Iraqi Journal of Science, 2019, Vol. 60, No.1, pp: 127-134
DOI: 10.24996/ijs.2019.60.1.14

ISSN: 0067-2904

Estimate the Two Parameters of Gamma Distribution Under Entropy Loss Function

Loaiy F. Naji[*] , Huda A. Rasheed
Department of Mathematics, College of Science, Mustansiriyah University, Baghdad, Iraq

Abstract

In this paper, Bayes estimators for the shape and scale parameters of Gamma distribution under the Entropy loss function have been obtained, assuming Gamma and Exponential priors for the shape and scale parameters respectively. Moment, Maximum likelihood estimators and Lindley's approximation have been used effectively in Bayesian estimation. Based on Monte Carlo simulation method, those estimators are compared depending on the mean squared errors (MSE's). The results show that, the performance of the Bayes estimator under Entropy loss function is better than other estimates in all cases.

Keywords: Gamma distribution, Maximum likelihood estimator, Entropy loss function, Lindley's approximation.

تقدير معلمتي توزيع كاما تحت دالة خسارة الانتروبي

لؤي فائق ناجي[*] ، هدى عبدالله رشيد
قسم الرياضيات، كلية العلوم، الجامعه المستنصرية، بغداد، العراق

الخلاصة

في هذا البحث، تم الحصول على مقدري بيز لمعلمتي القياس والشكل لتوزيع كاما تحت دالة خسارة الانتروبي، بافتراض دالتي أسبقية كاما والأسي لكل من معلمتي القياس والشكل على التوالي. مقدرات العزوم والإمكان الأعظم وتقريب ليندلي تم استخدامها بكفاءة في التقدير البيزي. استناداً الى طريقة مونت كارلو للمحاكاة فإن هذه المقدرات، تمت مقارنتها بالاعتماد على متوسط مربعات الخطأ (MSE's). أظهرت النتائج أن أداء مقدر بيز تحت دالة الخسارة الأنتروبي أفضل من التقديرات الاخرى لجميع الحالات .

1. Introduction

The gamma distribution plays a very important role in statistical inferential problems. It is widely used in reliability analysis and life testing and as a conjugate prior in Bayesian statistics.
It is a good alternative to the popular Weibull distribution, also, it is a flexible distribution that commonly offers a good fit to any variable such as in environmental, meteorology, climatology and other physical situation [1]. There are many applications for Gamma distribution in real life, for example, in bacterial gene expression, the copy number of a constitutively expressed protein often follows the gamma distribution, where the scale and shape parameter are, respectively, the mean number of bursts per cell cycle and the mean number of protein molecules produced by a single mRNA during its lifetime.

*Email: louy.faeq@gmail.com

The probability density function of the Gamma distribution is defined as follow [2]

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)} \quad ; \quad x>0 \ , \quad \alpha>0 \ , \quad \beta>0 \qquad (1)$$

Where,

α and β are often called the shape and scale parameters, respectively. The Gamma function is

$\Gamma(\alpha)=\int_0^{\infty} x^{\alpha-1}e^{-x}\,dx$, for $\alpha>0$

The cumulative distribution function (CDF) is:

$$F(x;\alpha,\beta)=\int_0^{x}\frac{\beta^{\alpha}}{\Gamma(\alpha)}\,u^{\alpha-1}\,e^{-u\beta}du$$

This function is called incomplete Gamma Function. The formula for the cumulative distribution can be written as

$$F(x;\alpha,\beta)=1-\sum_{j=0}^{\alpha-1}\frac{(\beta x)^j}{j!}e^{-\beta x}=\sum_{j=\alpha}^{\infty}\frac{(\beta x)^j}{j!}e^{-\beta x}$$

Therefore, the reliability function for $\Gamma(\alpha,\beta)$ is:[2]

$$R(x;\alpha,\beta)=\sum_{j=0}^{\alpha-1}\frac{(\beta x)^j}{j!}e^{-\beta x}$$

2. Estimation Methods

The moment estimators are used as primary estimators for maximum likelihood estimators of each of α and β. On the other hand, the maximum likelihood estimators are used to derive Bayesian estimators.

