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New Generalizations of Soft $LC-$Spaces

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Abstract

In this article, we introduce a new type of soft spaces namely, soft $L(NpC)-$spaces as a generalization of soft $LC-$spaces. Also, we study the weak forms of soft $L(NpC)-$ spaces, namely, soft NpL_1- spaces, soft NpL_2- spaces, soft NpL_3- space, and soft NpL_4- spaces. The characterizations and fundamental properties related to these types of soft spaces and the relationships among them are also discussed.

Keywords: Soft $K(NpC)-$space, soft $L(NpC)-$space, soft $\tilde{F}_\sigma - Np-$closed set, soft NpL_k- spaces, for $k=1,2,3,4$, soft $NP-$space, and soft $NpQ-$set space.

تعميمات جديدة لفضاءات – LC الناعمة

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الخلاصة

في هذة المقالة قدمنا نوعا جديدا من لفضاءات الناعمة اسميناها بالفضاءات – $L(NpC)$ الناعمة كتعميم للفضاءات – LC الناعمة ايضا درسنا الصيغ الضعيفة لفضاءات – $L(NpC)$ الناعمة وهي الفضاءات الناعمة من النمط – NpL_1 والفضاءات الناعمة من النمط – NpL_2 والفضاءات الناعمة من النمط – NpL_3 والفضاءات الناعمة من النمط NpL_4 . – المكافئات والخصائص الأساسية المتعلقة بهذه الانواع من الفضاءات الناعمة وعلاقاتهم مع بعضهم ايضا قد نوقشت.

Introduction

Molodtsov [1] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. He has shown several applications of this theory in solving many practical problems in economics, engineering, medical science, social science, etc. Shabir and Naz [2] presented the notion of soft topological spaces which are defined over an initial universe set with a fixed set of parameters, and they studied some concepts such as soft open sets, soft closed sets, soft closure and soft separation axioms. Arockiarani and Arokia [3], Mahmood and Ail [4] and Rong [5] introduced and studied soft $pre-$open sets, soft $N-pre-$open sets, soft $N-pre-$Lindelöf spaces and soft Lindelöf spaces respectively. Wilansky [6] presented the notion of $KC-$spaces in topological spaces and studied some relationships between $KC-$spaces and separation axioms. The concept of Lindelöf spaces

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was introduced by Alexandrof and Urysohn [7]. We know that there is no relationship between the concept of Lindelöf and closed subsets, so this point motivated some researchers to introduce a new concept that combines between Lindelöf and closed subsets, namely, $\mathrm{LC}-$spaces. The notion of $\mathrm{LC}-$spaces was first introduced by Mukherji and Sarkar [8]. Salih [9] introduced the concept of soft $\mathrm{KC}-$ spaces in soft topological spaces. Ali and Mahmood [10] generalized the notion of soft $\mathrm{KC}-$ spaces into soft $\mathrm{K(NpC)}-$ spaces and they gave several concepts that relate to these soft spaces. Mahmood [11] generalized the concept of an $\mathrm{LC}-$ spaces to soft $\mathrm{LC}-$ spaces and studied some relationships between soft $\mathrm{LC}-$spaces and each of soft $\mathrm{KC}-$ spaces and soft separation axioms. The aim of this paper is to study soft $\mathrm{L(NpC)}-$ spaces and weak forms of soft $\mathrm{L(NpC)}-$ spaces in soft topological spaces and prove some of their characterizations and basic properties

1. Preliminaries

Definition 1.1:[1]: A pair (E,G) is said to be a soft set over M if $\mathrm{E}: \mathrm{G} \to \mathrm{P(M)}$ is a function from the set of parameters G into $\mathrm{P(M)}$, and we can expressed by: $(\mathrm{E},\mathrm{G}) = \{(\mathrm{g},\mathrm{E(g)}): \mathrm{g} \in \mathrm{G} \text{ and } \mathrm{E(g)} \in \mathrm{P(M)}\}$.

Definition 1.2:[12]: A soft set (E,G) over M is called:
(i) A null soft set denoted by $\widetilde{\emptyset}$, if $\mathrm{E(g)} = \emptyset$, for each $\mathrm{g} \in \mathrm{G}$.
(ii) An absolute soft set denoted by $\widetilde{\mathrm{M}}$, if $\mathrm{E(g)} = \mathrm{M}$, for each $\mathrm{g} \in \mathrm{G}$.

Definition 1.3:[12,[13]: Let $(\mathrm{E}_1,\mathrm{G}_1)$ and $(\mathrm{E}_2,\mathrm{G}_2)$ be two soft sets over a common universe M. Then:
(i) The soft union of $(\mathrm{E}_1,\mathrm{G}_1)$ and $(\mathrm{E}_2,\mathrm{G}_2)$ is the soft set (E,G), where $\mathrm{G} = \mathrm{G}_1 \cup \mathrm{G}_2$, and for all $\mathrm{g} \in \mathrm{G}$,

$$\mathrm{E(g)} = \begin{Bmatrix} \mathrm{E}_1(\mathrm{g}) & \text{if } \mathrm{g} \in \mathrm{G}_1 - \mathrm{G}_2 \\ \mathrm{E}_2(\mathrm{g}) & \text{if } \mathrm{g} \in \mathrm{G}_2 - \mathrm{G}_1 \\ \mathrm{E}_1(\mathrm{g}) \cup \mathrm{E}_2(\mathrm{g}) & \text{if } \mathrm{g} \in \mathrm{G}_1 \cap \mathrm{G}_2 \end{Bmatrix}$$

We write $(\mathrm{E},\mathrm{G}) = (\mathrm{E}_1,\mathrm{G}_1)\widetilde{\cup}(\mathrm{E}_2,\mathrm{G}_2)$.
(ii) The soft intersection of $(\mathrm{E}_1,\mathrm{G}_1)$ and $(\mathrm{E}_2,\mathrm{G}_2)$ is the soft set (E,G), where $\mathrm{G} = \mathrm{G}_1 \cap \mathrm{G}_2$, and for all $\mathrm{g} \in \mathrm{G}$,
$\mathrm{E(g)} = \mathrm{E}_1(\mathrm{g}) \cap \mathrm{E}_2(\mathrm{g})$. We write $(\mathrm{E},\mathrm{G}) = (\mathrm{E}_1,\mathrm{G}_1)\widetilde{\cap}(\mathrm{E}_2,\mathrm{G}_2)$.

Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe set M with a fixed set of parameters G as follows:

Definition 1.4:[2]: A family $\widetilde{\sigma}$ of soft sets over M is said to be a soft topology on M if:
(i) $\widetilde{\mathrm{M}}, \widetilde{\emptyset} \,\widetilde{\in}\, \widetilde{\sigma}$.
(ii) If $(\mathrm{E}_1,\mathrm{G}), (\mathrm{E}_2,\mathrm{G}) \,\widetilde{\in}\, \widetilde{\sigma}$, then $(\mathrm{E}_1,\mathrm{G})\widetilde{\cap}(\mathrm{E}_2,\mathrm{G}) \,\widetilde{\in}\, \widetilde{\sigma}$.
(iii) If $(\mathrm{E}_\mathrm{i},\mathrm{G}) \,\widetilde{\in}\, \widetilde{\sigma}$, for all $\mathrm{i} \in \Psi$, then $\widetilde{\cup}\{(\mathrm{E}_\mathrm{i},\mathrm{G}): \mathrm{i} \in \Psi\} \,\widetilde{\in}\, \widetilde{\sigma}$.

