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On Skew Left $*$-n-Derivations of $*$-Ring

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Abstract
In this paper, the commutativity of *-ring and some related results are obtained by introducing the new concept which is called skew left $*$-n-derivations.

Keywords: Prime $*$-ring, semiprime $*$-ring, $*$-n-derivation, permuting mapping, skew left $*$-n-derivation.

حول المشتقات اليسارية الملتوية من النمط-n-*للحلقات-*

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الخلاصة
في هذا البحث، ابدالية الحلقات-* و بعض النتائج المتعلقة بها قد تم الحصول عليها من خلال اعطاء مفهوم جديد يسمى المشتقة اليسارية الملتوية من النمط -n-* .

1. Introduction

Throughout this paper $\mathcal{R}$ will represent an associative ring with center $\mathcal{Z}(\mathcal{R})$. A ring $\mathcal{R}$ is said to be n-torsion free if na=0 with $a\in\mathcal{R}$ then a=0, where n is nonzero integer [1]. For any υ, $\gamma\in\mathcal{R}$, the commutator $\upsilon\gamma$-$\gamma\upsilon$ will be denoted by $[\upsilon,\gamma]$ and the anti-commutator $\upsilon\circ\gamma$ will be denoted by $\upsilon\gamma+\gamma\upsilon$ [2]. Recall that a ring $\mathcal{R}$ is said to be prime if $a\mathcal{R}b$=0 implies that either a=0 or b=0 for all $a,b\in\mathcal{R}$ [3] and it is semiprime if $a\mathcal{R}a$=0 implies that a=0 for all $a\in\mathcal{R}$ [1]. An additive mapping $\xi:\mathcal{R}\rightarrow\mathcal{R}$ is called a derivation if $\xi(\upsilon\gamma)=\xi(\upsilon)\gamma+\upsilon\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ [4]. An additive mapping $\xi:\mathcal{R}\rightarrow\mathcal{R}$ is called a left derivation if $\xi(\upsilon\gamma)=\gamma\xi(\upsilon)+\upsilon\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ [5], it is clear that the concepts of derivation and left derivation are identical whenever $\mathcal{R}$ is commutative. A map $\mathcal{F}:\mathcal{R}\rightarrow\mathcal{R}$ is said to be commuting (resp. centralizing) on $\mathcal{R}$ if $[\mathcal{F}(\upsilon),\upsilon]$=0 (resp. $[\mathcal{F}(\upsilon),\upsilon]\in\mathcal{Z}(\mathcal{R})$) for all $\upsilon\in\mathcal{R}$ [6]. An additive mapping $\upsilon\rightarrow\upsilon^*$ of $\mathcal{R}$ into itself is called an involution if the following conditions are satisfied (i) $(\upsilon\gamma)^*=\gamma^*\upsilon^*$ (ii) $(\upsilon^*)^*=\upsilon$ for all $\upsilon,\gamma\in\mathcal{R}$ [2]. A ring equipped with an involution is known as ring with involution or $*$-ring. Let $\mathcal{R}$ be a $*$-ring. An additive mapping $\xi:\mathcal{R}\rightarrow\mathcal{R}$ is called a $*$-derivation if $\xi(\upsilon\gamma)=\xi(\upsilon)\gamma^*+\upsilon\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ [7]. An additive mapping $\xi:\mathcal{R}\rightarrow\mathcal{R}$ is called a left $*$-derivation $\xi(\upsilon\gamma)=\gamma^*\xi(\upsilon)+\upsilon\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ [8]. There are many works dealing with the commutativity of prime and semi prime rings admitting certain types of derivations [4,5,9,10,11]. Bresar and Vukman [7] studied the notion of a $*$-derivation of $\mathcal{R}$. Ali [12] defined symmetric $*$-biderivation and studied some properties of prime $*$-rings and semiprime $*$-rings. Recently Ashraf [13] defined the concept of $*$-n-derivation in prime $*$-rings and semiprime $*$-rings and studied the commutativity and some of their properties. In the present paper we introduce the notion of skew left $*$-n-derivation and study the commutativity and some related results involving skew left $*$-n-derivations in $*$-rings.

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2. Preliminaries

Some definitions and fundamental facts of skew left $*$-n-derivations are recalled in this section, which are principals of skew left $*$-n-derivation.

