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Study of the Structure of Exotic ^{52,54,56,58}Ca Isotopes Using OXBASH Code

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Abstract

This study dedicates to provide an information of shell model calculations, limited to fp-shell with an accuracy and applicability. The estimations depend on the evaluation of Hamiltoian's eigenvalues, that's compatible with positive parity of energy levels up to (10MeV) for most isotopes of *Ca*, and the Hamiltonian eigenvectors transition strength probability and inelastic electron-nucleus scattering. The Hamiltonian is effective in the regions where we have experimented. The known experimental data of the same were confirmed and proposed a new nuclear level for others.

The calculations are done with the help of OXBASH code. The results show good agreement with experimental energy states for $^{52,54}Ca$ isotopes while a new result are gotten for $^{56,58}Ca$ isotopes as it is a modern nucleus investigation. For ^{58}Ca , the results are given a smallest 2^+ excitation energy with strong B(E2) confirmed by large-"scale shell model in *fp*-shell space that cannot describe the small energy $E(2^+)$ besides that, the reduce transition probability B(E2) in each interaction is different.

Keywords: Exotic nuclei, OXBASH, Binding Energy, Separation energy, Deformation factor.

دراسة تركيب نظائر الكالسيوم ^{52,54,56,58}Ca الغريبة باستخدام برنا مج الاوكسباش

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الخلاصة

خصصت هذه الدراسة لتوفير معلومات دقيقة وقابلة للتطبيق عن طريق اجراء حسابات نموذج القشرة لمنطقة محددة هي قشرة fp. تعتمد التقديرات على تقيم القيم الذاتيه للهاملتونيان نلك التي تتوافق مع التناظر الزوجي لمستويات الطاقه حتى (10MeV) لمعظم نظائر الكالسيوم المدروسة، والدوال الذاتية للهاملتونيان لاحتماليه الانتقال القوية واستطارة الالكترون غير المرنة من النواة. الهاميلتونيان في هذه المناطق التي اختبرناها يكون فعالا. البيانات التجريبيه المعروفه لعدد من نظائر الكالسوم المتوفرة تم تاكيدها مع اقتراح مستويات طاقة جديد للانويه الاخرى .

عملنا الحسابات باستخدام برنامج الاوكسباش. النتائج اظهرت توافقا جيدا مع مستويات الطاقه التجريبية للنظائر (^{52,54}Ca) بينما حصلنا على نتائج جديده للنظائر (^{56,58}Ca) لانها نوى حديثة. نتائج النواة ^{58}Ca لطاقه التهيج (2^+) كانت صغيرة مع احتمالية انتقال مختزلة $B(E_2)$ قويه اولتي تم تاكيدها بواسطه نموذج القشرة واسع المدى ضمن نطاق القشرة–fp التي لا يمكنها وصف طاقه تهيج صغيرة $E(2^+)$ ، بالاضافة الى ان احتمالية الانتقال المختزلة $B(E_2)$ كانت مختلفه في كل تفاعل تم استخدامه.

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1.Introduction

Mayer [1] and by Haxel, Jensen, and Suess [2] introduced the shell model, with very effective in describing nuclei and its properties with little valence nucleons [3]. By the effective one- and two-body interaction one can study nuclei in the nuclear shell model. Last 30 years, many approaches proposed the effective interaction of nuclei in the "*s-d* and *f-p* shells. Kuo and Brown in the 1960s used the" Hamada-Johnston potential [4]in the systematic road to calculate the effective two-body interaction in the s-d [5] and "*f-p"*[6] shells. Later, this interactions two-body enhancement using the folded diagram [7].

In experimental techniques richter *et al.* [8] used similar approaches that introduced by Wildenthal [9] named nonlinear fit process to calculation 195 two-body matrix elements and four single particle energies in the *fp*-shell. By this method the interaction FPD6 obtained [10]

Recent advanced experimental techniques have extreme research of nuclear structures known as "exotic nuclei". These nuclei" have been the subject of a large number of theoretical studies" to understand its structures based on number of theoretical models. "Exotic feature in nuclear physics is of great interest not only because it constitutes a stringent test for the available nuclear models, but also because it opens up new research fields in nuclear science".