2.1 Moment Method

Suppose that, X be a random variable has a Gamma distribution defined by (1), and let $x_1, x_2, \ldots, x_n$ be a random sample of size n from X. Defining the first k sample moments about origin as

$m'_r=\frac{1}{n}\sum_{i=1}^{n} {x_i}^r$, r = 1, 2, . . . , k.

The first k population moments about origin are given by $\mu'_r=E(X^r)$.

Now, equaling these moments, that is

$\mu'_r=m'_r$, r = 1, 2, . . . , k

The solutions to the above equations denote by $\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_k$, yields the moment estimators of $\theta_1, \theta_2, \ldots, \theta_k$ [3][4]

The moment method for estimating the two–parameter Gamma distribution can be derived as follows

$$m_1=\frac{\sum_{i=1}^{n}x_i}{n}=\bar{x}$$

$$m_2=\frac{\sum_{i=1}^{n}x_i^2}{n}$$

$$\mu'_1=E(X)=\frac{\alpha}{\beta}$$

$$\mu'_2=E(X^2)=\frac{\alpha}{\beta^2}+\left(\frac{\alpha}{\beta}\right)^2$$

From $m_1=\mu'_1$, $m_2=\mu'_2$, we get

$$\hat{\alpha}=\frac{n\bar{x}^2}{\sum_{i=1}^{n}x_i^2-n\bar{x}^2} \qquad (2)$$

$$\hat{\beta}=\frac{n\bar{x}}{\sum_{i=1}^{n}x_i^2-n\bar{x}^2} \qquad (3)$$

2.2 Maximum Likelihood Method

The maximum likelihood method is one of the best methods of obtaining a point estimator of a parameter is proposed by R.A. Fisher (1912), and this technique was developed in the 1920s by a famous British statistician, Sir R. A. Fisher. As the name implies, the estimator will be the value of the parameter that maximizes the likelihood function.[5]

This method is the most popular procedure in estimating the parameter θ which specifies a probability function f(x,θ), based on the observations $x_1,x_2,\ldots,x_n$ which were independent sample from the distribution . The maximum likelihood estimator $\hat{\theta}$ of the parameter θ which maximizes the likelihood function will be as follows [6][7]

$$L(x_i;\theta)=\pi_{i=1}^{n}f(x_i;\theta)$$

The likelihood function for Gamma distribution, when two parameters are unknown is

$$L(x_1, x_2, \dots, x_n ; \alpha, \beta) = \frac{\beta^{n\alpha}}{(\Gamma(\alpha))^n} (\pi_{i=1}^{n} x_i)^{\alpha-1} e^{-\beta \sum_{i=1}^{n} x_i} \quad (4)$$

Taking the logarithm for the likelihood function, yields

$$\text{Ln L} = -\text{n}\ln\Gamma(\alpha) + \text{n}\alpha\ln\beta + (\alpha - 1)\sum_{i=1}^{n} lnx_i - \beta \sum_{i=1}^{n} x_i$$

The parameters that maximize the likelihood function are the solution of the equations

$$\frac{\partial lnL}{\partial \alpha} = -n\Psi(\alpha) + nln\beta + \sum_{i=1}^{n} lnx_i \quad (5)$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n\alpha}{\beta} - \sum_{i=1}^{n} x_i \quad (6)$$

Observe that, the two equations (5) and (6) are difficult and complicated to solve, then it is impossible to find MLE for α and β analytically, we can use the numerical analysis (numerical procedure) to obtain and estimate α and β that maximize the likelihood function. One of these numerical procedures is Newton-Raphson method and using Hessian matrix, which can be written as follows [3]

$$g_1(\alpha) = -n\Psi(\alpha) + nln\beta + \sum_{i=1}^{n} lnx_i$$

$$g_2(\beta) = \frac{n\alpha}{\beta} - n\bar{x}$$

The partial derivatives of $g_1(\alpha)$ with respect to unknown parameters α and β are

$$\frac{\partial g_1(\alpha)}{\partial \alpha} = -n\Psi'(\alpha)$$

Where $\Psi'(\alpha)$ is the derivative of $\Psi(\alpha)$ which is called as tri-gamma

$$\frac{\partial g_1(\alpha)}{\partial \beta} = \frac{n}{\beta}$$

The partial derivatives of $g_2(\beta)$ with respect to unknown parameters α and β are

