Iraqi Journal of Science, 2017, Vol. 58, No.2C, pp: 1094-1106
DOI: 10.24996.ijs.2017.58.2C.13

ISSN: 0067-2904

On Weakly Soft Omega Open Functions and Weakly Soft Omega Closed Functions in Soft Topological Spaces

Sabiha I. Mahmood
Department of Mathematics, College of Science, Al-Mustansiriyah University, Baghdad, Iraq.

Abstract

The main purpose from this paper is to introduce a new kind of soft open sets in soft topological spaces called soft omega open sets and we show that the collection of every soft omega open sets in a soft topological space $(X, \tilde{\tau}, E)$ forms a soft topology $\tilde{\tau}_{\omega}$ on X which is soft finer than $\tilde{\tau}$. Moreover we use soft omega open sets to define and study new classes of soft functions called weakly soft omega open functions and weakly soft omega closed functions which are weaker than weakly soft open functions and weakly soft closed functions respectively. We obtain their basic properties, their characterizations, and their relationships with other kinds of soft functions between soft topological spaces.

Keywords: Soft Omega Open Set, Soft Omega Closed Set, Weakly Soft Omega Open Function, Weakly Soft Omega Closed Function, Almost Soft Omega Open Function, Almost Soft Omega Closed Function, Contra Soft Omega Open Function, Contra Soft Omega Closed Function.

حول الدوال المفتوحة اوميكا الميسرة الضعيفة والدوال المغلقة اوميكا الميسرة الضعيفة في الفضاءات التبولوجية الميسرة

صبيحة ابراهيم محمود
قسم الرياضيات، كلية العلوم، الجامعة المستنصرية، بغداد، العراق.

الخلاصة

ان الغرض الرئيسي من هذا البحث هو تقديم نوع جديد من المجموعات المفتوحة الميسرة في الفضاءات التبولوجية الميسرة اسميناها بالمجموعات المفتوحة اوميكا الميسرة ثم اثبتنا ان مجموعة كل المجموعات الجزئية المفتوحة اوميكاالميسرة في الفضاء التبولوجي الميسر تشكل تبولوجيا ميسرة . فضلا عن ذلك استخدمنا المجموعات المفتوحة اوميكا الميسرة في تعريف ودراسة اصناف جديدة من الدوال الميسرة اسميناها بالدوال المفتوحة اوميكا الميسرة الضعيفة والدوال المغلقة اوميكا الميسرة الضعيفة والتي هي اضعف من الدوال المفتوحة الميسرة الضعيفة والدوال المغلقة الميسرة الضعيفة على التوالي. وقد حصلنا على خصائصهم الاساسية، ومكافئاتهم وعِلاقاتهم بانواعِ أخرى مِن الدوال الميسرة بين الفضاءات التبولوجية الميسرة.

Introduction

Molodtsov [1] introduced and studied the concept of soft set theorem to solve complicated problems in the economics, environment and engineering. Shabir and Naz [2] introduced the concept of soft topological spaces which are defined over a universe set with a fixed set of parameters. Akdag

*Email: s.alzubaidy.2015@gmail.com

and Ozkan [3], Arockiarani and Arokia Lancy [4], Yuksel and et al. [5], and Georgiou and Megaritis [6] defined and study soft α-open sets, soft pre-open sets, soft regular open sets, soft Θ-interior points and soft Θ-cluster points in soft topological spaces respectively. Also, soft open functions and soft closed functions were first introduced by Nazmul and Samanta [7]. In the present paper, we define and study a new type of soft open sets in soft topological spaces called soft omega open sets and we prove that the set of all soft omega open sets in a soft topological space $(X,\tilde{\tau},E)$ forms a soft topology $\tilde{\tau}_{\omega}$ on X which is soft finer than $\tilde{\tau}$. Moreover we use soft omega open sets to define and study new classes of soft functions called weakly soft omega open functions and weakly soft omega closed functions as generalization of weakly soft open functions and weakly soft closed functions respectively. We obtain their basic properties, their characterizations, and their relationships with other types of soft functions between soft topological spaces.

1. Preliminaries

In this paper, X refers to an initial universe set, $P(X)$ is the power set of X and E is the set of parameters for X. Now, we recall the following definitions.

Definition (1.1) [1]: A soft set over X is an ordered pair (S,E), where S is a function given by $S:E\rightarrow P(X)$ and E is the set of parameters for X.

Definition (1.2)[7]: If (S,E) is a soft set over X, then $\tilde{s}=(e,\{s\})$ is called a soft point of (S,E) if $e\in E$ and $s\in S(e)$, and is denoted by $\tilde{s}\,\tilde{\in}\,(S,E)$.

Definition (1.3) [2]: If $\tilde{\tau}$ is a family of soft sets over X. Then $\tilde{\tau}$ is called a soft topology over X if $\tilde{\tau}$ has the following properties:

(i) $\tilde{X}\,\tilde{\in}\,\tilde{\tau}$ and $\tilde{\phi}\,\tilde{\in}\,\tilde{\tau}$.

(ii) If $(S_1,E),(S_2,E)\,\tilde{\in}\,\tilde{\tau}$, then $(S_1,E)\,\tilde{\bigcap}\,(S_2,E)\,\tilde{\in}\,\tilde{\tau}$.

(iii) If $(S_\alpha,E)\,\tilde{\in}\,\tilde{\tau}, \forall\alpha\in\Lambda$, then $\bigcup_{\alpha\in\Lambda}(S_\alpha,E)\,\tilde{\in}\,\tilde{\tau}$.

The triple $(X,\tilde{\tau},E)$ is called a soft topological space over X. Any members of $\tilde{\tau}$ is called a soft open set in $\tilde{X}$ .The complement of a soft open set is called soft closed.

Definition (1.4)[8]: If (S,E) is a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then:

(i) The soft closure of (S,E) is the intersection of all soft closed sets in $\tilde{X}$ which contains (S,E) and is denoted by $cl(S,E)$.

(ii) The soft interior of (S,E) is the union of all soft open sets in $\tilde{X}$ which are contained in (S,E) and is denoted by $int(S,E)$

Definitions (1.5): A soft subset (S,E) of a soft topological space $(X,\tilde{\tau},E)$ is called:

(i) Soft α-open [3] if $(S,E)\,\tilde{\subseteq}\,int(cl(int(S,E)))$.

(ii) Soft pre-open [4] if $(S,E)\,\tilde{\subseteq}\,int(cl(S,E))$.

(iii) soft regular open [5] if $(S,E)=int(cl(S,E))$.

Definition (1.6)[6]: Let (S,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. A soft point $\tilde{x}\,\tilde{\in}\,(S,E)$ is called a soft Θ-interior point of (S,E), if there exists a soft open set (O,E) of $\tilde{x}$ such that $cl(O,E)\,\tilde{\subseteq}\,(S,E)$. The soft set of all soft Θ-interior point of (S,E) is denoted by $int_\theta(S,E)$.

Definition (1.7)[6]: Let (S,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. A soft point $\tilde{x}\,\tilde{\in}\,\tilde{X}$ is called a soft Θ-cluster point of (S,E) if $cl(O,E)\,\tilde{\bigcap}\,(S,E)\neq\tilde{\phi}$ for every soft open set (O,E) containing $\tilde{x}$. The soft set of all soft Θ-cluster point of (S,E) is denoted by $cl_\theta(S,E)$.

2. Soft Omega Open Sets

In this section we introduce new type of soft open sets in soft topological spaces called soft omega open sets, and we show that the collection of every soft omega open sets in $(X,\tilde{\tau},E)$ forms a soft topology $\tilde{\tau}_\omega$ on $\tilde{X}$ which is soft finer than $\tilde{\tau}$. Also, we study the basic properties of soft omega open sets.

Definition (2.1): A soft set (S, E) is called a countable soft set if the set $S(e)$ is countable $\forall e \in E$.

Definition (2.2): A soft subset (W, E) of a soft topological space $(X, \tilde{\tau}, E)$ is called soft omega open (briefly soft ω-open) if for each $\tilde{x} \tilde{\in} (W, E)$, there exists $(O, E) \tilde{\in} \tilde{\tau}$ such that $\tilde{x} \tilde{\in} (O, E)$ and $(O, E) - (W, E)$ is a countable soft set. The complement of a soft ω-open set is called soft omega closed (briefly soft ω-closed). The collection of every soft ω-open sets in $(X, \tilde{\tau}, E)$ is denoted by $\tilde{\tau}_\omega$.

Clearly, every soft open set is soft omega open, but the converse is not true in general we can see in the following example:

Example (2.3): Let $X = \{a, b, c, d\}$, $E = \{e_1, e_2\}$, and $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}\}$ be a soft topology over X. Then $(W, E) = \{(e_1, W(e_1)), (e_2, W(e_2))\} = \{(e_1, \{a, b, c\}), (e_2, \{a, b, c\})\}$ is a soft ω-open set in $\tilde{X}$, but is not soft open.

