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On The Class of (K-N)* Quasi-N-Normal Operators on Hilbert Space

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Abstract
In this paper, we will give another class of normal operator which is (K-N)* quasi-n-normal operator in Hilbert space, and give some properties of this concept as well as discussion the relation between this class with another class of normal operators.

Keywords: quasi-normal, (K-N) quasi-normal, (K-N) quasi-n-normal operator, (K-N)* quasi-n-normal operator

حول صفوف المؤثرات n السوية *(K–N) على فضاءات هلبرت

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الخلاصة
في هذا البحث سوف نعطي صف اخر من المؤثر الطبيعي، والذي يسمى (K–N)*quasi n–normal في فضاء هلبرت. واعطاء بعض الخواص ومناقشة علاقته مع انواع اخرى من المؤثر الطبيعي.

1- Introduction

Let H be complex Hilbert space, and $B(H)$ the space of all bounded linear operator from H in to it self, the quasi-normal operator was introduced at first by Brown A. in 1953 [1] and given some properties of this operator, also by hamiti V.R. [2] given another type of operators which is N-quasi normal operator. And Ahmed O. [3] given special type of operators which is n-power quasi normal operator with its properties. Also by Salim D.M. and Ahmed M.K. [4] given more general operator its called (K-N) quasi normal operator which is generalized in to (K-N) quasi n-normal operator by Sivakumar N and Bavithra V. [5].

In this article we introduce the generalization of above operator its call the (K-N)* quasi-n-normal operator, and given some basic properties and relations with some other types of operators.

2- Basic Concepts

Here, we recall fundamental concepts of this paper.

Definitions (2.1):

i. An operator $T : H \to H$ is said to be orthogonal operator if and only if $TT^* = I$, where I is identity operator. [6]

ii. An operator $T : H \to H$ is said to be idimpotent operator if $T^2 = T$.[6]

iii. An operator $T : H \to H$ is said to be normal operator if and only if $TT^* = T^*T$.[7]

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iv. An operator $T:H \to H$ is said to be quasi normal operator if T and T^*T are commute.[7]

Next, we recall the generalized of normal operator by the following definition.

Definition (2.2):

i. An operator $T:H \to H$ is said to be N-quasi normal operator if and only if $T(T^*T) = N(T^*T)T$.[2]

ii. An operator $T:H \to H$ is said to be K-quasi normal operator if and only if $T(T^*T)^K = (T^*T)^K T$.[8]

iii. An operator $T:H \to H$ is said to be n-power quasi-normal operator if and only if $T^n T^* T = T^* T T^n$.[3]

iv. An operator $T:H \to H$ is said to be quasi n-normal operator if and only if $T(T^*T^n) = (T^*T^n)T$.[5]

v. An operator $T:H \to H$ is said to be (K-N) quasi-normal operator if and only if $T^K(T^*T) = N(T^*T)T^K$.[4]

vi. An operator $T:H \to H$ is said to be (K-N) quasi-n-normal operator if and only if $T(T^*T^n) = N(T^*T^n)T$.[5]

3- Properties of (K-N)* quasi n-normal operator

At first, we give the definition of (K-N)* quasi-n-normal operator, this definition is generalized to definition appear in [5].

Definition (3.1):

Let T be a bounded operator from a complex Hilbert space H to it self, then T is said to be (K-N)* quasi-n-normal operator if satisfy the condition $T^K(T^*T^n)^K = N(T^*T^n)^K T^K$, where K and n are positive integer and N is bounded operator from a complex Hilbert space H to it self.

Next, can be introduce the relation between (K-N)* quasi-n-normal operator and other classes by the following remark.

Remarks (3.2):

1- if $N = I$ then T is power of quasi n-normal operator.

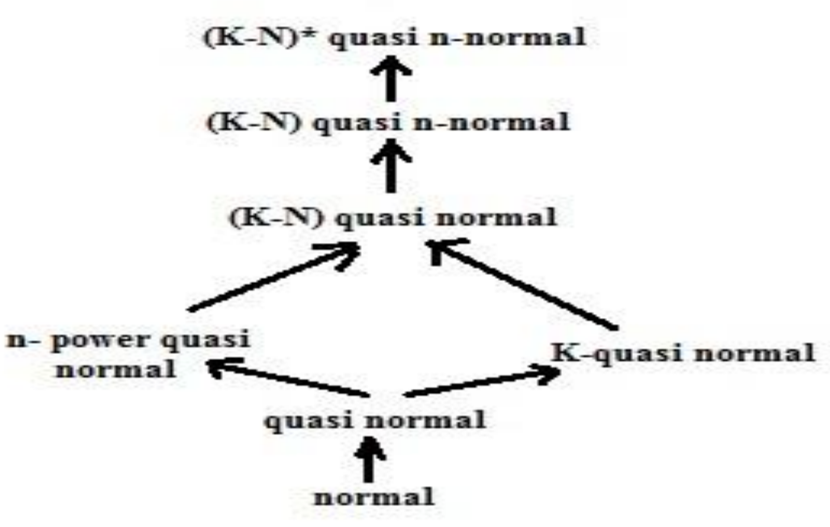
2- if $K = 1, n = 1$ then T is N quasi normal operator.

