DOI: 10.24996/ijs.2023.64.1.35

ISSN: 0067-2904

On Soft Somewhere Dense Open Functions and Soft Baire Spaces

Baravan A. Asaad [1,2]*, Tareq M. Al-shami[3], Zanyar A. Ameen[4]

[1]Deptartment of Mathematics, Faculty of Science, University of Zakho, Zakho 42002, Iraq
[2]Department of Computer Science, College of Science, Cihan University-Duhok, Iraq
[3]Department of Mathematics, Sana'a University, P.O.Box 1247 Sana'a, Yemen
[4]Department of Mathematics, College of Science, University of Duhok, Duhok 42001, Iraq

Received: 4/11/2021 Accepted: 4/11/2022 Published: 30/1/2023

Abstract

The paper starts with the main properties of the class of soft somewhere dense open functions and follows their connections with other types of soft open functions. Then preimages of soft sets with Baire property and images of soft Baire spaces under certain classes of soft functions are discussed. Some examples are presented that support the obtained results. Further properties of somewhere dense open functions related to different types of soft functions are found under some soft topological properties.

Keywords: soft open; soft semiopen; soft somewhat open; soft somewhere dense open; Baire space; fuzzy Baire space; soft Baire space.

على الدوال المفتوحة الكثيفة اللينة في مكان ما وفضاءات البير اللينة

برفان عبدالمحسن اسعد[1,2]* , طارق محمد الشامي[3], زانيار انور امين[4]

[1]قسم الرياضيات، كلية العلوم، جامعة زاخو، زاخو، العراق
[2]قسم علوم الحاسبات، كلية العلوم، جامعة جيهان، دهوك، العراق
[3]قسم الرياضيات، كلية العلوم، جامعة صنعاء، صنعاء، اليمن
[4]قسم الرياضيات، كلية العلوم، جامعة دهوك، دهوك، العراق

الخلاصة

تبدأ الورقة بالخصائص الرئيسية لفئة الدوال المفتوحة الكثيفة اللينة في مكان ما وتتبعها من خلال اتصالاتها بأنواع أخرى من دوال الفتح اللين .ثم تم مناقشة الصور المعكوسة للمجموعات اللينة ذات خاصية البير وصور الفضاءات البيراللينة ضمن فئات معينة من الدوال اللينة .تم تقديم بعض الأمثلة التي تدعم النتائج التي تم الحصول عليها .تم العثور على المزيد من الخصائص للدوال المفتوحة الكثيفة في مكان ما وذات صلة بأنواع مختلفة من الدوال اللينة ضمن بعض الخصائص التوبولوجية اللينة.

* Email: baravan.asaad@uoz.edu.krd

1. Introduction

To deal with uncertainties, there are some theories such as fuzzy set theory, vague set theory, and rough set theory. In 1999, Molodtsov [1] proposed soft sets as a tool for modelling mathematical problems that deal with uncertainties. In soft set theory, there is no restricted condition to the description of objects, so researchers are free to select the form of parameters they require, which significantly simplifies the decision-making process and makes the method more productive in the absence of partial data. soft set theory can be applied in many directions as reported by Molodtsov himself. Maji et al. [2] studied a (detailed) theoretical structure of Soft Set Theory. In particular, he established several operators for this theory. After Maji's contribution, some other mathematical structures have been studied, for example, soft group theory, soft ring theory, etc..

In 2011, the concept of soft (general) topology was defined by Shabir and Naz [3] and by Çağman et al. in [4]. In 2013, Nazmul and Samanta [5] defined soft continuity of functions. Then various generalized types of soft continuity and soft openness of functions appeared in the literature. Namely: soft semicontinuous [6], soft semi-open [6], soft β-continuous [7], soft β-open [7], soft somewhat continuous [8], soft somewhat open functions [8] and so on. In a similar direction, the notions of compactness [9], connectedness [10, 11], separability [12], separation axioms [13, 14, 15, 16], etc., have been introduced in soft settings.

A significant outcome in general topology and functional analysis is the Baire Category Theorem (BCT), which gives sufficient conditions for a topological space to be a Baire space. The concept of Baire spaces has been studied extensively in general topology in [17, 18, 19, 20, 21]. In 2013, Thangaraj and Anjalmose [22] studied Baire spaces in the context of Fuzzy heory. In 2017, Riaz and Fatima [23] studied the concept of soft Baire spaces in soft metric spaces. According to our knowledge, very few is known about Baire spaces in soft set theoretic approach. In particular, the topic of soft Baire spaces being preserved under soft functions. From Theorem 2.38 in [24], one can conclude that soft Baire spaces can be preserved under soft open soft continuous images. This conclusion may fail when preserving soft Baire space under soft open soft continuous preimages without imposing any extra condition. In [25], Ameen and Khalaf worked on the soft Baire invariance via soft semicontinuous, soft somewhat continuous and soft somewhat open functions. Recently, Ameen et al. [26] have studied more properties of soft somewhere dense continuous functions in order to consider the preservation of soft Baire space and property Baire of soft sets. This paper continues in the same line. In particular, the images of soft Baire spaces and preimages of soft sets with Baire property are discussed along with the class of soft SD-open functions defined by Al-shami in [27].

2.Preliminaries

This section presents some basic definitions and notations that will be used in the sequel. Henceforth, we mean by X an initial universe, E a set of parameters and $\mathcal{P}(X)$ the power set of X.

Definition 2.1 [1] A pair $(F, E) = \{(e, F(e)): e \in E\}$ is said to be a soft set over X, where $F: E \to \mathcal{P}(X)$ is a (crisp) map. The class of all soft sets on X is symbolized by $\mathcal{P}(X, E)$. If $A \subseteq E$, then it will be symbolized by $\mathcal{P}(X, A)$.

Definition 2.2 [5, 28] A soft set (F, E) over X is called

i. a soft element if $F(e) = \{x\}$ for all $e \in E$, where $x \in X$. It is denoted by $(\{x\}, E)$.

ii. a soft point if $e \in E$ and $x \in X$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \neq e$, $e' \in E$. It is denoted by $p(e, x)$. An expression $p(e, x) \,\tilde{\in}\, (F, E)$ means that $x \in F(e)$.

Definition 2.3 [29] The complement of (F, E) is a soft set $(X, E)\backslash(F, E)$ (or simply $(F, E)^c$), where $F^c: E \to \mathcal{P}(X)$ is given by $F^c(e) = X\backslash F(e)$ for all $e \in E$.