$$\frac{\partial g_2(\beta)}{\partial \alpha} = \frac{n}{\beta}$$

$$\frac{\partial g_2(\beta)}{\partial \beta} = -\frac{n\alpha}{\beta^2}$$

Hence,

$$J_k = \begin{bmatrix} \frac{\partial g_1(\alpha)}{\partial \alpha} & \frac{\partial g_1(\alpha)}{\partial \beta} \\ \frac{\partial g_2(\beta)}{\partial \alpha} & \frac{\partial g_2(\beta)}{\partial \beta} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The Jacobian matrix must be a non-singular symmetric matrix so its inverse can be found as

$$J_K^{-1} = \frac{1}{|J|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$\begin{bmatrix} \alpha_{k+1} \\ \beta_{k+1} \end{bmatrix} = \begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} - J_{k_i}^{-1} \begin{bmatrix} g_1(\alpha) \\ g_2(\beta) \end{bmatrix}$$

$$\begin{bmatrix} \alpha_{k+1} \\ \beta_{k+1} \end{bmatrix} = \begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} - \frac{\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} -n\Psi(\alpha_k) + nln\beta_k + \sum_{i=1}^{n} lnx_i \\ \frac{n\alpha_k}{\beta_k} - n\bar{x} \end{bmatrix}$$

$$\begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} - \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} -na_{22}\Psi(\alpha_k) + na_{22}ln\beta_k + a_{22}\sum_{i=1}^{n} lnx_i - \frac{na_{12}\alpha_k}{\beta_k} + na_{12}\bar{x} \\ na_{21}\Psi(\alpha_k) - na_{21}ln\beta_k - a_{21}\sum_{i=1}^{n} lnx_i + \frac{na_{11}\alpha_k}{\beta_k} - na_{11}\bar{x} \end{bmatrix}$$

The absolute value for the difference between the new value for α and β in new iterative value with previous value for α and β in last iterative represent the error term, it's symbol is ε , which is a very small and assumed value.

Then, error term is formulated as

$$\begin{bmatrix} \varepsilon_{k+1}(\alpha) \\ \\ \varepsilon_{k+1}(\beta) \end{bmatrix} = \left| \begin{bmatrix} \alpha_{k+1} \\ \\ \beta_{k+1} \end{bmatrix} - \begin{bmatrix} \alpha_k \\ \\ \beta_k \end{bmatrix} \right| \tag{7}$$

Where α_k and β_k are the initial values for α and β respectively, for which are assumed.

3. Bayes Estimation

3.1 Posterior Density Functions Using Gamma and Exponential Priors

To estimate α and β parameters for Gamma distribution, we assume that α has a prior $\pi_1(\cdot)$, which follows Gamma(a, b). Also, we assume that, the prior on β is $\pi_2(\cdot)$ and the density function of $\pi_2(\cdot)$ is Exponential and it is independent of $\pi_1(\cdot)$.

$$\pi_1(\alpha) = \begin{cases} \frac{(b)^a(\alpha)^{a-1}e^{-b\alpha}}{\Gamma(a)} & ; \quad a > 0,\ b > 0, \alpha > 0 \\ 0 & \quad 0.w \end{cases} \tag{8}$$

$$\pi_2(\beta) = \begin{cases} c\, e^{-\beta c} & ; \quad c > 0, \beta \geq 0 \\ 0 & ; \quad o.w \end{cases} \tag{9}$$

The equations (8) and (9) are prior distribution for α and β respectively.

The joint p.d.f is given by

$$J(x_1, x_2, \dots, x_n; \alpha, \beta) = L(x_1, x_2, \dots, x_n; \alpha, \beta)\ \pi_1(\alpha)\, \pi_2(\beta)$$

$$= \frac{\beta^{n\alpha}}{(\Gamma(\alpha))^n}\ \pi_{i=1}^{n} x_i^{\alpha-1} e^{-\beta \sum_{i=1}^{n} x_i}\ \frac{(b)^a(\alpha)^{a-1}e^{-b\alpha}}{\Gamma(a)}\ c\, e^{-\beta c}$$