The triple $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is said to be a soft topological space. Any member of $\widetilde{\sigma}$ is said to be soft open and its
complement is soft closed.

Definition 1.5:[14]: If $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft topological space, and $\widetilde{\emptyset} \neq (\mathrm{H},\mathrm{G}) \,\widetilde{\subseteq}\, \widetilde{\mathrm{M}}$. Then $\widetilde{\sigma}_{(\mathrm{H},\mathrm{G})} = \{(\mathrm{O},\mathrm{G})\widetilde{\cap}(\mathrm{H},\mathrm{G}): (\mathrm{O},\mathrm{G}) \,\widetilde{\in}\, \widetilde{\sigma}\}$ is called a relative soft topology on (H,G) and $((\mathrm{H},\mathrm{G}),\widetilde{\sigma}_{(\mathrm{H},\mathrm{G})},\mathrm{G})$ is called a soft subspace of $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$.

Definition 1.6: Let $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ be a soft topological space, and $(\mathrm{E}, \mathrm{G}) \widetilde{\subseteq} \widetilde{\mathrm{M}}$. Then (E, G) is called:
(i) Soft $\mathrm{pre}-\mathrm{open}$ (briefly soft $\mathrm{p}-\mathrm{open}$) **[3]** if $(\mathrm{E}, \mathrm{G}) \widetilde{\subseteq} \operatorname{int}(\mathrm{cl}((\mathrm{E}, \mathrm{G})))$.
(ii) Soft $\mathrm{N}-\mathrm{pre}-\mathrm{open}$ (briefly soft $\mathrm{Np}-\mathrm{open}$) **[4]** if for all $\widetilde{\mathrm{m}} \widetilde{\in} (\mathrm{E}, \mathrm{G})$, there exists a soft $\mathrm{p}-\mathrm{open}$ set (O, G) in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ with $\widetilde{\mathrm{m}} \widetilde{\in} (\mathrm{O}, \mathrm{G})$ and $(\mathrm{O}, \mathrm{G})-(\mathrm{E}, \mathrm{G})$ is finite.

Definition 1.7:[10]: If $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft topological space, and $(\mathrm{E}, \mathrm{G}) \widetilde{\subseteq} \widetilde{\mathrm{M}}$. Then:
(i) Soft $\mathrm{Np}-\mathrm{closure}$ of (E, G) is defined by:

$$\mathrm{Npcl}((\mathrm{E}, \mathrm{G}))=\widetilde{\bigcap}\{(\mathrm{F}, \mathrm{G}):(\mathrm{E}, \mathrm{G}) \widetilde{\subseteq}(\mathrm{F}, \mathrm{G}) \ \&\ (\mathrm{F}, \mathrm{G}) \text{ is soft } \mathrm{Np}-\text{closed in } \widetilde{\mathrm{M}}\}$$

(ii) (E, G) is soft $\mathrm{Np}-\mathrm{dense}$ in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ if $\mathrm{Npcl}((\mathrm{E}, \mathrm{G}))=\widetilde{\mathrm{M}}$.

Proposition 1.8:[10]: If $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft topological space, and $(\mathrm{E}, \mathrm{G}) \widetilde{\subseteq} \widetilde{\mathrm{M}}$. Then:
(i) $(\mathrm{E}, \mathrm{G}) \widetilde{\subseteq} \mathrm{Npcl}\big((\mathrm{E}, \mathrm{G})\big)$,
(ii) $\mathrm{Npcl}((\mathrm{E}, \mathrm{G}))$ is soft $\mathrm{Np}-\mathrm{closed}$ set in $\widetilde{\mathrm{M}}$.
(iii) (E, G) is soft $\mathrm{Np}-\mathrm{closed}$ if and only if $\mathrm{Npcl}((\mathrm{E}, \mathrm{G}))=(\mathrm{E}, \mathrm{G})$.

Proposition 1.9:[4]: Let $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$ be a soft open subspace of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$. Then
(i) $(\mathrm{E}, \mathrm{G}) \widetilde{\bigcap} \widetilde{\mathrm{H}}$ is soft $\mathrm{Np}-\mathrm{closed}$ (resp. soft $\mathrm{Np}-\mathrm{open}$) subset of $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$ whenever (E, G) is soft $\mathrm{Np}-\mathrm{closed}$ (resp. soft $\mathrm{Np}-\mathrm{open}$) subset of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$.
(ii) If $(\mathrm{E}, \mathrm{G}) \widetilde{\subseteq} \widetilde{\mathrm{H}}$. Then (E, G) is soft $\mathrm{Np}-\mathrm{open}$ in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ if and only if (E, G) is soft $\mathrm{Np}-\mathrm{open}$ in $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$.

Proposition 1.10:[4]: If (E, G) is soft $\mathrm{Np}-\mathrm{closed}$ in $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$ and $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$ is soft clopen subspace of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$, then (E, G) is soft $\mathrm{Np}-\mathrm{closed}$ in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$.

Definition 1.11: A soft topological space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is called:
(i) Soft $\mathrm{Np}-$ Lindelöf **[4]** (resp. soft Lindelöf **[5]**) if any cover of $\widetilde{\mathrm{M}}$ by soft $\mathrm{Np}-open$ (resp. soft open) subsets of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ contains a countable subcover.
(ii) Soft $\widetilde{\mathrm{T}}_2-\mathrm{space}$ **[15]** if for all $\tilde{\mathrm{a}}, \tilde{\mathrm{b}} \widetilde{\in} \widetilde{\mathrm{M}}$, and $\tilde{\mathrm{a}} \neq \tilde{\mathrm{b}}$, there exist soft open subsets $(\mathrm{H}_1, \mathrm{G})$ and $(\mathrm{H}_2, \mathrm{G})$ of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ such that $\tilde{\mathrm{a}} \widetilde{\in} (\mathrm{H}_1, \mathrm{G}), \tilde{\mathrm{b}} \widetilde{\in} (\mathrm{H}_2, \mathrm{G})$, and $(\mathrm{H}_1, \mathrm{G}) \widetilde{\bigcap} (\mathrm{H}_2, \mathrm{G})=\widetilde{\emptyset}$.
(iii) Soft $\mathrm{K}(\mathrm{NpC})-$ space **[10]** if any soft compact subset of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\mathrm{Np}-\mathrm{closed}$.
(iv) Soft $\mathrm{LC}-\mathrm{space}$ **[11]** (resp. soft $\mathrm{KC}-\mathrm{space}$ **[9]**) if any soft Lindelöf (resp. soft compact) subset of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft closed**.**

Proposition 1.12:[4]: If $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\mathrm{Np}-$ Lindelöf, then any soft $\mathrm{Np}-\mathrm{closed}$ subset of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\mathrm{Np}-$ Lindelöf.