Theorem (2.4): The collection of every soft omega open sets in $(X, \tilde{\tau}, E)$ forms a soft topology $\tilde{\tau}_\omega$ over $\tilde{X}$.

Proof: (i) Since $\tilde{\phi}, \tilde{X} \tilde{\in} \tilde{\tau} \Rightarrow \tilde{\phi}, \tilde{X} \in \tilde{\tau}_\omega$.

(ii) Suppose that $(W_1, E), (W_2, E) \tilde{\in} \tilde{\tau}_w$. To show that $(W_1, E) \tilde{\bigcap} (W_2, E) \tilde{\in} \tilde{\tau}_\omega$.

Let $\tilde{x} \tilde{\in} (W_1, E) \tilde{\bigcap} (W_2, E) \Rightarrow \tilde{x} \tilde{\in} (W_1, E)$ and $\tilde{x} \tilde{\in} (W_2, E)$. Since (W_1, E) is soft ω-open $\Rightarrow$ $\exists\ (O_1, E) \tilde{\in} \tilde{\tau}$ such that $\tilde{x} \tilde{\in} (O_1, E)$ and $(O_1, E) - (W_1, E)$ is countable. Since (W_2, E) is soft ω-open $\Rightarrow \exists\ (O_2, E) \tilde{\in} \tilde{\tau}$ such that $\tilde{x} \tilde{\in} (O_2, E)$ and $(O_2, E) - (W_2, E)$ is countable.

Since $\tilde{x} \tilde{\in} (O_1, E)$ and $\tilde{x} \tilde{\in} (O_2, E) \Rightarrow \tilde{x} \tilde{\in} (O_1, E) \tilde{\bigcap} (O_2, E)$ and $(O_1, E) \tilde{\bigcap} (O_2, E)$ is soft open.

To show that $((O_1, E) \tilde{\bigcap} (O_2, E)) - ((W_1, E) \tilde{\bigcap} (W_2, E))$ is countable.

$$((O_1, E) \tilde{\bigcap} (O_2, E)) - ((W_1, E) \tilde{\bigcap} (W_2, E)) = ((O_1, E) \tilde{\bigcap} (O_2, E)) \tilde{\bigcap} ((W_1, E) \tilde{\bigcap} (W_2, E))^c$$
$$= ((O_1, E) \tilde{\bigcap} (O_2, E)) \tilde{\bigcap} ((W_1, E)^c \tilde{\bigcup} (W_2, E)^c)$$
$$= [((O_1, E) \tilde{\bigcap} (O_2, E)) \tilde{\bigcap} (W_1, E)^c] \tilde{\bigcup} [(O_1, E) \tilde{\bigcap} (O_2, E)) \tilde{\bigcap} (W_2, E)^c)]$$
$$= [((O_1, E) \tilde{\bigcap} (O_2, E)) - (W_1, E)] \tilde{\bigcup} [(O_1, E) \tilde{\bigcap} (O_2, E)) - (W_2, E))]$$

But $((O_1, E) \tilde{\bigcap} (O_2, E)) - (W_1, E)$ and $((O_1, E) \tilde{\bigcap} (O_2, E)) - (W_2, E)$ are countable soft sets, then so is $[((O_1, E) \tilde{\bigcap} (O_2, E)) - (W_1, E)] \tilde{\bigcup} [(O_1, E) \tilde{\bigcap} (O_2, E)) - (W_2, E))]$. Therefore $((O_1, E) \tilde{\bigcap} (O_2, E)) - ((W_1, E) \tilde{\bigcap} (W_2, E))$ is a countable soft set. Thus $(W_1, E) \tilde{\bigcap} (W_2, E) \tilde{\in} \tilde{\tau}_\omega$.

(iii) Let $(W_\alpha, E) \tilde{\in} \tilde{\tau}_\omega, \forall \alpha \in \Lambda$. To show that $\bigcup_{\alpha \in \Lambda} (W_\alpha, E) \tilde{\in} \tilde{\tau}_\omega$.

Let $\tilde{x} \tilde{\in} \bigcup_{\alpha \in \Lambda} (W_\alpha, E) \Rightarrow \tilde{x} \tilde{\in} (W_{\alpha_0}, E)$ for some $\alpha_0 \in \Lambda$. Since $(W_{\alpha_0}, E) \tilde{\in} \tilde{\tau}_\omega \Rightarrow \exists\ (O, E) \tilde{\in} \tilde{\tau}$ such that $\tilde{x} \tilde{\in} (O, E)$ and $(O, E) - (W_{\alpha_0}, E)$ is countable. But $(W_{\alpha_0}, E) \tilde{\subseteq} \bigcup_{\alpha \in \Lambda} (W_\alpha, E) \Rightarrow (\bigcup_{\alpha \in \Lambda} (W_\alpha, E))^c \tilde{\subseteq}$ $(W_{\alpha_0}, E)^c \Rightarrow (O, E) \tilde{\bigcap} (\bigcup_{\alpha \in \Lambda} (W_\alpha, E))^c \tilde{\subseteq} (O, E) \tilde{\bigcap} (W_{\alpha_0}, E)^c \Rightarrow (O, E) - (\bigcup_{\alpha \in \Lambda} (W_\alpha, E)) \tilde{\subseteq}$ $(O, E) - (W_{\alpha_0}, E)$. But $(O, E) - (W_{\alpha_0}, E)$ is a countable soft set, then so is $(O, E) - (\bigcup_{\alpha \in \Lambda} (W_\alpha, E))$

$\Rightarrow \bigcup_{\alpha \in \Lambda} (W_\alpha, E) \tilde{\in} \tilde{\tau}_\omega \Rightarrow (X, \tilde{\tau}_w, E)$ is a soft topological space.

Definition (2.5): If $(X, \tilde{\tau}, E)$ is a soft topological space and $(S, E) \tilde{\subseteq} \tilde{X}$ .Then:

(i) The soft omega closure (briefly soft ω-closure) of (S, E) is the intersection of all soft ω-closed sets in $\tilde{X}$ which contains (S, E) and is denoted by $cl_\omega(S, E)$.

(ii) The soft omega interior (briefly soft ω-interior) of (S, E) is the union of all soft ω-open sets in $\tilde{X}$ which are contained in (S, E) and is denoted by $int_\omega(S, E)$.

Theorem (2.6): Let $(X,\tilde{\tau},E)$ be a soft topological space and $(S,E),(T,E)\tilde{\subseteq}\tilde{X}$. Then:

(i) $(S,E)\tilde{\subseteq}cl_{\omega}(S,E)\tilde{\subseteq}cl(S,E)$ and $int(S,E)\tilde{\subseteq}int_{\omega}(S,E)\tilde{\subseteq}(S,E)$.

(ii) If (S_{α},E) is soft ω-open set in $\tilde{X}$ for each $\alpha\in\Lambda$, then $\bigcup_{\alpha\in\Lambda}(S_{\alpha},E)$ is also soft ω-open set in $\tilde{X}$

(iii) If (S_{α},E) is soft ω-closed set in $\tilde{X}$ for each $\alpha\in\Lambda$, then $\bigcap_{\alpha\in\Lambda}(S_{\alpha},E)$ is also soft ω-closed set in $\tilde{X}$.

(iv) $cl_{\omega}(S,E)$ is a soft ω-closed set in $\tilde{X}$ and $int_{\omega}(S,E)$ is a soft ω-open set in $\tilde{X}$.

(v) (S,E) is soft ω-closed iff $cl_{\omega}(S,E)=(S,E)$ and (S,E) is soft ω-open iff $int_{\omega}(S,E)=(S,E)$.

(vi) $cl_{\omega}(cl_{\omega}(S,E))=cl_{\omega}(S,E)$ and $int_{\omega}(int_{\omega}(S,E))=int_{\omega}(S,E)$.

(vii) $\tilde{X}-cl_{\omega}(G,E)=int_{\omega}(\tilde{X}-(G,E))$ and $\tilde{X}-int_{\omega}(G,E)=cl_{\omega}(\tilde{X}-(G,E))$.

(viii) If $(S,E)\tilde{\subseteq}(T,E)$, then $cl_{\omega}(S,E)\tilde{\subseteq}cl_{\omega}(T,E)$ and $int_{\omega}(S,E)\tilde{\subseteq}int_{\omega}(T,E)$.

(ix) $cl_{\omega}((S,E)\tilde{\bigcup}(T,E))=cl_{\omega}(S,E)\tilde{\bigcup}cl_{\omega}(T,E)$ and $int_{\omega}((S,E)\tilde{\bigcap}(T,E))=int_{\omega}(S,E)\tilde{\bigcap}int_{\omega}(T,E)$.

(x) $\tilde{x}\tilde{\in}int_{\omega}(S,E)$ iff there is a soft ω-open set (O,E) in $\tilde{X}$ s.t $\tilde{x}\tilde{\in}(O,E)\tilde{\subseteq}(S,E)$.