Nuclei in these exotic regions typically have very short half-lives, on the order of a little hundred milliseconds or less, and they can have significantly different structural properties than those nuclei located near the valley of stability. Because of the rapid decay of exotic nuclei, it is rather difficult to make targets with them, therefore, experiments have been done in inverse kinematics with a beam of exotic nuclei incident on a target [11]. Exotic nuclei can be produced by nuclear reactions. Researcher Horoi et al. 2002, [12] applied the exponential convergence method to calculate the binding energies of "the shell model" of the shell nucleus $0f_{7/2}$ relative to the nucleus ^{40}Ca. Majeed and . A. Auda [13] studied the level schemes and transition rates B(E2;↑) with FPD6 and GXPF1 effective interactions. Their results agreed with the first 2^+ level for all isotopes. Hangen et al. [14], employed chiral effective field theory to calculate the interaction, binding energies and low lying excitation of $^{53,54,55,61}Ca$. Ali and Rasool [15] studied exotic nuclei in the *f7*-shell region for the nuclei ^{42}Ca, ^{43}Ca and ^{44}Ca by employing the effective interactions, *f742*pn and *f7*cdpn using the shell model code OXBASH for windows by applying spin-parity of valence nucleons. Bespalova et al. (2005)[16] matching experimental data for ^{40,42,44,46,48}Ca isotopes to find the single particle energies and transition probabilities. The data are analyzed within the dispersive optical model predicting a new" magic number N=34 for Z=20 nuclei.

It can be understood of the main objectives of this work as follows: first objective is to determine the accuracy effective of different effective interaction that derived from Hamiltonian for excited states of even-even calcium isotopes that described as a very deformed nucleus ($^{52\text{-}54\text{-}56\text{-}58}Ca$). Second objective to present a new theoretical data that calculated in the *fp*-space model for samples that have not any experimental data. A new version of OXBASH that characteristics with some nuclear structure in shell model are used

1. Shell model Calculation and Discussion:

2. 1 Binding Energy:

With increasingly large N/Z ratios, the atomic mass and radii combined give results for evidence of appearing the magic numbers. For *Ca* isotopes, when excited and to be back to ground state the multiple neutron emission and multistep process will deal with i) ß-decay when the parent nucleus excited, ii) γ -emission or neutron emission to be back to ground state. Binding energy (BE) brings determined information about the nuclide configuration as minimizes the ground state energy. For our samples binding energy of the neutron orbital's in valence space across Z=20 and N=32,34,36 and 38 will has a configuration $f_{7/2}, p_{3/2}, f_{5/2}, p_{1/2}$ with extended gaps, Figure-1, decreasing as neutron increasing and that means decreasing pairing force effects (pairing correlation at fermi level) and the "residual attractive interaction" between two "paired nucleons" when their "angular" momentum couple to zero. Binding energy will be small and the valence neutrons are more easily knocked out when neutron number increases. From the binding energy one can find the neutron pairing energy of *Ca*-isotopes and have value as the effective interactions made a too much strong pairing.

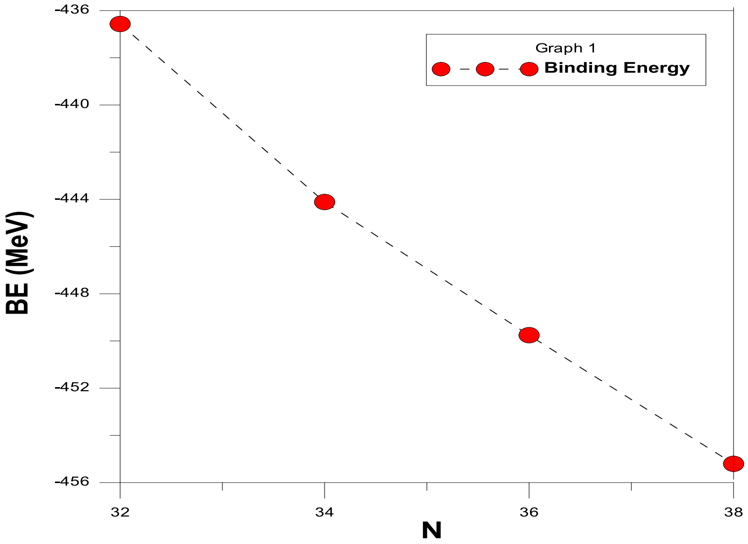

Figure 1-The binding energies for the *Ca* isotopes decreasing as neutron number increase.