3- if $K = 1, N = I$ then T is quasi n-normal operator.

4- if $n = 1, N = I$ then T is power of quasi normal operator.

5- if $K = 1, n = 1, N = I$ then T is quasi normal operator.

To illustrate these remarks, we will introduce the following diagram.



The following theorems give some properties of (K-N)* quasi n-normal operator.

Theorem (3.3):

Let $T \in B(H)$ is an operator if C is commutes with U and V, and $C^2T^K = NC^2T^K$ then T is (K-N)* quasi normal.

Where, $B^2 = (T^nT^*)^K$, $C^2 = (T^*T^n)^K$, $U = \operatorname{Re}T^K = \left(\frac{T+T^*}{2}\right)^K$ and $V = \operatorname{Im}T^K = \left(\frac{T-T^*}{2i}\right)^K$

Proof:

Since $CU = UC$, $CV = VC$ so, $C^2U = UC^2$, $C^2V = VC^2$
then

$$T^*T^nT + T^*T^nT^* = TT^*T^n + T^*T^*T^n$$
$$T^*T^nT - T^*T^nT^* = TT^*T^n - T^*T^*T^n$$
$$TT^*T^n = T^*T^nT$$
$$T^K(T^*T^n)^K = (T^*T^n)^KT^K$$

This gives $T^KC^2 = C^2T^K$, and by using the condition $C^2T^K = NC^2T^K$ so we get: $T^K(T^*T^n)^K = N(T^*T^n)^KT^K$ then, T is (K-N)* quasi n-normal operator.

more properties give by the following theorem.

Theorem (3.4):

If $T \in B(H)$ is an operator such that $C^2U = \frac{1}{N}UC^2$, $C^2V = \frac{1}{N}VC^2$ then T is (K-N)* quasi n-normal operator.

Proof:

Since $C^2U = \frac{1}{N}UC^2$, $C^2V = \frac{1}{N}VC^2$ then we have

$C^2(U+iV) = \frac{1}{N}(U+iV)C^2$ and we have $C^2T^K = \frac{1}{N}T^KC^2$ therefore;

$$(T^*T^n)^KT^K = \frac{1}{N}T^K(T^*T^n)^K$$

so, $T^K(T^*T^n)^K = N(T^*T^n)^KT^K$ then, T is (K-N)* quasi n-normal operator.

The operation on (K-N)* quasi n-normal operator have been given by the following theorem.

Theorem (3.5):

Let T_1, T_2 be two (K-N)* quasi n-normal operator from H to H, such that $T_1^*T_2^n = T_2^*T_1^n = T_1T_2^* = T_2T_1^* = T_1T_2 = T_2T_1 = 0$ then $T_1 + T_2$ is (K-N)* quasi n-normal operator.

Proof:

$$(T_1+T_2)^K[(T_1+T_2)^*(T_1+T_2)^n]^K = (T_1+T_2)^K[(T_1^*+T_2^*)^K(T_1+T_2)^{nK}]$$
$$= (T_1^K+T_2^K)\cdot\left((T_1^*)^K+(T_2^*)^K\right)\cdot(T_1^{nK}+T_2^{nK})$$
$$= (T_1^K+T_2^K)\cdot\left((T_1^*)^KT_1^{nK}+(T_1^*)^KT_2^{nK}+(T_2^*)^KT_1^{nK}+(T_2^*)^KT_2^{nK}\right)$$
$$= (T_1^K+T_2^K)\cdot\left((T_1^*)^KT_1^{nK}+(T_2^*)^KT_2^{nK}\right)$$
$$= T_1^K(T_1^*)^KT_1^{nK}+T_1^K(T_2^*)^KT_2^{nK}+T_2^K(T_1^*)^KT_1^{nK}+T_2^K(T_2^*)^KT_2^{nK}$$
$$= T_1^K(T_1^*)^KT_1^{nK}+T_2^K(T_2^*)^KT_2^{nK}$$
$$= T_1^K(T_1^*T_1^n)^K+T_2^K(T_2^*T_2^n)^K$$
$$= N\left((T_1^*T_1^n)^KT_1^K\right)+N\left((T_2^*T_2^n)^KT_2^K\right)$$

Hence $T_1 + T_2$ is (K-N)* quasi n-normal operator.

From above theorem, we can get the corollary its proof easy can be omitted it.