Definition 2.4 [1] A soft subset (F, E) over X is called
i. null if $F(e) = \emptyset$ for any $e \in E$.
ii. absolute if $F(e) = X$ for any $e \in E$.
The null and absolute soft sets are respectively symbolized by $\tilde{\Phi}$ and $\tilde{X}$. Clearly, $\tilde{X}^c = \tilde{\Phi}$ and $\tilde{\Phi}^c = \tilde{X}$.

Definition 2.5 [2] Let $A, B \subseteq E$. It is said that (G, A) is a soft subset of (H, B) (written by $(G, A) \,\tilde{\subseteq}\, (H, B)$) if $A \subseteq B$ and $F(e) \subseteq G(e)$ for any $e \in A$. We say $(G, A) = (H, B)$ if $(G, A) \,\tilde{\subseteq}\, (H, B)$ and $(H, B) \,\tilde{\subseteq}\, (G, A)$.

The definitions of soft union and soft intersection of two soft sets with respect to arbitrary subsets of E was given by Maji et al. [2]. But it turns out that these definitions are misleading and ambiguous as reported by Ali et al. [29] which we follow in this manuscript.

Definition 2.6 Let $\{(F_\alpha, E): \alpha \in \Lambda\}$ be a collection of soft sets over X, where Λ is any index set.
i. The intersection of (F_α, E), for $\alpha \in \Lambda$, is a soft set (G, E) such that $G(e) = \bigcap_{\alpha\in\Lambda} F_\alpha(e)$ for each $e \in E$ and denoted by $(G, E) = \widetilde{\bigcap_{\alpha\in\Lambda}} (F_\alpha, E)$.
ii. The union of (F_α, E), for $\alpha \in \Lambda$, is a soft set (G, E) such that $G(e) = \bigcup_{\alpha\in\Lambda} F_\alpha(e)$ for each $e \in E$ and denoted by $(G, E) = \widetilde{\bigcup_{\alpha\in\Lambda}} (F_\alpha, E)$.
iii. The symmetric difference of (F_{α_1}, E) and $(F_{\alpha_2}, E)$is defined by

$$(F_{\alpha_1}, E)\tilde{\Delta}(F_{\alpha_2}, E) = ((F_{\alpha_1}, E)\backslash(F_{\alpha_2}, E)) \,\tilde{\cup}\, ((F_{\alpha_2}, E)\backslash(F_{\alpha_1}, E)).$$

Definition 2.7 [3] A subfamily $\mathcal{T}$ of $\mathcal{P}(X, E)$ is called a soft topology on X if
i. $\tilde{\Phi}$ and $\tilde{X}$ belong to $\mathcal{T}$,
ii. finite intersection of sets from $\mathcal{T}$ belongs to $\mathcal{T}$, and
iii. any union of sets from $\mathcal{T}$ belongs to $\mathcal{T}$.
Terminologically, we call $(X, \mathcal{T}, E)$ a soft topological space on X. The elements of $\mathcal{T}$ are called soft open sets, and their complements are called soft closed sets.
Henceforward, $(X, \mathcal{T}, E)$ means a soft topological space.

Definition 2.8 [4] A subfamily $\mathcal{B} \subseteq \mathcal{T}$ is called a soft base for the soft topology $\mathcal{T}$ if each element of $\mathcal{T}$ is a union of elements of $\mathcal{B}$.

Definition 2.9 [3] Let (Y, E) be a non-null soft subset of $(X, \mathcal{T}, E)$. Then $\mathcal{T}_Y := \{(G, E)\tilde{\cap}(Y, E): (G, E) \,\tilde{\in}\, \mathcal{T}\}$ is called a soft relative topology on Y and $(Y, \mathcal{T}_Y, E)$ is a soft subspace of $(X, \mathcal{T}, E)$.

Definition 2.10 [3] Let (F, E) be a soft subset of $(X, \mathcal{T}, E)$. The soft interior of (F, E) is the largest soft open set contained in (F, E) and denoted by $Int_X((F, E))$ (or shortly $Int((F, E))$). The soft closure of (F, E) is the smallest soft closed set which contains (F, E) and denoted by $Cl_X((F, E))$ (or simply $Cl((F, E))$).

Lemma 2.11 [30] For a soft subset (G, E) of $(X, \mathcal{T}, E)$,

$$Int((G, E)^c) = (Cl((G, E)))^c \text{ and } Cl((G, E)^c) = (Int((G, E)))^c.$$

The concepts in the following definition have been taken from the following references [7, 8, 23, 24, 31, 32].

Definition 2.12 A soft subset (G, E) of $(X, \mathcal{T}, E)$ is called

1. soft dense if $Cl((G, E)) = \tilde{X}$.
2. soft co-dense if $Int((G, E)) = \widetilde{\Phi}$.
3. soft nowhere dense if $Int(Cl((G, E))) = \widetilde{\Phi}$.
4. soft meager if it is a countable union of soft nowhere dense sets.
5. soft comeager if it is the complement of soft meager set.
6. soft G_δ if it is a countable intersection of soft open sets.
7. soft regular closed if $(G, E) = Cl(Int((G, E)))$.
8. soft semiopen if $(G, E) \widetilde{\subseteq} Cl(Int((G, E)))$.
9. soft β-open if $(G, E) \widetilde{\subseteq} Cl(Int(Cl((G, E))))$.
10. soft somewhat open (briefly, soft SW-open) if $Int((G, E)) \neq \widetilde{\Phi}$ or $(G, E) = \widetilde{\Phi}$.
11. soft somewhere dense (briefly, soft SD-open) if $Int(Cl((G, E))) \neq \widetilde{\Phi}$ or $(G, E) = \widetilde{\Phi}$.

Definition 2.13 [33] A soft topological spaces $(X, \mathcal{T}, E)$ is called soft Hausdorff if every distinct soft points are separated by two disjoint soft open sets.

3. Soft somewhere dense open functions and their relations

The concepts in the following definition have been taken from the following references [5, 6, 7, 8, 27].

Definition 3.1 Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. A soft function $f: (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is called

1. soft continuous (resp., soft semicontinuous, soft SW-continuous, soft SD-continuous, and soft β-continuous) if the inverse image of each soft open subset of $(Y, \mathcal{S}, E')$ is a soft open (resp., soft semiopen, soft SW-open, soft SD-open, and β-open) subset of $(X, \mathcal{T}, E)$.
2. soft open (resp., soft semiopen, soft SW-open, soft SD-open, and soft β-open) if the image of each soft open subset of $(X, \mathcal{T}, E)$ is a soft open (resp., soft semiopen, soft SW-open, soft SD-open, and β-open) subset of $(Y, \mathcal{S}, E')$.
3. soft homeomorphism (resp. soft SW-homeomorphism) if it is a soft open and soft continuous (resp. soft SW-open and soft SW-continuous) bijection from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$.