2.2 Two neutron separation energy:

Binding energy and two-nucleon separation energies are different but they provide important information on the stability of “nuclei and shell gaps” or new magicity in terms of neutron separation energies. For our sample of nuclei these two neutron separation energy can be described by mass differences and given by [17]:

$$S_{2n} = BE(N - Z, 2) - B(N, Z)$$

where “*BE(N,Z)* is the binding energy” for even-even nuclei.

This energyhas a good role to predict sub shells for a nucleus. Since the pairing dominates, this energy for even number of nucleon must be higher than for odd number of one nucleon. From the Figure-2, one can observe that this energy decreases as neutron number increases i.e., the line drops as mass increases. The change of line slope of two nucleon separation energies is a bluk of this decrement at a shell-gap consists mainly of twice the difference of two effective single particle energies, plus a pairing, as in ^{56}Ca and ^{58}Ca. fig (2) the neutron pairing energy Δ_{2n} are determined as a difference between the two-neutron separation energy S_{2n} in nucleons *(N,Z)* and given by:

$$\Delta_{2n} = S_{2n}(N, Z) - 2S_n(N - 1, Z)$$

$$\Delta_{2n} = S_n(N, Z) - S_n(N - 1, Z)$$

where the energy eigenvalues depend on (n, ℓ) quantum number, but it’s degenerate in (*m*) quantum number magic an large energy gap above each shell.

To explain the decreasing of the gap between energy levels as move far away from the nucleons one can deal that with the required to more the nucleons to gather in nucleons and these Δ_{2n} for each nuclei $^{52,54,56,58}Ca$ are agreed with the energies of 8^+ state, for 52,*54Ca* in *fpd6*-interaction and in *GX1A* for ^{56}Ca but it is far for 6^+ state far ^{58}Ca.The differences due to uncertainties in these nuclei spin-parity assignments.

Table 1-Isotopes of Calcium with two neutron separation energies and shell gaps comparing with experimental [18]

Nucleus	Δ_{2n}		**S_{2n} (MeV)**	
	E_{the}	E_{exp}	E_{the}	E_{exp}
^{52}Ca	2.90	4.11	10.55	9.35
^{54}Ca	0.209	1.468	9.03	7.83
^{56}Ca	-0.48	1.124	8.12	6.03
^{58}Ca	0.127	-	6.61	4.66

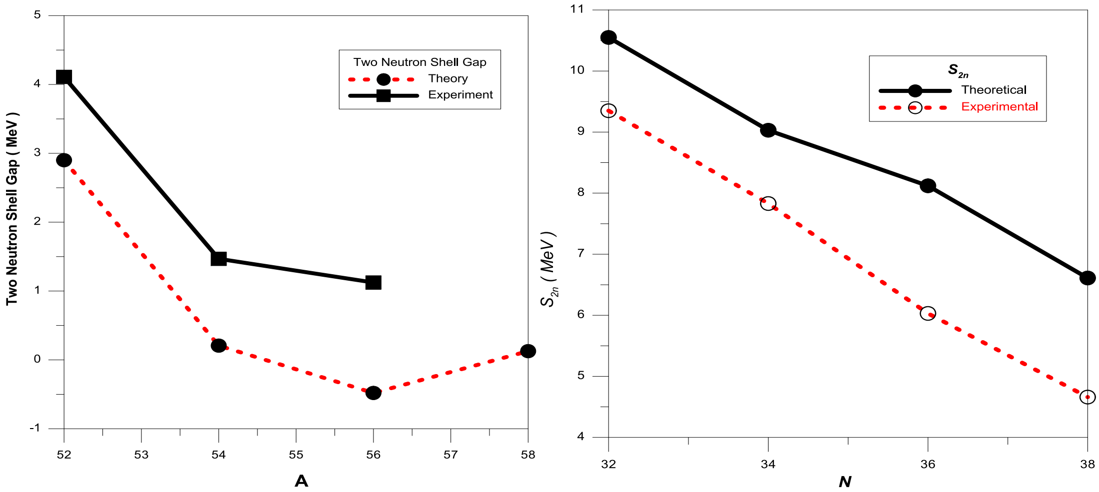

Figure 2-Variation of separation energies and shell gap of the ground state of the *Ca* isotopes by the *GX1A* Hamiltonian. Experimental data from [18]

