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Pairwise Neutrosophic Simply *b*-Open Set via Neutrosophic Bi-topological Spaces

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Abstract

In this article an attempt has been made to procure the concept of pairwise neutrosophic simply open set, pairwise neutrosophic simply continuous mapping, pairwise neutrosophic simply open mapping, pairwise neutrosophic simply compactness, pairwise neutrosophic simply *b*-open set, pairwise neutrosophic simply *b*-continuous mapping, pairwise neutrosophic simply *b*-open mapping and pairwise neutrosophic simply *b*-compactness via neutrosophic bi-topological spaces (in short NBTS). Besides, we furnish few illustrative examples on them via NBTS. Further, we investigate some basic properties of them, and formulate several results on NBTSs.

Keywords: Neutrosophic Set; NBTS; Pairwise Neutrosophic Simply Open Set.

1. Introduction

In 1998, Smarandache [1] grounded the concept of Neutrosophic Set (in short NS) as an extension of Fuzzy Set [2] and Intuitionistic Fuzzy Set [3]. From then it becomes very useful in the area of topology, bi-topology, decision making, artificial intelligence, etc. In 2020, Das et al. [4] established a decision making model using single-valued NS. The notion of Neutrosophic Topology (in short NT) on NSs was presented by Salama and Alblowi [5] in 2012. The notion of generalized NS and generalized NT was also studied by Salama and Alblowi [6]. Afterwards, Arokiarani et al. [7] defined neutrosophic semi-open (in short NSO) function via Neutrosophic Topological Space (in short NTS). Later on, Rao and Srinivasa [8] defined neutrosophic pre-open (in short NPO) sets and pre-closed sets via NTSs. The concept of neutrosophic semi-closed set and neutrosophic semi-open set via NTSs was grounded by Iswaraya and Bageerathi [9] in 2016. The idea of generalized neutrosophic closed sets via NTSs was presented by Dhavaseelan and Jafari [10]. Dhavaseelan et al. [11] established the neutrosophic α^m-continuity via NTSs. Later on, Imran et al. [12] grounded the neutrosophic semi-α-open sets via NTSs. Imran et al. [13] also defined the notion of neutrosophic generalized alpha generalized continuity via NTSs. The notion of neutrosophic generalized closed sets via NTSs was grounded by Pushpalatha and Nandhini [14] in 2019. In 1996, Andrijevic [15] introduced the notion of *b*-open set in general topology. Later on, Ebenanjar et al. [16] introduced the neutrosophic *b*-open (in short N-*b*-O) sets in NTSs. In 2020, Page and Imran [17] established the neutrosophic generalized homeomorphism via NTSs. The idea of neutrosophic generalized *b*-closed sets in NTS was grounded by Maheswari et al. [18]. Afterwards, Maheswari and Chandrasekar [19] studied the concept of neutrosophic *gb*-continuity via NTSs. Thereafter, Bageerathi and Jeya Puvaneswari [20] defined the neutrosophic feebly connectedness and compactness of NTSs. The idea of generalized N-*b*-O

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set via NTSs was presented by Das and Pramanik [21] in 2020. Afterwards, Das and Pramanik [22] grounded the concept of neutrosophic Φ-open sets and neutrosophic Φ-continuous functions via NTSs. In 2021, Das and Tripathy [23] presented the notion of neutrosophic simply *b*-open set via NTSs. In 2019, Noori and Yousif [24] presented the soft simply compact space via soft topological spaces. Afterwards, the idea of neutrosophic simply soft open set and neutrosophic simply soft compactness via Neutrosophic Soft Topological Spaces was introduced by Das and Pramanik [25] in 2020. Das and Tripathy [26] grounded the notion of NT on Neutrosophic Multiset, and introduced the Neutrosophic Multiset Topological Space. In 2021, Das et al. [27] studied the concept of Quadripartitioned Neutrosophic Topological Spaces. In 1963, Kelly [28] grounded the concept of bi-topological space. Afterwards, Cooke and Reilly [29] established the notion of compactness via bi-topological space. In 2011, Tripathy and Sarma [30] studied on *b*-locally open sets via bi-topological spaces. The idea of pairwise *b*-locally open, pairwise *b*-locally closed function via bi-topological spaces was presented by Tripathy and Sarma [31]. In 2013, Tripathy and Sarma [32] also grounded the weakly *b*-continuous mapping via bi-topological spaces. Later on, Kandil and El-Shafee [33] established the idea of Fuzzy Bi-topological Space (in short FBTS). Thereafter, Ibedou [34] presented the notion of separation axioms via FBTSs. Recently, Gani and Bhanu [35] introduced the idea of fuzzy pairwise gamma semi-continuity and fuzzy pairwise gamma semi-compactness via FBTSs. In 2019, Ozturk and Ozkan [36] grounded the idea of Neutrosophic Bi-topological Space (in short NBTS) by extending the concept of Fuzzy Bi-topological Spaces [34]. In 2020, Das and Tripathy [37] introduced the idea of pairwise N-*b*-O set via NBTSs, and studied their different properties. In 2020, Mwchahary and Basumatary [38] further studied the NBTSs. Later on, Tripathy and Das [39] also presented the notion of pairwise neutrosophic *b*-continuous mapping via NBTSs. In 2021, AL-Nafee et al. [40] presented the notion of neutrosophic soft bi-topological space. Ganesan and Smarandache [41] introduced the notion of neutrosophic biminimal α-open set via neutrosophic biminimal space.

Research gap: No study on pairwise neutrosophic simply open set, pairwise neutrosophic simply continuous function, pairwise neutrosophic simply compactness, pairwise neutrosophic simply *b*-open set, pairwise neutrosophic simply *b*-continuous function and pairwise neutrosophic simply *b*-compactness via NBTSs has been reported in the recent literature.

Motivation: To fill the research gap, we introduce the notion of pairwise neutrosophic simply open set, pairwise neutrosophic simply continuous function, pairwise neutrosophic simply compactness, pairwise neutrosophic simply *b*-open set, pairwise neutrosophic simply *b*-continuous function and pairwise neutrosophic simply *b*-compactness via NBTSs.
The remaining part of the article has been divided into the following sections:
Section 2 presents the preliminaries and definitions on NS, NTS and NBTS. In Section 3, we introduce the concept of pairwise neutrosophic simply open set, pairwise neutrosophic simply continuous function, pairwise neutrosophic simply *b*-open set and pairwise neutrosophic simply *b*-continuous function via NBTSs, and formulate several interesting results on them. In Section 4, we introduce the concept of pairwise neutrosophic simply compactness, pairwise neutrosophic simply *b*-compactness via NBTSs. Besides, we formulate some interesting results on them in the form of theorems, propositions, remarks, etc. via NBTSs. Finally, in Section 5, we conclude the paper by stating some future directions of research.

2. Preliminaries and Definitions

In this section, we provide some basic definitions and results on NS, NTS and NBTS.

Definition 2.1. [40]. Let Ψ be a fixed set. Then, an NS L over Ψ is defined by:
$L=\{(\theta, T_L(\theta), I_L(\theta), F_L(\theta)): \theta\in\Psi\}$, where T_L, I_L and F_L denotes the truth, indeterminacy and false membership function, respectively from Ψ to [0, 1], and so $0 \leq T_L(\theta)+I_L(\theta)+F_L(\theta) \leq 3$, for all $\theta\in\Psi$.

Definition 2.2. [5]. The null NS (0_N) and whole NS (1_N) over Ψ are defined as follows:
$0_N=\{(\theta, 0, 0, 1): \theta\in\Psi\}$ & $1_N=\{(\theta, 1, 0, 0): \theta\in\Psi\}$.
Clearly, $0_N \subseteq L \subseteq 1_N$, for any NS L over Ψ.
The NSs 0_N and 1_N also has three other representations. They are given as follows:
(*i*) $0_N=\{(\theta, 0, 0, 0): \theta\in\Psi\}$ & $1_N=\{(\theta, 1, 1, 1): \theta\in\Psi\}$,
(*ii*) $0_N=\{(\theta, 0, 1, 0): \theta\in\Psi\}$ & $1_N=\{(\theta, 1, 0, 1): \theta\in\Psi\}$,
(*iii*) $0_N=\{(\theta, 0, 1, 1): \theta\in\Psi\}$ & $1_N=\{(\theta, 1, 1, 0): \theta\in\Psi\}$.