For the definition of soft functions between collections of all soft sets, we refer the reader to [34]. Henceforward, by the word "function" we mean "soft function".

The following diagram has been taken from [8] describing a connection between the soft types of open functions which was defined above.

$$
\begin{tikzcd}
\text{soft semiopen} \arrow[r] \arrow[d] & \text{soft } \beta\text{-open} \arrow[d] \\
\text{soft } SW\text{-open} \arrow[r] & \text{soft } SD\text{-open}
\end{tikzcd}
$$

Diagram I: Relationship between some generalized soft open functions

The above arrows mean inclusions and their reverses are generally false.

Example 3.2 Let $X = Y = \mathbb{R}$ be the set of real numbers, $\mathbb{Q}$ be the set of rational numbers and $E = \{e\}$ be a set of parameters. If $\mathcal{T} = \{(U,E): (U,E) = (G,E)\widetilde{\cup}[(H,E)(\mathbb{Q},E)]\}$ is a soft topology on X, where $(G,E),(H,E) \widetilde{\in} \mathcal{S}$, and $\mathcal{S}$ is the soft topology on Y generated by $\{(e,B(e)): B(e) = (a,b); a,b \in \mathbb{R}; a < b\}$, then the identity function $f: (X,\mathcal{T},E) \to (Y,\mathcal{S},E)$ is soft β-open (consequently, soft SD-open) but not soft SW-open (consequently, soft semiopen).

Example 3.3 Consider the soft topological space $(Y,\mathcal{S},E)$ which is given in Example 3.2. Define a function $f: (Y,\mathcal{S},E) \to (Y,\mathcal{S},E)$ by

$$f(p(e,x)) = \begin{cases} p(e,x), & \text{if } p(e,x) \widetilde{\notin} (\{0,1\},E); \\ p(e,0), & \text{if } p(e,x) = p(e,1); \\ p(e,1), & \text{if } p(e,x) = p(e,0). \end{cases}$$

One can easily show f is soft SW-open (consequently, soft SD-open) but not soft β-open (consequently, is not soft semiopen).

The definition of a soft somewhere dense open (SD-open) function for a single soft point can be stated as follows:

Proposition 3.4 Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces. A function $f: (X,\mathcal{T},E) \to (Y,\mathcal{S},E')$ is soft SD-open at $p(e,x) \widetilde{\in} X$ if for each soft open set (U,E) over X containing $p(e,x)$, there exists a soft SD-open set (V,E') over Y such that $f(p(e,x)) \widetilde{\in} (V,E') \widetilde{\subseteq} f((U,E))$.

Now, we give some results on soft SD-open functions.

Theorem 3.5 Let $(X,\mathcal{T},E),(Y,\mathcal{S},E')$ be soft topological spaces and let $f: (X,\mathcal{T},E) \to (Y,\mathcal{S},E')$ be a bijection. The following are equivalent:

1. f is soft SD-open;
2. for each non-null soft open set (G,E) over X, there exists a non-null soft open set (H,E') over Y such that $(H,E') \widetilde{\subseteq} Cl(f((G,E)))$;
3. for each soft closed set (F,E) over X with $f((F,E)) \neq \tilde{Y}$, there exists a proper soft closed set (K,E') over Y such that $Int(f((F,E))) \widetilde{\subseteq} (K,E')$.

Proof. (1) ⇒ (2) Follows from the definition.

(2) ⇒ (3) Let (F,E) be a soft closed over X with $f((F,E)) \neq \tilde{Y}$. This implies $\tilde{X}\setminus(F,E)$ is a non-null soft open set over X. By (1), there exists a soft open set (H,E') over Y such that $\widetilde{\Phi} \neq (H,E') \widetilde{\subseteq} Cl(f(\tilde{X}\setminus(F,E)))$. Therefore, $Int(f((F,E))) = \tilde{Y}\setminus(Cl(f(\tilde{X}\setminus(F,E)))) \widetilde{\subseteq} \tilde{Y}\setminus(H,E')$. Set $(K,E') = \tilde{Y}\setminus(H,E')$. So (K,E') is a soft closed set over Y that satisfies the required property.

(3) ⇒ (1) [27, Proposition 4.4] and [24, Theorem 2.5].

Here shall state that the above result cannot proved without bijectivity, and counterexamples showing this are not difficult to obtain.

Theorem 3.6 Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces and let (G,E) be a soft open subspace over X. If $f: (X,\mathcal{T},E) \to (Y,\mathcal{S},E')$ is soft SD-open over X, then $f|_{(G,E)}$ is a soft SD-open over (G,E).

Proof. If (U,E) is any soft open in (G,E), then (U,E) is also soft open over X because (G,E) is soft open. By assumption, we have $f((U,E))$ is SD-open and hence we get the result.

Theorem 3.7 Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces and let (D,E) be a soft dense subspace over X. *If* $f:(D,E)\to(Y,\mathcal{S},E')$ is a soft SD-open function, then each extension of f is soft SD-open over X.

Proof. Let g be any extension of f and let (U,E) be a soft open set over X. Since (D,E) is soft dense over X, so $(U,E)\tilde{\sqcap}(D,E)$ is a non-null soft open set in (D,E). By assumption and Theorem 3.5, there exists a non-null soft SD-open set (V,E') over Y such that $(V,E')\tilde{\sqsubseteq} f((U,E)\tilde{\sqcap}(D,E))=g((U,E)\tilde{\sqcap}(D,E))\tilde{\sqsubseteq} g((U,E))$. Thus, by Theorem 3.5, g is a soft SD-open function over X.

Theorem 3.8 Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces. Let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a function and $\{(G_\alpha,E):\alpha\in\Lambda\}$ be any soft cover over X. Then f is soft SD-open, if $f|_{(G_\alpha,E)}$ is soft SD-open for each $\alpha\in\Lambda$.

Proof. Let (U,E) be a (non-null) soft open set over X. Then $(U,E)\tilde{\sqcap}(G_\alpha,E)$ is a non-null soft open set in (G_α,E) for each α. By assumption, $f\big((U,E)\tilde{\sqcap}(G_\alpha,E)\big)$ is a soft SD-open set in Y. But $f((U,E))=\tilde{\bigsqcup} f\big((U,E)\tilde{\sqcap}(G_\alpha,E)\big)$, which is a union of soft SD-open sets and by [24, Theorem 2.19], $f((U,E))$ is a soft SD-open set over Y. Hence, f is a soft SD-open function.