Proposition (2.1) [2]

Let $\mathcal{R}$ be a ring, then for all $v, \gamma, z \in \mathcal{R}$ we have

1- $[v, \gamma z] = \gamma[v, z] + [v, \gamma]z$
2- $[v\gamma, z] = v[\gamma, z] + [v, z]\gamma$
3- $v \circ (\gamma z) = (v \circ \gamma)z - \gamma[v, z] = \gamma(v \circ z) + [v, \gamma]z$
4- $(v\gamma) \circ z = v(\gamma \circ z) - [v, z]\gamma = (v \circ z)\gamma + v[\gamma, z]$

Definition (2.2) [6]

A map $\xi: \mathcal{R}^n \to \mathcal{R}$ is called permuting (or symmetric) if the equation $\xi(v_1, v_2, \dots, v_n) = \xi(v_{\pi(1)}, v_{\pi(2)}, \dots, v_{\pi(n)})$ holds, for all $v_i \in \mathcal{R}$ and for every permutation $\{\pi(1), \pi(2), \dots, \pi(n)\}$.

Definition (2.3) [13]

An n-additive mapping $\xi: \mathcal{R}^n \to \mathcal{R}$ is said to be a $*$-n-derivation if the following equations are identical:

$\xi(v_1\gamma, v_2, \dots, v_n) = \xi(v_1, v_2, \dots, v_n)\gamma^* + v_1\xi(\gamma, v_2, \dots, v_n)$
$\xi(v_1, v_2\gamma, \dots, v_n) = \xi(v_1, v_2, \dots, v_n)\gamma^* + v_2\xi(v_1, \gamma, \dots, v_n)$
•
•
•
$\xi(v_1, v_2, \dots, v_n\gamma) = \xi(v_1, v_2, \dots, v_n)\gamma^* + v_n\xi(v_1, v_2, \dots, \gamma)$, for all $v_1, \gamma, v_2, \dots, v_n \in \mathcal{R}$.

Now we introduce the concept of skew left $*$-n-derivation to get our main results.

Definition (2.4)

Let $\mathcal{R}$ be a $*$-ring. An n-additive symmetric mapping $\xi: \mathcal{R}^n \to \mathcal{R}$ is said to be a skew left $*$-n-derivation if

$\xi(v_1\gamma, v_2, \dots, v_n) = \gamma^*\, \xi(v_1, v_2, \dots, v_n) + v_1\xi(\gamma, v_2, \dots, v_n)$
$\xi(v_1, v_2\gamma, \dots, v_n) = \gamma^*\, \xi(v_1, v_2, \dots, v_n) + v_2\xi(v_1, \gamma, \dots, v_n)$
•
•
•
$\xi(v_1, v_2, \dots, v_n\gamma) = \gamma^*\xi(v_1, v_2, \dots, v_n) + v_n\xi(v_1, v_2, \dots, \gamma)$, for all $v_1\gamma, v_2, \dots, v_n \in \mathcal{R}$.

The following example explains the notion of skew left $*$-n-derivation.

Example (2.5):

Let $\mathcal{R} = \left\{\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} \mid a \in \mathbb{R}\right\}$ be a ring, and $\mathbb{R}$ be a ring of real numbers. A map $\xi: \mathcal{R}^n \to \mathcal{R}$ define by

$\xi\left(\begin{pmatrix} 0 & a_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_2 \\ 0 & 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 & a_n \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & a_1a_2 \dots a_n \\ 0 & 0 \end{pmatrix}$, for all

$\begin{pmatrix} 0 & a_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_2 \\ 0 & 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 & a_n \\ 0 & 0 \end{pmatrix} \in \mathcal{R}$. And $r \to r^*$ such that $\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & -a \\ 0 & 0 \end{pmatrix}$

Then it easy to check that ξ is skew left $*$-n-derivation.

3. The Main Results

We investigate the commutativity of $*$-ring and some related results by using the notion of skew left $*$-n-derivations.