Definition 2.3.[40]. Suppose that $W=\{(\theta, T_W(\theta), I_W(\theta), F_W(\theta)): \theta\in\Psi\}$ and $K=\{(\theta, T_K(\theta), I_K(\theta), F_K(\theta)): \theta\in\Psi\}$ be two NSs over Ψ. Then,
(*i*) $W^c=\{(\theta, 1-T_W(\theta), 1-I_W(\theta), 1-F_W(\theta)): \theta\in\Psi\}$,
(*ii*) $W\subseteq K \Leftrightarrow T_W(\theta) \leq T_K(\theta), I_W(\theta) \geq I_K(\theta), F_W(\theta) \geq F_K(\theta)$, for all $\theta\in\Psi$,
(*iii*) $W\cup K=\{(\theta, \max\{T_W(\theta), T_K(\theta)\}, \min\{I_W(\theta), I_K(\theta)\}, \min\{F_W(\theta), F_K(\theta)\}): \theta\in\Psi\}$,
(*iv*) $W\cap K=\{(\theta, \min\{T_W(\theta), T_K(\theta)\}, \max\{I_W(\theta), I_K(\theta)\}, \max\{F_W(\theta), F_K(\theta)\}): \theta\in\Psi\}$.

Definition 2.4.[5]. Assume that τ be a family of NSs over a fixed set Ψ. Then, τ is called an Neutrosophic Topology (in short NT) on Ψ if the following condition holds:
(*i*) $0_N, 1_N \in\tau$,
(*ii*) $W_1, W_2 \in\tau \Rightarrow W_1\cap W_2 \in\tau$,
(*iii*) $\cup_{i\in\Delta} W_i \in\tau$, for every $\{W_i : i\in\Delta\} \subseteq \tau$.
If τ is an NT, then (Ψ, τ) is called an Neutrosophic Topological Space (in short NTS). If $W\in\tau$, then W is said to be an neutrosophic open set (in short NOS) and W^c is said to be an neutrosophic closed set (in short NCS) in (Ψ, τ).

Definition 2.5.[36]. Suppose that (Ψ, τ_1) and (Ψ, τ_2) are two NTSs such that τ_1 and τ_2 are different. Then, the triplet (Ψ, τ_1, τ_2) is called an Neutrosophic Bi-topological Space (in short NBTS).

Definition 2.6.[37]. Let (Ψ, τ_1, τ_2) be an NBTS. Then P, an NS over Ψ is called as
(*i*) τ_{ij} N-*b*-O set if and only if $P \subseteq N_{cl}^{i}N_{int}^{j}(P) \cup N_{int}^{j}N_{cl}^{i}(P)$,
(*ii*) τ_{ij} NSO set if and only if $P \subseteq N_{cl}^{i}N_{int}^{j}(P)$,
(*iii*) τ_{ij} NPO set if and only if $P \subseteq N_{int}^{j}N_{cl}^{i}(P)$.

Theorem 2.7.[37]. Suppose that (Ψ, τ_1, τ_2) is an NBTS.
(*i*) Every NOS in (Ψ, τ_j) ($j = 1, 2$) is a τ_{ij} NSO set in (Ψ, τ_1, τ_2).
(*ii*) Every NOS in (Ψ, τ_j) ($j = 1, 2$) is a τ_{ij} NPO set in (Ψ, τ_1, τ_2).

Definition 2.8.[37]. Suppose that (Ψ, τ_1, τ_2) is an NBTS. Then H, an NS over Ψ is said to be a pairwise NOS (in short PNOS) in (Ψ, τ_1, τ_2) if there exist an NOS W_1 in τ_1 and another NOS W_2 in τ_2 such that $H = W_1 \cup W_2$.

Definition 2.9.[37]. An NS H is called a pairwise NSO set (in short PNSO-set) in an NBTS (Ψ, τ_1, τ_2) if $H = W \cup L$, where W is a τ_{ij} NSO set and L is a τ_{ji} NSO set in (Ψ, τ_1, τ_2).

Definition 2.10.[37]. An NS H is called a pairwise NPO set (in short PNPO-set) in an NBTS (Ψ, τ_1, τ_2) if $H = W \cup L$, where W is a τ_{ij} NPO set and L is a τ_{ji} NPO set in (Ψ, τ_1, τ_2).

Theorem 2.11.[37]. Assume that (Ψ, τ_1, τ_2) be an NBTS.
(*i*) Every PNOS in (Ψ, τ_1, τ_2) is a PNSO-set in (Ψ, τ_1, τ_2);
(*ii*) Every PNOS in (Ψ, τ_1, τ_2) is a PNPO-set in (Ψ, τ_1, τ_2).

Definition 2.12.[37]. An NS H is called a pairwise N-*b*-O set (in short PN-*b*-OS) in an NBTS (Ψ, τ_1, τ_2) if $H = W \cup L$, where W is a τ_{ij} N-*b*-O set and L is a τ_{ji} N-*b*-O set in (Ψ, τ_1, τ_2). If H is a PN-*b*-OS, then H^c is called a pairwise neutrosophic *b*-closed set (in short PN-*b*-CS).

Theorem 2.13.[37]. In an NBTS (Ψ, τ_1, τ_2),
(*i*) every τ_{ij} N-*b*-O set is a PN-*b*-OS;
(*ii*) every τ_{ij} N-*b*-C set is a PN-*b*-CS;
(*iii*) every PNSO-set is also a PN-*b*-OS;
(*iv*) every PNPO-set is also a PN-*b*-OS.

3. Pairwise Neutrosophic Simply *b*-Open Set

In this section, we procure the concept of pairwise neutrosophic simply open set (in short PNSOS), pairwise neutrosophic simply continuous mapping (in short PNS-C-mapping), pairwise neutrosophic simply *b*-open set (in short PNS-*b*-OS) and pairwise neutrosophic simply *b*-continuous function (in short PNS-*b*-C-mapping) via NBTSs.

Definition 3.1. Let (Ψ, τ_1, τ_2) be an NBTS. Then X, an NS over Ψ is called a τ_{ij} neutrosophic simply open set (in short τ_{ij} NSOS) if and only if $N_{int}^{i}(N_{cl}^{j}(X)) \subseteq N_{cl}^{j}(N_{int}^{i}(X))$. The complement of X is called a τ_{ij} neutrosophic simply closed set (in short τ_{ij} NSCS).

Example 3.2. Let Ψ={a, b} be a fixed set. Let τ_1={0_N, 1_N, X_1, X_2}, τ_2={0_N, 1_N, Y_1, Y_2 } be two different NTs on Ψ such that X_1={(a,0.5,0.2,0.1), (b,0.6,0.1,0.1)}, X_2={(a,0.9,0.1, 0.1), (b,0.8,0.1,0.1)}, Y_1={(a,0.4,0.3,0.2), (b,0.7,0.4,0.2)}, Y_2={(a,0.6,0.1,0.2), (b,0.9,0.3,0.1)}. Therefore, (Ψ, τ_1, τ_2) is an NBTS. Clearly, X={(a,0.1,0.8,0.8), (b,0.2,0.8,0.9)} is a τ_{12} NSOS in (Ψ, τ_1, τ_2).

Remark 3.3. Let (Ψ, τ_1, τ_2) be an NBTS. Then, every τ_{ij} NSOS may not be a τ_{ji} NSOS, which follows from the following example.

Example 3.4. Let (Ψ, τ_1, τ_2) be an NBTS as shown in Example 3.2. Then, X={(a,0.1,0.8,0.8), (b,0.2,0.8,0.9)} is a τ_{12} NSOS in (Ψ, τ_1, τ_2) but it is not a τ_{21} NSOS in (Ψ, τ_1, τ_2).

Proposition 3.5. Let (Ψ, τ_1, τ_2) be an NBTS. Then, the NSs 0_N and 1_N are both τ_{ij} NSOS and τ_{ji} NSOS in (Ψ, τ_1, τ_2).
Proof. The proof is so easy, so omitted.

Theorem 3.6. Let (Ψ, τ_1, τ_2) be an NBTS.
(*i*) If X is an NOS in (Ψ, τ_i), then X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2);

(*ii*) If X is an NOS in (Ψ, τ_j), then X is a τ_{ji} NSOS in (Ψ, τ_1, τ_2).

Proof. (*i*) Let (Ψ, τ_1, τ_2) be an NBTS. Suppose that X is an NOS in (Ψ, τ_i). Therefore, $N^i_{int}(X) = X$. It is known that, $N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(X)$ and $N^j_{cl}(X) = N^j_{cl}(N^i_{int}(X))$. This implies, $N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(N^i_{int}(X))$. Hence, X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2).
(*ii*) The proof is similar to the proof of the first part of this theorem, so omitted.

Remark 3.7. Suppose that (Ψ, τ_1, τ_2) is an NBTS. Every NOS in (Ψ, τ_j) may not be a τ_{ij} NSOS in (Ψ, τ_1, τ_2). This follows from the following example.

Example 3.8. Let (Ψ, τ_1, τ_2) be an NBTS as shown in Example 3.2. Then, the NS $Y=\{(a,0.4, 0.3,0.2), (b,0.7,0.4,0.2)\}$ is an NOS in (Ψ, τ_2), but it is not a τ_{12} NSOS in (Ψ, τ_1, τ_2).

Theorem 3.9. If X is an NCS in (Ψ, τ_j) and a τ_{ji} NSO set in (Ψ, τ_1, τ_2) $(i, j = 1, 2; i \neq j)$, then X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2).