Theorem 3.9 Let $(X,\mathcal{T},E),(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a function. The following are equivalent:

1. f is soft SD-open;
2. for each soft open dense set (D,E') over Y, then $f^{-1}((D,E'))$ is soft dense over X;
3. for each soft (closed) nowhere dense set (D,E') over Y, then $f^{-1}((D,E'))$ is soft co-dense over X.

Proof. (1) $\Rightarrow$ (2) Let (D,E') be a soft open dense set over Y. Suppose otherwise that $f^{-1}((D,E'))$ is not soft dense over X. Then, there is a soft closed (K,E) over X such that $f^{-1}((D,E'))\tilde{\sqsubseteq}(K,E)\neq\tilde{X}$. But $\tilde{X}\backslash(K,E)$ is non-null soft open over X so, by (1), there exists a soft open set (V,E') over Y such that $\tilde{\Phi}\neq(V,E')\tilde{\sqsubseteq} Cl(f(\tilde{X}\backslash(K,E)))$. Therefore, $(V,E')\tilde{\sqsubseteq} Cl(f(\tilde{X}\backslash(K,E)))\tilde{\sqsubseteq} Cl(f(f^{-1}(\tilde{Y}\backslash(D,E'))))\tilde{\sqsubseteq} Cl(\tilde{Y}\backslash(D,E'))=\tilde{Y}\backslash(D,E'$. Thus, $(D,E')\tilde{\sqsubseteq}\tilde{Y}\backslash(V,E')\neq\tilde{\Phi}$. But $\tilde{Y}\backslash(V,E')$ is soft closed over Y which contradicts the soft density of (D,E') over Y. Hence, $f^{-1}((D,E'))$ must be soft dense over X.

(2)⟺(3) Are complementary of each other.

(2) $\Rightarrow$ (1) Let (U,E) be a non-null soft open set over X. We need to prove that $Int(Cl(f((U,E))))\neq\tilde{\Phi}$. Assume $Int(Cl(f((U,E))))=\tilde{\Phi}$. Then $Cl(Int(\tilde{Y}\backslash f((U,E))))=\tilde{Y}$. By (2) $\tilde{X}=Cl(f^{-1}(Int(\tilde{Y}\backslash f((U,E)))))\tilde{\sqsubseteq} Cl(f^{-1}(\tilde{Y}\backslash f((U,E))))$. But $f^{-1}(\tilde{Y}\backslash f((U,E)))\tilde{\sqsubseteq}\tilde{X}\backslash(U,E)$ and $\tilde{X}\backslash(U,E)$ is a soft closed set over X. Therefore, $\tilde{X}=Cl(f^{-1}(Y\backslash f((U,E))))\tilde{\sqsubseteq}\tilde{X}\backslash(U,E)$. This means that $(U,E)=\tilde{\Phi}$, which is a contradiction. Thus, $Int\left(f\big((U,E)\big)\right)\neq\tilde{\Phi}$, and hence f is soft SD-open.

Lemma 3.10 [26] *Let* (G,E) be a non-null soft subset of $(X,\mathcal{T},E)$. *If* (G,E) is soft β-open, then $Int(Cl((G,E)))\neq\tilde{\Phi}$.

Theorem 3.11 Let $(X,\mathcal{T},E),(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a function. If f is soft semicontinuous, the following are equivalent:

1. f is soft SD-open;
2. for each soft (closed) nowhere dense set (M,E') over Y, $f^{-1}((M,E'))$ is soft nowhere dense over X;
3. for each soft SD-open set (F,E) over X, $f((F,E))$ is soft SD-open over Y;
4. f is soft β-open.

Proof. (1) $\Rightarrow$(2) Let (M,E') be a soft closed nowhere dense set over Y. Then, $Int\left(Cl\left(f^{-1}\left((M,E')\right)\right)\right) \tilde{\subseteq} f^{-1}\left((M,E')\right)$. As f is soft semicontinuous, we have $Int(Cl(f^{-1}((M,E')))) = Int(f^{-1}((M,E')))$. By Theorem 3.9, $Int(f^{-1}((M,E'))) = \tilde{\Phi}$. Thus, $Int(Cl(f^{-1}((M,E')))) = \tilde{\Phi}$, and hence, $f^{-1}((M,E'))$ is soft nowhere dense over X.
(2) $\Leftrightarrow$ (3) Suppose (3) is not true. There exists a soft SD-open set (F,E) over X such that $f((F,E'))$ is not a soft SD-open, which means that $f((F,E))$ is soft nowhere dense over Y. Therefore, $Cl(f\left((F,E)\right))$ is also soft nowhere dense (because the soft closure of soft nowhere dense set is soft nowhere dense). By (2), $f^{-1}(\mathrm{Cl}(f((F,E))))$ is soft nowhere dense and $(F,E) \tilde{\subseteq} f^{-1}(f((F,E))) \tilde{\subseteq} f^{-1}(\mathrm{Cl}(f((F,E))))$, which implies that (F,E) is not a soft SD-set over X. This is a contradiction. Hence (3) must be true. The converse can be proved similarly.
(2) $\Rightarrow$ (4) Let (G,E) be a soft open set over X. In order to show that $f((G,E))$ is a β-open set over Y, we let $p(e,y) \tilde{\notin} Cl(Int(Cl(f((G,E)))))$. Then there is a soft open set (H,E') over Y containing $p(e,y)$ such that $Int(Cl(f((G,E))))\tilde{\cap}(H,E') = \tilde{\Phi}$ and so

$$\tilde{\Phi} = Int(Cl(f((G,E)))) \tilde{\cap} Int(Cl((H,E'))) \tilde{\supseteq} Int(Cl(f((G,E)) \tilde{\cap} (H,E'))).$$