Theorem (3.1): let $\mathcal{R}$ be a prime $*$-ring and ξ be a skew left $*$-n-derivation. Then $\mathcal{R}$ is commutative ring or $\xi = 0$

Proof:

Since ξ is a skew left $*$-n-derivation, then

$\xi((v_1\gamma)z, v_2, \dots, v_n) = z^*\xi(v_1\gamma, v_2, \dots, v_n) + v_1\gamma\xi(z, v_2, \dots, v_n)$
$= z^*\gamma^*\xi(v_1, v_2, \dots, v_n) + z^*v_1\xi(\gamma, v_2, \dots, v_n) + v_1\gamma\xi(z, v_2, \dots, v_n)$ ……………………………......… (1)

Also we have

$\xi(v_1(\gamma z), v_2, \dots, v_n) = (\gamma z)^*\xi(v_1, v_2, \dots, v_n) + v_1\xi(\gamma z, v_2, \dots, v_n)$
$= z^*\gamma^*\xi(v_1, v_2, \dots, v_n) + v_1z^*\xi(\gamma, v_2, \dots, v_n) + v_1\gamma\xi(z, v_2, \dots, v_n)$, for all $v, \gamma, z, v_2, \dots, v_n \in \mathcal{R}$. ………. (2)

Combining equations (1) and (2), to have

$z^*\upsilon_1\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=\upsilon_1 z^*\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$
Putting z instead of z^* in the last equation, we obtain
$z\upsilon_1\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=\upsilon_1 z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$ ………………………………….. (3)
$[z,\upsilon_1]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$ ………………………………….. (4)
Replacing $z=z\mathfrak{r}$ in equation (4) and using it, to get $[z,\upsilon_1]\mathfrak{r}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$
Let $\gamma=\upsilon_1$ in above equation, then
$[z,\upsilon_1]\mathcal{R}\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)=0$
Then either $[z,\upsilon_1]=0$, which mean that $\mathcal{R}$ is commutative or $\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)=0$.

Theorem (3.2): Let $\mathcal{R}$ be a 2-torsion free prime $*$-ring and ξ_1 be a skew left $*$-n-derivation and ξ_2 be a $*$-n-derivation such that if $\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)=0$ for all $\upsilon_1,\upsilon_2,\dots,\upsilon_n,\gamma_1,\gamma_2,\dots,\gamma_n\in\mathcal{R}$ then either $\xi_1=0$ or $\xi_2=0$.

Proof:

Since $\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)=0$ ………… … (1)
Replacing $\upsilon_1=\upsilon_1 z$ in equation (1) to get
$0=\xi_1(\upsilon_1 z,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+\xi_2(\upsilon_1 z,\upsilon_2,\dots,\upsilon_n)\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)$
$=z^*\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)\upsilon_1\xi_1(z,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+$
$z^*\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)+\upsilon_1\xi_2(z,\upsilon_2,\dots,\upsilon_n)\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)$
$=z^*\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)z^*\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)\}+$
$\upsilon_1\{\xi_1(z,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+\xi_2(z,\upsilon_2,\dots,\upsilon_n)\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)$ ……………..… (2)
From equation (1) and equation (2)
$z^*\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)z^*\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)=0$
Let $z^*=z$ in above equation to obtain
$z\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)+\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)z\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)=0$ ………………… (3)
Multiplying equation (3) from the right by $p\xi_1(\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_n)$ to get
$z\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)p\xi_1(\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_n)+$
$\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)z\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)p\xi_1(\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_n)=0$,for all $\upsilon_1,\upsilon_2,\dots,\upsilon_n,\gamma_1,\gamma_2,\dots,\gamma_n,\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_n\in\mathcal{R}$ (4)
Equation (3) gives us
$z\xi_1(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\xi_2(\gamma_1,\gamma_2,\dots,\gamma_n)=-\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)z\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)$
Since $\mathcal{R}$ is a 2-torsion, then equation (4) becomes
$\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\mathcal{R}\xi_1(\gamma_1,\gamma_2,\dots,\gamma_n)\mathcal{R}\xi_1(\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_n)=0$
By $*$-primeness of $\mathcal{R}$, either
$\xi_2(\upsilon_1,\upsilon_2,\dots,\upsilon_n)=0$ or $\xi_1(\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_n)=0$, for all $\upsilon_1,\upsilon_2,\dots,\upsilon_n,\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_n\in\mathcal{R}$. That is, either $\xi_1=0$ or $\xi_2=0$.

Theorem (3.3): Let $\mathcal{R}$ be a semiprime $*$-ring admitting a non-zero skew left $*$-n-derivation ξ.Then $\xi(\mathcal{R},\mathcal{R},\dots,\mathcal{R})\subseteq\mathcal{Z}$.