Proof. Suppose that (Ψ, τ_1, τ_2) is an NBTS. Let X be an NCS in (Ψ, τ_j) and a τ_{ji} NSO set in (Ψ, τ_1, τ_2) $(i, j = 1, 2; i \neq j)$. Since X is an NCS in (Ψ, τ_i), so $N^j_{cl}(X) = X$. Further, since X is a τ_{ji} NSO set in (Ψ, τ_1, τ_2), so $X \subseteq N^j_{cl}(N^i_{int}(X))$ (1) Now,
$N^j_{cl}(X) = X$
$\Rightarrow N^j_{cl}(X) \subseteq X$
$\Rightarrow N^j_{cl}(X) \subseteq X \subseteq N^j_{cl}(N^i_{int}(X))$ [Using equation(1)]
$\Rightarrow N^j_{cl}(X) \subseteq N^j_{cl}(N^i_{int}(X))$
$\Rightarrow N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(X) \subseteq N^j_{cl}(N^i_{int}(X))$ [Since $N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(X)$]
$\Rightarrow N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(N^i_{int}(X))$
Therefore, X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2).

Theorem 3.10. Let X be an NOS in (Ψ, τ_i) and a τ_{ji} NPC set in (Ψ, τ_1, τ_2). Then, X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2).

Proof. Suppose that (Ψ, τ_1, τ_2) is an NBTS. Let X be an NOS in (Ψ, τ_i) and a τ_{ji} NPC set in (Ψ, τ_1, τ_2) $(i, j = 1, 2; i \neq j)$. Since X is an NOS in (Ψ, τ_i), so $N^i_{int}(X) = X$. Further, since X is a τ_{ji} NPC set in (Ψ, τ_1, τ_2), so $N^i_{int}(N^j_{cl}(X)) \subseteq X$ (2)
From equation (2), we have
$N^i_{int}(N^j_{cl}(X)) \subseteq X$
$\Rightarrow N^i_{int}(N^j_{cl}(X)) \subseteq X = N^i_{int}(X)$ [Since $X = N^i_{int}(X)$]
$\Rightarrow N^i_{int}(N^j_{cl}(X)) \subseteq N^i_{int}(X)$
$\Rightarrow N^i_{int}(N^j_{cl}(X)) \subseteq N^i_{int}(X) \subseteq N^j_{cl}(N^i_{int}(X))$ [Since $N^i_{int}(X) \subseteq N^j_{cl}(N^i_{int}(X))$]
$\Rightarrow N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(N^i_{int}(X))$
Therefore, X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2).
Theorem 3.11. If X is both τ_{ji} NPO set and τ_{ij} NSOS in an NBTS (Ψ, τ_1, τ_2), then X is also a τ_{ji} NSO set in (Ψ, τ_1, τ_2).
Proof. Let X be both τ_{ji} NPO set and τ_{ij} NSOS in an NBTS (Ψ, τ_1, τ_2). Since X is a τ_{ji} NPO set, so $X \subseteq N^i_{int}(N^j_{cl}(X))$.
(3)

Further, since X is a τ_{ij} NSOS, so $N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(N^i_{int}(X))$. (4)
Therefore, from equations (3) and (4), we have $X \subseteq N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(N^i_{int}(X))$.
Which implies $X \subseteq N^j_{cl}(N^i_{int}(X))$. Hence, X is a τ_{ji} NSO set in (Ψ, τ_1, τ_2).

Theorem 3.12. If X is both τ_{ji} N-*b*-O set and τ_{ij} NSOS in an NBTS (Ψ, τ_1, τ_2), then X is also a τ_{ji} NSO set in (Ψ, τ_1, τ_2).

Proof. Suppose that (Ψ, τ_1, τ_2) is an NBTS. Let X be both τ_{ji} N-*b*-O set and τ_{ij} NSOS in (Ψ, τ_1, τ_2). Since X is a τ_{ji} N-*b*-O set, so $X \subseteq N^j_{cl}(N^i_{int}(X)) \cup N^i_{int}(N^j_{cl}(X))$. (5)
Further, since X is a τ_{ij} NSOS, so $N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(N^i_{int}(X))$. (6)
Now, from equations (5) and (6), we have $X \subseteq N^j_{cl}(N^i_{int}((X)) \cup N^i_{int}(N^j_{cl}(X)) \subseteq N^j_{cl}(N^i_{int}(X))$.
This means $X \subseteq N^j_{cl}(N^i_{int}(X))$. Therefore, X is a τ_{ji} NSO set in (Ψ, τ_1, τ_2).

Definition 3.13. Suppose that (Ψ, τ_1, τ_2) is an NBTS. Then, an NS X over Ψ is said to be a τ_{ij} neutrosophic simply *b*-open set (in short NS*b*OS) in (Ψ, τ_1, τ_2) if and only if it is both τ_{ji} N-*b*-O set and τ_{ij} NSOS in (Ψ, τ_1, τ_2). If X is a τ_{ij} NS*b*OS in (Ψ, τ_1, τ_2), then X^c is called a τ_{ij} neutrosophic simply *b*-closed set (in short NS*b*CS) in (Ψ, τ_1, τ_2).

Theorem 3.14. Every τ_{ij} NS*b*OS is also a τ_{ij} NSOS (τ_{ji} N-*b*-O set) in (Ψ, τ_1, τ_2).

Proof. Let (Ψ, τ_1, τ_2) be an NBTS. Let X be a τ_{ij} NS*b*OS in (Ψ, τ_1, τ_2). By Definition 3.13, X is both τ_{ji} N-*b*-O set and τ_{ij} NSOS. Therefore, X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2). Hence, every τ_{ij} NS*b*OS is a τ_{ij} NSOS.
Similarly, it can be shown that, every τ_{ij} NS*b*OS is also a τ_{ji} N-*b*-O set in (Ψ, τ_1, τ_2).

Proposition 3.15. Let (Ψ, τ_1, τ_2) be an NBTS. Then, both 0_N and 1_N are τ_{ij} NS*b*OS and τ_{ji} NS*b*OS in (Ψ, τ_1, τ_2).

Proof. The proof is so easy, so omitted.

Lemma 3.16. Let (Ψ, τ_1, τ_2) be an NBTS.
(*i*) If X is an NOS in (Ψ, τ_i), then X is a τ_{ij} NS*b*OS in (Ψ, τ_1, τ_2);
(*ii*) If X is an NOS in (Ψ, τ_j), then X is a τ_{ji} NS*b*OS in (Ψ, τ_1, τ_2).

Remark 3.17. Every NOS in (Ψ, τ_j) may not be a τ_{ij} NS*b*OS in (Ψ, τ_1, τ_2), which follows from the following example.

Example 3.18. Let (Ψ, τ_1, τ_2) be an NBTS as shown in Example 3.2. Then, Y={(a,0.4,0.3, 0.2), (b,0.7,0.4,0.2)} is an NOS in (Ψ, τ_2), but it is not a τ_{12} NS*b*OS in (Ψ, τ_1, τ_2).

Theorem 3.19. Every τ_{ij} NS*b*OS is also a τ_{ji} NSO set in (Ψ, τ_1, τ_2).
Proof. Let (Ψ, τ_1, τ_2) be an NBTS. Let X be a τ_{ij} NS*b*OS in (Ψ, τ_1, τ_2). By Definition 3.13, X is both τ_{ji} N-*b*-O set and τ_{ij} NSOS in (Ψ, τ_1, τ_2). Further, by Theorem 3.12, it is clear that X is a τ_{ji} NSO set in (Ψ, τ_1, τ_2). Hence, every τ_{ij} NS*b*OS is also a τ_{ji} NSO set in (Ψ, τ_1, τ_2).

Theorem 3.20. If X is an NCS in (Ψ, τ_j) and a τ_{ji} NSO set in (Ψ, τ_1, τ_2), then X is a τ_{ij} NS*b*OS in (Ψ, τ_1, τ_2).

Proof. Let X be an NCS in the NTS (Ψ, τ_j) and a τ_{ji} NSO set in the NBTS (Ψ, τ_1, τ_2). By Theorem 3.9, X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2). Since every τ_{ji} NSO set is a τ_{ji} N-b-O set in (Ψ, τ_1, τ_2), so X is a τ_{ji} N-b-O set in (Ψ, τ_1, τ_2). Therefore, X is both τ_{ji} N-b-O set and τ_{ij} NSOS in (Ψ, τ_1, τ_2). Hence, X is a τ_{ij} NSbOS in (Ψ, τ_1, τ_2).

Theorem 3.21. If X is an NOS in (Ψ, τ_i) and a τ_{ji} NPC set in (Ψ, τ_1, τ_2), then X is a τ_{ij} NSbOS in (Ψ, τ_1, τ_2).