Therefore, $f((G,E)) \tilde{\cap} (H,E')$ is soft nowhere dense over Y. By (2), $f^{-1}(f((G,E)) \tilde{\cap} (H,E'))$ is soft nowhere dense over X. But $f^{-1}(f((G,E)) \tilde{\cap} (H,E')) \tilde{\supseteq} (G,E) \tilde{\cap} f^{-1}((H,E'))$. Hence, $(G,E) \tilde{\cap} f^{-1}((H,E'))$ is soft nowhere dense over X. This implies that

$$Int\left((G,E) \tilde{\cap} f^{-1}\left((H,E')\right)\right) = (G,E) \tilde{\cap} Int\left(f^{-1}\left((H,E')\right)\right) = \tilde{\Phi}$$

and so $(G,E) \tilde{\cap} Cl(Int(f^{-1}((H,E')))) = \tilde{\Phi}$. Since f is soft semicontinuous, then $f^{-1}((H,E')) \tilde{\subseteq} Cl(Int(f^{-1}((H,E'))))$. Therefore, $(G,E) \tilde{\cap} f^{-1}((H,E')) = \tilde{\Phi}$ and then $f((G,E)) \tilde{\cap} (H,E') = \tilde{\Phi}$. Thus, $p(e,y) \tilde{\notin} f((G,E))$. This proves that $f((G,E)) \tilde{\subseteq} Cl(Int(Cl(f((G,E)))))$ and so f is a soft β-open function.
(4) $\Rightarrow$ (1) Let (U,E) be a soft open set over X. If $(U,E) = \tilde{\Phi}$, by default, $f((U,E))$ is a soft SD-set. Suppose that $(U,E) \neq \tilde{\Phi}$. By (4), we have $f((U,E)) \tilde{\subseteq} Cl(Int(Cl(f((U,E)))))$, and by Lemma 3.10, we have $Int(Cl(f((U,E)))) \neq \tilde{\Phi}$. Thus, f is a soft SD-open.

4.Preimages of soft sets with Baire property and images soft Baire spaces

Definition 4.1 [23] A soft topological space $(X,\mathcal{T},E)$ is called soft Baire if the intersection of each countable collection of soft open dense sets over X is a soft dense. Equivalently, each non-null soft open set over X is soft non-meager.

Definition 4.2 [26] Let $(X,\mathcal{T},E)$ be a soft topological space. A set (F,E) over X is said to have the soft Baire property if it is can be represented as $(F,E) = (G,E)\tilde{\Delta}(M,E)$, where (G,E) is soft open and (M,E) is a soft meager set.

Proposition 4.3 Let $(X,\mathcal{T},E),(Y,\mathcal{S},E')$ be a soft topological spaces and let $f:(X,\mathcal{T},E) \to (Y,\mathcal{S},E')$ be a soft semicontinuous soft SD-open function. If (M,E') is a soft meager set over Y, then $f^{-1}((M,E'))$ is a soft meager over X.
Proof. Let (M,E') be a soft meager set over Y. Then, $(M,E') = \widetilde{\bigcup_{i=1}^{\infty}} (N_i,E')$ such that (N_i,E') is a soft nowhere set over Y for $i = 1,2,\cdots$. Therefore,
$f^{-1}((M,E')) = f^{-1}\left(\widetilde{\bigcup_{i=1}^{\infty}} (N_i,E')\right) = \widetilde{\bigcup_{i=1}^{\infty}} f^{-1}((N_i,E'))$.
By Theorem 3.11 (2), we obtain $f^{-1}((N_i,E'))$ is soft nowhere dense for each i. Hence, $f^{-1}((M,E))$ is soft meager set over X.
Lemma 4.4 [26] Each soft semiopen set is of soft Baire property.

Theorem 4.5 Let $(X,\mathcal{T},E),(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a soft semicontinuous soft SD-open function. If a soft set (F,E') over Y have a soft Baire property, then $f^{-1}((F,E'))$ have a soft Baire property.
Proof. Let $(F,E')\tilde{\subseteq}\tilde{Y}$ be a set having a soft Baire property. Then, $(F,E')=(H,E')\tilde{\Delta}(N,E')$ for some soft open (H,E') and soft meager (N,E') subsets over Y. Now,

$$f^{-1}((F,E'))=f^{-1}((H,E'))\tilde{\Delta}f^{-1}((N,E')).$$

Then by Proposition 4.3, we obtain $f^{-1}((N,E'))$ is a soft meager set over X. It is enough to show that $f^{-1}((H,E'))$ has the soft Baire property. Since (H,E') is soft open and f is soft semicontinuous, so $f^{-1}((H,E'))$ is a soft semiopen set over X. Hence, by Lemma 4.4, $f^{-1}((H,E'))$ has the soft Baire property. Thus, $f^{-1}((F,E'))$ has the soft Baire property, as required.

Theorem 4.6 Let $(X,\mathcal{T},E),(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a soft continuous and a soft SD-open surjection. If (C,E) is a soft comeager set over X, then $f((C,E))$ is soft comeager over X.
Proof. Without loss of generality, let (C,E) be a soft dense G_δ-set over X. One can easily conclude that (C,E) have Baire property. By Theorem 4.5, $f((C,E))$ is having a Baire property. So $f((C,E))=(H,E')\tilde{\Delta}(M,E')$, where (H,E') and (M,E') are respectively soft open and soft meager. Now, it is enough to show that (H,E') is soft dense over Y. Suppose not. Then, there exists a non-null soft open set (V,E') such that $(H,E')\tilde{\sqcap}(V,E')=\tilde{\Phi}$. Therefore, $f((C,E))\tilde{\sqcap}(V,E')\tilde{\subseteq}(M,E')$ which implies that $f((C,E))\tilde{\sqcap}(V,E')$ is a soft meager set over Y. Since f is soft SD-open, then by Proposition 4.3, we have $f^{-1}(f((C,E))\tilde{\sqcap}(V,E'))$ is soft meager over X. But $(C,E)\tilde{\sqcap}f^{-1}((V,E'))\tilde{\subseteq}f^{-1}(f((C,E))\tilde{\sqcap}(V,E'))$. This means that $(C,E)\tilde{\sqcap}f^{-1}((V,E'))$ is soft meager, which is not possible as (C,E) is soft dense and $f^{-1}((V,E'))$ is non-null soft open. Hence, $f((C,E))$ is soft comeager.

Theorem 4.7 Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a function. If f is soft semicontinuous, the following are equivalent:
1. f is soft SD-open;
2. for each soft open dense set (D,E') over Y, $Int(f^{-1}((D,E')))$ is soft dense over X.

Proof. (1) ⇒ (2) Let (D,E') be a soft open dense set over Y. By Theorem 3.9 (2), we have $f^{-1}((D,E'))$ is soft dense over X. Since f is soft semicontinuous, then $f^{-1}((D,E'))\tilde{\subseteq}Cl(Int(f^{-1}((D,E'))))$ and so by [8, Lemma 3.16], $\tilde{X}=Cl(f^{-1}((D,E')))=Cl(Int(f^{-1}((D,E'))))$. Thus, $Int(f^{-1}((D,E')))$ is soft dense over X.
(2) ⇒ (1) It follows from Theorem 3.9.