Proof:

Replacing υ_1 by $\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\mathfrak{r}$ in equation (3) of theorem (3.1), to get
$z\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\mathfrak{r}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\mathfrak{r}z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$
That is,
$0=z\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\mathfrak{r}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)-\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\mathfrak{r}z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$
$=[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\mathfrak{r}]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$
$=[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathfrak{r}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)+\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)[z,\mathfrak{r}]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$
By using equation (4) of theorem (3.1) in the last equation to get
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathfrak{r}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$ ... (1)
Multiply equation (1) from the right by z, to get
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathfrak{r}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)z=0$ … (2)
Replacing $\mathfrak{r}=\mathfrak{r}z$ in equation (1) where $\mathfrak{r},z\in\mathcal{R}$, to get
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathfrak{r}z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)z=0$ … (3)
Comparing equation (2) and (3) to obtain
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathfrak{r}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)z=[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathfrak{r}z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$
This means that
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathfrak{r}[z,\xi(\gamma,\upsilon_2,\dots,\upsilon_n)]=0$, for all $z,\gamma,\mathfrak{r},\upsilon_1,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$. Now put $\gamma=\upsilon_1$ to get
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]\mathcal{R}[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]=0$

This gives
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]^*\mathcal{R}[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]^*$=0
By the $*$-semiprime of $\mathcal{R}$, yields that
$[z,\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)]$=0 and this means that, $\xi(\mathcal{R},\mathcal{R},\dots,\mathcal{R})\subseteq\mathcal{Z}$.
Recall that an additive mapping $\xi:\mathcal{R}\to\mathcal{R}$ is called a left multiplier if $\xi(\upsilon\gamma)=\xi(\upsilon)\gamma$ [12].

Theorem (3.4): Let $\mathcal{R}$ be a semiprime $*$-ring and ξ be a skew left $*$-n-derivation such that $\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\gamma_1=\upsilon_1\xi(\gamma_1,\gamma_2,\dots,\gamma_n)$ for all $\upsilon_1,\upsilon_2,\dots,\upsilon_n,\gamma_1,\gamma_2,\dots,\gamma_n\in\mathcal{R}$. Then ξ is a left multiplier.

Proof:

By hypothesis,
$\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\gamma_1=\upsilon_1\xi(\gamma_1,\gamma_2,\dots,\gamma_n)$, for all $\upsilon_1,\upsilon_2,\dots,\upsilon_n,\gamma_1,\gamma_2,\dots,\gamma_n\in\mathcal{R}$. …………………….. (1)
Replacing γ_1 by $\gamma_1 z$ in equation (1) and since ξ is a skew left $*$-n-derivation, then
$\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\gamma_1 z=\upsilon_1 z^*\xi(\gamma_1,\gamma_2,\dots,\gamma_n)+\upsilon_1\gamma_1\xi(z,\gamma_2,\dots,\gamma_n)$
Again by using equation (1) in the last equation
$\upsilon_1\xi(\gamma_1,\gamma_2,\dots,\gamma_n)z=\upsilon_1 z^*\xi(\gamma_1,\gamma_2,\dots,\gamma_n)+\upsilon_1\gamma_1\xi(z,\gamma_2,\dots,\gamma_n)$
Using z instead of z^* to obtain
$\upsilon_1\xi(\gamma_1,\gamma_2,\dots,\gamma_n)z=\upsilon_1 z\xi(\gamma_1,\gamma_2,\dots,\gamma_n)+\upsilon_1\gamma_1\xi(z,\gamma_2,\dots,\gamma_n)$
By applying equation (1) on left side of last equation to get
$\upsilon_1\gamma_1\xi(z,\gamma_2,\dots,\gamma_n)=\upsilon_1 z\xi(\gamma_1,\gamma_2,\dots,\gamma_n)+\upsilon_1\gamma_1\xi(z,\gamma_2,\dots,\gamma_n)$ and this mean that
$\upsilon_1 z\xi(\gamma_1,\gamma_2,\dots,\gamma_n)$=0, for all $\upsilon_1,z,\gamma_1,\gamma_2,\dots,\gamma_n\in\mathcal{R}$ …………………..… (2)
Replacing υ_1 by $\xi(\gamma_1,\gamma_2,\dots,\gamma_n)$ in equation (2) then
$\xi(\gamma_1,\gamma_2,\dots,\gamma_n)z\xi(\gamma_1,\gamma_2,\dots,\gamma_n)$=0, for all $\gamma_1,\gamma_2,\dots,\gamma_n,z\in\mathcal{R}$ and this gives
$\xi(\gamma_1,\gamma_2,\dots,\gamma_n)\,\mathcal{R}\,\xi(\gamma_1,\gamma_2,\dots,\gamma_n)$=0, for all $\gamma_1,\gamma_2,\dots,\gamma_n,\in\mathcal{R}$
This implies that
$\xi(\gamma_1,\gamma_2,\dots,\gamma_n)^*\mathcal{R}\,\xi(\gamma_1,\gamma_2,\dots,\gamma_n)^*$=0
Using $*$-semiprimeness leads to ξ is a left multiplier.