Proof. Let X be an NOS in the NTS (Ψ, τ_i) and a τ_{ji} NPC set in the NBTS (Ψ, τ_1, τ_2). By Theorem 3.10, X is a τ_{ij} NSOS in (Ψ, τ_1, τ_2). Since every NOS in (Ψ, τ_i) is a τ_{ji} N-b-O set in (Ψ, τ_1, τ_2), so X is a τ_{ji} N-b-O set in (Ψ, τ_1, τ_2). Therefore, X is both τ_{ji} N-b-O set and τ_{ij} NSOS in (Ψ, τ_1, τ_2). Hence, X is a τ_{ij} NSbOS in (Ψ, τ_1, τ_2).

Theorem 3.22. If X is both τ_{ji} NPO set and τ_{ij} NSbOS in (Ψ, τ_1, τ_2), then X is a τ_{ji} NSO set in (Ψ, τ_1, τ_2).

Proof. Let (Ψ, τ_1, τ_2) be an NBTS. Let X be both τ_{ji} NPO set and τ_{ij} NSbOS in (Ψ, τ_1, τ_2). Since, every τ_{ij} NSbOS is a τ_{ij} NSOS, so X is τ_{ij} NSOS in (Ψ, τ_1, τ_2). Now, by Theorem 3.11, X is a τ_{ji} NSO set in (Ψ, τ_1, τ_2).

Definition 3.23. An NS H is called a pairwise neutrosophic simply open set (in short PNSOS) in an NBTS (Ψ, τ_1, τ_2) if and only if $H=K\cup L$, where K is a τ_{ij} NSOS and L is a τ_{ji} NSOS in (Ψ, τ_1, τ_2). The complement of a PNSOS is called a pairwise neutrosophic simply closed set (in short PNSCS) in (Ψ, τ_1, τ_2).
Clearly, the NSs 0_N and 1_N are both PNSOSs and PNSCSs in (Ψ, τ_1, τ_2).

Theorem 3.24. Every τ_{ij} NSOS is a PNSOS in (Ψ, τ_1, τ_2).

Proof. Let (Ψ, τ_1, τ_2) be an NBTS. Let X be a τ_{ij} NSOS in (Ψ, τ_1, τ_2). Now, X can be expressed by $X=X\cup 0_N$, where X is a τ_{ij} NSOS and 0_N is a τ_{ji} NSOS. Hence, X is a PNSOS in (Ψ, τ_1, τ_2). Therefore, every τ_{ij} NSOS is a PNSOS in (Ψ, τ_1, τ_2).

Theorem 3.25. Let (Ψ, τ_1, τ_2) be an NBTS. Then, every PNOS in (Ψ, τ_1, τ_2) is also a PNSOS in (Ψ, τ_1, τ_2).

Proof. Let S be a PNOS in an NBTS (Ψ, τ_1, τ_2). Therefore, there exists an NOS E in (Ψ, τ_i) and another NOS F in (Ψ, τ_j) such that $S=E\cup F$. Since every NOS in (Ψ, τ_i) is a τ_{ij} NSOS, so E is a τ_{ij} NSOS in (Ψ, τ_1, τ_2). Further, since every NOS in (Ψ, τ_j) is a τ_{ji} NSOS, so E is a τ_{ji} NSOS in (Ψ, τ_1, τ_2). Therefore, S is the union of a τ_{ij} NSOS and a τ_{ji} NSOS in (Ψ, τ_1, τ_2). Hence, S is a PNSOS in (Ψ, τ_1, τ_2).

Definition 3.26. Suppose that (Ψ, τ_1, τ_2) be an NBTS. An NS H is called a pairwise neutrosophic simply b-open set (in short PNSbOS) in (Ψ, τ_1, τ_2) if and only if H can be expressed as $H=W\cup L$, where W is a τ_{ij} NSbOS and L is a τ_{ji} NSbOS in (Ψ, τ_1, τ_2). If H is a PNSbOS in (Ψ, τ_1, τ_2), then H^c is called a pairwise neutrosophic simply b-closed set (in short PNSbCS) in (Ψ, τ_1, τ_2).

Remark 3.27. In an NBTS (Ψ, τ_1, τ_2), the NSs 0_N and 1_N are both PNSbOS and PNSbCS in (Ψ, τ_1, τ_2).

Theorem 3.28. Let (Ψ, τ_1, τ_2) be an NBTS. Then,
(i) every τ_{ij} NSbOS is also a PNSbOS in (Ψ, τ_1, τ_2);
(ii) every τ_{ji} NSbOS is also a PNSbOS in (Ψ, τ_1, τ_2).

Proof. (i) Let us consider an NBTS (Ψ, τ_1, τ_2). Assume that X be a τ_{ij} NSbOS in (Ψ, τ_1, τ_2). Now, X can be expressed by $X=X\cup 0_N$, where X is a τ_{ij} NSbOS and 0_N is a τ_{ji} NSbOS in (Ψ, τ_1, τ_2). Therefore, X is a PNSbOS in (Ψ, τ_1, τ_2). Hence, every τ_{ij} NSbOS in (Ψ, τ_1, τ_2) is also a PNSbOS in (Ψ, τ_1, τ_2).
(ii) The proof is similar to the proof of the first part of this theorem, so omitted.

Theorem 3.29. Let (Ψ, τ_1, τ_2) be an NBTS. Then, every PNOS in (Ψ, τ_1, τ_2) is also a PNSbOS (Ψ, τ_1, τ_2).

Proof. Suppose that S be a PNOS in an NBTS (Ψ, τ_1, τ_2). Therefore, there exists an NOS E in (Ψ, τ_i) and another NOS F in (Ψ, τ_j) such that $S=E\cup F$. Since every NOS in (Ψ, τ_i) is a τ_{ij} NSbOS, so E is a τ_{ij} NSbOS in (Ψ, τ_1, τ_2). Further, since every NOS in (Ψ, τ_j) is a τ_{ji} NSbOS, so E is a τ_{ji} NSbOS in (Ψ, τ_1, τ_2). Therefore, S is the union of a τ_{ij} NSbOS and a τ_{ji} NSbOS in (Ψ, τ_1, τ_2). Hence, S is a PNSbOS in (Ψ, τ_1, τ_2).

Theorem 3.30. Let (Ψ, τ_1, τ_2) be an NBTS. Then, every PNSbOS in (Ψ, τ_1, τ_2) is also a PNSOS (Ψ, τ_1, τ_2).

Proof. Let S be a PNSbOS in an NBTS (Ψ, τ_1, τ_2). Therefore, there exists a τ_{ij} NSbOS E and another τ_{ij} NSbOS F such that $S=E\cup F$. Since every τ_{ij} NSbOS is a τ_{ij} NSOS, so E is a τ_{ij} NSOS in (Ψ, τ_1, τ_2). Further, since every τ_{ji} NSbOS is a τ_{ji} NSOS, so E is a τ_{ji} NSOS in (Ψ, τ_1, τ_2). Therefore, S is the union of a τ_{ij} NSOS and a τ_{ji} NSOS in (Ψ, τ_1, τ_2). Hence, S is a PNSOS in (Ψ, τ_1, τ_2).

Definition 3.31. Suppose that (Ψ, τ_1, τ_2) and $(\Omega, \delta_1, \delta_2)$ be two NBTSs. Then, a one to one and onto mapping $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is called as
(i) pairwise neutrosophic continuous mapping (in short PN-C-mapping) if and only if $\xi^{-1}(L)$ is a PNOS in Ψ, whenever L is a PNOS in Ω.
(ii) pairwise neutrosophic semi-continuous mapping (in short PN-Semi-C-mapping) if and only if $\xi^{-1}(L)$ is a PNSO set in Ψ, whenever L is a PNOS in Ω.
(iii) pairwise neutrosophic pre-continuous mapping (in short PN-Pre-C-mapping) if and only if $\xi^{-1}(L)$ is a PNPO set in Ψ, whenever L is a PNOS in Ω.
(iv) pairwise neutrosophic b-continuous mapping (in short PN-b-C-mapping) if and only if $\xi^{-1}(L)$ is a PN-b-OS in Ψ, whenever L is a PNOS in Ω.
(v) pairwise neutrosophic simply continuous mapping (in short PN-Simply-C-mapping) if and only if $\xi^{-1}(L)$ is a PNSOS in Ψ, whenever L is a PNOS in Ω.
(vi) pairwise neutrosophic simply b-continuous mapping (in short PN-Simply-b-C-mapping) if and only if $\xi^{-1}(L)$ is a PNSbOS in Ψ, whenever L is a PNOS in Ω.

Theorem 3.32. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ and $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ be two PN-C-mappings. Then, the composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is also a PN-C-mapping.