(xi) $\tilde{x}\tilde{\in}cl_{\omega}(S,E)$ iff for every soft ω-open set (O,E) containing $\tilde{x}$, $(O,E)\tilde{\bigcap}(S,E)\neq\tilde{\phi}$.

(xii) $\bigcup_{\alpha\in\wedge}cl_{\omega}(S_{\alpha},E)\tilde{\subseteq}cl_{\omega}(\bigcup_{\alpha\in\wedge}(S_{\alpha},E))$ and $\bigcup_{\alpha\in\wedge}int_{\omega}(S_{\alpha},E)\tilde{\subseteq}int_{\omega}(\bigcup_{\alpha\in\wedge}(S_{\alpha},E))$.

Proof:(ix) Since $(S,E)\tilde{\subseteq}(S,E)\tilde{\bigcup}(T,E)$ and $(T,E)\tilde{\subseteq}(S,E)\tilde{\bigcup}(T,E)$
$\Rightarrow$ $cl_{\omega}(S,E)\tilde{\subseteq}cl_{\omega}((S,E)\tilde{\bigcup}(T,E))$ and $cl_{\omega}(T,E)\tilde{\subseteq}cl_{\omega}((S,E)\tilde{\bigcup}(T,E))$
$\Rightarrow$ $cl_{\omega}(S,E)\tilde{\bigcup}cl_{\omega}(T,E)\tilde{\subseteq}cl_{\omega}((S,E)\tilde{\bigcup}cl_{\omega}(T,E))$. To show that
$cl_{\omega}((S,E)\tilde{\bigcup}(T,E))\tilde{\subseteq}cl_{\omega}(S,E)\tilde{\bigcup}cl_{\omega}(T,E)$. Since $(S,E)\tilde{\subseteq}cl_{\omega}(S,E)$ and $(T,E)\tilde{\subseteq}cl_{\omega}(T,E)$
$\Rightarrow$ $(S,E)\tilde{\bigcup}(T,E)\tilde{\subseteq}cl_{\omega}(S,E)\tilde{\bigcup}cl_{\omega}(T,E)$. Since $cl_{\omega}(S,E)$ and $cl_{\omega}(T,E)$ are soft ω-closed sets in $\tilde{X}$, then so is $cl_{\omega}(S,E)\tilde{\bigcup}cl_{\omega}(T,E)$. But $cl_{\omega}((S,E)\tilde{\bigcup}(T,E))$ is the smalles soft ω-closed set which contains $(S,E)\tilde{\bigcup}(T,E)$,therefore $cl_{\omega}((S,E)\tilde{\bigcup}(T,E))\tilde{\subseteq}cl_{\omega}(S,E)\tilde{\bigcup}cl_{\omega}(T,E)$. Hence $cl_{\omega}((S,E)\tilde{\bigcup}(T,E))=cl_{\omega}(S,E)\tilde{\bigcup}cl_{\omega}(T,E)$. From theorem (2.4) and definition (2.5), it is easy to prove other cases.

3. Weakly Soft Omega Open Functions

In this section we introduce and study new kinds of soft functions in soft topological spaces called weakly soft omega open functions, soft omega open functions, almost soft omega open functions, and weakly soft open functions which are weaker than soft open functions. Further, we study the characteristics and basic properties of weakly soft omega open functions.

Definition (3.1): A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is called:

(i) Weakly soft omega open (briefly weakly soft ω-open) if $f((O,E))\tilde{\subseteq}int_{\omega}(f(cl(O,E)))$ for every soft open set (O,E) in $\tilde{X}$.

(ii) Soft omega open (briefly soft ω-open) if $f((O,E))$ is soft ω-open set in $\tilde{Y}$ for every soft open set (O,E) in $\tilde{X}$.

(iii) Weakly soft open if $f((O,E))\tilde{\subseteq}int(f(cl(O,E)))$ for every soft open set (O,E) in $\tilde{X}$.

Clearly, every weakly soft open function as well as soft ω-open function is weakly soft ω-open function, but the converse is not generally true we can see in the following examples:.

Examples (3.2):(i): Let $X=Y=N$, $E=\{e\}$, $\tilde{\tau}=\{\tilde{N},\tilde{\phi},(O_1,E),(O_2,E)\}$ be a soft topology over X and $\tilde{\sigma}=\{\tilde{N},\tilde{\phi},(O_2,E)\}$ be a soft topology over Y, where $(O_1,E)=\{(e,\{1\})\}$ and $(O_2,E)=\{(e,N-\{1\})\}$. Let $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E)$ be a soft function defined by: $f(\tilde{x})=\tilde{x}$, $\forall\ \tilde{x}\tilde{\in}\tilde{X}$. Then f is a weakly soft ω-open function, but is not weakly soft open, since (O_1,E) is a soft open set in $\tilde{X}$, but

$(O_1, E) = f((O_1, E)) \tilde{\not\subseteq} \operatorname{int}(f(\operatorname{cl}(O_1, E))) = \operatorname{int}((O_1, E)) = \tilde{\phi}$.

(ii): Let $X = Y = \Re$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{\Re}, \tilde{\phi}, (O_1, E), (O_2, E)\}$ be a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (O_2, E)\}$ be a soft topology over Y, where $(O_1, E) = \{(e, \{1\})\}$ and $(O_2, E) = \{(e, \Re - \{2\})\}$. Let $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ be a soft function defined by: $f(\tilde{x}) = \tilde{x}$, $\forall \tilde{x} \tilde{\in} \tilde{X}$. Then f is a weakly soft ω-open function, but is not soft ω-open, since (O_1, E) is a soft open set in $\tilde{X}$, but $f((O_1, E)) = (O_1, E)$ is not soft ω-open in $\tilde{Y}$.

Remarks (3.3): (i): Every soft open function is weakly soft open function and weakly soft ω-open function, but the converse is not true in general. In example (3.2), no. (ii), f is weakly soft open function as well as weakly soft ω-open function, but is not soft open function.

(ii): Every soft open function is soft ω-open function, but the converse is not true in general. In example (3.2), no. (i), f is soft ω-open function, but is not soft open function.

Theorem (3.4): For a soft function $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E')$ the following statements are equivalent:

(i) f is weakly soft ω-open.

(ii) $f(\operatorname{int}_\theta(A, E)) \tilde{\subseteq} \operatorname{int}_\omega(f((A, E)))$ for every soft subset (A, E) of $\tilde{X}$.

(iii) $\operatorname{int}_\theta(f^{-1}((B, E'))) \tilde{\subseteq} f^{-1}(\operatorname{int}_\omega(B, E'))$ for every soft subset (B, E') of $\tilde{Y}$.

(iv) $f^{-1}(\operatorname{cl}_\omega(B, E')) \tilde{\subseteq} \operatorname{cl}_\theta(f^{-1}((B, E')))$ for every soft subset (B, E') of $\tilde{Y}$.

(v) For each $\tilde{x} \tilde{\in} \tilde{X}$ and each soft open set (O, E) in $\tilde{X}$ containing $\tilde{x}$, there exists a soft ω-open set (W, E') in $\tilde{Y}$ containing $f(\tilde{x})$ such that $(W, E') \tilde{\subseteq} f(\operatorname{cl}(O, E))$.

(vi) $f(\operatorname{int}(F, E)) \tilde{\subseteq} \operatorname{int}_\omega(f((F, E)))$ for every soft closed set (F, E) in $\tilde{X}$.

(vii) $f(\operatorname{int}(\operatorname{cl}(O, E))) \tilde{\subseteq} \operatorname{int}_\omega(f(\operatorname{cl}(O, E)))$ for every soft open set (O, E) in $\tilde{X}$.

(viii) $f((O, E)) \tilde{\subseteq} \operatorname{int}_\omega(f(\operatorname{cl}(O, E)))$ for every soft pre-open set (O, E) in $\tilde{X}$.

(ix) $f((O, E)) \tilde{\subseteq} \operatorname{int}_\omega(f(\operatorname{cl}(O, E)))$ for every soft α-open set (O, E) in $\tilde{X}$.

Proof: $(i) \Rightarrow (ii)$. Let (A, E) be any soft subset of $\tilde{X}$ and $\tilde{x} \tilde{\in} \operatorname{int}_\theta(A, E)$. Then, there exists a soft open set (O, E) in $\tilde{X}$ such that $\tilde{x} \tilde{\in} (O, E) \tilde{\subseteq} \operatorname{cl}(O, E) \tilde{\subseteq} (A, E)$. Hence $f(\tilde{x}) \tilde{\in} f((O, E)) \tilde{\subseteq} f(\operatorname{cl}(O, E)) \tilde{\subseteq} f((A, E))$. Since f is weakly soft ω-open, then $f((O, E)) \tilde{\subseteq} \operatorname{int}_\omega(f(\operatorname{cl}(O, E))) \tilde{\subseteq} \operatorname{int}_\omega(f((A, E)))$. It implies that $f(\tilde{x}) \tilde{\in} \operatorname{int}_\omega(f((A, E)))$. This shows that $\tilde{x} \tilde{\in} f^{-1}(\operatorname{int}_\omega(f((A, E))))$. Therefore $\operatorname{int}_\theta(A, E) \tilde{\subseteq} f^{-1}(\operatorname{int}_\omega(f((A, E))))$, and so, $f(\operatorname{int}_\theta(A, E)) \tilde{\subseteq} \operatorname{int}_\omega(f((A, E)))$ for every soft subset (A, E) of $\tilde{X}$.

