,3 represent the numeric values of the linguistic variables $NL, NM, NS, Z, PS, PM, PL$ respectively.

Fuzzy rule-base of the form if- then rules is used to describe the relationships (w_{ij}) between two concepts of fuzzy cognitive map proposed to evaluate water quality in the distribution networks of Baghdad city. For example, if- then rules for relation between f_1 and f_3 are:

R_1: $if\ f_{1(AS)}\ is$ PS and $f_{3(CL)}\ is\ PL\ then\ B_{R_1}\ is\ PL,$

R_2: $if\ f_{1(AS)}\ is$ Z and $f_{3(CL)}\ is\ PL\ then\ B_{R_2}\ is\ PL$
we take the minimum value of the membership of f_1 and f_3 (minimum operation between two fuzzy set)
$\mu_{B_{R_1}} = \{ 0, 0, \dots, 0, 0.125, 0.125, 0.125, \dots\dots, 0.125\}$,
$\mu_{B_{R_2}} = \{0,0, \dots\dots, 0, 0.125, 0.25, 0.375, 0.5, 0.625.0.75, 0.875, \dots\dots, 0.875\}$.
$\mu_{B_{R_1}}$ is the fuzzy value of the output of the rule R_1 in the universe of discourse of PL.
$\mu_{B_{R_2}}$ is the fuzzy value of the output for the rule R_2 in the universe of discourse of PL .
By using max composition method, we obtain the output of rules R_1 and R_2(degree of activation of these rule) in the single fuzzy value. Therefore,
$B = \{0, \dots, 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 0.875, \dots\dots\ 0.875\}$.
Finally, using the center of gravity (COG) defuzzification method

$$COG(B) = \frac{\sum_{i=1}^{N}(B(I_i)I_i}{\sum_{i=1}^{N} B(I_i)} \Rightarrow COG(B) = \frac{477.75}{9.625} \simeq 49.636$$

where N is the number of quantization used to discrete the membership function B of the fuzzy output
Then,

$$w_{13} = \frac{49.636}{100} = 0.49636 \simeq 0.496$$

similarly, we can determine all the weights of the FCM model proposed for the water quality failures in distribution network. Thus, the initial weight matrix W_{ij} , $i, j = 1,2,3,4,5$, with $W_{ii} = 0$, is obtained

$$W^{initial} = \begin{bmatrix} 0 & 0 & 0.496 & 0 & 0 \\ 0 & 0 & 0.611 & 0.196 & 0.196 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

After applying inference algorithm of FCM, fuzzy cognitive map reaches a fixed equilibrium point $\boldsymbol{f}(\mathrm{t}) = \boldsymbol{f}(\mathrm{t}-1)$ after only five iteration. The equilibrium fixed point is $\boldsymbol{f}(5)$ = [0.5, 0.5, 0.6349, 0.6249, 0.6249].

4.2 Fuzzy measures and fuzzy integral based on bi-element set to evaluate water quality

Fuzzy measures based on bi-element sets are used to account for the interconnectedness of these factors. Let the fuzzy measure based on bi-element for sample space (CH-M)WQF is

$$g = [0.25 \quad 0.25 \quad 0.6 \quad 0.45 \quad 0.45]$$

The value of $\lambda = -0.9033$. The values of $\lambda-$fuzzy measure based on bi-element for all subset of g, it is calculated using equation 2. To apply Choquet integral, rearrange the values of the state vector at five iterations, which correspond to the measurable function in increasing order as follows:

$$0.5 \leq 0.5 \leq 0.6024 \leq 0.6024 \leq 0.6349$$

Using equation 3, the activation level for (CH-M) WQF can be determined as follows :

$$ChI_{(CH-M)WQF} = \sum_{i=1}^{5}[f(x_{\sigma(i^+)}) - f(x_{\sigma(i-1)^+})]\ g(A_{\sigma(i^+)})$$

$$ChI_{(CH-M)WQF} = [0.5-0](1) + [0.5-0.5](0.922) + [0.6024-0.5](0.796) + [0.6024-0.6024](0.7099) + [0.6349-0.6024](0.6) \simeq 0.601$$

Therefore, the activated level of CH-M)WQF $\simeq$ 0.601,this mean that the rate of chemical and microbiological contamination in distribution networks of Bagdad city is 0.601.
Now apply Sugeno integral based on bi-element sets**:**

$$SuI_{(CH-M)WQF} = \bigvee_{i=1}^{5}\left(f(x_{\sigma(i^+)}) \bigwedge g(A_{\sigma(i^+)})\right)$$

$SuI_{(CH-M)WQF} = (0.5 \wedge 1) \vee (0.5 \wedge 0.922) \vee (0.6024 \wedge 0.796) \vee (0.6024 \wedge 0.7099) \vee (0.6349 \wedge 0.6)$
$= 0.5 \vee 0.5 \vee 0.6024 \vee 0.6024 \vee 0.6 \Rightarrow SuI_{(CH-M)WQE} = 0.6$

Therefore, the activated level of (CH-M) WQF $\simeq$ 0.6, this mean that the rate of chemical and microbiological contamination in distribution networks of Bagdad city is 0.6

In traditional methods are examined samples of water to calculate the ratio of pollution to some factors often chlorine and bacteria. In this model, the totally ratio of pollution are calculated by using the mathematical model.

6. Conclusion

In this paper, we have proposed a mathematical model that combines fuzzy cognitive map, fuzzy logic, fuzzy measure, and fuzzy integral to evaluate water quality in the distribution networks of Baghdad city. The results represented the evaluation of water pollution of some chemical and microbiological contaminants, which are difficult to evaluation through traditional methods. Furthermore, ratio of Arsenic and Dichloromethane in the water distribution network of Baghdad City are calculated which are not computed in the traditional methods in water department of Municipality of Baghdad city.

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