Theorem 4.8 Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a soft semicontinuous soft SD-open surjection. If $(X,\mathcal{T},E)$ is a soft Baire space, then $(Y,\mathcal{S},E')$ is a soft Baire space.
Proof. If $(Y,\mathcal{S},E')$ is not a soft Baire space, then there is a non-null soft open set (H,E') over Y that is meager. By Proposition 4.3, $f^{-1}((H,E'))$ is a soft meager set over X. Since f is soft semicontinuous, then $Cl(f^{-1}((H,E')))=Cl(Int(f^{-1}((H,E'))))$ and so $Int(f^{-1}((H,E')))\neq\tilde{\Phi}$. Contradiction to the assumption that $(X,\mathcal{T},E)$ is soft Baire. Hence, $(Y,\mathcal{S},E')$ must be soft Baire.

From Theorem 4.8 and Diagram I, one can have the following:

Corollary 4.9 [25] Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a soft semicontinuous a soft SW-open surjection. If $(X,\mathcal{T},E)$ is a soft Baire space, then $(Y,\mathcal{S},E')$ is a soft Baire space.

It is worth remarking that the Example 3.2 justifies that Theorem 4.8 is a natural extension of [25, Theorem 3.7], because both given spaces $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E)$ are soft Baire, while the function defined between them satisfies only hypothesis of Theorem 4.8. The next example shows that the soft semicontinuity of a function f cannot be canceled in Theorem 4.8:

Example 4.10 Let $X=Y=\mathbb{R}$ be the set of real numbers and let $E=\{e\}$ be a set of parameters. Suppose $\mathcal{T}=\{(e,G(e)): G(e)=\emptyset \text{ or } \mathbb{R}\setminus G(e) \text{ is finite}\}$ is the soft topology on X and $\mathcal{S}$ is the soft topology on Y generated by $\{(e,B(e)): B(e)=(a,\infty); a\in\mathbb{R}\}$. Define $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E)$ to be the identity function. Then, f is soft SD-open but not soft semicontinuous. On the other hand, $(Y,\mathcal{S},E)$ is a soft Baire space but $(X,\mathcal{T},E)$ is not.

5.Further results related to different types of soft functions

Theorem 5.1 Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces and let $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ be a soft continuous and a soft closed surjection. *If* $(X,\mathcal{T},E)$ is soft regular, then the following are equivalent:
1. f is soft SD-open;
2. f is soft SW-open;
3. For each soft open set (G,E) over X, there is a soft open set (O,E) over X with $\widetilde{\Phi}\neq(O,E)\widetilde{\sqsubseteq}(G,E)$ such that $f((O,E))$ is soft open;
4. For each soft dense set (D,E') over Y, $f^{-1}((D,E'))$ is soft dense over X.

Proof. (1) ⇒ (2) Let (G,E) be a soft open set over X. By soft regularity of X, there is a non-null soft open set (U,E) such that $Cl((U,E))\widetilde{\sqsubseteq}(G,E)$. Since f is soft closed, then $f(Cl((U,E)))$ is soft closed. By soft SD-openness of f, we have that $\widetilde{\Phi}\neq Int(f(Cl((U,E))))\widetilde{\sqsubseteq} Int(f((G,E)))$. Thus, f is soft SW-open.
(2) ⇒ (3) Given any non-null soft open set (G,E) over X, by (2), there exists a non-null soft open set (H,E') over Y such that $(H,E')\widetilde{\sqsubseteq} f((G,E))$. Put $(W,E)=(H,E')\widetilde{\sqcap} f^{-1}((G,E))$. Then, (W,E) shall not be a null set and $f((W,E))=(H,E')$, which proves (3).
(3) ⇒ (4) Suppose (D,E') is a soft dense set over Y such that $f^{-1}((D,E'))$ is not soft dense. Then there is a non-null soft open set (G,E) over X such that $(G,E)\widetilde{\sqcap} f^{-1}((D,E'))=\widetilde{\Phi}$. By (3), there is a soft open set (O,E) with $\widetilde{\Phi}\neq(O,E)\widetilde{\sqsubseteq}(G,E)$ such that $f((O,E))$ is soft open, which means that $(D,E')\widetilde{\sqcap} f((O,E))=\widetilde{\Phi}$. Thus, (D,E') cannot be soft dense, which is a contradiction.
(4) ⇒ (1) By [8, Theorem 5.9] and Diagram I.

Lemma 5.2 [35, Theorem 3.1.6] Let f be a function from a soft compact space $(X,\mathcal{T},E)$ onto a soft space $(Y,\mathcal{S},E')$. If f is soft continuous, then $(Y,\mathcal{S},E')$ is soft compact.

Lemma 5.3 [33, Theorem 5.3] Let (C,E) be a subset of a soft Hausdorff space $(X,\mathcal{T},E)$. If (C,E) is soft compact, then (C,E) is soft closed.

Remark 5.4 It should be noted that the above lemmas do not hold in the cases of soft Hausdorff spaces given in [3, 13, 15].

Theorem 5.5 Let f be a soft continuous function from a soft locally compact space $(X,\mathcal{T},E)$ into a soft Hausdorff space $(Y,\mathcal{S},E')$. Then, f is soft SD-open if and only if f is soft SW-open.
Proof. By default, each soft SW-open function is soft SD-open.

Conversely, let (G, E) be a soft open set over X. By soft locally compactness of X, there is a non-null soft open set (U, E) such that $Cl((U, E)) \tilde{\subseteq} (G, E)$ and $Cl((U, E))$ is soft compact. Since f is soft continuous, by Lemma 5.2 and Lemma 5.3, we have $f(Cl((U, E)))$ is soft closed. As the soft SD-openness of f guarantees that $\tilde{\Phi} \neq Int\left(f\left(Cl\big((U, E)\big)\right)\right) \tilde{\subseteq} Int\left(f\big((G, E)\big)\right)$, we obtain that f is soft SW-open.

Lemma 5.6 [36] Let (A, E) and (B, E) be subsets of $(X, \mathcal{T}, E)$. If (A, E) is soft open and (B, E) is soft β-open, then $(A, E)\tilde{\cap}(B, E)$ is soft β-open in (A, E).