Theorem (3.5): Let $\mathcal{R}$ be a semiprime $*$-ring and If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}^n$, then ξ a maps from $\mathcal{R}^n$ to $\mathcal{Z}(\mathcal{R})$.

Proof:

By hypothesis $\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$ ……………………..… (1)
Let $\gamma=\gamma z$ in equation (1), to get
$\xi(\upsilon\gamma z,\upsilon_2,\dots,\upsilon_n)=(\gamma z)^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma z,\upsilon_2,\dots,\upsilon_n)$
$=z^*\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+z^*\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)+\upsilon\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)$, for all $\upsilon,\gamma,z,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$. ………… (2)
On the other hand
$\xi(\upsilon\gamma z,\upsilon_2,\dots,\upsilon_n)=z^*\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)+\upsilon\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)$
$=z^*\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+z^*\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)+\upsilon\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)$ ………………………….. (3)
Comparing equations (2) and (3) to have
$[\upsilon,z^*]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0
Replacing $z^*=z$ in last equation to obtain
$[\upsilon,z]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0, for all $\upsilon,\gamma,z,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ ………………………… … (4)
Replacing $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon$ instead of υ in equation (4) and using it then
$[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 ………………………… … (5)
Let $\upsilon=\upsilon z$ in equation (5). Then
$[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]\upsilon z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 …………………………… (6)
Now, multiplying equation (5) from the right side by z
$[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)z$=0 ……………………………. (7)
Comparing equations (6) and (7) to get
$[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]\upsilon[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]$=0, hence
$[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]\mathcal{R}[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]$=0. Since $\mathcal{R}$ is semiprime $*$-ring $[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),z]$=0, for all $\gamma,z,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$. Hence ξ is a map $\mathcal{R}^n$ into $\mathcal{Z}(\mathcal{R})$.

Theorem (3.6): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}^n$ such that $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\neq\upsilon$ and $\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$ for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$, then ξ =0.

Proof:

By hypothesis
$\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\,\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$ ……………… (1)
Let $\upsilon=\upsilon z$ in equation (1) to get
$\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\xi(z,\upsilon_2,\dots,\upsilon_n)+\upsilon z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\xi(z,\upsilon_2,\dots,\upsilon_n)\,\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=$
$\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\,\xi(z\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\{\gamma^*\xi(z,\upsilon_2,\dots,\upsilon_n)+z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\}$
This implies that
$[\gamma^*,\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)]\xi(z,\upsilon_2,\dots,\upsilon_n)+(\upsilon-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$
By Theorem (3.5) the above equation becomes
$(\upsilon-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$, for all $\gamma,z,\upsilon,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$
That is, $(\upsilon-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))\mathcal{R}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$. Since $\mathcal{R}$ is prime $*$-ring then either $(\upsilon-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))=0$ or $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$. But $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\neq\upsilon$, then $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$ for all $\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.7): Let $\mathcal{R}$ be a prime $*$-ring and If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}^n$ such that $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\neq\upsilon^*$and $\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\,\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$, then $\xi=0$.

Proof:
By hypothesis
$\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ … (1)
Replacing $\gamma=\upsilon\gamma$ in equation (1) to get
$\gamma^*\upsilon^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=\{\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\}\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$
This implies that
$\gamma^*\upsilon^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)-\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$
$\gamma^*(\upsilon^*-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$
Applying Theorem (3.5) to get
$(\upsilon^*-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$
Hence, $(\upsilon^*-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))\mathcal{R}\,\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$. Since $\mathcal{R}$ is prime $*$-ring then $(\upsilon^*-\xi(\upsilon,\upsilon_2,\dots,\upsilon_n))=0$ or $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$. But $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\neq\upsilon^*$, then $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$ for all $\upsilon,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.8): Let $\mathcal{R}$ be a prime $*$-ring and $a\in\mathcal{R}$. If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}^n$ and $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),a]=0$, then either $\xi(a)=0$ or $a\in\mathcal{Z}(\mathcal{R})$.