Proof. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ and $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ be two PN-C-mappings. Assume that Q be a PNOS in $(\Pi, \theta_1, \theta_2)$. Since, $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-C-mapping, so $\zeta^{-1}(Q)$ is a PNOS in $(\Omega, \delta_1, \delta_2)$. Further, since $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a PN-C-mapping, so $\xi^{-1}(\zeta^{-1}(Q))=(\zeta\circ\xi)^{-1}(Q)$ is a PNOS in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is a PNOS in (Ψ, τ_1, τ_2) whenever Q is a PNOS in $(\Pi, \theta_1, \theta_2)$. Therefore, $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-C-mapping.

Theorem 3.33. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a PN-Simply-C-mapping, and $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ be a PN-C-mapping. Then, the composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-Simply-C-mapping.

Proof. Suppose that S is a PNOS in $(\Pi, \theta_1, \theta_2)$. Since, $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-C-mapping, so $\zeta^{-1}(S)$ is a PNOS in $(\Omega, \delta_1, \delta_2)$. Further, since $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a PN-Simply-C-mapping, so $\xi^{-1}(\zeta^{-1}(S))=(\zeta\circ\xi)^{-1}(S)$ is a PNSOS in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is a PNSOS in (Ψ, τ_1, τ_2), whenever Q is a PNOS in $(\Pi, \theta_1, \theta_2)$. Therefore, $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-Simply-C-mapping.

Theorem 3.34. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a PN-Simply-*b*-C-mapping and $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ be a PN-C-mapping. Then, the composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-Simply-*b*-C-mapping.

Proof. Suppose that S is a PNOS in $(\Pi, \theta_1, \theta_2)$. Since, $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-C-mapping, so $\zeta^{-1}(S)$ is a PNOS in $(\Omega, \delta_1, \delta_2)$. Further, since $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a PN-Simply-*b*-C-mapping, so $\xi^{-1}(\zeta^{-1}(S))=(\zeta\circ\xi)^{-1}(S)$ is a PNS*b*OS in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is a PNS*b*OS in (Ψ, τ_1, τ_2), whenever Q is a PNOS in $(\Pi, \theta_1, \theta_2)$. Therefore, $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a PN-Simply-*b*-C-mapping.

Theorem 3.35.
(*i*) Every pairwise neutrosophic continuous mapping is a pairwise neutrosophic pre-continuous mapping.
(*ii*) Every pairwise neutrosophic continuous mapping is a pairwise neutrosophic semi-continuous mapping.
(*iii*) Every pairwise neutrosophic pre-continuous mapping is a pairwise neutrosophic *b*-continuous mapping.
(*iv*) Every pairwise neutrosophic semi-continuous mapping is a pairwise neutrosophic *b*-continuous mapping.
(*v*) Every pairwise neutrosophic continuous mapping is a pairwise neutrosophic *b*-continuous mapping.
(*vi*) Every pairwise neutrosophic continuous mapping is a pairwise neutrosophic simply continuous mapping.
(*vii*) Every pairwise neutrosophic continuous mapping is a pairwise neutrosophic simply *b*-continuous mapping.
(*viii*) Every pairwise neutrosophic simply *b*-continuous mapping is a pairwise neutrosophic simply continuous mapping.

Proof. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a bijective mapping from an NBTS (Ψ, τ_1, τ_2) to another NBTS $(\Omega, \delta_1, \delta_2)$.
(*i*) Suppose that ξ be a pairwise neutrosophic continuous mapping and L be a pairwise N-O-S in Ω. Since ξ is a pairwise neutrosophic continuous mapping, so $\xi^{-1}(L)$ is a PNOS in Ψ. Further,

since every PNOS is again a PNPO set, so $\xi^{-1}(L)$ is a PNPO set in (Ψ, τ_1, τ_2). Therefore, $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic pre-continuous mapping.

(*ii*) Assume that ξ be a pairwise neutrosophic continuous mapping. Let L be a PNOS in Ω. Since ξ is a pairwise neutrosophic continuous mapping, so $\xi^{-1}(L)$ is a PNOS in Ψ. Further, since every PNOS is again a PNSO set, so $\xi^{-1}(L)$ is a PNSO set in (Ψ, τ_1, τ_2). Therefore, $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic semi-continuous mapping.

(*iii*) Suppose that ξ is a pairwise neutrosophic pre-continuous mapping and L be a PNOS in $(\Omega, \delta_1, \delta_2)$. Since ξ is a pairwise neutrosophic pre-continuous mapping, so $\xi^{-1}(L)$ is a PNPO set in (Ψ, τ_1, τ_2). Since every PNPO set is a PN-*b*-OS, so $\xi^{-1}(L)$ is a PN-*b*-OS in Ψ. Therefore, $\xi^{-1}(L)$ is a PN-*b*-OS in Ψ whenever L is a PNOS in Ω. Hence, $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic *b*-continuous mapping.

(*iv*) Let ξ be a pairwise neutrosophic semi-continuous function. Let L be a PNOS in Ω. Since ξ is a pairwise neutrosophic semi-continuous function, so $\xi^{-1}(L)$ is a PNSO set in Ψ. Since every PNSO set is a PN-*b*-OS, so $\xi^{-1}(L)$ is a PN-*b*-OS in Ψ. Hence, $\xi^{-1}(L)$ is a PN-*b*-OS in Ψ whenever L is a PNOS in Ω. Therefore, $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic *b*-continuous mapping.

(*v*) Let ξ be a pairwise neutrosophic continuous mapping, and L be a PNOS in Ω. Since ξ is a pairwise neutrosophic continuous mapping, so $\xi^{-1}(L)$ is a PNOS in Ψ. Again, since every PNOS is a PN-*b*-OS, so $\xi^{-1}(L)$ is a PN-*b*-OS in (Ψ, τ_1, τ_2). Therefore, $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic *b*-continuous mapping.

(*vi*) Let L be a PNOS in $(\Omega, \delta_1, \delta_2)$. By hypothesis, $\xi^{-1}(L)$ is a PNOS in (Ψ, τ_1, τ_2). Since every PNOS is a PNSOS, so $\xi^{-1}(L)$ is a PNSOS in (Ψ, τ_1, τ_2). Therefore, $\xi^{-1}(L)$ is a PNSOS in (Ψ, τ_1, τ_2) whenever L is a PNOS in $(\Omega, \delta_1, \delta_2)$. Hence, the mapping $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic simply-continuous mapping.

(*vii*) Suppose that L is a PNOS in $(\Omega, \delta_1, \delta_2)$. By hypothesis, $\xi^{-1}(L)$ is a PNOS in (Ψ, τ_1, τ_2). Since every PNOS is a PNS*b*OS, so $\xi^{-1}(L)$ is a PNS*b*OS in (Ψ, τ_1, τ_2). Therefore, $\xi^{-1}(L)$ is a PNS*b*OS in (Ψ, τ_1, τ_2) whenever L is a PNOS in $(\Omega, \delta_1, \delta_2)$. Hence, the mapping $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic simply *b*-continuous mapping.

(*viii*) Assume that L be a PNOS in $(\Omega, \delta_1, \delta_2)$. By hypothesis, $\xi^{-1}(L)$ is a PNS*b*OS in (Ψ, τ_1, τ_2). Since every PNS*b*OS is a PNSOS, so $\xi^{-1}(L)$ is a PNSOS in (Ψ, τ_1, τ_2). Therefore, $\xi^{-1}(L)$ is a PNSOS in (Ψ, τ_1, τ_2) whenever L is a PNOS in $(\Omega, \delta_1, \delta_2)$. Hence, the mapping $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic simply continuous mapping. Therefore, every pairwise neutrosophic simply *b*-continuous mapping is a pairwise neutrosophic simply continuous mapping.

Definition 3.36. Let ξ be a mapping from an NBTS (Ψ, τ_1, τ_2) to another NBTS $(\Omega, \delta_1, \delta_2)$. Then, ξ is called as

(*i*) pairwise neutrosophic open mapping if and only if $\xi(K)$ is a PNOS in Ω, whenever K is a PNOS in Ψ;

(*ii*) pairwise neutrosophic pre-open mapping if and only if $\xi(K)$ is a PNPO set in Ω, whenever K is a PNOS in Ψ;

(*iii*) pairwise neutrosophic semi-open mapping if and only if $\xi(K)$ is a PNSO set in Ω, whenever K is a PNOS in Ψ;

(*iv*) pairwise neutrosophic *b*-open mapping if and only if $\xi(K)$ is a PN-*b*-OS in Ω, whenever K is a PNOS in Ψ;

(*v*) pairwise neutrosophic simply-open mapping if and only if $\xi(K)$ is a PNSOS in Ω, whenever K is a PNOS in Ψ;

(*vi*) pairwise neutrosophic simply *b*-open mapping if and only if $\xi(K)$ is a PNS*b*OS in Ω, whenever K is a PNOS in Ψ.

Theorem 3.37. If $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ and $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ be two pairwise neutrosophic open mappings, then their composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is also a pairwise neutrosophic open mapping.