Theorem 5.7 Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. A function $f: (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft β-open if and only if f is soft SD-open on each soft open set over Y.

Proof. Suppose that f is a soft β-open function and (V, E') is any soft open set over Y. Let (G, E) be soft open over X. By Lemma 5.6, $f((G, E))\tilde{\cap}(V, E')$ is soft β-open in (V, E'). If it is null, then by definition, $f((G, E))\tilde{\cap}(V, E')$ is soft SD-open. Otherwise, by [24, Proposition 2.8], $f((G, E))\tilde{\cap}(V, E')$ is soft SD-open in (V, E'). Thus, f is soft SD-open on each soft open set over Y.

Conversely, assume that f is a soft SD-open function on each soft open set over Y. Given $y_{e'} \in \tilde{Y}$ and let (G, E) be soft open over X such that $y_{e'} \tilde{\in} f((G, E))$. If (H, E') is a soft open set containing $y_{e'}$, then $f((G, E))\tilde{\cap}(H, E') \neq \tilde{\Phi}$. By assumption, we have

$$\tilde{\Phi} \neq Int(Cl(f((G, E)) \tilde{\cap} (H, E'))) = Int(Cl(f((G, E)))) \tilde{\cap} (H, E').$$

This implies that $y_{e'} \tilde{\in} Cl(Int(Cl(f((G, E)))))$. Hence, f is soft β-open.

Theorem 5.8 Let $(X, \mathcal{T}, E)$ be a soft regular space and let $(Y, \mathcal{S}, E')$ be any soft space. Let $f: (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ be a soft continuous and a soft closed surjection. Then, the following are equivalent:

1. f is soft SD-open;
2. $f((R, E))$ is a soft regular closed set over Y for each soft regular closed (R, E) over X.

Proof. (1) $\Rightarrow$ (2) Suppose otherwise that there is a soft open set (H, E') over Y such that $(H, E') \tilde{\cap} f\big((R, E)\big) \neq \tilde{\Phi}$, but $(H, E') \tilde{\cap} Int(f((R, E))) = \tilde{\Phi}$. Now, setting $(W, E) = f^{-1}((H, E')) \tilde{\cap} Int((R, E))$, then we get a non-null soft open set (W, E). On the other hand,

$$\begin{aligned} Int(f((W, E))) &= Int((H, E'))\tilde{\cap}Int(f(Int((R, E)))) \\ &\tilde{\subseteq} Int((H, E')) \tilde{\cap} Int(f((R, E))) = \tilde{\Phi}. \end{aligned}$$

This is impossible from Theorem 5.1 (2). Hence, $f((R, E))$ must be soft regular closed.

(2) $\Rightarrow$ (1) Assume that (G, E) is a non-null soft open set over X such that $Int(Cl(f((G, E)))) = \tilde{\Phi}$. By soft continuity of f, we have $f(Cl((G, E))) \tilde{\subseteq} Cl(f((G, E)))$ and so $Int(f(Cl((G, E)))) = \tilde{\Phi}$. But $Cl((G, E))$ is soft regular closed, which is a contradiction to (2). Thus, f is soft SD-open.

We shall note that for condition (2) $\Rightarrow$ (1), we have only used "soft continuity of f".

Theorem 5.9 Let $(X, \mathcal{T}, E)$ be a soft regular and let $(Y, \mathcal{S}, E')$ be any soft space. If $f: (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is a soft continuous and a soft SD-open surjection, then $f|_{(R,E)}: (R, E) \to f((R, E))$ is soft SD-open, where (R, E) is a soft regular closed set over X.

Proof. Since every soft continuous function is soft semicontinuous, we now apply Theorem 4.7. Let (D, E') be a soft open dense set over $f((R, E))$. By Theorem 5.8, $f((R, E))$ is soft regular closed over X. Set $(F, E') = ((D, E') \tilde{\cap} Int(f((R, E))))\tilde{\cup}(\tilde{Y} \backslash f((R, E)))$, then (F, E') is a soft open dense over Y. By Theorem 4.7, we have $f^{-1}((F, E'))$ is soft open dense over X, and so

$$(f|_{(R,E)})^{-1}((D,E')) = f^{-1}((D,E')) \tilde{\cap} (R,E) \tilde{\supseteq} f^{-1}((F,E')) \tilde{\cap} (R,E)$$

is soft open dense in (R,E). Hence, we obtain the result.

Theorem 5.10 Let (X,d,E) be a soft Baire metric space and let $(Y,\mathcal{S},E')$ be any soft space. Let $f:(X,d,E) \to (Y,\mathcal{S},E')$ be a soft continuous and a soft closed surjection. Then, the following are equivalent:

1. f is soft SD-open;
2. $f|_{(D,E)}$ is soft open, for some soft dense G_δ set (D,E) over X;
3. $f|_{(D,E)}$ is soft open, for some soft dense set (D,E) over X;
4. $f|_{(D,E)}$ is soft SD-open, for some soft dense set (D,E) over X.

Proof. (1) $\Rightarrow$ (2) Let $\mathcal{G} = \{(G,E): \tilde{\Phi} \neq (G,E) \tilde{\subseteq} \tilde{X}$ and $f((G,E))$ is soft open$\}$. This is possible by Theorem 5.1 (3). For each $n \in \mathbb{N}$, we define $\mathcal{G}_n = \{(G,E) \tilde{\in} \mathcal{G}: \sup\{d(x_e, y_{e'}): x_e, y_{e'} \tilde{\in} (G,E)\} < 1/n\}$. Then, $\tilde{\bigcup}\mathcal{G}_n$ is soft open and Theorem 5.1 (3) guarantees that $\tilde{\bigcup}\mathcal{G}_n$ is soft dense. Since (X,d,E) is soft Baire, then $(D,E) = \widetilde{\bigcap_{n=1}^{\infty}} \tilde{\bigcup}\mathcal{G}_n$ is a soft dense G_δ set over X. The soft openness of $f|_{(D,E)}$ follows from the definition of $\mathcal{G}$.

(2) $\Rightarrow$ (3) Clear.

(3) $\Rightarrow$ (4) Diagram I.

(4) $\Rightarrow$ (1) Theorem 3.7.

6.Conclusion

This paper contributes to the area of soft topology introduced by Shabir and Naz [3] in 2011. We have described soft somewhere dense open functions using some celebrated types of soft continuous and soft open functions. Then, we have defined soft sets that have Baire property and soft comeager sets. We determine some conditions under which the preimage of soft sets with Baire property are preserved. On the other hand, under such conditions, the image of soft comeager sets (resp. soft Baire spaces) are preserved. To support the obtained results and relationships, we have provided a couple of examples.