$(ii) \Rightarrow (iii)$. Let (B, E') be any soft subset of $\tilde{Y}$. Then by (ii), we get $f(\operatorname{int}_\theta(f^{-1}((B, E')))) \tilde{\subseteq} \operatorname{int}_\omega(f(f^{-1}((B, E'))))$. Thus $\operatorname{int}_\theta(f^{-1}((B, \tilde{E}))) \tilde{\subseteq} f^{-1}(\operatorname{int}_\omega(B, E'))$ for every soft subset (B, E') of $\tilde{Y}$.

$(iii) \Rightarrow (iv)$. Let (B, E') be any soft subset of $\tilde{Y}$. By (iii), we have $\tilde{X} - \operatorname{cl}_\theta(f^{-1}((B, E'))) = \operatorname{int}_\theta(\tilde{X} - f^{-1}((B, E'))) = \operatorname{int}_\theta(f^{-1}(\tilde{Y} - (B, E'))) \tilde{\subseteq} f^{-1}(\operatorname{int}_\omega(\tilde{Y} - (B, E'))) = f^{-1}(\tilde{Y} - \operatorname{cl}_\omega(B, E')) = \tilde{X} - f^{-1}(\operatorname{cl}_\omega(B, E'))$. Therefore, we obtain $f^{-1}(\operatorname{cl}_\omega(B, E')) \tilde{\subseteq} \operatorname{cl}_\theta(f^{-1}((B, E')))$ for every soft subset (B, E') of $\tilde{Y}$.

$(iv) \Rightarrow (i)$. Let (O, E) be any soft open set in $\tilde{X}$ and $(B, E') = \tilde{Y} - f(\operatorname{cl}(O, E))$. By (iv), we have $f^{-1}(\operatorname{cl}_\omega(\tilde{Y} - f(\operatorname{cl}(O, E)))) \tilde{\subseteq} \operatorname{cl}_\theta(f^{-1}(\tilde{Y} - f(\operatorname{cl}(O, E))))$. Thus we obtain $f^{-1}(\tilde{Y} - \operatorname{int}_\omega(f(\operatorname{cl}(O, E)))) \tilde{\subseteq} \operatorname{cl}_\theta(\tilde{X} - f^{-1}(f(\operatorname{cl}(O, E)))) \tilde{\subseteq} \operatorname{cl}_\theta(\tilde{X} - \operatorname{cl}(O, E)) = \tilde{X} - \operatorname{int}_\theta(\operatorname{cl}(O, E))$. Therefore $(O, E) \tilde{\subseteq} \operatorname{int}_\theta(\operatorname{cl}(O, E)) \tilde{\subseteq} f^{-1}(\operatorname{int}_\omega(f(\operatorname{cl}(O, E))))$, and hence $f((O, E)) \tilde{\subseteq} \operatorname{int}_\omega(f(\operatorname{cl}(O, E)))$. Thus f is weakly soft ω-open function.

$(i) \Rightarrow (v)$. Let $\tilde{x} \tilde{\in} \tilde{X}$ and (O, E) be a soft open set in $\tilde{X}$ such that $\tilde{x} \tilde{\in} (O, E)$. Since f is weakly soft ω-open, then $f(\tilde{x}) \tilde{\in} f((O, E)) \tilde{\subseteq} \operatorname{int}_\omega(f(\operatorname{cl}(O, E)))$. Let $(W, E') = \operatorname{int}_\omega(f(\operatorname{cl}(O, E)))$. Hence (W, E') is a soft ω-open set in $\tilde{Y}$ containing $f(\tilde{x})$ such that $(W, E') \tilde{\subseteq} f(\operatorname{cl}(O, E))$.

$(v) \Rightarrow (i)$. Let (O,E) be any soft open set in $\tilde{X}$ and let $\tilde{y} \tilde{\in} f((O,E))$. Then there is $\tilde{x} \tilde{\in} (O,E)$ such that $f(\tilde{x}) = \tilde{y}$. By (v), there exists a soft ω-open set (W,E') in $\tilde{Y}$ containing $f(\tilde{x})$ such that $(W,E') \tilde{\subseteq} f(cl(O,E))$. Hence we have $\tilde{y} \in (W,E') \tilde{\subseteq} int_{\omega} f(cl(O,E))$. This shows that $f((O,E)) \tilde{\subseteq} int_{\omega}(f(cl(O,E)))$, i.e., f is a weakly soft ω-open function.

$(i) \Rightarrow (vi)$. Let (F,E) be any soft closed set in $\tilde{X}$, then $int(F,E)$ is a soft open set in $\tilde{X}$. By (i), we get $f(int(F,E)) \tilde{\subseteq} int_{\omega}(f(cl(int(F,E)))) \tilde{\subseteq} int_{\omega}(f(cl(F,E))) = int_{\omega}(f((F,E)))$. Hence $f(int(F,E)) \tilde{\subseteq} int_{\omega}(f((F,E)))$ for every soft closed set (F,E) in $\tilde{X}$.

$(vi) \Rightarrow (vii)$. This is obvious.

$(vii) \Rightarrow (viii)$. Since (O,E) is a soft pre-open set in $\tilde{X}$, then $(O,E) \tilde{\subseteq} int(cl(O,E))$. Since $int(cl(O,E))$ is a soft open set in $\tilde{X}$, then by (vii), we get $f(int(cl(int(cl(O,E))))) \tilde{\subseteq} int_{\omega}(f(cl(int(cl(O,E)))))$. But $int(cl(O,E))$ is a soft regular open set in $\tilde{X}$, then $f(int(cl(O,E))) = f(int(cl(int(cl(O,E))))) \tilde{\subseteq} int_{\omega}(f(cl(int(cl(O,E))))) \tilde{\subseteq} int_{\omega}(f(cl(O,E)))$. Thus $f((O,E)) \tilde{\subseteq} f(int(cl(O,E))) \tilde{\subseteq} int_{\omega}(f(cl(O,E)))$.

Therefore $f((O,E)) \tilde{\subseteq} int_{\omega}(f(cl(O,E)))$ for every soft pre-open set (O,E) in $\tilde{X}$.

$(viii) \Rightarrow (ix) \Rightarrow (i)$. This is obvious.

Theorem (3.5): Let $(X,\tilde{\tau},E)$ be a soft regular space. Then for a soft function $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E')$ the following statements are equivalent:

(i) f is weakly soft ω-open.

(ii) f is soft ω-open.

(iii) For any soft subset (B,E') of $\tilde{Y}$ and any soft closed set (F,E) in $\tilde{X}$ containing $f^{-1}((B,E'))$, there exists a soft ω-closed set (W,E') in $\tilde{Y}$ containing (B,E') such that $f^{-1}((W,E')) \tilde{\subseteq} (F,E)$.

Proof: $(i) \Rightarrow (ii)$. Let (O,E) be any non-null soft open set in $\tilde{X}$. Since $(X,\tilde{\tau},E)$ is soft regular, then for each $\tilde{x} \tilde{\in} (O,E)$, there exists a soft open set $(O,E)_{\tilde{x}}$ in $\tilde{X}$ such that $\tilde{x} \tilde{\in} (O,E)_{\tilde{x}} \tilde{\subseteq} cl((O,E)_{\tilde{x}}) \tilde{\subseteq} (O,E)$. Hence we obtain that $(O,E) = \tilde{\bigcup}\{(O,E)_{\tilde{x}} : \tilde{x} \tilde{\in} (O,E)\} = \tilde{\bigcup}\{cl((O,E)_{\tilde{x}}) : \tilde{x} \tilde{\in} (O,E)\}$, and $f((O,E)) = \tilde{\bigcup}\{f((O,E)_{\tilde{x}}) : \tilde{x} \tilde{\in} (O,E)\} \tilde{\subseteq} \tilde{\bigcup}\{int_{\omega}(f(cl((O,E)_{\tilde{x}}))) : \tilde{x} \tilde{\in} (O,E)\} \tilde{\subseteq} int_{\omega}(f(\tilde{\bigcup}cl((O,E)_{\tilde{x}})) : \tilde{x} \tilde{\in} (O,E)\}$ $= int_{\omega}(f((O,E)))$. Thus f is soft ω-open.