Proof. Assume that Q be a PNOS in (Ψ, τ_1, τ_2). Since, $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic open mapping, so $\xi(Q)$ is a PNOS in $(\Omega, \delta_1, \delta_2)$. Further, since $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a pairwise neutrosophic open mapping, so $\zeta(\xi(Q))=(\zeta\circ\xi)(Q)$ is a PNOS in $(\Pi, \theta_1, \theta_2)$. Hence, $(\zeta\circ\xi)(Q)$ is a PNOS in $(\Pi, \theta_1, \theta_2)$, whenever Q is a PNOS in (Ψ, τ_1, τ_2). Therefore, $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a pairwise neutrosophic open mapping.

Theorem 3.38. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a pairwise neutrosophic open mapping and $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ be a pairwise neutrosophic simply open mapping. Then, the composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is also a pairwise neutrosophic simply open mapping.

Proof. Let Q be a PNOS in (Ψ, τ_1, τ_2). Since $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a pairwise neutrosophic open mapping, so $\xi(Q)$ is a PNOS in $(\Omega, \delta_1, \delta_2)$. Further, since $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a pairwise neutrosophic simply open mapping, so $\zeta(\xi(Q))=(\zeta\circ\xi)(Q)$ is a PNSOS in $(\Pi, \theta_1, \theta_2)$. Hence, $(\zeta\circ\xi)(Q)$ is a PNSOS in $(\Pi, \theta_1, \theta_2)$, whenever Q is a PNOS in (Ψ, τ_1, τ_2). Therefore, the composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a pairwise neutrosophic simply open mapping.

Theorem 3.39. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a pairwise neutrosophic open mapping and $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ be a pairwise neutrosophic simply *b*-open mapping. Then, the composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is also a pairwise neutrosophic simply *b*-open mapping.

Proof. Suppose that Q is a PNOS in (Ψ, τ_1, τ_2). Since, $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic open mapping, so $\xi(Q)$ is a PNOS in $(\Omega, \delta_1, \delta_2)$. Further, since $\zeta:(\Omega, \delta_1, \delta_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a pairwise neutrosophic simply *b*-open mapping, so $\zeta(\xi(Q))=(\zeta\circ\xi)(Q)$ is a PNS*b*OS in $(\Pi, \theta_1, \theta_2)$. Hence, $(\zeta\circ\xi)(Q)$ is a PNS*b*OS in $(\Pi, \theta_1, \theta_2)$, whenever Q is a PNOS in (Ψ, τ_1, τ_2). Therefore, the composition mapping $\zeta\circ\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Pi, \theta_1, \theta_2)$ is a pairwise neutrosophic simply *b*-open mapping.

Theorem 3.40.
(*i*) Every pairwise neutrosophic open mapping is a pairwise neutrosophic pre-open mapping.
(*ii*) Every pairwise neutrosophic open mapping is a pairwise neutrosophic semi-open mapping.
(*iii*) Every pairwise neutrosophic pre-open mapping is a pairwise neutrosophic *b*-open mapping.
(*iv*) Every pairwise neutrosophic semi-open mapping is a pairwise neutrosophic *b*-open mapping.
(*v*) Every pairwise neutrosophic open mapping is a pairwise neutrosophic *b*-open mapping.
(*vi*) Every pairwise neutrosophic open mapping is a pairwise neutrosophic simply open mapping.
(*vii*) Every pairwise neutrosophic open mapping is a pairwise neutrosophic simply *b*-open mapping.

(*viii*) Every pairwise neutrosophic simply *b*-open mapping is a pairwise neutrosophic simply open mapping.

Proof. Suppose that ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) is a one to one and onto mapping from an NBTS (Ψ, τ_1, τ_2) to another NBTS (Ω, δ_1, δ_2).
(*i*) Let ξ be a pairwise neutrosophic open mapping. Assume that *L* be a PNOS in Ψ. Since ξ is a pairwise neutrosophic open mapping, so ξ(*L*) is a PNOS in Ω. Further, since every PNOS is also a PNPO set, so ξ(*L*) is a PNPO set in Ω. Therefore, ξ(*L*) is a PNPO set in Ω whenever *L* be a PNOS in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) is a pairwise neutrosophic pre-continuous mapping.
(*ii*) Let ξ be a pairwise neutrosophic open mapping. Assume that *L* be a PNOS in Ψ. Since ξ is a pairwise neutrosophic open mapping, so ξ(*L*) is a PNOS in Ω. Further, since every PNOS is also a PNSO set, so ξ(*L*) is a PNSO set in Ω. Therefore, ξ(*L*) is a PNSO set in Ω whenever *L* be a PNOS in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) is a pairwise neutrosophic semi-continuous mapping.
(*iii*) Suppose that ξ is a pairwise neutrosophic pre-open mapping. Let *L* be a PNOS in (Ψ, τ_1, τ_2). Since ξ is a pairwise neutrosophic pre-open mapping, so ξ(*L*) is a PNPO set in (Ω, δ_1, δ_2). It is known that, every PNPO set is a PN-*b*-OS. So ξ(*L*) is a PN-*b*-OS in (Ω, δ_1, δ_2). Therefore, ξ(*L*) is a PN-*b*-OS in (Ω, δ_1, δ_2) whenever *L* is a PNOS in (Ψ, τ_1, τ_2). Hence, ξ is a pairwise neutrosophic *b*-open mapping.
(*iv*) Let ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) be a pairwise neutrosophic semi-open mapping. Let *L* be a PNOS in (Ψ, τ_1, τ_2). Since ξ is a pairwise neutrosophic semi-open mapping, so ξ(*L*) is a PNSO set in (Ω, δ_1, δ_2). It is known that, every PNSO set is a PN-*b*-OS. So ξ(*L*) is a PN-*b*-OS in (Ω, δ_1, δ_2). Therefore, ξ(*L*) is a PN-*b*-OS in (Ω, δ_1, δ_2) whenever *L* is a PNOS in (Ψ, τ_1, τ_2). Hence, ξ is a pairwise neutrosophic *b*-open mapping.
(*v*) Let ξ be a pairwise neutrosophic open mapping. Assume that *L* be a PNOS in Ψ. Since ξ is a pairwise neutrosophic open mapping, so ξ(*L*) is a PNOS in Ω. Further, since every PNOS is also a PN-*b*-OS, so ξ(*L*) is a PN-*b*-OS in Ω. Therefore, ξ(*L*) is a PN-*b*-OS in Ω whenever *L* be a PNOS in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) is a pairwise neutrosophic *b*-continuous mapping.
(*vi*) Let ξ be a pairwise neutrosophic open mapping. Assume that *L* be a PNOS in Ψ. By hypothesis, ξ(*L*) is a PNOS in Ω. Further, since every PNOS is also a PNSOS, so ξ(*L*) is a PNSOS in Ω. Therefore, ξ(*L*) is a PNSOS in Ω whenever *L* be a PNOS in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) is a pairwise neutrosophic simply-continuous mapping.
(*vii*) Assume that ξ be a pairwise neutrosophic open mapping. Let *L* be a PNOS in Ψ. By hypothesis, ξ(*L*) is a PNOS in Ω. Further, since every PNOS is also a PNS*b*OS, so ξ(*L*) is a PNS*b*OS in Ω. Therefore, ξ(*L*) is a PNS*b*OS in Ω whenever *L* be a PNOS in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) is a pairwise neutrosophic simply *b*-continuous mapping.
(*viii*) Suppose that ξ be a pairwise neutrosophic simply *b*-open mapping. Let *L* be a PNOS in Ψ. By hypothesis, so ξ(*L*) is a PNS*b*OS in Ω. Further, since every PNS*b*OS is also a PNSOS, so ξ(*L*) is a PNSOS in Ω. Therefore, ξ(*L*) is a PNOS in Ω whenever *L* be a PNOS in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)$\rightarrow$(Ω, δ_1, δ_2) is a pairwise neutrosophic simply continuous mapping.

4. Pairwise Neutrosophic Simply *b*-Compactness

In this section, an attempt is made to procure the concept of pairwise neutrosophic compact (in short PN-compact) set, pairwise neutrosophic simply compact (in short PNS-compact) set, and pairwise neutrosophic simply *b*-compact (in short PNS-*b*-compact) set via NBTS.

Definition 4.1. A family $\{X_\alpha:\alpha\in\Delta\}$, where Δ is an index set and X_α is a PNOS in (Ψ, τ_1, τ_2), for each $\alpha\in\Delta$, is called a pairwise neutrosophic open cover of an NS X if $X\subseteq\cup_{\alpha\in\Delta}X_\alpha$.

Definition 4.2. An NBTS (Ψ, τ_1, τ_2) is called a PN-compact space if each pairwise neutrosophic open cover of 1_N has a finite sub-cover.

Definition 4.3. An NS B over Ψ is called a PN-compact relative to (Ψ, τ_1, τ_2) if every pairwise neutrosophic open cover of B has a finite pairwise neutrosophic open sub-cover.