Investigation of the structural properties of these isotopes is done by calculating their quadrupole deformations. We observe that the nucleus increasing in deformation and less bound. FP-shell nuclei testing in a shell model came from that have the 2+ state energy and transition rates B(E2) by the effective interaction. In ^{56}Ca, the 2+ is predicted by the *KB3G* interaction at 0.92MeV. Therefore, at N=34 there will be no peak in the 2+ energy marking a magic neutron number, as in Figure-3. Though, all calculations made with the different effective interactions predict that the ground state of ^{54}Ca is dominated to better than 90% by the neutron configuration $(0f_{7/2})8:(1p_{3/2})4:(1p_{1/2})2$ that is predicted it's the doubly magic or it has a closure of the $1p_{1/2}$ orbit that has a weak neutron- neutron interaction between orbits.

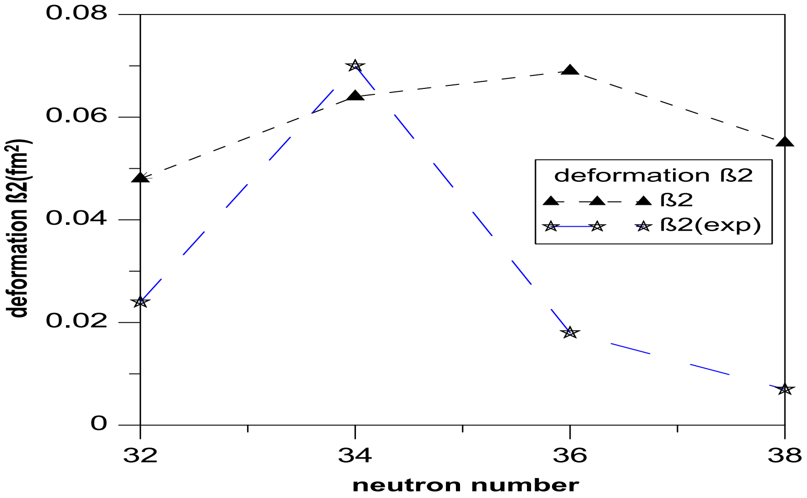

Figure 3-Deformation factor as a function of neutron number.

2.3 Excitation energies

The theoretical levels schemes of selected state of each nuclei are presented in fig. (4) for our calculation in FP-space model given for different effective interaction *GX1A*, *KB3G*, *FPD6* and *GX1* and although there are many levels experimentally identified in ^{52}Ca as it available, their spins and their associations energies are often included. Spin spectra for effective interactions are made up to for $^{52\text{-}54}Ca$ and for $^{56\text{-}58}Ca$. The core is considered as ^{40}Ca for each nucleus with different neutron valance outside core.

Because of the spin-orbit splitting, the sizeable energy “gap” will rise in fp-shell between $f_{7/2}$ and other orbits $p_{3/2}$,$p_{1/2}$,and $f_{5/2}$. The calculation of energy levels showed main difference in the size of the shell gaps. As sample of calcium isotopes are even-even nuclei that enable us to investigate the shell structure. The well-known pairing interaction leads to the lowering of ‘the energy of 0+ states. From the Figures-4, J^{π} = 0+ ground state and the low-energy excitation spectrum with first excited state 2+ in almost all nuclei. One can conclude that the structure of odd-even nuclei is much more complicated than their odd-odd neighbors.

The energies of the lowest 2+ states in the even-even Ca isotopes are shown in Figure-4. The high excitation energies at the neutron magic numbers Z=20 and N=32 are "clearly visible for ^{52}Ca, while the lowest energies occur in case of the nuclei ^{54}Ca, ^{56}Ca and ^{58}Ca. The" high excitation energy of the 2+ state" in ^{52}Ca is due to the filling of the orbital $v2p_{3/2}$ that consists of a sub-shell and agrees with the experimental evidence that obtain by [19].

For 0+ states, the states of different nucleus are in the same sequence of levels, so the symmetry is not taken into account. More, for all *J* values, ^{52}Ca has more common features as discussed below. The large scale calculation for $J^{\pi}T$ will do since isospin is conserved quantity that makes the possibility to define nucleus levels.