And the marginal p.d.f. of $(x_1, x_2, \dots, x_n)$ is given by

$$f(x_1, x_2, \dots, x_n) = \int_0^\infty \int_0^\infty L(x_1, x_2, \dots, x_n; \alpha, \beta) \pi_1(\alpha)\, \pi_2(\beta) \mathrm{d}\alpha\, \mathrm{d}\beta$$

The posterior density functions of α and β is defined as follows

$$h(\alpha, \beta | x_1, x_2, \dots, x_n) = \frac{L(x_1, x_2, \dots, x_n; \alpha, \beta)\ \pi_1(\alpha)\, \pi_2(\beta)}{\int_0^\infty \int_0^\infty L(x_1, x_2, \dots, x_n; \alpha, \beta)\pi_1(\alpha)\, \pi_2(\beta) \mathrm{d}\alpha \mathrm{d}\beta}$$

$$= \frac{\frac{\beta^{n\alpha}}{(\Gamma(\alpha))^n} \pi_{i=1}^{n} x_i^{\alpha-1} e^{-\beta \sum_{i=1}^{n} x_i}\ \frac{(b)^a(\alpha)^{a-1}e^{-b\alpha}}{\Gamma(a)}\, c\, e^{-\beta c}}{\int_0^\infty \int_0^\infty \frac{\beta^{n\alpha}}{(\Gamma(\alpha))^n} \pi_{i=1}^{n} x_i^{\alpha-1} e^{-\beta \sum_{i=1}^{n} x_i}\ \frac{(b)^a(\alpha)^{a-1}e^{-b\alpha}}{\Gamma(a)}\, c\, e^{-\beta c}\ d\alpha\, d\beta}$$

3.2 Bayes Estimators for α and β under Entropy Loss Function

Entropy loss function is one of asymmetric loss functions [3]. It was first introduced by James and Stein for estimation of the Variance-Covariance (i.e., Dispersion) matrix of the multivariate normal distribution [8]. Entropy loss function can be written as the following form.[9]

$$L(\hat{\theta}, \theta) = \frac{\hat{\theta}}{\theta} - ln\frac{\hat{\theta}}{\theta} - 1 \tag{10}$$

Therefore, the Bayes estimator under the entropy loss $\hat{\theta}_{BE}$, is given by

$$\hat{\theta}_{BE} = [E(\theta^{-1}|X)]^{-1} \tag{11}$$

Bayesian estimators of the shape parameter α and scale parameter β of the Gamma distribution has been obtained under Entropy loss function as follows

$$E[\mathrm{u}(\alpha, \beta)] = \int_0^\infty \int_0^\infty \mathrm{u}(\alpha, \beta)\, \mathrm{h}(\alpha, \beta | \mathrm{x}_1, \dots \mathrm{x}_\mathrm{n})\ d\alpha d\beta$$

Where u(α,β) be any function for α and β.

After substitution, yields

$$E[\mathrm{u}(\alpha, \beta)] = \frac{\int_0^\infty \int_0^\infty \mathrm{u}(\alpha,\beta) L(x_1,x_2,\dots,x_n;\alpha,\beta)\ \pi_1(\alpha)\ \pi_2(\beta)\ d\alpha d\beta}{\int_0^\infty \int_0^\infty L(x_1,x_2,\dots,x_n;\alpha,\beta)\ \pi_1(\alpha)\ \pi_2(\beta)\ d\alpha d\beta}$$

i) Bayesian Estimation for the Shape Parameter α under Entropy Loss Function

To obtain Bayesian estimation for α, under Entropy loss function, assume that

$u(\alpha,\beta)=\frac{1}{\alpha}$

Therefore, $E\left((\frac{1}{\alpha})\middle|\underline{x}\right)=\frac{\int_0^\infty\int_0^\infty \frac{1}{\alpha}L(x_1,x_2,\dots,x_n;\alpha,\beta)\pi_1(\alpha)\,\pi_2(\beta)\,d\alpha d\beta}{\int_0^\infty\int_0^\infty L(x_1,x_2,\dots,x_n;\alpha,\beta)\pi_1(\alpha)\,\pi_2(\beta)\,d\alpha\,d\beta}$