$(ii) \Rightarrow (iii)$. Let (B,E') be any soft subset of $\tilde{Y}$ and (F,E) be any soft closed set in $\tilde{X}$ such that $f^{-1}((B,E')) \tilde{\subseteq} (F,E) \Rightarrow \tilde{X} - (F,E) \tilde{\subseteq} \tilde{X} - f^{-1}((B,E')) = f^{-1}(\tilde{Y} - (B,E')) \Rightarrow f(\tilde{X} - (F,E)) \tilde{\subseteq}$ $\tilde{Y} - (B,E') \Rightarrow (B,E') \tilde{\subseteq} \tilde{Y} - f(\tilde{X} - (F,E)) = (W,E')$. Hence (W,E') is a soft ω-closed set in $\tilde{Y}$ containing (B,E') such that $f^{-1}((W,E')) \tilde{\subseteq} (F,E)$.

$(iii) \Rightarrow (i)$. Let (B,E') be any soft set in $\tilde{Y}$, and let $(F,E) = cl(f^{-1}((B,E')))$. Then (F,E) is a soft closed set in $\tilde{X}$, and $f^{-1}((B,E')) \tilde{\subseteq} (F,E)$. By (iii) , there exists a soft ω-closed set (W,E') in $\tilde{Y}$ containing (B,E') such that $f^{-1}((W,E')) \tilde{\subseteq} (F,E)$. Since (W,E') is a soft ω-closed set in $\tilde{Y}$, then $f^{-1}(cl_{\omega}(B,E')) \tilde{\subseteq} f^{-1}((W,E')) \tilde{\subseteq} (F,E) = cl(f^{-1}((B,E'))) \tilde{\subseteq} cl_{\theta}(f^{-1}((B,E')))$ for every soft subset (B,E') of $\tilde{Y}$. Therefore by theorem (3.4), no. (iv), f is weakly soft ω-open.

Theorem (3.6): Let $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E')$ be a bijective soft function. Then the following statements are equivalent:

(i) f is weakly soft ω-open.

(ii) $cl_{\omega}(f(int(F,E))) \tilde{\subseteq} f((F,E))$ for every soft closed set (F,E) in $\tilde{X}$.

(iii) $cl_{\omega}(f((O,E))) \tilde{\subseteq} f(cl(O,E))$ for every soft open set (O,E) in $\tilde{X}$.

Proof: (i) $\Rightarrow$ (ii). Let (F,E) be a soft closed set in $\tilde{X}$. Since f is a bijective soft function, then we have $\tilde{Y}-f((F,E))=f(\tilde{X}-(F,E))\tilde{\subseteq}\operatorname{int}_{\omega}(f(cl(\tilde{X}-(F,E)))=\operatorname{int}_{\omega}(f(\tilde{X}-\operatorname{int}(F,E)))=\operatorname{int}_{\omega}(\tilde{Y}-f(\operatorname{int}(F,E)))$ $=\tilde{Y}-cl_{\omega}(f(\operatorname{int}(F,E)))$. Thus $cl_{\omega}(f(\operatorname{int}(F,E)))\tilde{\subseteq}f((F,E))$ for every soft closed set (F,E) in $\tilde{X}$.

(ii) $\Rightarrow$ (iii). Let (O,E) be a soft open set in $\tilde{X}$. Since $cl(O,E)$ is a soft closed set in $\tilde{X}$ and $(O,E)\tilde{\subseteq}\operatorname{int}(cl(O,E))$, then by (ii), we have, $cl_{\omega}(f((O,E)))\tilde{\subseteq}cl_{\omega}(f(\operatorname{int}(cl(O,E))))\tilde{\subseteq}f(cl(O,E))$.

(iii) $\Rightarrow$ (i). Let (O,E) be a soft open set in $\tilde{X}$. Then by (iii), we have $\tilde{Y}-\operatorname{int}_{\omega}(f(cl(O,E)))=cl_{\omega}(\tilde{Y}-f(cl(O,E)))=cl_{\omega}(f(\tilde{X}-cl(O,E)))\tilde{\subseteq}f(cl(\tilde{X}-cl(O,E)))=f(\tilde{X}-\operatorname{int}(cl(O,E)))\tilde{\subseteq}f(\tilde{X}-(O,E))=\tilde{Y}-f((O,E))$. Therefore, we have $f((O,E))\tilde{\subseteq}\operatorname{int}_{\omega}(f(cl(O,E)))$ for every soft open set (O,E) in $\tilde{X}$. Thus f is weakly soft ω-open.

Definition (3.7): A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is called strongly soft continuous if $f(cl(S,E))\tilde{\subseteq}f((S,E))$ for every soft subset (S,E) of $\tilde{X}$.

Theorem (3.8): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is weakly soft ω-open and strongly soft continuous, then f is soft ω-open.

Proof: Let (O,E) be a soft open set in $\tilde{X}$. Since f is weakly soft ω-open, then $f((O,E))\tilde{\subseteq}\operatorname{int}_{\omega}(f(cl(O,E))$. But f is strongly soft continuous, then $f((O,E))\tilde{\subseteq}\operatorname{int}_{\omega}(f((O,E)))$ and therefore $f((O,E))$ is soft ω-open.

A soft ω-open function need not be strongly soft continuous in general we can see in the following example:

Example (3.9): Let $X=\Re$, $E=\{e\}$, $\tilde{\tau}=\{\tilde{\Re},\tilde{\phi}\}$ be a soft topology over X. Then the identity soft function of $(X,\tilde{\tau},E)$ onto $(X,\tilde{\tau},E)$ is a soft ω-open function, but is not strongly soft continuous, since if $\tilde{\phi}\neq(S,E)\tilde{\subset}\tilde{X}$, then $f(cl(S,E))=f(\tilde{X})=\tilde{X}\tilde{\not\subset}f((S,E))=(S,E)$.

Definition (3.10): A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is called relatively weakly soft open if $f((O,E))$ is soft open in $f(cl(O,E))$ for every soft open set (O,E) in $\tilde{X}$.

Theorem (3.11): A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is soft ω-open if, f is weakly soft ω-open and relatively weakly soft open.

Proof: Assume that f is weakly soft ω-open and relatively weakly soft open. Let (O,E) be a soft open set in $\tilde{X}$ and let $\tilde{y}\tilde{\in}f((O,E))$. Since f is relatively weakly soft open, there is a soft open set (V,E') in $\tilde{Y}$ such that $f((O,E))=f(cl(O,E))\tilde{\cap}(V,E')$. Because f is weakly soft ω-open, it follows that $f((O,E))\tilde{\subseteq}\operatorname{int}_{\omega}(f(cl(O,E)))$. Then $\tilde{y}\in\operatorname{int}_{\omega}(f(cl(O,E)))\tilde{\cap}(V,E')\tilde{\subseteq}f(cl(O,E))\tilde{\cap}(V,E')=f((O,E))$ and therefore by theorem (2.6), no. (x), $f((O,E))$ is soft ω-open.

Definition (3.12): A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is called:

(i) Contra soft closed if $f((O,E))$ is soft open in $\tilde{Y}$ for every soft closed set (O,E) in $\tilde{X}$.

(ii) Contra soft omega closed (briefly contra soft ω-closed) if $f((O,E))$ is soft ω-open in $\tilde{Y}$ for every soft closed set (O,E) in $\tilde{X}$.

Remark (3.13): Every Contra soft closed function is contra soft ω-closed function, but the converse is not true in general. In examples (3.2), no. (i), f is contra soft ω-closed function, but is not contra soft closed function, since $(O_1,E)=\{(e,\{1\})\}$ is a soft closed set in $\tilde{X}$, but $f((O_1,E))=(O_1,E)$ is not soft open in $\tilde{Y}$.

Theorem (3.14):

(i): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is a contra soft ω-closed function, then f is weakly soft ω-open.

(ii): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is a contra soft closed function, then f is weakly soft open.

Proof: **(i):** Let (O,E) be a soft open set in $\tilde{X}$, then we have $f((O,E)) \tilde{\subseteq} f(cl(O,E)) = int_{\omega}(f(cl(O,E)))$.

(ii): It is obvious.

Remark (3.15): The converse of theorem (3.14) , no. (i), (ii) may not be true in general. In examples (3.2), no. (ii), f is weakly soft ω-open function (resp. weakly soft open function), but is not contra soft ω-closed function, since $(F,E) = \{(e,\{2\})\}$ is a soft closed set in $\tilde{X}$, but $f((F,E)) = (F,E)$ is not soft ω-open in $\tilde{Y}$.

Definition (3.16): A soft function $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E')$ is called complementary weakly soft omega open (briefly complementary weakly soft ω-open or c.w.s.ω.o) if for each soft open set (O,E) in $\tilde{X}$, $f(b(O,E))$ is a soft ω-closed set in $\tilde{Y}$, where $b(O,E)$, denotes the boundary of (O,E).

Weakly soft ω-open functions and complementary weakly soft ω-open functions are independent we can see in the following examples:

Examples (3.17):(i): In examples (3.2), no. (ii), f is a weakly soft ω-open function, but is not c.w.s.ω.o , since $(O_1,E) = \{(e,\{1\})\}$ is a soft open set in $\tilde{X}$ and $b(O_1,E) = \{(e,\Re - \{1\})\}$, but $f(b(O_1,E)) = \{(e,\Re - \{1\})\}$ is not sot ω-closed in $\tilde{\Re}$.