Proof:
By hypothesis
$O=[\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n),a]=0$
$=[\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n),a]=0$, for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ ……………………….…. (1)
Hence $[\gamma^*,a]\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+[\upsilon,a]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$ ………………………..…. (2)
Replacing υ by a and γ^* by γ in equation (2) to get
$[\gamma,a]\xi(a,\upsilon_2,\dots,\upsilon_n)=0$ …………………………… (3)
Replacing $\gamma=\upsilon\gamma$ in equation (3) and using it to get
$[\upsilon,a]\gamma\xi(a,\upsilon_2,\dots,\upsilon_n)=0$, and this implies that $[\upsilon,a]\mathcal{R}\xi(a,\upsilon_2,\dots,\upsilon_n)=0$, since $\mathcal{R}$ is prime then either $a\in\mathcal{Z}(\mathcal{R})$ or $\xi(a,\upsilon_2,\dots,\upsilon_n)=0$ for all $a,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.9): Let $\mathcal{R}$ be a semiprime $*$-ring. If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}$ then $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]=0$ for all $\upsilon,z,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Proof:
By hypothesis $\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(y,\upsilon_2,\dots,\upsilon_n)$ ……………………..…. (1)
Substituting $\gamma=\gamma z$ in equation (1) we get
$\xi(\upsilon\gamma z,\upsilon_2,\dots,\upsilon_n)=(\gamma z)^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon\xi(\gamma z,\upsilon_2,\dots,\upsilon_n)$
$=z^*\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+\upsilon z^*\xi(\gamma,\upsilon_2,\dots,\upsilon_n)+\upsilon\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)$ ………………………… (2)
Also we have
$\xi(\upsilon\gamma z,\upsilon_2,\dots,\upsilon_n)=z^*\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)+\upsilon\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)$
$=z^*\gamma^*\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)+z^*\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)+\upsilon\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)$ ………………………..…. (3)
Comparing equations (2) and (3) to get $[\upsilon,z^*]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$
Let $z^*=z$ in above equation to get
$[\upsilon,z]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)=0$ ………………………… … (4)

Replacing υ by $\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)\upsilon$ in equation (4) and using it to get
$[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]\upsilon\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ ……………………………… (5)
Let $\upsilon=\upsilon z$ in equations (5) then
$[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]\upsilon z\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ ……………………………… (6)
Now, multiplying equation (5) from the right side by z we have
$[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)z=0$ ……………………………… (7)
Comparing equations (6) and (7) to get $[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]\upsilon[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]=0$
Hence $[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]\mathcal{R}[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]=0$. Since $\mathcal{R}$ is semiprime $*$-ring then $[\xi(\gamma,\upsilon_2,\ldots,\upsilon_n),z]=0$ for all $\gamma,z,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$.

Theorem (3.10): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}^n$ such that $\xi([\upsilon,\gamma],\upsilon_2,\ldots,\upsilon_n)=0$ for all $\upsilon,\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$ then $\xi=0$ or $\mathcal{R}$ is commutative.

Proof:

By hypothesis $\xi([\upsilon,\gamma],\upsilon_2,\ldots,\upsilon_n)=0$ ……………………………… (1)
Let $\upsilon=\upsilon\gamma$ in equation (1) and using it to get
$[\upsilon,\gamma]\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ ……………………………… (2)
Replacing $\upsilon=\upsilon z$ in equation (2) then
$[\upsilon,\gamma]z\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)+\upsilon[z,\gamma]\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$
By using equation (2) the last equation to get
$[\upsilon,\gamma]z\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ and this implies that $[\upsilon,\gamma]\mathcal{R}\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$. Since $\mathcal{R}$ is prime then $[\upsilon,\gamma]=0$ and this means that $\mathcal{R}$ is commutative, or $\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ for all $\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$.

Theorem (3.11): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}^n$ such that $[\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n),\gamma]=[\upsilon,\gamma]$ for all $\upsilon,\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$ then $\xi=0$ or $\mathcal{R}$ is commutative.