Notice that, it is difficult to find the solution of the ratio of two integrals. Therefore, the approximation form of Lindley will be used to get $E\left(\frac{1}{\alpha}\middle|\underline{x}\right)$ as follows

$$E\left((\frac{1}{\alpha})\middle|\underline{x}\right)=\frac{1}{\hat{\alpha}}+\frac{1}{2}(u_{11}\sigma_{11})+p_1u_1\sigma_{11}+\frac{1}{2}(L_{30}u_1\sigma_{11}^2)+\frac{1}{2}(L_{12}u_1\sigma_{11}\sigma_{22}) \qquad (12)$$

Where,

$L_{ij}=\frac{\partial^{i+j}}{\partial\alpha^i\partial\beta^j}lnL(\alpha,\beta)$; i , j = 0, 1, 2, 3

$lnL(\alpha,\beta)=n\alpha\ln\beta-n\ln\Gamma(\alpha)-\beta\sum_{i=1}^n x_i+(\alpha-1)\sum_{i=1}^n\ln x_i$

$L_{12}=\frac{\partial^3 lnL(\alpha,\beta)}{\partial\alpha\partial\beta^2}=-\frac{n}{\beta^2}$, $L_{21}=\frac{\partial^3\ln L(\alpha,\beta)}{\partial^2\partial\beta}=0$

$L_{03}=\frac{\partial^3\ln L(\alpha,\beta)}{\partial\beta^3}=\frac{2n\alpha}{\beta^3}$, $L_{30}=\frac{\partial^3\ln L(\alpha,\beta)}{\partial\alpha^3}=-n\,\Psi''(\alpha)$

$L_{20}=\frac{\partial^2\ln L(\alpha,\beta)}{\partial\alpha^2}=-n\Psi'(\alpha)$, $L_{02}=\frac{\partial^2\ln L(\alpha,\beta)}{\partial\beta^2}=\frac{-n\alpha}{\beta^2}$

$\sigma_{11}=-\frac{1}{L_{20}}=\frac{1}{n\Psi'(\alpha)}$, $\sigma_{22}=-\frac{1}{L_{02}}=\frac{\beta^2}{n\alpha}$

$u_1=\frac{\partial u(\alpha,\beta)}{\partial\alpha}=-\alpha^{-2}$, $u_{11}=\frac{\partial^2 u(\alpha,\beta)}{\partial\alpha^2}=2\alpha^{-3}$

$u_2=\frac{\partial u(\alpha,\beta)}{\partial\beta}=0$, $u_{22}=\frac{\partial^2 u(\alpha,\beta)}{\partial\beta^2}=0$

We assumed that α and β are independent. Therefore, the joint p.d.f of α and β is given by

$\pi(\alpha,\beta)=\frac{(b)^a(\alpha)^{a-1}e^{-b\alpha}}{\Gamma(a)}\;ce^{-c\beta}$

$p=ln\pi(\alpha,\beta)=(a-1)ln\alpha+alnb-b\alpha-ln\Gamma(\alpha)+lnc-c\beta$

$p_1=\frac{\partial p}{\partial\alpha}=\frac{a-1}{\alpha}-b$, $p_2=\frac{\partial p}{\partial\beta}=-c$

Now, we can apply Lindley's form (12), as follows

$$E\left(\frac{1}{\alpha}\middle|\underline{x}\right)=\frac{1}{\hat{\alpha}}+\frac{1}{\hat{\alpha}^3 n\Psi'(\hat{\alpha})}-\frac{1}{\hat{\alpha}^2 n\Psi'(\hat{\alpha})}\left(\frac{a-1}{\hat{\alpha}}-b\right)+\frac{1}{2}\left(\frac{n\,\Psi''(\hat{\alpha})}{\hat{\alpha}^2\left(n\Psi'(\hat{\alpha})\right)^2}\right)+\frac{1}{2}\left(\frac{1}{\hat{\alpha}^3 n\Psi'(\alpha)}\right) \qquad (13)$$

Now, Substituting (13) into (11) yields,

$$\hat{\alpha}_{BE}=\frac{1}{\frac{1}{\hat{\alpha}}+\frac{1}{\hat{\alpha}^3 n\Psi'(\hat{\alpha})}-\frac{1}{\hat{\alpha}^2 n\Psi'(\hat{\alpha})}\left(\frac{a-1}{\hat{\alpha}}-b\right)+\frac{1}{2}\left(\frac{n\,\Psi''(\hat{\alpha})}{\hat{\alpha}^2\left(n\Psi'(\hat{\alpha})\right)^2}\right)+\frac{1}{2}\left(\frac{1}{\hat{\alpha}^3 n\Psi'(\hat{\alpha})}\right)}$$