2. New Generalizations of Soft LC − Spaces

In this section, we define and study soft $\mathrm{L}(\mathrm{NpC})-$ spaces and the four weak forms of the soft $\mathrm{L}(\mathrm{NpC})-$ spaces, namely, soft $\mathrm{NpL}_{\mathrm{k}}-$ spaces, for $\mathrm{k}=1,2,3,4$. We obtain several characterizations about these soft spaces as well as we also discuss the relationships among themselves.

Definition 2.1: A soft topological space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is called a soft $\mathrm{L}(\mathrm{NpC})-\mathrm{space}$ if any soft Lindelöf set in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\mathrm{Np}-\mathrm{closed}$.

Remark 2.2: Each soft $\mathrm{LC}-\mathrm{space}$ is a soft $\mathrm{L}(\mathrm{NpC})-\mathrm{space}$, but the converse may not be true.

Examples 2.3: If $M = \mathfrak{R}$, $G = \{g_1, g_2\}$, $(g_1, \{5\}) = \tilde{m} \tilde{\in} \tilde{\mathfrak{R}}$, and $\tilde{\sigma}_{Inc.} = \{(E,G) \tilde{\subseteq} \tilde{\mathfrak{R}}: \tilde{m} \tilde{\in} (E,G)\} \tilde{\cup} \{\tilde{\emptyset}\}$ is the included soft point topology on $\mathfrak{R}$. Then $(\mathfrak{R}, \tilde{\sigma}_{Inc.}, G)$ is soft $L(NpC)-$ space, because, if (D,G) is soft Lindelöf subset of $\tilde{\mathfrak{R}}$, then $(D,G)^c$ is soft $Np-$open subset of $\tilde{\mathfrak{R}}$, since for all $\tilde{x} \tilde{\in} (D,G)^c$, there exist $\{\tilde{x}, \tilde{m}\}$ is a soft $p-$open subset of $\tilde{\mathfrak{R}}$ such that $\tilde{x} \tilde{\in} \{\tilde{x}, \tilde{m}\}$ and $\{\tilde{x}, \tilde{m}\} - (D,G)^c$ is finite, so (D,G) is soft $Np-$closed. But $(\mathfrak{R}, \tilde{\sigma}_{Inc.}, G)$ is not soft $LC-$space, because $(S,G) = \{(g_1,\{5\}), (g_2,\{3\})\}$ is soft Lindelöf, but is not soft closed.

Theorem 2.4: A soft topological space $(M, \tilde{\sigma}, G)$ is a soft $L(NpC)-$ space iff every soft point in $(M, \tilde{\sigma}, G)$ has a soft clopen neighborhood which is a soft $L(NpC)-$subspace.

Proof: Suppose $(M, \tilde{\sigma}, G)$ is a soft $L(NpC)-$space, so for any $\tilde{m} \tilde{\in} \tilde{M}$, $\tilde{M}$ is itself a soft clopen neighborhood which is a soft $L(NpC)-$subspace. Conversely, Assume that (S,G) is soft Lindelöf in $(M, \tilde{\sigma}, G)$ and $\tilde{m} \tilde{\notin} (S,G)$. Choose a soft clopen neighborhood $(H_{\tilde{m}}, G)$ of $\tilde{m}$ with $((H_{\tilde{m}}, G), \tilde{\sigma}_{(H_{\tilde{m}},G)}, G)$ is a soft $L(NpC)-$subspace of $(M, \tilde{\sigma}, G)$. Thus $(H_{\tilde{m}}, G) \tilde{\cap} (S,G)$ is soft Lindelöf in $((H_{\tilde{m}}, G), \tilde{\sigma}_{(H_{\tilde{m}},G)}, G)$. But $((H_{\tilde{m}}, G), \tilde{\sigma}_{(H_{\tilde{m}},G)}, G)$ is soft $L(NpC)-$space, hence $(H_{\tilde{m}}, G) \tilde{\cap} (S,G)$ is soft $Np-$closed in $((H_{\tilde{m}}, G), \tilde{\sigma}_{(H_{\tilde{m}},G)}, G)$, and so soft $Np-$closed in $(M, \tilde{\sigma}, G)$ by Proposition (1.8). Therefore, $(H_{\tilde{m}}, G) - [(H_{\tilde{m}}, G) \tilde{\cap} (S,G)] = (H_{\tilde{m}}, G) - (S,G)$ is soft $Np-$open in $(M, \tilde{\sigma}, G)$ such that $\tilde{m} \tilde{\in} (H_{\tilde{m}}, G) - (S,G)$ and $[(H_{\tilde{m}}, G) - (S,G)] \tilde{\cap} (S,G) = \tilde{\emptyset}$, thus (S,G) is soft $Np-$closed. Hence $(M, \tilde{\sigma}, G)$ is a soft $L(NpC)-$space.

Definition 2.5: If $(M, \tilde{\sigma}, G)$ is a soft topological space, and $(E,G) \tilde{\subseteq} \tilde{M}$, then (E,G) is called soft $\tilde{F}_\sigma - Np-$closed (resp. soft $\tilde{G}_\sigma - Np-$open) if (E,G) is a countable soft union (resp. soft intersection) of soft $Np-$closed (resp. soft $Np-$open) sets.

Remark 2.6: Any soft $Np-$closed (resp. soft $Np-$open) set is soft $\tilde{F}_\sigma - Np-$closed (resp. soft $\tilde{G}_\sigma - Np-$open), but the converse may not be true.

Example 2.7: If $M = Z$, $G = \{g_1, g_2, g_3\}$, and $\tilde{\sigma}_{cof.} = \{(D,G) \tilde{\subseteq} \tilde{Z}: (D,G)^c \text{ is finite}\} \tilde{\cup} \{\tilde{\emptyset}\}$ is the soft cofinite topology on Z. Then $(E,G) = \{(g_1, Z-\{5\}), (g_2, Z-\{5\}), (g_3, Z-\{5\})\}$ is soft $\tilde{F}_\sigma - Np-$closed in $(Z, \tilde{\sigma}_{cof.}, G)$, but not soft $Np-$closed, since $(E,G)^c = \{(g_1,\{5\}), (g_2,\{5\}), (g_3,\{5\})\}$ is not soft $Np-$open. Also, $(T,G) = \{(g_1,\{5\}), (g_2,\{5\}), (g_3,\{5\})\}$ is soft $\tilde{G}_\sigma - Np-$open in $(Z, \tilde{\sigma}_{cof,}, G)$, but is not soft $Np-$open.

Definition 2.8: A soft topological space $(M, \tilde{\sigma}, G)$ is said to be a soft $NP-$space if any soft $\tilde{G}_\sigma - Np-$open
subset of $(M, \tilde{\sigma}, G)$ is soft $Np-$open.

Now, we present generalizations of soft $L(NpC)-$spaces.