Proof:

By hypothesis $[\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n),\gamma]=[\upsilon,\gamma]$ ……………………… (1)
Let $\upsilon=\upsilon z$ in equation (1) to get
$[\xi(\upsilon z,\upsilon_2,\ldots,\upsilon_n),\gamma]=[\upsilon z,\gamma]$
$[z^*,\gamma]\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n)+z^*[\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n),\gamma]+[\upsilon,\gamma]\xi(z,\upsilon_2,\ldots,\upsilon_n)+\upsilon[\xi(z,\upsilon_2,\ldots,\upsilon_n),\gamma]=[\upsilon,\gamma]z+\upsilon[z,\gamma]$
By using equation (1), the last equation can be reduced to
$[z^*,\gamma]\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n)+z^*[\upsilon,\gamma]+[\upsilon,\gamma]\xi(z,\upsilon_2,\ldots,\upsilon_n)+\upsilon[z,\gamma]=[\upsilon,\gamma]z+\upsilon[z,\gamma]$ ……………………… (2)
Replacing $\upsilon=\gamma$ and $z^*=z$ in equation (2) to get
$[z,\gamma]\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ ……………………………..(3)
Replacing z by $z\mathfrak{r}$ in equation (3) and using it to get
$[z,\gamma]\mathfrak{r}\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ for all $\gamma,\mathfrak{r},z,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$.
This implies that $[z,\gamma]\mathcal{R}\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$. Since $\mathcal{R}$ is prime then $\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ for all $\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$, or $\mathcal{R}$ is commutative.

Theorem (3.12): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}^n$ such that $\xi((\upsilon\circ\gamma),\upsilon_2,\ldots,\upsilon_n)=0$ for all $\upsilon,\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$ then $\xi=0$ or $\mathcal{R}$ is commutative.

Proof:

By hypothesis $\xi((\upsilon\circ\gamma),\upsilon_2,\ldots,\upsilon_n)=0$ ………......................................(1)
Let $\upsilon=\upsilon\gamma$ in equation (1) and using it to get
$(\upsilon\circ\gamma)\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ …………..(2)
Replacing $\upsilon=s\upsilon$ in equation (2) then $(s\circ\gamma)\upsilon\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$
Hence $(s\circ\gamma)\mathcal{R}\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$. Since $\mathcal{R}$ is prime $*$-ring then $(s\circ\gamma)=0$, replace $s=sz$ we get $s[z,\gamma]=0$. Now let $s=\upsilon s$ then we have $\upsilon s[z,\gamma]=0$, that $\upsilon\mathcal{R}[z,\gamma]=0$ for $0\neq\upsilon\in\mathcal{R}$ and since $\mathcal{R}$ is prime $*$-ring then $\mathcal{R}$ is commutative, or $\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ for all $\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$.

Theorem (3.13): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a skew left $*$-n-derivation ξ of $\mathcal{R}$ such that $\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n)\circ\gamma=0$ for all $\upsilon,\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$ then $\xi=0$ or $\mathcal{R}$ is commutative.

Proof:

By hypothesis $\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n)\circ\gamma=0$ ……………………………… ..… (1)
Replacing $\upsilon=\upsilon z$ in equation (1) and using it to get
$[\gamma,z^*]\xi(\upsilon,\upsilon_2,\ldots,\upsilon_n)-[\upsilon,\gamma]\xi(z,\upsilon_2,\ldots,\upsilon_n)=0$ ……………………………………..(2)
Let $\upsilon=\gamma$ and $z^*=z$ in equation (2) then
$[\gamma,z]\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)=0$ ……………………………………… (3)
Replacing $z=\upsilon z$ in equation (3) and using it to get

$[\gamma, \upsilon]z\xi(\gamma, \upsilon_2, \dots, \upsilon_n)$=0 for all $\upsilon, \gamma, z, \upsilon_2, \dots, \upsilon_n \in \mathcal{R}$
This implies that $[\gamma, \upsilon]\mathcal{R}\xi(\gamma, \upsilon_2, \dots, \upsilon_n)$=0, since $\mathcal{R}$ is prime then $\xi(\gamma, \upsilon_2, \dots, \upsilon_n)$=0 for all $\gamma, \upsilon_2, \dots, \upsilon_n \in \mathcal{R}$, or $\mathcal{R}$ is commutative.

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