Theorem 4.4. Every pairwise neutrosophic closed sub-set of a PN-compact space (Ψ, τ_1, τ_2) is PN-compact relative to Ψ.

Proof. Assume that (Ψ, τ_1, τ_2) be a PN-compact space. Suppose that K is a pairwise neutrosophic closed sub-set of (Ψ, τ_1, τ_2). Therefore, K^c is a PNOS in (Ψ, τ_1, τ_2). Suppose that $U=\{U_i : i\in\Delta$ and U_i is a PNOS in $\Psi\}$ be a pairwise neutrosophic open cover of K. Then, $\mathcal{H}=\{K^c\}\cup U$ is a pairwise neutrosophic open cover of 1_N. Since, (Ψ, τ_1, τ_2) is a PN-compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \ldots\ldots, H_n, K^c\}$. This implies, $\{H_1, H_2, H_3, \ldots\ldots, H_n\}$ is a finite pairwise neutrosophic open cover of K. Hence, K is a PN-compact set relative to Ψ.

Theorem 4.5. If $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a PN-C-mapping, then for each PN-compact set Q relative to Ψ, $\xi(Q)$ is a PN-compact set relative to Ω.

Proof. Let $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ be a PN-C-mapping. Let Q be a PN-compact set relative to Ψ. Suppose that $\mathcal{H}=\{H_i : i\in\Delta$, and H_i is a PNOS in $\Omega\}$ is a pairwise neutrosophic open cover of $\xi(Q)$. By hypothesis, $\xi^{-1}(\mathcal{H})=\{\xi^{-1}(H_i) : i\in\Delta$ and $\xi^{-1}(H_i)$ is a PNOS in $\Psi\}$ is a pairwise neutrosophic open cover of $\xi^{-1}(\xi(Q))=Q$. Since, Q is a PN-compact set relative to Ψ, so there exists a finite pairwise neutrosophic simply open sub-cover of Q say $\{H_1, H_2, H_3,\ldots.., H_n\}$ such that $Q \subseteq \cup_i\{H_i : i = 1, 2, \ldots., n\}$. Now, $Q \subseteq \cup_i\{H_i : i = 1, 2, \ldots., n\}$,
which implies $\xi(Q) \subseteq \cup_i\{\xi(H_i) : i = 1, 2, \ldots., n\}$. Therefore, there exist a finite pairwise neutrosophic open sub-cover $\{\xi(H_1), \xi(H_2), \xi(H_3), \ldots., \xi(H_n)\}$ of $\xi(Q)$ such that $\xi(Q)\subseteq \cup_i\{\xi(H_i) : i = 1, 2, \ldots., n\}$. Hence, $\xi(Q)$ is a PN-compact set relative to Ω.

Theorem 4.6. If $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic open function and $(\Omega, \delta_1, \delta_2)$ is a PN-compact space, then (Ψ, τ_1, τ_2) is also a PN-compact space.
Proof. Assume that $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ be a pairwise neutrosophic open function. Let $(\Omega, \delta_1, \delta_2)$ be a PN-compact space. Suppose that $\mathcal{H}=\{H_i: i\in\Delta$ and H_i is a PNOS in $\Psi\}$ be a pairwise neutrosophic open cover of Ψ. Therefore, $\xi(\mathcal{H})=\{\xi(H_i): i\in\Delta$ and $\xi(H_i)$ is a PNOS in $\Omega\}$ is a pairwise neutrosophic open cover of Ω. Since, $(\Omega, \delta_1, \delta_2)$ is a PN-compact space, so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \ldots\ldots, \xi(H_n)\}$ such that $1_N \subseteq\cup\{\xi(H_i): i=1, 2, \ldots, n\}$. This implies, $\{H_1, H_2, \ldots, H_n\}$ is a finite sub-cover for Ψ. Hence, (Ψ, τ_1, τ_2) is a PN-compact space.

Definition 4.7. A family $\{X_\alpha: \alpha\in\Delta\}$, where Δ is an index set and X_α is a PNSOS in (Ψ, τ_1, τ_2), for each $\alpha\in\Delta$, is called a pairwise neutrosophic simply open cover of a NS X if $X \subseteq \cup_{\alpha\in\Delta}X_\alpha$.

Definition 4.8. An NBTS (Ψ, τ_1, τ_2) is said to be a PNS-compact space if each pairwise neutrosophic simply-open cover of 1_N has a finite sub-cover.

Definition 4.9. An NS B of an NBTS (Ψ, τ_1, τ_2) is called a PNS-compact relative to Ψ if every pairwise neutrosophic simply-open cover of B has a finite sub-cover.

Theorem 4.10. Every pairwise neutrosophic simply-closed subset of a PNS-compact space (Ψ, τ_1, τ_2) is PNS-compact relative to Ψ.

Proof. Suppose that (Ψ, τ_1, τ_2) is a PNS-compact space, and K be a pairwise neutrosophic simply-closed sub-set of (Ψ, τ_1, τ_2). So, K^c is a PNSOS in (Ψ, τ_1, τ_2). Assume that $G=\{G_i : i\in\Delta$ and G_i is a PNSOS in $\Psi\}$ be a pairwise neutrosophic simply-open cover of K. Then, $\mathcal{H}=\{K^c\}\cup G$ is a pairwise neutrosophic simply-open cover of 1_N. Since, (Ψ, τ_1, τ_2) is a PNS-compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \ldots, H_n, K^c\}$. This implies, $\{H_1, H_2, H_3, \ldots, H_n\}$ is a finite pairwise neutrosophic simply-open cover of K. Hence, K is a PNS-compact set relative to Ψ.

Theorem 4.11. Every PNS-compact space is a PN-compact space.
Proof. Suppose that (Ψ, τ_1, τ_2) be a PNS-compact space. Therefore, every pairwise neutrosophic simply-open cover of 1_N has a finite sub-cover. Suppose that (Ψ, τ_1, τ_2) may not be a PN-compact space. Then, there exists a pairwise neutrosophic open cover $\mathcal{H}$ (say) of 1_N, which has no finite sub-cover. Since, every PNOS is a PNSOS, so we have a pairwise neutrosophic simply-open cover $\mathcal{H}$ of 1_N, which has no finite sub-cover. This contradicts the fact that (Ψ, τ_1, τ_2) is a PNS-compact space. Hence, the NBTS (Ψ, τ_1, τ_2) is a PN-compact space.

Theorem 4.12. If $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a PN-Simply-C-mapping, then for each PNS-compact set Q relative to Ψ, $\xi(Q)$ is a PN-compact set relative to Ω.

Proof. Assume that $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a PN-Simply-C-mapping, and Q be a PNS-compact set relative to Ψ. Suppose that $\mathcal{H}=\{H_i : i\in\Delta$, and H_i is a PNOS in $\Omega\}$ be a pairwise neutrosophic open cover of $\xi(Q)$. By our hypothesis, $\xi^{-1}(\mathcal{H})=\{\xi^{-1}(H_i)$: where $i\in\Delta$ and $\xi^{-1}(H_i)$ is a PNSOS in $\Psi\}$ is a pairwise neutrosophic simply open cover of $\xi^{-1}(\xi(Q))=Q$. Since, Q is a PNS-compact set relative to Ψ, so there exists a finite pairwise neutrosophic simply open sub-cover of Q say $\{\xi^{-1}(H_1), \xi^{-1}(H_2), \xi^{-1}(H_3),\ldots, \xi^{-1}(H_n)\}$ such that $Q\subseteq\cup_i\{\xi^{-1}(H_i): i = 1, 2, \ldots, n\}$. Now, $Q\subseteq\cup_i\{\xi^{-1}(H_i) : i = 1, 2, \ldots, n\}$, so we have $\xi(Q)\subseteq\cup_i\{\xi(\xi^{-1}(H_i)) : i = 1, 2, \ldots,n\}$,which means $\xi(Q)\subseteq\cup_i\{H_i : i = 1, 2, \ldots, n\}$.
Therefore, there exist a finite pairwise neutrosophic open sub-cover $\{H_1, H_2, H_3, \ldots, H_n\}$ of $\xi(Q)$ such that $\xi(Q)\subseteq\cup_i\{H_i : i = 1, 2, \ldots, n\}$. Hence, $\xi(Q)$ is a PN-compact set relative to Ω.

Theorem 4.13. If $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic simply open function and $(\Omega, \delta_1, \delta_2)$ is a PNS-compact space, then (Ψ, τ_1, τ_2) is a PN-compact space.