Corollary (3.6):

Let T_1, T_2 be two (K-N)* quasi n-normal operator from H to H, such that $T_1^*T_2^n = T_2^*T_1^n = T_1T_2^* = T_2T_1^* = T_1T_2 = T_2T_1 = 0$ and $T_2 = -T_2$ then $T_1 - T_2$ is (K-N)* quasi n-normal operator.

Theorem (3.7):

Let T_1 be (K-N)* quasi n-normal operator and T_2 quasi n-normal operator. Then there product T_1T_2 is (K-N)* quasi n-normal operator if the following conditions are satisfied:

(i) $T_1T_2 = T_2T_1$

(ii) $T_1T_2^* = T_2^*T_1$

(iii) $T_1^*T_2 = T_2T_1^*$

Proof:

$$(T_1T_2)^K\left((T_1T_2)^*(T_1T_2)^n\right)^K = T_1^KT_2^K\left((T_2T_1)^*(T_1T_2)^n\right)^K$$
$$= T_1^KT_2^K\left(T_1^*T_2^*T_1^nT_2^n\right)^K$$
$$= T_1^KT_2^K(T_1^*)^K(T_2^*)^KT_1^{nK}T_2^{nK}$$
$$= T_1^K(T_1^*)^KT_2^KT_1^{nK}(T_2^*)^KT_2^{nK}$$
$$= T_1^K(T_1^*)^KT_1^{nK}T_2^K(T_2^*)^KT_2^{nK}$$
$$= T_1^K(T_1^*T_1^n)^K \cdot T_2^K(T_2^*T_2^n)^K$$
$$= N(T_1^*T_1^n)^KT_1^K \cdot (T_2^*T_2^n)^KT_2^K$$
$$= N(T_1^*)^KT_1^{nK}T_1^K \cdot (T_2^*)^KT_2^{nK}T_2^K$$
$$= N(T_1^*)^KT_1^{nK}(T_2^*)^KT_1^KT_2^{nK}T_2^K$$
$$= N(T_1^*)^K(T_2^*)^KT_1^{nK}T_2^{nK}T_1^KT_2^K$$
$$= N(T_1^*T_2^*)^K(T_1^nT_2^n)^K(T_1T_2)^K$$
$$= N\left((T_1T_2)^*(T_1T_2)^n\right)^K(T_1T_2)^K$$

Hence, the product T_1T_2 is (K-N)* quasi n-normal operator.

Theorem (3.8):

A power of (K-N)* quasi n-normal operator is again (K-N)* quasi n-normal operator.

Proof:

let T be (K-N)* quasi n-normal operator, we prove that be using mathematical induction, therefore T is (K-N)* quasi n-normal operator, then the result is true for $m = 1$.

That is, $T^K(T^*T^n)^K = N(T^*T^n)^KT^K$ (1)

Now, we assume that the result is true for $m = n$

$$\left[T^K(T^*T^n)^K\right]^n = \left[N(T^*T^n)^KT^K\right]^n \quad (2)$$

Let us prove the result for $m = n+1$

That is, $\left[T^K(T^*T^n)^K\right]^{n+1} = \left[N(T^*T^n)^KT^K\right]^{n+1}$

$$\left[T^K(T^*T^n)^K\right]^{n+1} = \left[T^K(T^*T^n)^K\right]^n \cdot T^K(T^*T^n)^K$$
$$= \left[N(T^*T^n)^KT^K\right]^n \cdot \left[N(T^*T^n)^KT^K\right] \quad \text{by (1) and (2).}$$

Then, $\left[T^K(T^*T^n)^K\right]^{n+1} = \left[N(T^*T^n)^KT^K\right]^{n+1}$.

Also, there are another relation among (K-N)* quasi n-normal operator and another operators.

Theorem (3.9):

If $T \in B(H)$ is an invertible orthogonal operator then $N = I$, where T is (K-N)* quasi n-normal operator.

Proof:

Since T is (K-N)* quasi n-normal operator and invertible orthogonal operator

Then, $T^K (T^* T^n)^K = N(T^* T^n)^K T^K$

So, $T^K (T^{n-1})^K = N(T^{n-1})^K T^K$

$(T^n)^K = N(T^n)^K$

$N = I$

Theorem (3.10):

Let $T \in B(H)$ is an invertible idempotent operator then $N = T$, where T is (K-N)* quasi n-normal operator.

Proof:

Since T is (K-N)* quasi n-normal operator, then $T^K (T^* T^n)^K = N(T^* T^n)^K T^K$

And by using T is an invertible idempotent operator. Then, $T(T^K)^* T^K = N(T^K)^* T^K T$

$\left[TT^* T = NT^* T\right] \cdot T^{-1} (T^*)^{-1}$

So, $N = T$.

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