For future work, our plan is to study further properties of soft somewhere dense sets and soft sets with Baire property.

References

[1] D Molodtsov, Soft set theory first results, *Comput Math Appl*, vol. 37, pp. 19-31, 1999.

[2] P K Maji, R Biswas, A R Roy, Soft set theory, *Computers and Mathematics with Applications*, vol. 45, pp. 555-562. 2003.

[3] M Shabir, M Naz, On soft topological spaces, *Comput Math Appl*, vol. 61, pp. 1786-1799, 2011.

[4] N Çağman, S Karataş, S Enginoglu, Soft topology, *Comput Math Appl*, vol. 62, pp. 351-358, 2011.

[5] S K Nazmul, S K Samanta, Neighbourhood properties of soft topological spaces, *Ann. Fuzzy Math. Inform.*, vol. 6, pp. 1-15, 2013.

[6] J Mahanta, P K Das, On soft topological space via semiopen and semiclosed soft sets, *Kyungpook Math J.*, vol. 4, pp. 221-23, 2014.

[7] Y Yumak and A K Kaymakci, Soft β-open sets and their applications, *J. New Theory*, vol. 4 , pp. 80-89, 2015.

[8] Z A Ameen, B A Asaad, T M Al-shami, Soft somewhat continuous and soft somewhat open functions, *Accepted in TWMS J. of Apl. & Eng. Math*., 2021.

[9] A Aygünoğlu, H. Aygün, Some notes on soft topological spaces, *Neural Comput & Applic*, vol. 21, pp. 113-119, 2012.

[10] M E El-Shafei and T M Al-shami, Some operators of a soft set and soft connected spaces using soft somewhere dense sets, *Journal of Interdisciplinary Mathematics*, 2021.

[11] F. Lin, Soft connected spaces and soft paracompact spaces, *Int. J. Eng. Math*., vol. 7, no. 2, pp. 1-7, 2013.
[12] W. Rong, The countabilities of soft topological spaces, *International Journal of Computational and Mathematical Sciences*, vol. 6, pp. 159-162, 2012.
[13] T M Al-shami, M E El-Shafei, Partial belong relation on soft separation axioms and decision-making problem, two birds with one stone, *Soft computing*, vol. 24, pp. 5377-5387, 2020.
[14] M E El-Shafei, M Abo-Elhamayel, T M Al-shami, Partial soft separation axioms and soft compact spaces, *Filomat*, vol. 32, no. 13, pp. 4755–4771, 2018.
[15] M E El-Shafei, T M Al-shami, Applications of partial belong and total non-belong relations on soft separation axioms and decision-making problem, *Computational and Applied Mathematics*, vol. 39, no. 3, pp. 1–17, 2020.
[16] M Terepeta, On separating axioms and similarity of soft topological spaces. *Soft Computing*, vol. 23, pp. 1049-1057, 2019.
[17] J Dobos, A note on the invariance of Baire spaces under mappings, *Časopis pěst. mat.,* vol. 108, pp. 409-411, 1983.
[18] J Dobos, Z Piotrowski, I Reilly, Preimages of Baire spaces, *Math. Bohem.* Vol. 119, pp. 373-379, 1994.
[19] Z Frolik, Baire spaces and some generalizations of complete metric spaces, *Czech. Math. J.*, vol. 11, pp. 237-248, 1961.
[20] Z Frolik, Remarks concerning the invariance of Baire spaces under mappings, *Czech. Math. J.* 11 (1961), 381-385.
[21] R Haworth and R McCoy, Baire spaces, Dissertationes Math, vol. 141, pp. 1-77, 1977.
[22] G Thangaraj, S Anjalmose, On fuzzy Baire spaces, *J. Fuzzy Math*., vol. 21, no. 3, pp. 667-676, 2013.
[23] M. Riaz, and F. Zain, Certain properties of soft metric spaces, *Journal of Fuzzy Mathematics*, vol. 25. no. 3, pp. 543-560, 2017.
[24] T M Al-shami, Soft somewhere dense sets on soft topological spaces, *Commun. Korean Math. Soc,* vol. 33, no. 4, pp. 1341-1356. 2018.
[25] Z A Ameen, A. B. Khalaf, The invariance of soft Baire spaces under soft weak functions, *Journal of Interdisciplinary Mathematics*, vol 25, no. 5, pp. 1295-1306, 2022.
[26] Z A Ameen, T M Al-shami, B A Asaad, Further properties of soft somewhere dense continuous functions and soft Baire spaces, *To appear in Fuzzy Information and Engineering*, 2021.
[27] T M Al-shami, I Alshammari, B A Asaad, Soft maps via soft somewhere dense sets, *Filomat*, vol. 34, no. 10, pp. 3429–3440, 2020.
[28] A Allam, T Ismail, and R Muhammed, A new approach to soft belonging, *Ann. Fuzzy Math. Inform,* vol. 13, no. 1, pp. 145-152, 2017.
[29] M Ali, F Feng, X Liu, W K Min, M Shabir, On some new operations in soft set theory. *Comput Math Appl,* vol. 57, pp. 1547-1553, 2009.
[30] S Hussain, B Ahmad, Some properties of soft topological spaces, *Comput Math Appl*, vol. 62, pp. 4058-4067, 2011.
[31] B Chen, Soft semi-open sets and related properties in soft topological spaces, *Appl. Math. Inf. Sci.*, vol. 7, pp. 287-294, 2013.
[32] S A Yuksel, Naime Tozlu, and Zehra Guzel Ergul, Soft regular generalized closed sets in soft topological spaces, *Int. Journal of Math. Analysis*, vol. 8, no. 8, pp. 355-367, 2014.
[33] S. Bayramov and C. Gunduz. A new approach to separability and compactness in soft topological spaces. *TWMS Journal of Pure and Applied Mathematics*, vol. 9, no. 21, pp. 82–93, 2018.
[34] A Kharal, B Ahmad, Mappings of soft classes, *New Math. Nat. Comput.*, vol. 7, no. 3, pp. 471-481, 2011.
[35] S. A. Jubair. *On soft function in soft topological spaces*, Master's thesis, College of College of Computer of sciences and Mathematics, Al-Qadisiyah University, 2016.
[36] T M Al-shami, Z A Ameen, B A Asaad, Soft bi-continuity and related soft functions, *Accepted in Journal of Mathematics and Computer Science*, 2022.