For $^{52\text{-}54}Ca$ nuclei, the changing in nuclear structure as a function of neutron number still give good information of the nuclei and the appearance a new shell gap. Due to filling of orbital, the shell closure for ^{54}Ca cannot be predicted by the *KB3G* and *FPD6* Hamiltonian, just by *GX1A*, as shown in the Figure- 4. As gap increasing, pairing is broken and the energy of 2+ state increases with the gap, so the magic property depends on shell gaps between $\upsilon p_{1/2}$ and $\upsilon f_{7/2}$. The appearance of sub-shell rises because of the evolution of neutron $f_{5/2}$ orbital and weakened interaction between protons and neutrons.

Now the spectra of $^{56\text{-}58}Ca$ showing the interaction between the valence neutrons and dearly that light agreement results for *KB3G,GX1A* and *FPD6* Interactions while its higher value in GX1 (3.64MeV) for 2+ state calculation energy of 2+ state for ^{56}Ca is close to the 2+ energy state of ^{46}Ca.

There is no experimental data available for these nuclei (^{56}Ca *and* ^{58}Ca) that allow comparing with. We note that the difference between excited state 2+,4+,6+ and 8+ are close to equality except for *FPD6* interaction. The same results are obtained for 2+,4+ and 6+ level with difference close to equality for *GX1A* and *GX1* rather than for *FPD6* and *KB3G*.

The first excited state of ^{52}Ca relatively has high excitation energy (2.56 MeV) indicating to a sub shell. By the different effective interaction, the first 2+ may be able to study as particle-hole (p-h) excitation for a shell with energy gap about (2.4 MeV).

Table 2- Excitation energies calculate by different effective interaction. Experimental data form [20] as available

^{52}Ca						
#	J^+	E_{oxbash} **(MeV)**				E_{exp}**(MeV)** **Ref. [20]**
		GX1A	**FPD6**	**KB3G**	**GX1**	
1	0^+	0	0	0	0	0
2	2^+	4.430	2.963	3.942	4.112	1.810
3	4^+	5.320	5.074	5.935	5.474	3.272
4	6^+	8.705	6.245	7.624	8.775	5.122
5	8^+	11.341	10.012	10.178	11.268	–
^{54}Ca						
#	J^+	E_{oxbash} **(MeV)**				E_{exp}**(MeV)**
		GX1A	**FPD6**	**KB3G**	**GX1**	
1	0^+	0	0	0	0	–
2	2^+	5.403	2.469	2.419	5.489	–
3	4^+	6.245	3.631	3.792	6.747	–
4	6^+	8.569	5.239	5.447	8.658	–
5	8^+	12.133	9.307	9.374	12.384	–
^{56}Ca						
#	J+	E_{oxbash} **(MeV)**				E_{exp}**(MeV)**
		GX1A	**FPD6**	**KB3G**	**GX1**	
1	0^+	0	0	0	0	–

2	2^+	2.766	2.041	1.844	3.640	–
3	4^+	3.314	2.945	2.772	4.333	–
4	6^+	5.700	5.618	5.357	5.856	–
5	8^+	8.755	9.725	8.951	8.952	–

^{58}Ca						
#	J+	E_{oxbash} (MeV)				E_{exp}(MeV)
		GX1A	FPD6	KB3G	GX1	
1	0^+	0	0	0	0	–
2	2^+	2.570	2.692	2.371	3.398	–
3	4^+	4.705	4.557	4.911	4.832	–
4	6^+	7.340	8.626	8.224	7.507	–