Where $\hat{\alpha}$ are the maximum likelihood estimators.

ii) Bayesian estimation for the scale parameter β under Entropy loss function

Assume that, $u(\alpha,\beta)=\frac{1}{\beta}$, then,

$u_1=\frac{\partial u(\alpha,\beta)}{\partial\alpha}=0$, $u_{11}=\frac{\partial^2 u(\alpha,\beta)}{\partial\alpha^2}=0$

$u_2=\frac{\partial u(\alpha,\beta)}{\partial\beta}=-\beta^{-2}$, $u_{22}=\frac{\partial^2 u(\alpha,\beta)}{\partial\beta^2}=2\beta^{-3}$

Thus, $E\left(\frac{1}{\beta}\right)=\frac{1}{\hat{\beta}}+\frac{1}{2}(u_{22}\sigma_{22})+p_2u_2\sigma_{22}+\frac{1}{2}(L_{03}u_2\sigma_{22}^2)+\frac{1}{2}(L_{21}u_2\sigma_{11}\sigma_{22})$

$$=\frac{1}{\hat{\beta}}+\frac{1}{2}\left(\frac{2\hat{\beta}^2}{\hat{\beta}^3 n\hat{\alpha}}\right)+\frac{c\hat{\beta}^2}{\hat{\beta}^2 n\hat{\alpha}}+\frac{1}{2}\left(\frac{-2n\hat{\alpha}}{\hat{\beta}^3}\,\frac{1}{\hat{\beta}^2}\,\frac{\hat{\beta}^4}{n^2\hat{\alpha}^2}\right)$$

$$=\frac{1}{\hat{\beta}}+\frac{c}{n\hat{\alpha}} \qquad (14)$$

After Substituting (13) into (11) yields,

$$\hat{\beta}_{BE}=\frac{1}{\frac{1}{\hat{\beta}}+\frac{c}{n\hat{\alpha}}}$$

Where $\hat{\alpha}$, $\hat{\beta}$ are the maximum likelihood estimators.

4. Simulation Study

In this section, Monte-Carlo simulation is employed to compare the performance of three estimates (moment, Maximum likelihood and Bayes estimators under Entropy loss function) for unknown shape and scale parameters based on mean squared errors (MSE's) as follows

$$MSE(\theta) = \frac{\sum_{i=1}^{I}\left(\hat{\theta}_i-\theta\right)^2}{I}$$

Where, I is the number of replications.

We generated I = 3000 samples of size n = 20, 30, 50, and 100 to represent small, moderate and large sample sizes from Gamma distribution with α = 2, 3 and β =0.5, 1.

The parameters for the prior distribution of α were chosen as a=3, b= 3 and for β's prior parameters are c = 4.

5. Discussion

The results are summarized and tabulated in Tables (1-6) which contains the expected values and (MSE's) for estimating α and β, and we have observed that

1. The performance of Bayes estimates under Entropy loss function for each of α and β is the best, since it gives smallest mean square error, as indicated for all combination of initial values of parameters.

2. It is observed that, MSE's of all estimators of shape parameter is increasing with the increase of the value of shape parameter. Also, MSE values for all estimates are increasing with the increase of the scale parameter value for all cases.

Table 1- The expected values of different estimators for unknown shape parameter α of Gamma distribution when α = 2

Method / n	$\hat{\alpha}_{MO}$		$\hat{\alpha}_{ML}$		$\hat{\alpha}_{BE}$	
	β=0.5	β=1	β=0.5	β=1	β=0.5	β=1
20	2.486393	2.486393	2.33479	2.334791	2.11877	2.11877
30	2.298321	2.298321	2.194657	2.194658	2.05860	2.05860
50	2.183145	2.183145	2.118412	2.118412	2.03897	2.03897
100	2.090724	2.090724	2.055311	2.055311	2.01663	2.01663

Table 2-The expected values of different estimators for unknown shape parameter α of Gamma distribution when α = 3