(ii): Let $X = Y = \Re$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{\Re}, \tilde{\phi}, (O_1,E), (O_2,E)\}$ be a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (O_2,E)\}$ be a soft topology over Y, where $(O_1,E) = \{(e,\{1\})\}$ and $(O_2,E) = \{(e,\Re - \{1\})\}$. Let $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E)$ be a soft function defined by: $f(\tilde{x}) = \tilde{x}$, $\forall\ \tilde{x} \tilde{\in} \tilde{X}$. Then f is a c.w.s.ω.o function, but is not weakly soft ω-open, since (O_1,E) is soft open in $\tilde{X}$, but $f((O_1,E)) = (O_1,E)$ $\tilde{\not\subset} int_{\omega}(f(cl(O_1,E))) = int_{\omega}(f((O_1,E))) = int_{\omega}(O_1,E) = \tilde{\phi}$.

Theorem (3.18): If $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E')$ is bijective weakly soft ω-open and complementary weakly soft ω-open, then f is soft ω-open.

Proof: Let (O,E) be a soft open set in $\tilde{X}$ with $\tilde{x} \tilde{\in} (O,E)$. Since f is weakly soft ω-open, then by theorem (3.4), no. (v), there exists a soft ω-open set (W,E') containing $f(\tilde{x}) = \tilde{y}$ such that (W,E') $\tilde{\subseteq} f(cl(O,E))$. Now, $b(O,E) = cl(O,E) - (O,E)$ and hence $\tilde{x} \tilde{\notin} b(O,E)$. Thus $\tilde{y} \tilde{\notin} f(b(O,E))$, and therefore $\tilde{y} \tilde{\in} (W,E') - f(b(O,E))$. Put $(W,E')_{\tilde{y}} = (W,E') - f(b(O,E))$. Since f is complementary weakly soft ω-open, then $(W,E')_{\tilde{y}}$ is a soft ω-open set. Since $\tilde{y} \tilde{\in} (W,E')_{\tilde{y}}$, then $\tilde{y} \tilde{\in} f(cl(O,E))$. But $\tilde{y} \tilde{\notin} f(b(O,E)) = f(cl(O,E)) - f((O,E))$ which implies that $\tilde{y} \tilde{\in} f((O,E))$. Therefore $f((O,E)) = \tilde{\bigcup}\{(W,E')_{\tilde{y}} : (W,E')_{\tilde{y}} \tilde{\in} \tilde{\tau}_{\omega},\ \tilde{y} \tilde{\in} f((O,E))\}$. Thus f is soft ω-open.

Definition (3.19): A soft function $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E')$ is called almost soft omega open (briefly almost soft ω-open) if $f((O,E))$ is soft ω-open in $\tilde{Y}$ for every soft regular open set (O,E) in $\tilde{X}$.

Theorem (3.20): If $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E')$ is an almost soft ω-open function, then it is a weakly soft ω-open function.

Proof: Let (O,E) be a soft open set in $\tilde{X}$. Since f is an almost soft ω-open function and $int(cl(O,E))$ is soft regular open, then $f(int(cl(O,E)))$ is soft ω-open in $\tilde{Y}$, and hence $f((O,E)) \tilde{\subseteq} f(int(cl(O,E))) \tilde{\subseteq} int_{\omega}(f(cl(O,E)))$. Thus f is weakly soft ω-open.

The converse of theorem (3.20) may not be true in general we can see in the following example:

Example (3.21): Let $X = Y = \Re$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{\Re}, \tilde{\phi}, (O_1,E), (O_2,E), (O_3,E)\}$ be a soft topology over X and $\tilde{\sigma} = \{\tilde{\Re}, \tilde{\phi}, (V_1,E), (V_2,E), (V_3,E)\}$ be a soft topology over Y, where $(O_1,E) = \{(e,\{1\})\}$, $(O_2,E) = \{(e,\{3\})\}$, $(O_3,E) = \{(e,\{1,3\})\}$, $(V_1,E) = \{(e,\Re - \{1\})\}$, $(V_2,E) = \{(e,\Re - \{3\})\}$, and $(V_3,E) = \{(e,\Re - \{1,3\})\}$. Let $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E)$ be a soft function defined by: $f(\tilde{x}) = \tilde{x}$,

$\forall \tilde{x} \tilde{\in} \tilde{X}$. Then f is a weakly soft ω-open function, but is not almost soft ω-open, since (O_1, E) is a soft regular open set in $\tilde{X}$, but $f((O_1, E)) = (O_1, E)$ is not soft ω-open in $\tilde{Y}$.

It is clear that every soft ω-open function is an almost soft ω-open function, but the converse is not true in general we can see in the following example:

Example (3.22): Let $X = Y = \Re$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{\Re}, \tilde{\phi}, (O_1, E), (O_2, E), (O_3, E), (O_4, E)\}$ be a soft topology over X and $\tilde{\sigma} = \{\tilde{\Re}, \tilde{\phi}, (O_1, E), (O_2, E)\}$ be a soft topology over Y, where $(O_1, E) = \{(e, \{1\})\}$, $(O_2, E) = \{(e, \Re - \{1\})\}$, $(O_3, E) = \{(e, \{2\})\}$, and $(O_4, E) = \{(e, \{1,2\})\}$. Let $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ be a soft function defined by: $f(\tilde{x}) = \tilde{x}$, $\forall \tilde{x} \tilde{\in} \tilde{X}$. Then f is an almost soft ω-open function, but is not soft ω-open, since (O_3, E) is a soft open set in $\tilde{X}$, but $f((O_3, E)) = (O_3, E)$ is not soft ω-open in $\tilde{Y}$.

Remark (3.23): Almost soft ω-open function and contra soft ω-closed function are independent we can see in the following examples:

Examples (3.24):(i): In examples (3.21), f is a contra soft ω-closed function, but is not almost soft ω-open.

(ii): Let $X = Y = \Re$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{\Re}, \tilde{\phi}, (O_1, E), (O_2, E), (O_3, E), (O_4, E)\}$ be a soft topology over X and $\tilde{\sigma} = \{\tilde{\Re}, \tilde{\phi}, (O_1, E), (O_2, E)\}$ be a soft topology over Y, where $(O_1, E) = \{(e, \{1\})\}$, $(O_2, E) = \{(e, \Re - \{1\})\}$, $(O_3, E) = \{(e, \Re - \{2\})\}$, and $(O_4, E) = \{(e, \Re - \{1,2\})\}$. Let $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ be a soft function defined by: $f(\tilde{x}) = \tilde{x}$, $\forall \tilde{x} \tilde{\in} \tilde{X}$. Then f is an almost soft ω-open function, but is not contra soft ω-closed, since $(F, E) = \{(e, \{2\})\}$ is a soft closed set in $\tilde{X}$, but $f((F, E)) = (F, E)$ is not soft ω-open in $\tilde{Y}$.

Diagram (1) show the relation between weakly soft ω-open functions and each of soft open functions, soft ω-open functions, almost soft ω-open functions, weakly soft open functions, contra soft ω-closed functions, and contra soft closed functions

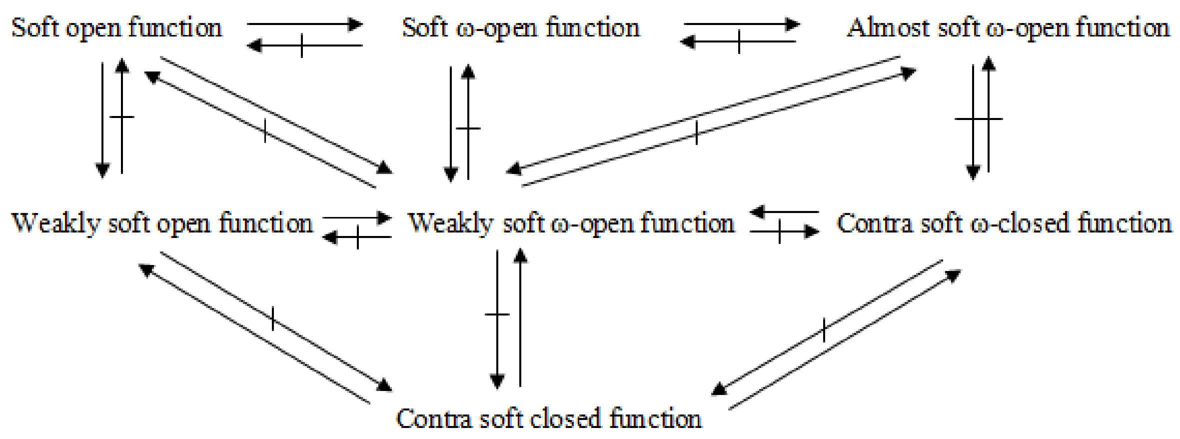

Diagram (1)

4. Weakly Soft Omega Closed Functions

In this section we introduce and study new kinds of soft functions in soft topological spaces called weakly soft omega closed functions, soft omega closed functions, almost soft omega closed functions, and weakly soft closed functions which are weaker than soft closed functions. Further, we study the characteristics and basic properties of weakly soft omega closed functions.