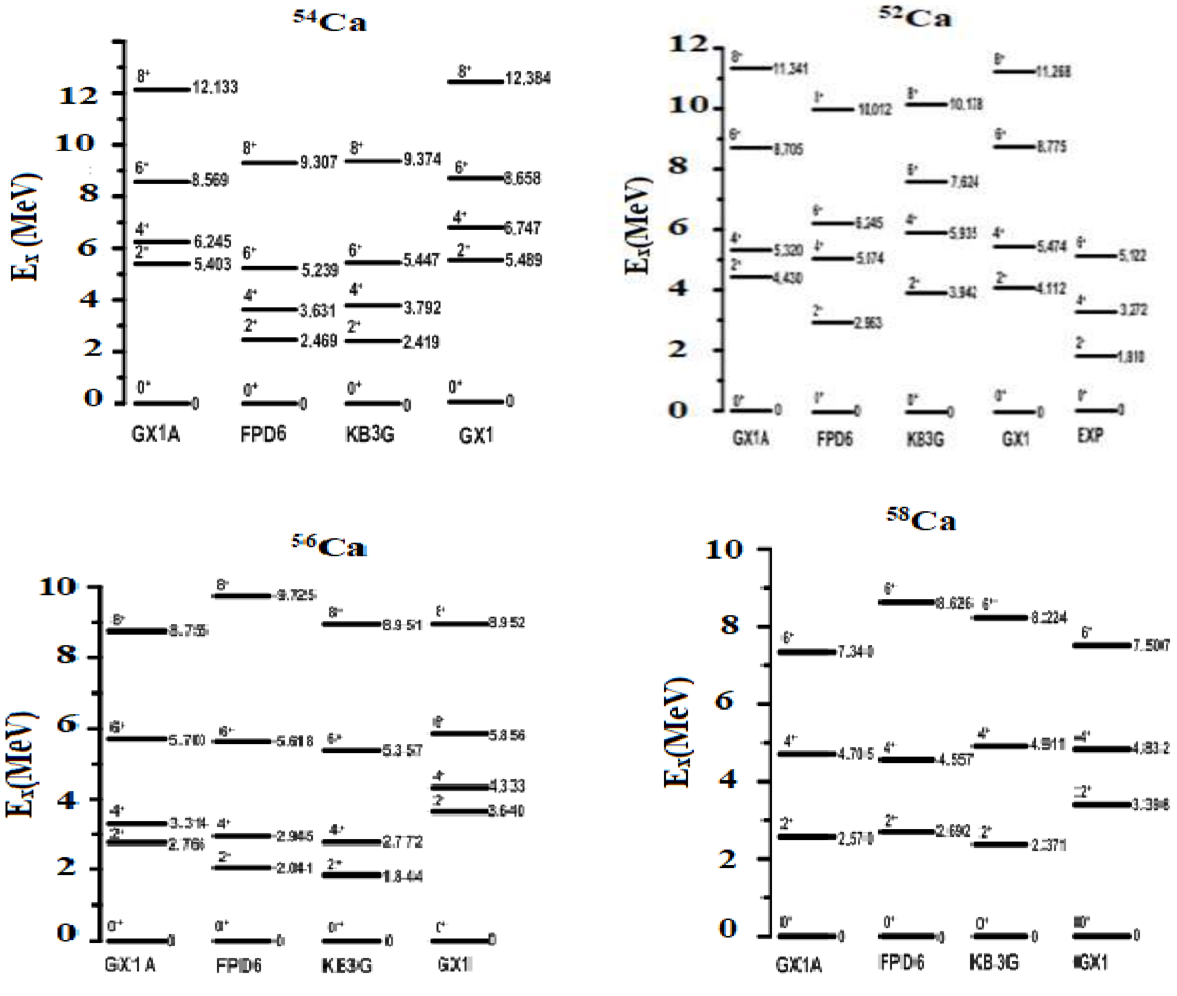

Figure 4-Excitation Energy levels of shell-model calculations with four effective interactions *GX1A, FPD6, KB3G* and *GX1*.

3. Conclusions:

The heavy *Ca* isotopes ($^{52,54,56,58}Ca$) that are neutron-rich nuclei have been studied by means of *OXBASH* code in the framework of shell model. The following results that developed our understanding of nuclear systems by mean of describing the motion of individual nucleons and their interactions.

Pairing interaction play an important role near the drip line of neutrons and away from shell closure.

Neutron single-particle energies of valance state and occupation percentage found that any changing in energy gap for *1p* and *2p* state will happen due to spin-orbit coupling, and for energy that calculated showed when mass number grows from 52 to 58 one can see the increasing between $\upsilon\, 1f_{5/2}$ and $\upsilon\, 2p_{1/2}$state for 2^+ state reaching in value about 4MeV. This gap energy is proportional to gap between $\upsilon\, 1f_{7/2}$ and $\upsilon\, 1p_{3/2}$ in ^{54}Ca .

The fact of increasing of gap means that nuclei is stable as filled states to be magic nuclei for *Z*=20 for *N*=32,34.

For exotic nuclei it is possible to study the gamma-decay [γ] of excited states populated either in the radioactive decay of the mother nucleus or in the decay of a long-lived isomeric state populated in the production reaction of the exotic nucleus.

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