Definitions 2.9: A soft topological space $(M, \tilde{\sigma}, G)$ is said to be:
(i) Soft NpL_1-space if any soft Lindelöf $\tilde{F}_\sigma - Np-$closed subset of $(M, \tilde{\sigma}, G)$ is soft $Np-$closed.
(ii) Soft NpL_2-space if $Npcl((E,G))$ is soft Lindelöf whenever (E,G) is soft Lindelöf subset of $(M, \tilde{\sigma}, G)$.
(iii) Soft NpL_3-space if any soft Lindelöf subset of $(M, \tilde{\sigma}, G)$ is soft $\tilde{F}_\sigma - Np-$closed.

(iv) Soft $\mathrm{NpL}_4 -$ space if (E,G) is soft Lindelöf subset of $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$, then there exists a soft Lindelöf $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed subset (S,G) of $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ with $(\mathrm{E},\mathrm{G}) \widetilde{\subseteq} (\mathrm{S},\mathrm{G}) \widetilde{\subseteq} \mathrm{Npcl}((\mathrm{E},\mathrm{G}))$.

Theorem 2.10: Each soft $\mathrm{L}(\mathrm{NpC}) -$ space is a soft $\mathrm{NpL}_{\mathrm{k}} -$ space, $\mathrm{k} = 1,2,3,4$.

Proof: It is followed from Definition (2.9).

The converse of Theorem (2.10) may not be true as in the following examples:

Examples 2.11:

(i) If $\mathrm{M} = \mathfrak{R}$, $\mathrm{G} = \{\mathrm{g}\}$ and $\widetilde{\sigma}_{\mathrm{u}}$ is the soft usual topology on $\mathfrak{R}$. Then $(\mathfrak{R},\widetilde{\sigma}_{\mathrm{u}},\mathrm{G})$ is a soft hereditarily Lindelöf $\mathrm{NpL}_2 -$ space, and thus soft $\mathrm{NpL}_4 -$ space, but neither soft $\mathrm{NpL}_3 -$ space, since $(\mathrm{E},\mathrm{G}) = \{(\mathrm{g},(0,1))\}$ is soft Lindelöf in $(\mathfrak{R},\widetilde{\sigma}_{\mathrm{u}},\mathrm{G})$, but it is not soft $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed nor a soft $\mathrm{NpL}_1 -$ space, since $(\mathrm{H},\mathrm{G}) = \{(\mathrm{g},(0,1])\}$ $= \widetilde{\bigcup}\{(\mathrm{g},[{}^{1}/_{\mathrm{n}},1]) : \mathrm{n} = 2,3,\ldots\}$ is soft Lindelöf $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed in $(\mathfrak{R},\widetilde{\sigma}_{\mathrm{u}},\mathrm{G})$ which is not soft $\mathrm{Np} -$ closed. Therefore, $(\mathfrak{R},\widetilde{\sigma}_{\mathrm{u}},\mathrm{G})$ is not a soft $\mathrm{L}(\mathrm{NpC}) -$ space.

(ii) Let $\mathrm{M} = \mathfrak{R}$, $\mathrm{G} = \{\mathrm{g}_1,\mathrm{g}_2\}$, $\left(\mathrm{g}_1,\{\sqrt{2}\}\right) = \widetilde{\mathrm{m}} \widetilde{\in} \widetilde{\mathfrak{R}}$, and let $\widetilde{\sigma}_{\mathrm{Exc.}} = \left\{(\mathrm{H},\mathrm{G}) \widetilde{\subseteq} \widetilde{\mathfrak{R}} : \widetilde{\mathrm{m}} \widetilde{\notin} (\mathrm{H},\mathrm{G})\right\} \widetilde{\cup} \left\{\widetilde{\mathfrak{R}}\right\}$ be the excluded soft point topology on $\mathfrak{R}$. Then $(\mathfrak{R},\widetilde{\sigma}_{\mathrm{Exc.}},\mathrm{G})$ is a soft $\mathrm{NpL}_3 -$ space, because if (E,G) is soft Lindelöf subset of $(\mathfrak{R},\widetilde{\sigma}_{\mathrm{Exc.}},\mathrm{G})$, then either $\widetilde{\mathrm{m}} \widetilde{\in} (\mathrm{E},\mathrm{G})$ or $\widetilde{\mathrm{m}} \widetilde{\notin} (\mathrm{m},\mathrm{G})$. If $\widetilde{\mathrm{m}} \widetilde{\in} (\mathrm{E},\mathrm{G})$, then (E,G) is soft closed. Hence, soft $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed, and if $\widetilde{\mathrm{m}} \widetilde{\notin} (\mathrm{E},\mathrm{G})$, then (E,G) is countable, so (E,G) is soft $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed. But, $(\mathfrak{R},\widetilde{\sigma}_{\mathrm{Exc.}},\mathrm{G})$ is not soft $\mathrm{L}(\mathrm{NpC}) -$ space, because $(\mathrm{B},\mathrm{G}) = \{(\mathrm{g}_1,\mathrm{Q}),(\mathrm{g}_2,\mathrm{Q})\} \widetilde{\subseteq} \widetilde{\mathfrak{R}}$ is soft Lindelöf, but is not soft $\mathrm{Np} -$ closed.

Proposition 2.12: $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC}) -$ space if and only if it is a soft $\mathrm{NpL}_3 -$ space and a soft $\mathrm{NpL}_1 -$ space.

Proof: Assume (B,G) is soft Lindelöf in $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$, since $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is soft $\mathrm{NpL}_3 -$ space, so (B,G) is soft $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed, but $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{NpL}_1 -$ space, hence (B,G) is soft $\mathrm{Np} -$ closed in $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ that is $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC}) -$ space. The other direction follows from Theorem (2.10).

Proposition 2.13: Each soft space which is a soft NpL_1 and a soft $\mathrm{NpL}_4 -$ space is a soft $\mathrm{NpL}_2 -$ space.

Proof: Suppose (P,G) is soft Lindelöf in $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$, since $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is soft NpL_4, so there exists a soft Lindelöf $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed set (S,G) in $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ with $(\mathrm{P},\mathrm{G}) \widetilde{\subseteq} (\mathrm{S},\mathrm{G}) \widetilde{\subseteq} \mathrm{Npcl}((\mathrm{P},\mathrm{G}))$. But $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{NpL}_1 -$ space. Hence, (S,G) is soft $\mathrm{Np} -$ closed. Therefore, $\mathrm{Npcl}((\mathrm{P},\mathrm{G})) \widetilde{\subseteq} (\mathrm{S},\mathrm{G}) \widetilde{\subseteq} \mathrm{Npcl}((\mathrm{P},\mathrm{G}))$. Thus $(\mathrm{S},\mathrm{G}) = \mathrm{Npcl}(\mathrm{P},\mathrm{G})$ is soft Lindelöf. That is $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{NpL}_2 -$ space.

Proposition 2.14: Each soft $\mathrm{NpL}_2 -$ space (resp. soft $\mathrm{NpL}_3 -$ space) is a soft $\mathrm{NpL}_4 -$ space.