Definition (4.1): A soft function $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E')$ is called:

(i) Weakly soft omega closed (briefly weakly soft ω-closed) if $cl_\omega(f(int((F, E)))) \tilde{\subseteq} f(F, E)$ for every soft closed set (F, E) in $\tilde{X}$.

(ii) Soft omega closed (briefly soft ω-closed) if $f((F, E))$ is soft ω-closed set in $\tilde{Y}$ for every soft closed set (F,E) in $\tilde{X}$.

(iii) Weakly soft closed if $cl(f(int(F,E))) \tilde{\subseteq} f((F,E))$ for every soft closed set (F,E) in $\tilde{X}$.

Clearly, every weakly soft closed function as well as soft ω-closed function is weakly soft ω-closed function, but the converse is not generally true we can see in the following examples:

Examples (4.2): (i): In examples (3.2), no. (i), f is a weakly soft ω-closed function, but is not weakly soft closed, since $(F,E)=\{(e, N-\{1\})\}$ is a soft closed set in $\tilde{X}$, but $cl(f(int(F,E))) = cl(f((F,E))) = cl(F,E) = \tilde{N} \tilde{\not\subseteq} f((F,E)) = (F,E)$.

(ii): In examples (3.2), no. (ii), f is a weakly soft ω-closed function, but is not soft ω-closed, since $(F,E)=\{(e, \Re-\{1\})\}$ is a soft closed set in $\tilde{X}$, but $f((F,E)) = (F,E)$ is not soft ω-closed in $\tilde{Y}$.

Remark (4.3): (i): Every soft closed function is weakly soft closed function and weakly soft ω-closed function, but the converse is not true in general. In example (3.2), no. (ii), f is weakly soft closed function as well as weakly soft ω-closed function, but is not soft closed function.

(ii): Every soft closed function is soft ω-closed function, but the converse is not true in general. In **Example (3.2),** no. (i), f is soft ω-closed function, but is not soft closed function.

Theorem (4.4): For a soft function $f:(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},E')$ the following statements are equivalent:

(i) f is weakly soft ω-closed.

(ii) $cl_\omega(f((O,E))) \tilde{\subseteq} f(cl(O,E))$ for every soft open set (O,E) in $\tilde{X}$.

(iii) $cl_\omega(f(int(F,E))) \tilde{\subseteq} f((F,E))$ for every soft closed set (F,E) in $\tilde{X}$.

(iv) $cl_\omega(f(int(F,E))) \tilde{\subseteq} f((F,E))$ for every soft pre-closed set (F,E) in $\tilde{X}$.

(v) $cl_\omega(f(int(F,E))) \tilde{\subseteq} f((F,E))$ for every soft α-closed set (F,E) in $\tilde{X}$.

(vi) $cl_\omega(f((O,E))) \tilde{\subseteq} f(cl(O,E))$ for every soft regular open set (O,E) in $\tilde{X}$.

(vii) $cl_\omega(f(int(cl(O,E)))) \tilde{\subseteq} f(cl(O,E))$ for every soft set (O,E) in $\tilde{X}$.

(viii) $cl_\omega(f(int(cl_\theta(O,E)))) \tilde{\subseteq} f(cl_\theta(O,E))$ for every soft set (O,E) in $\tilde{X}$.

(ix) $cl_\omega(f((O,E))) \tilde{\subseteq} f(cl(O,E))$ for every soft pre-open set (O,E) in $\tilde{X}$.

Proof: $(i) \Rightarrow (ii)$. Let (O,E) be any soft open set in $\tilde{X}$. Then $cl(U,E)$ is soft closed in $\tilde{X}$. Hence by (i), we get $cl_\omega(f((O,E))) = cl_\omega(f(int(O,E))) \tilde{\subseteq} cl_\omega(f(int(cl(O,E)))) \tilde{\subseteq} f(cl(O,E))$.

$(ii) \Rightarrow (iii)$. Let (F,E) be any soft closed set in $\tilde{X}$. Then $int(F,E)$ is a soft open set in $\tilde{X}$. Therefore by (ii), we have $cl_\omega(f(int(F,E))) \tilde{\subseteq} f(cl(int(F,E))) \tilde{\subseteq} f(cl(F,E)) = f((F,E))$.

$(iii) \Rightarrow (iv)$. Since (F,E) is a soft pre-closed set in $\tilde{X}$, then $cl(int(F,E)) \tilde{\subseteq} (F,E)$. Since $cl(int(F,E))$ is soft closed in $\tilde{X}$, then by (iii), we get, $cl_\omega(f(int(F,E))) \tilde{\subseteq} cl_\omega(f(int(cl(int(F,E))))) \tilde{\subseteq} f(cl(int(F,E))) \tilde{\subseteq} f((F,E))$. Therefore $cl_\omega(f(int(F,E))) \tilde{\subseteq} f((F,E))$ for every soft pre-closed set (F,E) in $\tilde{X}$.

$(iv) \Rightarrow (v) \Rightarrow (i)$. It is obvious.

$(i) \Rightarrow (vii)$. Let (O,E) be any soft set in $\tilde{X}$. Then $cl(O,E)$ is soft closed in $\tilde{X}$. Hence by (i), we get $cl_\omega(f(int(cl(O,E)))) \tilde{\subseteq} f(cl(O,E))$.

$(vii) \Rightarrow (ix)$. Let (O,E) be any soft pre-open set in $\tilde{X}$. Then $(O,E) \tilde{\subseteq} int(cl(O,E)) \Rightarrow cl_\omega(f((O,E))) \tilde{\subseteq} cl_\omega(f(int(cl(O,E)))) \tilde{\subseteq} f(cl(O,E))$ for every soft pre-open set (O,E) in $\tilde{X}$.

$(ix) \Rightarrow (vi)$. It is obvious.

$(vi) \Rightarrow (i)$. Let (F,E) be any soft closed set in $\tilde{X} \Rightarrow cl(F,E) = (F,E) \Rightarrow int(cl(F,E)) = int(F,E)$. Since $int(cl(F,E))$ is soft regular open in $\tilde{X}$, then by (vi), we have $cl_\omega(f(int(F,E))) = cl_\omega(f(int(cl(F,E)))) \tilde{\subseteq} f(cl(int(F,E))) \tilde{\subseteq} f(cl(F,E)) = f((F,E))$ for every soft closed set (F,E) in $\tilde{X}$. Thus f is weakly soft ω-closed.

$(viii) \Rightarrow (ii)$. Let (O,E) be any soft open set in $\tilde{X}$. Then by (viii), we have $cl_\omega(f(int(cl_\theta(O,E)))) \tilde{\subseteq}$

$f(cl_\theta(O,E))$. Since (O,E) is a soft open set in $\tilde{X}$, then $cl(O,E)=cl_\theta(O,E) \Rightarrow int(cl(O,E))= int(cl_\theta(O,E))$. Therefore $cl_\omega(f((O,E)))\tilde{\subseteq} cl_\omega(f(int(cl(O,E))))=cl_\omega(f(int(cl_\theta(O,E))))\tilde{\subseteq} f(cl_\theta(O,E))=f(cl(O,E))$ for every soft open set (O,E) in $\tilde{X}$.

(ii) $\Rightarrow$ (viii). Let (O,E) be any soft set in $\tilde{X}$. Since $int(cl_\theta(O,E))$ is soft open in $\tilde{X}$, then by (ii), we have $cl_\omega(f(int(cl_\theta(O,E))))\tilde{\subseteq} f(cl(int(cl_\theta(O,E))))\tilde{\subseteq} f(cl(cl_\theta(O,E)))\tilde{\subseteq} f(cl_\theta(O,E))$ for every soft set (O,E) in $\tilde{X}$.

Theorem (4. 5): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is a bijective soft function. Then f is weakly soft ω-open if and only if f is weakly soft ω-closed.

Proof: $\Rightarrow$ Let (F,E) be any soft closed set in $\tilde{X}$, since f is weakly soft ω-open, then we have $\tilde{Y}-f((F,E))= f(\tilde{X}-(F,E))\tilde{\subseteq} int_\omega(f(cl(\tilde{X}-(F,E))))=int_\omega(f(\tilde{X}-int(F,E)))=int_\omega(\tilde{Y}-f(int(F,E))) =\tilde{Y}-cl_\omega(f(int(F,E)))\Rightarrow cl_\omega(f(int(F,E)))\tilde{\subseteq} f((F,E))$. Hence f is weakly soft ω-closed. Similarly, we can prove the converse.

Definition (4.6): A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is called:

(i) Contra soft open if $f((O,E))$ is soft closed in $\tilde{Y}$ for every soft open set (O,E) in $\tilde{X}$.