Proof. Suppose that $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a pairwise neutrosophic simply open function. Let $(\Omega, \delta_1, \delta_2)$ be a PNS-compact space. Let $\mathcal{H}=\{H_i: i\in\Delta$ and H_i is a PNOS in $\Psi\}$ be a pairwise neutrosophic open cover of Ψ. Therefore, $\xi(\mathcal{H})=\{\xi(H_i): i\in\Delta$ and $\xi(H_i)$ is a PNSOS in $\Omega\}$ is a pairwise neutrosophic simply open cover of Ω. Since, $(\Omega, \delta_1, \delta_2)$ is a PNS-compact space, so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \ldots, \xi(H_n)\}$ such that $1_N\subseteq\cup\{\xi(H_i): i=1, 2, \ldots, n\}$. This implies, $\{H_1, H_2, \ldots, H_n\}$ is a finite pairwise neutrosophic open sub-cover for Ψ. Hence, (Ψ, τ_1, τ_2) is a PN-compact space.

Definition 4.14. A family $\{X_\alpha: \alpha\in\Delta$ and X_α is a PNS*b*OS in $(\Psi, \tau_1, \tau_2)\}$, where Δ is an index set, is called a pairwise neutrosophic simply *b*-open cover of a N-set X if $X \subseteq \cup_{\alpha\in\Delta}X_\alpha$.

Definition 4.15. An NBTS (Ψ, τ_1, τ_2) is called a PNS-*b*-compact space if each pairwise neutrosophic simply *b*-open cover of 1_N has a finite sub-cover.

Definition 4.16. An neutrosophic subset B of an NBTS (Ψ, τ_1, τ_2) is said to be a PNS-*b*-compact relative to Ψ if every pairwise neutrosophic simply *b*-open cover of B has a finite sub-cover.

Theorem 4.17. Let (Ψ, τ_1, τ_2) be an NBTS. Then, every pairwise neutrosophic simply *b*-closed subset of a PNS-*b*-compact space (Ψ, τ_1, τ_2) is PNS-*b*-compact relative to Ψ.
Proof. Assume that (Ψ, τ_1, τ_2) be a PNS-*b*-compact space, and K be a pairwise neutrosophic simply *b*-closed sub-set of (Ψ, τ_1, τ_2). So, K^c is a pairwise neutrosophic simply *b*-open set in (Ψ, τ_1, τ_2). Suppose that $G =\{G_i : i\in\Delta$ and G_i is a PNS*b*OS in $\Psi\}$ is a pairwise neutrosophic simply *b*-open cover of K. Then, $\mathcal{H}=\{K^c\}\cup G$ is a pairwise neutrosophic simply *b*-open cover of 1_N. Since, (Ψ, τ_1, τ_2) is a PNS-*b*-compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \ldots.., H_n, K^c\}$. This implies, $\{H_1, H_2, H_3, \ldots.., H_n\}$ is a finite pairwise neutrosophic simply *b*-open cover of K. Hence, K is a PNS-*b*-compact set relative to Ψ.
Theorem 4.18. Every PNS-*b*-compact space is a PN-compact space.
Proof. Assume that (Ψ, τ_1, τ_2) be a PNS-*b*-compact space. Therefore, every pairwise neutrosophic simply *b*-open cover of 1_N has a finite sub-cover. Assume that (Ψ, τ_1, τ_2) may not be a PN-compact space. Then, there exists a pairwise neutrosophic open cover $\mathcal{H}$ (say) of 1_N, which has no finite sub-cover. Since, every PNOS is a PNS*b*OS, so we have a pairwise neutrosophic simply *b*-open cover $\mathcal{H}$ of 1_N, which has no finite sub-cover. This contradicts the fact that (Ψ, τ_1, τ_2) is a PNS-*b*-compact space. Hence, (Ψ, τ_1, τ_2) is a PN-compact space.
Theorem 4.19. Every PNS-compact space is a PNS-*b*-compact space.
Proof. Let (Ψ, τ_1, τ_2) be a PNS-compact space. Therefore, every pairwise neutrosophic simply open cover of 1_N has a finite sub-cover. Let (Ψ, τ_1, τ_2) may not be a PNS-*b*-compact space. Then, there exists a pairwise neutrosophic simply *b*-open cover $\mathcal{H}$ (say) of 1_N, which has no finite sub-cover. Since, every PNS*b*OS is a PNSOS, so we have a pairwise neutrosophic simply open cover $\mathcal{H}$ of 1_N, which has no finite sub-cover. This contradicts the fact that (Ψ, τ_1, τ_2) is a PNS-compact space. Hence, (Ψ, τ_1, τ_2) is a PNS-*b*-compact space.
Theorem 4.20. If $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ be a pairwise neutrosophic simply *b*-continuous function, then for each PNS-*b*-compact set Q relative to Ψ, $\xi(Q)$ is a PN-compact set relative to Ω.

Proof. Assume that $\xi:(\Psi, \tau_1, \tau_2)\to(\Omega, \delta_1, \delta_2)$ be a pairwise neutrosophic simply *b*-continuous function, and Q be a PNS-*b*-compact set relative to Ψ. Let $\mathcal{H}=\{H_i : i\in\Delta$, and H_i is a PNOS in $\Omega\}$ be a pairwise neutrosophic open cover of $\xi(Q)$. By hypothesis, $\xi^{-1}(\mathcal{H})=\{\xi^{-1}(H_i) : i\in\Delta$ and $\xi^{-1}(H_i)$ is a PNS*b*OS in $\Psi\}$ is a pairwise neutrosophic simply *b*-open cover of $\xi^{-1}(\xi(Q))=Q$. Since, Q is a PNS-compact set relative to Ψ, so there exists a finite pairwise neutrosophic simply *b*-open sub-cover of Q say $\{\xi^{-1}(H_1), \xi^{-1}(H_2), \xi^{-1}(H_3), \ldots, \xi^{-1}(H_n)\}$ such that $Q \subseteq \cup_i\{\xi^{-1}(H_i): i = 1, 2, \ldots, n\}$. Now, $Q \subseteq \cup_i\{\xi^{-1}(H_i) : i = 1, 2, \ldots, n\}$. This gvies $\xi(Q) \subseteq \cup_i\{\xi(\xi^{-1}(H_i)) : i = 1, 2, \ldots, n\}$, which implies $\xi(Q) \subseteq \cup_i\{H_i : i = 1, 2, \ldots, n\}$. Therefore, there exist a finite pairwise neutrosophic open sub-cover $\{H_1, H_2, H_3, \ldots, H_n\}$ of $\xi(Q)$ such that $\xi(Q) \subseteq \cup_i\{H_i : i = 1, 2, \ldots, n\}$. Hence, $\xi(Q)$ is a PN-compact set relative to Ω.

Theorem 4.21. If ξ:(Ψ, τ_1, τ_2)→(Ω, δ_1, δ_2) is a pairwise neutrosophic simply *b*-open function and (Ω, δ_1, δ_2) is a PNS-*b*-compact space, then (Ψ, τ_1, τ_2) is a PN-compact space.

Proof. Let ξ:(Ψ, τ_1, τ_2)→(Ω, δ_1, δ_2) be a pairwise neutrosophic simply *b*-open function. Suppose that (Ω, δ_1, δ_2) be a PNS-*b*-compact space. Let $\mathcal{H}$={H_i: $i\in\Delta$ and H_i is a pairwise N-O-S in Ψ} be a pairwise neutrosophic open cover of Ψ. Therefore, $\xi(\mathcal{H})$={$\xi(H_i)$: $i\in\Delta$ and $\xi(H_i)$ is a PNS*b*OS in Ω} is a pairwise neutrosophic simply *b*-open cover of Ω. Since, (Ω, δ_1, δ_2) is a PNS-*b*-compact space, so there exists a finite sub-cover say {$\xi(H_1)$, $\xi(H_2)$, ……., $\xi(H_n)$} such that $1_N\subseteq\cup\{\xi(H_i)$: i=1, 2, …, $n\}$. This implies, {H_1, H_2, …, H_n} is a finite pairwise neutrosophic open sub-cover for Ψ. Hence, (Ψ, τ_1, τ_2) is a PN-compact space.

5. Conclusions

In this paper, we have introduced the PNSOS, PNS-C-mapping, PNS-compactness, PNS*b*OS, PNS-*b*-C-mapping and PNS-*b*-compactness via NBTSs. By defining PNSOS, PNS-C-mapping, PNS-compactness, PNS*b*OS, PNS-*b*-C-mapping and PNS-*b*-compactness, we have established several results on NBTSs, and furnished few illustrative examples. In the future, we hope that based on these notions in NBTSs, many new investigations like pairwise neutrosophic simply separation axioms, pairwise neutrosophic simply connectedness, etc. can be carried out by the researchers around the globe via NBTSs.

Further, the proposed notions can be applied in the field of Quadripartitioned Neutrosophic Topological Space [27], Pentapartitioned Neutrosophic Topological Space [42], Rough Pentapartitioned Neutrosophic Topological Space [43], Neutrosophic Tri-topological Space [44], Neutrosophic Soft Bitopological Spaces [40]. Neutrosophic *d*-Algebra [45], etc. Also, it is claimed that, this study has no limitations at all.

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