Proof: Assume (B,G) is soft Lindelöf in $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$. Since $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{NpL}_2 -$ space, so $\mathrm{Npcl}((\mathrm{B},\mathrm{G}))$ is soft Lindelöf. That is $(\mathrm{B},\mathrm{G}) \widetilde{\subseteq} \mathrm{Npcl}((\mathrm{B},\mathrm{G})) \widetilde{\subseteq} \mathrm{Npcl}((\mathrm{B},\mathrm{G}))$. But, $\mathrm{Npcl}((\mathrm{B},\mathrm{G}))$ is soft $\mathrm{Np} - c$losed, hence there exists $(\mathrm{F},\mathrm{G}) = \mathrm{Npcl}((\mathrm{B},\mathrm{G}))$ which is a soft Lindelöf $\widetilde{\mathrm{F}}_{\sigma} - \mathrm{Np} -$ closed subset of $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ such that $(\mathrm{B},\mathrm{G}) \widetilde{\subseteq} (\mathrm{F},\mathrm{G}) \widetilde{\subseteq} \mathrm{Npcl}((\mathrm{B},\mathrm{G}))$. Thus $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{NpL}_4 -$ space. In the same way, we can prove that $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{NpL}_4 -$ space, if $(\mathrm{M},\widetilde{\sigma},\mathrm{G})$ is a soft $\mathrm{NpL}_3 -$ space.

Remark 2.15: The converse of Proposition (2.14) is not true. In Examples (2.11),(i), $(\mathfrak{R}, \widetilde{\sigma}_{\mathrm{u}}, \mathrm{G})$ is a soft NpL_4- space, but it is not soft NpL_3- space. Also, in Examples (2.11),(ii), $(\mathfrak{R}, \widetilde{\sigma}_{\mathrm{Exc.}}, \mathrm{G})$ is a soft NpL_4- space, but it is not soft NpL_2- space.

Proposition 2.16: Any soft $\mathrm{Np}-$ Lindelöf space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_2- space, and any soft NpL_2- space with soft Lindelöf $\mathrm{Np}-$ dense set is a soft Lindelöf.

Proof: Assume that (K, G) is soft Lindelöf in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$, since $\mathrm{Npcl}((\mathrm{K}, \mathrm{G}))$ is soft $\mathrm{Np}-$ closed in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$, then $\mathrm{Npcl}((\mathrm{K}, \mathrm{G}))$ is soft $\mathrm{Np}-$ Lindelöf in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ by Proposition (1.10), and so is soft Lindelöf. Therefore, $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft NpL_2- space. Now, if (H, G) is soft Lindelöf $\mathrm{Np}-$ dense set in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$, then $\mathrm{Npcl}((\mathrm{H}, \mathrm{G}))=\widetilde{\mathrm{M}}$. But $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_2- space, so $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft Lindelöf.

Proposition 2.17: Any soft $\mathrm{NP}-$ space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_1- space.

Proof: Suppose (E, G) is soft Lindelöf and soft $\tilde{\mathrm{F}}_\sigma-\mathrm{Np}-$ closed subset of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$, hence $(\mathrm{E}, \mathrm{G})^{\mathrm{c}}$ is soft $\widetilde{\mathrm{G}}_\sigma-\mathrm{Np}-$ open, but $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{NP}-$ space, so by Definition (2.8), $(\mathrm{E}, \mathrm{G})^{\mathrm{c}}$ is soft $\mathrm{Np}-$ open in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$. That is (B, G) is soft $\mathrm{Np}-$ closed. Therefore, $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_1- space.

Proposition 2.18: If $\{(\mathrm{E}_{\mathrm{i}}\mathrm{G}): \mathrm{i} \in \mathrm{N}\}$ is a countable family of soft $\mathrm{Np}-$ Lindelöf sets in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$. Then $\widetilde{\bigcup}\{(\mathrm{E}_{\mathrm{i}}, \mathrm{G}): \mathrm{i} \in \mathrm{N}\}$ is soft $\mathrm{Np}-$ Lindelöf.

Proposition 2.19: Each soft $\mathrm{Np}-$ Lindelöf NpL_1- space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{NP}-$ space.

Proof: Assume (H, G) is a soft $\tilde{\mathrm{F}}_\sigma-\mathrm{Np}-$closed subset of $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$. Hence $(\mathrm{H}, \mathrm{G})= \widetilde{\bigcup}\{(\mathrm{H}_{\mathrm{n}}, \mathrm{G}): \mathrm{n} \in \mathrm{N}\}$, where $(\mathrm{H}_{\mathrm{n}}, \mathrm{G})$ is soft $\mathrm{Np}-$ closed in $\widetilde{\mathrm{M}}$, $\forall\, \mathrm{n} \in \mathrm{N}$. But $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\mathrm{Np}-$ Lindelöf, so by Proposition (1.10), $(\mathrm{H}_{\mathrm{n}}, \mathrm{G})$ is soft $\mathrm{Np}-$ Lindelöf in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$, $\forall\, \mathrm{n} \in \mathrm{N}$. Thus, $(\mathrm{H}, \mathrm{G})=\widetilde{\bigcup}\{(\mathrm{H}_{\mathrm{n}}, \mathrm{G}): \mathrm{n} \in \mathrm{N}\}$ is soft $\mathrm{Np}-$ Lindelöf in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ by Proposition (2.18). Since $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_1- space, so (H, G) is soft $\mathrm{Np}-$ closed. Therefore, $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{NP}-$ space.

Corollary 2.20: A soft $\mathrm{Np}-$ Lindelöf space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_1- space if and only if it is a soft $\mathrm{NP}-$ space.

Proposition 2.21: A soft open subspace $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$ of a soft $\mathrm{L(NpC)}-$ space (resp. soft NpL_3- space) $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L(NpC)}-$ space (resp. soft NpL_3- space).

Proof: Suppose (E, G) is any soft Lindelöf subset of $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$, then (E, G) is a soft Lindelöf in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$. Since $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L(NpC)}-$ space, then (E, G) is soft $\mathrm{Np}-$ closed in $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$. By Proposition (1.7),(i), $(\mathrm{E}, \mathrm{G})=(\mathrm{E}, \mathrm{G}) \widetilde{\bigcap} \widetilde{\mathrm{H}}$ is soft $\mathrm{Np}-$ closed in $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$. Hence $(\mathrm{E}, \widetilde{\sigma}_{\mathrm{E}}, \mathrm{G})$ is a soft $\mathrm{L(NpC)}-$ space.

Proposition 2.22: A soft clopen subspace $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$ of a soft NpL_1- space (resp. soft NpL_2- space, soft NpL_4- space) $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_1-space (resp. soft NpL_2- space, soft NpL_4- space).