(ii) Contra soft ω-open if $f((O,E))$ is soft ω-closed in $\tilde{Y}$ for every soft open set (O,E) in $\tilde{X}$.

Remark (4.7): Every Contra soft open function is contra soft ω-open function, but the converse is not true in general. In examples (3.2), no. (i), f is contra soft ω-open function, but is not contra soft open function, since $(O_2,E)=\{(e,N-\{1\})\}$ is a soft open set in $\tilde{X}$, but $f((O_2,E))=(O_2,E)$ is not soft closed in $\tilde{Y}$.

Theorem (4.8):(i): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is a contra soft ω-open function, then f is weakly soft ω-closed.

(ii) If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is a contra soft open function, then f is weakly soft closed.

Proof: (i): Let (F,E) be a soft closed set in $\tilde{X}$, then $cl_\omega(f(int(F,E)))=f(int(F,E))\tilde{\subseteq} f((F,E))$.

(ii): It is obvious.

Remark (4.9): The converse of theorem (4.8) , no. (i), (ii) may not be true in general. In examples (3.2), no. (ii), f is weakly soft ω-closed function (resp. weakly soft closed function), but is not contra soft ω-open function, since $(O_2,E)=\{(e,\Re-\{2\})\}$ is a soft open set in $\tilde{X}$, but $f((O_2,E))=(O_2,E)$ is not soft ω-closed in $\tilde{Y}$.

Definition (4.10): A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is called almost soft omega closed (briefly almost soft ω-closed) if $f((F,E))$ is soft ω-closed in $\tilde{Y}$ for every soft regular closed set (F,E) in $\tilde{X}$.

Theorem (4.11): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is an almost soft ω-closed function, then it is a weakly soft ω-closed function.

Proof: Let (F,E) be a soft closed set in $\tilde{X}$. Since f is an almost soft ω-closed function and $cl(int(F,E))$ is soft regular closed, then $f(cl(int(F,E)))$ is soft ω-closed in $\tilde{Y}$ and hence $cl_\omega(f(int(F,E)))\tilde{\subseteq} f(cl(int(F,E))\tilde{\subseteq} f(cl(F,E))=f((F,E))$. Thus f is a weakly soft ω-closed function.

Remark (4.12): The converse of theorem (4.11) may not be true in general. In example (3.21), f is a weakly soft ω-closed function, but is not almost soft ω-closed, since $(F,E)=\{(e,\Re-\{1\})\}$ is a soft regular closed set in $\tilde{X}$, but $f((F,E))=(F,E)$ is not soft ω-closed in $\tilde{Y}$.

Theorem (4.13): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is a bijective weakly soft ω-closed function, then for every soft subset $(B,\tilde{E})$ of $\tilde{Y}$ and every soft open set (O,E) in $\tilde{X}$ with $f^{-1}((B,\tilde{E}))\tilde{\subseteq}(O,E)$, there exists a soft ω-closed set $(F,\tilde{E})$ in $\tilde{Y}$ such that $(B,E')\tilde{\subseteq}(F,E')$ and $f^{-1}((F,E')))\tilde{\subseteq} cl(O,E)$.

Proof: Let $(B,\tilde{E})$ be a soft subset of $\tilde{Y}$ and let (O,E) be a soft open set in $\tilde{X}$ such that $f^{-1}((B,E'))$ $\tilde{\subseteq}(O,E)$. Put $(F,E')=cl_\omega(f(int(cl(O,E))))$, then (F,E') is a soft ω-closed set in $\tilde{Y}$ such that (B,E')

$\tilde{\subseteq}(F,E')$, since $(B,'E)\tilde{\subseteq} f((O,E))\tilde{\subseteq} f(int(cl(O,E)))\tilde{\subseteq} cl_{\omega}(f(int(cl(O,E))))=(F,E')$. But f is weakly soft ω-closed, then $f^{-1}((F,E')))\tilde{\subseteq} cl(O,E)$.

Taking the soft set $(B,\tilde{E})$ in theorem (4.13) to be $\tilde{y}\tilde{\in}\tilde{Y}$ we obtain the following result:

Corollary (4.14): If $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is a bijective weakly soft ω-closed function, then for every soft point $\tilde{y}\tilde{\in}\tilde{Y}$ and every soft open set (O,E) in $\tilde{X}$ with $f^{-1}(\tilde{y})\tilde{\subseteq}(O,E)$, there exists a soft ω-closed set (F,E') in $\tilde{Y}$ containing $\tilde{y}$ such that $f^{-1}((F,E')))\tilde{\subseteq} cl(O,E)$.

Remark (4.15): It is clear that every soft ω-closed function is an almost soft ω-closed function, but the converse is not true in general. In example (3.22), f is an almost soft ω-closed function, but is not soft ω-closed, since $(F,E)=\{(e,\Re-\{2\})\}$ is a soft closed set in $\tilde{X}$, but $f((F,E))=(F,E)$ is not soft ω-closed in $\tilde{Y}$.

Remark (4.16): Almost soft ω-closed function and contra soft ω-open function are independent. In example (3.21), f is a contra soft ω-open function, but is not almost soft ω-closed. Also, in example (3.24), no. (ii), f is an almost soft ω-closed function, but is not contra soft ω-open, since (O_3,E) is a soft open set in $\tilde{X}$, but $f((O_3,E))=(O_3,E)$ is not soft ω-closed in $\tilde{Y}$.

Diagram (2) show the relation between weakly soft ω-closed functions and each of soft closed functions, soft ω-closed functions, almost soft ω-closed functions, weakly soft closed functions, contra soft ω-open functions, and contra soft open functions.

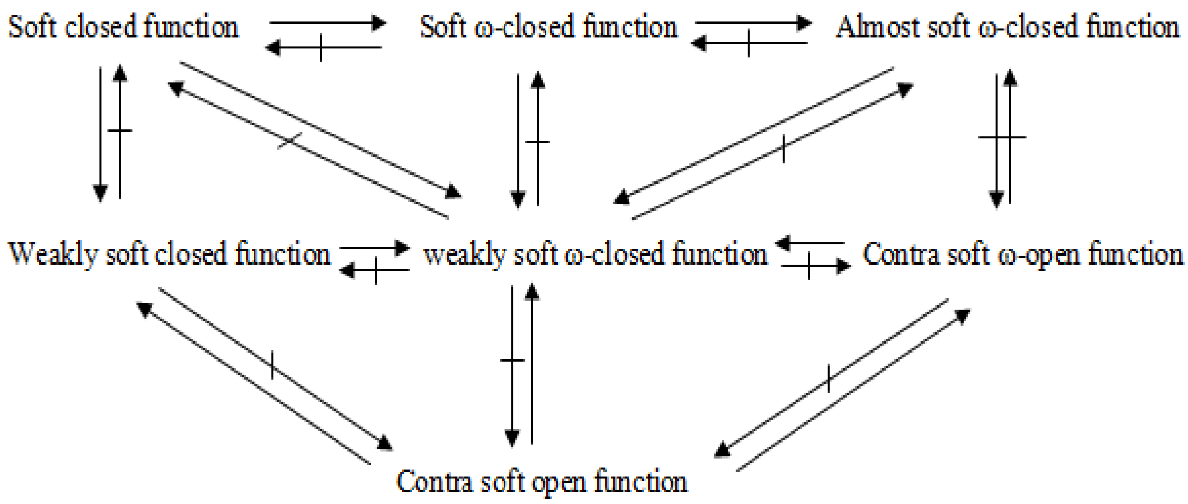

Diagram (2)

References

1. Molodtsov, D. **1999**. Soft Set theory-First results. *Computers and Mathematics with Applications*, **37**(4-5): 19-31.
2. Shabir, M. and Naz, M. **2011**. On soft topological spaces. *Computers and Mathematics with Applications*, **61**(7):1786-1799.
3. Akdag, M. and Ozkan, A. **2014**. Soft α-open sets and soft α-continuous functions. *Abstract and Applied Analysis, Article ID 891341*,pp: 1-7.
4. Arockiarani, I. and Arokia Lancy, A. **2013**. Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces. *International Journal of Mathematical Archive*, **4**(2): 17-23.
5. Yuksel, S.A., Tozlu, N. and Ergul, Z.G. **2014**. Soft Regular Generalized Closed Sets in Soft Topological Spaces. *International Journal of Mathematical Analysis*, **8**(8): 355-367.
6. Georgiou, D.N., Megaritis, A.C. and Petropoulos, V.I. **2013**. On Soft Topological Spaces. *Applied Mathematics and Information Sciences*, **7**(5):1889-1901.

7. Nazmul, Sk. and Samanta, S.K. **2012**. Neighbourhood properties of soft topological spaces. *Annals of Fuzzy Mathematics and Informatics*,:1- 15.

8. Cağman, N., Karataş, S. and Enginoglu, S. **2011**. Soft topology. *Computers and Mathematics with Applications*, **62**: 351-358.