Method / n	$\hat{\alpha}_{MO}$		$\hat{\alpha}_{ML}$		$\hat{\alpha}_{BE}$	
	β=0.5	β=1	β=0.5	β=1	β=0.5	β=1
20	3.600494	3.600494	3.447432	3.447433	3.08855	3.08855
30	3.405721	3.405721	3.299321	3.299319	3.06486	3.06486
50	3.255532	3.405721	3.18809	3.299319	3.04956	3.04956
100	3.126059	3.126059	3.089527	3.089528	3.02157	3.02157

Table 3-The MSE values of different estimators for unknown shape parameter α of Gamma distribution when α = 2

Method / n	$\hat{\alpha}_{MO}$		$\hat{\alpha}_{ML}$		$\hat{\alpha}_{BE}$	
	β=0.5	β=1	β=0.5	β=1	β=0.5	β=1
20	1.13161	1.13161	0.80765	0.80765	0.54285	0.54286
30	0.58915	0.58915	0.38833	0.38833	0.29458	0.29458
50	0.29714	0.29714	0.18510	0.18510	0.15440	0.15440
100	0.13609	0.13609	0.08313	0.08313	0.07594	0.07594

Table 4-The MSE values of different estimators for unknown shape parameter α of Gamma distribution when α = 3

Method / n	$\hat{\alpha}_{MO}$		$\hat{\alpha}_{ML}$		$\hat{\alpha}_{BE}$	
	β=0.5	β=1	β=0.5	β=1	β=0.5	β=1
20	2.06203	2.06203	1.62539	1.62540	1.08765	1.08765
30	1.11338	1.11338	0.84883	0.84883	0.63298	0.63298
50	0.61598	1.11338	0.44923	0.84883	0.37139	0.37139
100	0.25924	0.25924	0.18543	0.18543	0.16787	0.16787

Table 5-The expected values of different estimators for unknown scale parameter β of Gamma distribution when β = 0.5

Method / n	$\hat{\beta}_{MO}$		$\hat{\beta}_{ML}$		$\hat{\beta}_{BE}$	
	α=2	α=3	α=2	α=3	α=2	α=3
20	0.63802	0.61058	0.59870	0.58464	0.56879	0.56515
30	0.58456	0.57368	0.55831	0.55562	0.53969	0.54329
50	0.55128	0.54540	0.53514	0.53420	0.52443	0.52708
100	0.52472	0.52256	0.51588	0.51658	0.51073	0.51314

Table 6-The expected values of different estimators for unknown scale parameter β of Gamma distribution when β = 1

Method / n	$\hat{\beta}_{MO}$		$\hat{\beta}_{ML}$		$\hat{\beta}_{BE}$	
	α=2	α=3	α=2	α=3	α=2	α=3
20	1.27605	1.22115	1.19739	1.16928	1.08359	1.09387
30	1.16913	1.14735	1.11661	1.11124	1.04460	1.06298
50	1.10256	1.14735	1.07027	1.11124	1.02830	1.04032
100	1.04945	1.04511	1.03176	1.03317	1.01135	1.01949

Table 7-The MSE values of different estimators for unknown scale parameter β of Gamma distribution when β = 0.5

Method / n	$\hat{\beta}_{MO}$		$\hat{\beta}_{ML}$		$\hat{\beta}_{BE}$	
	α=2	α=3	α=2	α=3	α=2	α=3
20	0.09335	0.06904	0.06875	0.05533	0.05686	0.04880
30	0.04509	0.03596	0.03113	0.02778	0.02714	0.02533
50	0.02224	0.01936	0.01490	0.01465	0.01361	0.01381
100	0.01023	0.00824	0.00681	0.00627	0.00651	0.00608

Table 8-The MSE values of different estimators for unknown scale parameter β of Gamma distribution when β = 1

Method / n	$\hat{\beta}_{MO}$		$\hat{\beta}_{ML}$		$\hat{\beta}_{BE}$	
	α=2	α=3	α=2	α=3	α=2	α=3
20	0.37339	0.27617	0.27500	0.22131	0.19269	0.17422
30	0.18037	0.14383	0.12452	0.11114	0.09660	0.09322
50	0.08896	0.14383	0.05961	0.11114	0.05043	0.05236
100	0.04091	0.03295	0.02723	0.02508	0.02511	0.02363

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