Proof: Suppose that (B, G) is any soft Lindelöf $\tilde{\mathrm{F}}_\sigma-\mathrm{Np}-$ closed set in $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$. Hence, there exist $\{(\mathrm{B}_k, \mathrm{G}): \mathrm{k} \in \mathrm{N}\}$ is a countable family of soft $\mathrm{Np}-$ closed in $(\mathrm{H}, \widetilde{\sigma}_{\mathrm{H}}, \mathrm{G})$ such that

$(B,G)=\widetilde{\bigcup}\{(B_k,G): k\in N\}$. Since $(H,\widetilde{\sigma}_H,G)$ is a soft clopen subspace of $(M,\widetilde{\sigma},G)$, then by Proposition (1.8), (B_k,G) is soft $Np-$ closed in $(M,\widetilde{\sigma},G)$ for each $k\in N$. Hence, (B,G) is a soft Lindelöf $\widetilde{F}_\sigma - Np -$ closed set in $(M,\widetilde{\sigma},G)$. But, $(M,\widetilde{\sigma},G)$ is soft NpL_1, so (B,G) is a soft $Np-$ closed subset of $(M,\widetilde{\sigma},G)$. Thus $(B,G)=(B,G)\widetilde{\bigcap}\widetilde{H}$ is soft $Np-$ closed in $(H,\widetilde{\sigma}_H,G)$ by Proposition (1.7),(i). Therefore, $(H,\widetilde{\sigma}_H,G)$ is a soft NpL_1 $-$space.

Proposition 2.23: A soft topological space $(M,\widetilde{\sigma},G)$ is a soft $\widetilde{T}_2-$ space if and only if for every two distinct soft points $\widetilde{m}_1,\widetilde{m}_2 \widetilde{\in} \widetilde{M}$, there exists a soft open set (O,G) containing $\widetilde{m}_1$ such that $\widetilde{m}_2 \widetilde{\notin} cl((O,G))$.

Theorem 2.24: A soft $\widetilde{T}_2-$ space $(M,\widetilde{\sigma},G)$ is a soft $L(NpC)-$ space if and only if it is a soft NpL_1- space and a soft NpL_2- space.

Proof: The first direction is followed from Theorem (2.10).

Conversely, suppose (L,G) is soft Lindelöf in $(M,\widetilde{\sigma},G)$, and let $\tilde{a}\widetilde{\notin}(L,G)$. Since $(M,\widetilde{\sigma},G)$ is a soft $\widetilde{T}_2-$ space, so by Proposition (2.23), we have for any $\tilde{b}\widetilde{\in}(L,G)$, there exists $(E_{\tilde{b}},G)\widetilde{\in}\widetilde{\sigma}$ containing $\tilde{b}$ such that $\tilde{a}\widetilde{\notin} cl((E_{\tilde{b}},G))$. Since $(L,G)\widetilde{\subseteq}\widetilde{\bigcup}\{(E_{\tilde{b}},G):\tilde{b}\widetilde{\in}(L,G)\}$ and (L,G) is soft Lindelöf, then there exists $\{(E_{\tilde{b}_m},G)\}_{m\in N}$ which is a countable subcover, that is $(L,G)\widetilde{\subseteq}\widetilde{\bigcup}\{(E_{\tilde{b}_m},G): m\in N\}\widetilde{\subseteq}\widetilde{\bigcup}\{cl((E_{\tilde{b}_m},G)): m\in N\}$. Now, for each $m\in N$,
$cl((E_{\tilde{b}_m},G))\widetilde{\bigcap}(L,G)$ is a soft Lindelöf, and so $Npcl[cl((E_{\tilde{b}_m},G))\widetilde{\bigcap}(L,G)]$ is also soft Lindelöf, since $(M,\widetilde{\sigma},G)$ is a soft NpL_2- space. Put $(P,G)=\widetilde{\bigcup}\{Npcl[cl((E_{\tilde{b}_m},G))\widetilde{\bigcap}(L,G)]: m\in N\}$, so (P,G) is a soft Lindelöf and soft $\widetilde{F}_\sigma - Np-$ closed subset of $(M,\widetilde{\sigma},G)$. Since $(M,\widetilde{\sigma},G)$ is a soft NpL_1-space, then (P,G) is soft $Np-$ closed in $(M,\widetilde{\sigma},G)$ and $\tilde{a}\widetilde{\notin}(P,G)$, thus $\tilde{a}\widetilde{\notin} Npcl((L,G))$. This shows (L,G) is soft $Np-$closed in $\widetilde{M}$. That is $(M,\widetilde{\sigma},G)$ is a soft $L(NpC)-$ space.

Theorem 2.25: Each soft $\widetilde{T}_2-$ space $(M,\widetilde{\sigma},G)$ which is a soft $NP-$ space is a soft $L(NpC)-$ space.

Proof: Suppose (B,G) is soft Lindelöf subset of $(M,\widetilde{\sigma},G)$. If $\tilde{a}\widetilde{\in}(B,G)^c$, then for all $\tilde{b}\widetilde{\in}(B,G)$, we have $\tilde{a}\neq\tilde{b}$, but $(M,\widetilde{\sigma},G)$ is soft $\widetilde{T}_2$, so there exist soft open sets $(U,G)_{\tilde{a}}$ and $(V,G)_{\tilde{b}}$ in $(M,\widetilde{\sigma},G)$ with $\tilde{a}\widetilde{\in}(U,G)_{\tilde{a}}$, $\tilde{b}\widetilde{\in}(V,G)_{\tilde{b}}$ and $(U,G)_{\tilde{a}}\widetilde{\bigcap}(V,G)_{\tilde{b}}=\widetilde{\emptyset}$. Hence $\{(V,G)_{\tilde{b}}:\tilde{b}\widetilde{\in}(B,G)\}$ is a soft open cover of (B,G). But (B,G) is a soft Lindelöf, so there exists $\{(V,G)_{\tilde{b}_i}: i\in N\}$ which is a countable subcover. Put $(W,G)=\widetilde{\bigcup}\{(V,G)_{\tilde{b}_i}: i\in N\}$ and $(V,G)=\widetilde{\bigcap}\{(U,G)_{\tilde{a}_i}: i\in N\}$. Thus (V,G) is a soft $Np-$ open, since $(M,\widetilde{\sigma},G)$ is a soft $NP-$ space and also (W,G) is soft open, since (W,G) is a soft union of soft open sets. So $\tilde{a}\widetilde{\in}(V,G)$ and $(B,G)\widetilde{\subseteq}(W,G)$. To show that $(V,G)\widetilde{\bigcap}(W,G)=\widetilde{\emptyset}$. Since $(U,G)_{\tilde{a}_i}\widetilde{\bigcap}(V,G)_{\tilde{b}_i}=\widetilde{\emptyset}$, $\forall\ i\in N$, then $(V,G)\widetilde{\bigcap}(V,G)_{\tilde{b}_i}=\widetilde{\emptyset}$, $\forall\ i\in N$, Thus $(V,G)\widetilde{\bigcap}$
$(W,G)=\widetilde{\emptyset}$. Therefore, $(V,G)\widetilde{\bigcap}(B,G)=\widetilde{\emptyset}$, that is $\tilde{a}\widetilde{\in}(V,G)\widetilde{\subseteq}(B,G)^c$, so $(B,G)^c$ is soft $Np-$ open. Hence (B,G) is a soft $Np-$ closed set. Therefore, $(M,\widetilde{\sigma},G)$ is a soft $L(NpC)-$ space.

Remark 2.26: In Theorem (2.25), if $(M,\widetilde{\sigma},G)$ is not soft $NP-$ space, then the theorem is not true. In Example (2.11), (i), $(\mathfrak{R},\widetilde{\sigma}_u,G)$ is not soft $NP-$space, because $(H,G)=$

$\{(g,(0,1])\} = \widetilde{\cup}\{(g,[1/n,1]) : n = 2,3,\dots\}$ is soft $\tilde{F}_\sigma - \mathrm{Np} -$ closed in $(\mathfrak{R}, \widetilde{\sigma}_u, G)$ which is not soft $\mathrm{Np} -$ closed. While $(\mathfrak{R}, \widetilde{\sigma}_u, G)$ is not soft $\mathrm{L(NpC)} -$ space.

Corollary 2.27: If a soft $\mathrm{Np} -$ Lindelöf space $(M, \widetilde{\sigma}, G)$ is soft $\widetilde{T}_2$. Then $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{NP} -$ space iff it is a soft $\mathrm{L(NpC)} -$ space.

Proposition 2.28: Any soft $\mathrm{L(NpC)} -$ space is a soft $\mathrm{K(NpC)} -$ space.

Proof: Suppose $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{L(NpC)} -$ space and (E, G) is any soft compact subset of $(M, \widetilde{\sigma}, G)$, then (E, G) is soft Lindelöf in $(M, \widetilde{\sigma}, G)$. Since $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{L(NpC)} -$ space, then (E, G) is soft $\mathrm{Np} -$ closed. Thus $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{K(NpC)} -$ space.

Remark 2.29: The converse of Proposition (2.28) is not true. In examples (2.11), (ii), $(\mathfrak{R}, \widetilde{\sigma}_{\mathrm{Exc.}}, G)$ is a soft $\mathrm{K(NpC)} -$ space, but is not soft $\mathrm{L(NpC)} -$ space.

Corollary 2.30: If $(M, \widetilde{\sigma}, G)$ is soft NpL_1 and soft $\mathrm{NpL}_3 -$ space, then $(M, \widetilde{\sigma}, G)$ is soft $\mathrm{K(NpC)} -$ space

Proof: Since $(M, \widetilde{\sigma}, G)$ is soft NpL_1 and soft $\mathrm{NpL}_3 -$ space, then by Proposition (2.12), $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{L(NpC)} -$ space, and by Proposition (2.28), $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{K(NpC)} -$ space.

Definition 2.31: A soft topological space $(M, \widetilde{\sigma}, G)$ is called:
(i) Soft hereditarily Lindelöf if any soft set in $(M, \widetilde{\sigma}, G)$ is soft Lindelöf.
(ii) Soft anti $-$ Lindelöf if any soft Lindelöf set in $(M, \widetilde{\sigma}, G)$ is soft countable.
(iii) Soft $\mathrm{NpQ} -$ set space if any soft set in $(M, \widetilde{\sigma}, G)$ is soft $\tilde{F}_\sigma - \mathrm{Np} -$ closed.

Proposition 2.32: Each soft anti $-$ Lindelöf space with a finite set of parameters is soft NpL_3. Hence, each soft anti $-$ Lindelöf $\mathrm{NpL}_1 -$ space with a finite set of parameters is a soft $\mathrm{L(NpC)} -$ space.

Proof: Assume that (L, G) is a soft Lindelöf subset of a soft anti $-$ Lindelöf space $(M, \widetilde{\sigma}, G)$, so (L, G) is countable. Since $\{\widetilde{m}\}$ is soft $\mathrm{Np} -$ closed in $(M, \widetilde{\sigma}, G)$ for each $\widetilde{m} \widetilde{\in} \widetilde{M}$ and G is finite, then $(L, G) = \widetilde{\cup}\{\{\widetilde{m}_n\} : n \in N\}$, hence (L, G) is a soft $\tilde{F}_\sigma - \mathrm{Np} -$ closed set. Thus $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{NpL}_3 -$ space.

The converse of Proposition (2.32) is not true we can see that in the following example:

Example 2.33: The soft indiscrete topology $(\mathfrak{R}, \widetilde{\sigma}_i, G)$ on $\mathfrak{R}$ is a soft $\mathrm{NpL}_3 -$ space, but it is not soft anti $-$ Lindelöf, since $\widetilde{\mathfrak{R}} \widetilde{\subseteq} \widetilde{\mathfrak{R}}$ is soft Lindelöf, but it is not countable.

Proposition 2.34: Each soft $\mathrm{NpQ} -$ set space $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{NpL}_3 -$ space.

Proof: Assume that (L, G) is any soft Lindelöf subset of a soft $\mathrm{NpQ} -$ set space $(M, \widetilde{\sigma}, G)$, then (L, G) is soft $\tilde{F}_\sigma - \mathrm{Np} -$ closed in $(M, \widetilde{\sigma}, G)$, so $(M, \widetilde{\sigma}, G)$ is a soft $\mathrm{NpL}_3 -$ space.

Remark 2.35: The converse of Proposition (2.34) is not true. In Example (2.11),(ii), $(\mathfrak{R}, \widetilde{\sigma}_{\mathrm{Exc.}}, G)$ is a soft $\mathrm{NpL}_3 -$ space, but it is not soft $\mathrm{NpQ} -$ set space, because $(H, G) = \{(g_1, \{\mathfrak{R} - \{\sqrt{2}\}), (g_2, \mathfrak{R})\} \widetilde{\subseteq} \widetilde{\mathfrak{R}}$ is not soft $\tilde{F}_\sigma - \mathrm{Np} -$ closed, since if (H, G) is soft $\tilde{F}_\sigma - \mathrm{Np} -$ closed, then $(H, G) = \widetilde{\cup}\{(S_j, G) : j \in N\}$, where (S_j, G) is soft $\mathrm{Np} -$ closed in $\widetilde{\mathfrak{R}}$,

$\forall\, j \in N$. Since $\tilde{m} \tilde{\notin} (S_j, G), \forall\, j \in N$, then (S_j, G) is finite $\forall\, j \in N$. Therefore, (H, G) is countable. This is a contradiction.
Now, we set a condition that makes soft $NpL_3 -$ spaces imply soft $NpQ -$ set spaces.

Proposition 2.36: If $(M, \tilde{\sigma}, G)$ is soft hereditarily Lindelöf $NpL_3 -$ space, then $(M, \tilde{\sigma}, G)$ is soft $NpQ -$ set space.

Proof: Suppose that (L, G) is any soft subset of $(M, \tilde{\sigma}, G)$, since $(M, \tilde{\sigma}, G)$ is a soft hereditarily Lindelöf, then (L, G) is soft Lindelöf. Since $(M, \tilde{\sigma}, G)$ is a soft $NpL_3 -$ space, then (L, G) is a soft $\tilde{F}_\sigma - Np -$ closed set in $(M, \tilde{\sigma}, G)$. Therefore, $(M, \tilde{\sigma}, G)$ is a soft $NpQ -$ set space.

Corollary 2.37: A soft hereditarily Lindelöf space $(M, \tilde{\sigma}, G)$ is soft $NpQ -$ set space if and only if it is soft $NpL_3 -$ space.

Corollary 2.38: If $(M, \tilde{\sigma}, G)$ is a countable soft space with G is finite. Then $(M, \tilde{\sigma}, G)$ is a soft $NpL_3 -$ space if and only if it is a soft $NpQ -$ set space.

Proposition 2.39:
(i) Each soft $NpQ -$ set space which is a soft NpL_1 is a soft $L(NpC) -$ space.
(ii) Each soft $NpQ -$ set space which is a soft $NP -$ space is a soft $L(NpC) -$ **s**pace.
(iii) Each soft $NP -$ space which is a soft NpL_3 is a soft $L(NpC) -$ **s**pace.

Proof: **(i)** Assume $(M, \tilde{\sigma}, G)$ is a soft $NpQ -$ set space, then $(M, \tilde{\sigma}, G)$ is a soft $NpL_3 -$ space by Proposition (2.34). But $(M, \tilde{\sigma}, G)$ is a soft $NpL_1 -$ space, thus $(M, \tilde{\sigma}, G)$ is a soft $L(NpC) -$ space by Proposition (2.12)**.**
(ii) Suppose (P, G) is a soft Lindelöf subset of a soft $NpQ -$ set space $(M, \tilde{\sigma}, G)$, then (P, G) is soft $\tilde{F}_\sigma - Np -$
closed. Since $(M, \tilde{\sigma}, G)$ is a soft $NP -$ space, then (P, G) is soft $Np - c$losed. Hence, $(M, \tilde{\sigma}, G)$ is a soft $L(NpC)$
$-$ space.
(iii) Assume that $(M, \tilde{\sigma}, G)$ is a soft $NP -$ space, hence by Proposition (2.17), $(M, \tilde{\sigma}, G)$ is a soft $NpL_1 -$ space,
but $(M, \tilde{\sigma}, G)$ is a soft $NpL_3 -$ space, so by Proposition (2.12), $(M, \tilde{\sigma}, G)$ is a soft $L(NpC)$ $-$ space.

Corollary 2.40: **(i)** Each soft $NpQ -$ set space which is a soft $NpL_1 -$ space is a soft $NpL_2 -$ space.
(ii) Each soft $NP -$ space which is a soft $NpL_3 -$ space is a soft $NpL_2 -$space.

Proof: **(i)** If (K, G) is a soft Lindelöf subset of a soft $NpQ -$ set space $(M, \tilde{\sigma}, G)$, then (K, G) is soft $\tilde{F}_\sigma - Np -$ closed, but $(M, \tilde{\sigma}, G)$ is soft NpL_1, so (K, G) is soft $Np - c$losed. Thus $(K, G) = Npcl((K, G))$, and $Npcl((K, G))$ is soft Lindelöf. Therefore, $(M, \tilde{\sigma}, G)$ is a soft $NpL_2 -$ space.
(ii) Assume (K, G) is a soft Lindelöf subset of a soft $NpL_3 -$ space $(M, \tilde{\sigma}, G)$, so (K, G) is a soft $\tilde{F}_\sigma - Np -$
closed. Since $(M, \tilde{\sigma}, G)$ is a soft $NP - s$pace, thus (K, G) is soft $Np -$ closed, that is $Npcl((K, G)) = (K, G))$,
so $Npcl((K, G))$ is soft Lindelöf. Hence $(M, \tilde{\sigma}, G)$ is a soft $NpL_2 -$ space.

Proposition 2.41: If $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft hereditarily Lindelöf and soft $\mathrm{NP}-s$pace. Then $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpQ
$-$ set space iff $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space.

Proof: Suppose that $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{NpQ}-$ set space, since $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\mathrm{NP}-$ space, hence by Proposition (2.39),(ii), $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space. Conversely, suppose that $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space, then by Theorem (2.10), $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_3- space**,** and since $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft hereditarily Lindelöf, then $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{NpQ}-$ set space by Proposition (2.36).

Theorem 2.42: If $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\widetilde{\mathrm{T}}_2-$ space and a soft NpL_1- space. The following are equivalent:
(a) $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space.
(b) $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_4- space.
(c) $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_3- space.
(d) $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_2- space.
Proof: $(\mathbf{a}) \Rightarrow (\mathbf{b})$**:** By Theorem (2.10).
$(\mathbf{b}) \Rightarrow (\mathbf{a})$**:** According to Proposition (2.13) and Theorem (2.24).
$(\mathbf{b}) \Rightarrow (\mathbf{c})$: Assume that $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_4- space, since $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_1- space, then by Proposition (2.13), $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_2- space. But $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\widetilde{\mathrm{T}}_2$, so $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space by Theorem (2.24). Thus $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_3- space by Theorem (2.10).
$(\mathbf{c}) \Rightarrow (\mathbf{b})$: This is followed by Proposition (2.12), and Theorem (2.10).
$(\mathbf{c}) \Rightarrow (\mathbf{d})$: Assume that $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_3- space, since $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_1- space, then $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space by Theorem (2.12). So by Theorem (2.10), $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_2- space.
$(\mathbf{d}) \Rightarrow (\mathbf{c})$: Assume $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft NpL_2- space, since $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is soft $\widetilde{\mathrm{T}}_2$ and soft NpL_1- space, then by Theorem (2.24), $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space. Therefore, by Proposition (2.12), $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft NpL_3- space.

Conclusions
(i) Each soft $\mathrm{LC}-$ space (resp. soft $\mathrm{L}(\mathrm{NpC})-$ space, soft $\mathrm{NP}-$ space and soft $\mathrm{NpQ}-$ set space) are a soft $\mathrm{L}(\mathrm{NpC})-$ space (resp. soft $\mathrm{K}(\mathrm{NpC})-$ space, soft NpL_1- space and soft NpL_3- space), respectively.
(ii) Each soft $\mathrm{L}(\mathrm{NpC})-$ space is a soft $\mathrm{NpL}_\mathrm{k}-$ space, $\mathrm{k}=1,2,3,4$, but the converse is not true.
(iii) Each soft NpL_2- space (resp. soft NpL_3- space) is a soft NpL_4- space, but the converse is not true.
(iv) Each soft $\mathrm{Np}-$ Lindelöf NpL_1- space is a soft $\mathrm{NP}-$ space.
(v) A soft open subspace of a soft $\mathrm{L}(\mathrm{NpC})-$ space (resp. soft NpL_3- space) is a soft $\mathrm{L}(\mathrm{NpC})-$ space (resp.
soft NpL_3- space).
(vi) A soft clopen subspace of a soft NpL_1- space (resp. soft NpL_2- space, soft NpL_4- space) is a soft NpL_1-space (resp. soft NpL_2- space, soft NpL_4- space).
(vii) A soft $\widetilde{\mathrm{T}}_2-$ space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ is a soft $\mathrm{L}(\mathrm{NpC})-$ space if and only if it is a soft NpL_1- space and a soft NpL_2- space.
(viii) Each soft $\widetilde{\mathrm{T}}_2-$ space $(\mathrm{M}, \widetilde{\sigma}, \mathrm{G})$ which is a soft $\mathrm{NP}-$ space is a soft $\mathrm{L}(\mathrm{NpC})-$ space, but the converse is not true.
(ix) We can apply this research to the algebraic, dynamic and geometric topology.

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