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Influence of the Induced Magnetic and Rotation on Mixed Convection Heat Transfer for the Peristaltic Transport of Bingham plastic Fluid in an Asymmetric Channel

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Abstract
In this paper, the peristaltic flow under the impact of heat transfer, rotation and induced magnetic field of a two dimensional for the Bingham plastic fluid is discussed. The coupling among of momentum with rotational, energy and the induced magnetic field equations are achieved by the perturbation approximation method and the mathematica software to solve equations that are nonlinear partial differential equations. The fluid moves in an asymmetric channel, and assumption the long wavelength and low Reynolds number, approximation are used for deriving a solution of the flow. Expression of the axial velocity, temperature, pressure gradient, induced magnetic field, magnetic force, current density are developed the effect of rotation, a stress on the lower and upper channel and stream function. The quantities flow have been tested for variant parameters. The impact of the Bingham, Brinkman, Hartman and Grashof numbers are also tested for different values to indicate the effect on the move of flow fluid. The applications can be seen through many graphics.

Keywords: induced magnetic field, magnetic force, peristaltic transport, heat transfer, current density, rotation, velocity, stream function, stress.

تأثير المغناطيسية المستحثة والدوران على نقل الحرارة بالحمل المختلط للنقل التمعجي لسائل بلاستيك بينغهام في قناة غير متماثلة

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الخلاصة
في هذا البحث ، تمت مناقشة التدفق التمعجي تحت تأثير انتقال الحرارة والدوران والمجال المغناطيسي المستحث ثنائي الأبعاد لسائل بلاستيك بينغهام. يتم تحقيق اقتران الزخم مع معادلات الدوران والطاقة والمجال المغناطيسي المستحث بواسطة طريقة تقريب الاضطراب و برنامج ماثيمتكا لحل المعادلات التفاضلية الجزئية الغير خطية. يتحرك المائع في قناة غير متماثلة بافتراض الطول الموجي الطويل وعدد رينولدز المنخفض ، يتم استخدام التقريب لاشتقاق حلول التدفق. التعبير عن السرعة المحورية ودرجة الحرارة وتدرج الضغط والمجال

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المغناطيسي المستحث والقوة المغناطيسية وتطور كثافة التيار وتأثير الدوران والضغط على القناة السفلية والعلوية ووظيفة التدفق. تم اختبار تدفق الكميات لمعلمات متغيرة. تأثير اختبار أرقام Bingham و Brinkmanو Hartman و Grashof لقيم مختلفة ، لتبيان التأثير على حركة مائع التدفق. يمكن رؤية التطبيقات من خلال العديد من الرسومات.

1- Introduction

In the physiological and engineering applications, the peristaltic motion appears in a wide area such as urine transport in ureter, and swallowing of food through esophagus, etc. The wide applications of the peristaltic transport have been pulling the attention of researchers after the principle work of Latham [1]. Peristaltic pumping with long wavelength at low Reynolds number is used in [2]. Faraday and Richie are the first researchers that they have put the principle of magnetic fluid dynamic and the effects through their known experiment [3]. The influence of an induced magnetic field on peristaltic flow with the heat and mass transfer are discussed in [4-6]. Effects of rotation and initial stress on peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field are studied in [7,11,12,13,14]. The influence of induced magnetic on the peristaltic flow of nanofluid is also suited in [8,15-18]. In [16] , authors investigated the effect of Magnetic Field on Peristaltic of Bingham Plastic Fluid in a Symmetric Channel. In [17], authors mixed convective heat transfer analysis for the peristaltic transport of viscoplastic fluid. Effect of heat sink/source on peristaltic flow of Jeffrey fluid through a symmetric channel is investigated in[9]. In [10], authors studied Momentum and heat transfer behavior of Jeffrey, Maxwell and Oldroyd-B nanofluids past a stretching surface with non-uniform heat source/sink . Effects of velocity slip model and induced magnetic field on peristaltic transport of non-Newtonian fluid in the presence of double-diffusivity convection in nan fluid are discussed in [19]. The influence of MHD on mixed convective heat and mass transfer analysis for the peristaltic transport of viscoplastic fluid with porous medium in tapered channel is considered in [20]. Impacts of Porous Medium on Unsteady Helical Flows of Generalized Oldroyd-B Fluid with Two Infinite Coaxial Circular Cylinders is shown in [21-23]. Many researchers studied the combined effects of nonuniform temperature gradients and heat source and magneto convection in a porous fluid system for more details see [24,25,26].

In this paper, the impact the rotation, energy and induced magnetic field with the magnetic force and current density on the peristaltic flow are discussed. Different values of parameters when effect of the fluid move through two dimensional channel are also tested. We study Electrically conduction to fluid with existence of a magnetic field with energy and rotation. Finally , velocity, temperature, gradient pressure, stream function, magnetic force, stress at lower and upper channel, induced magnetic and current density are illustrated by many plots and graphics.

2- Mathematical Formulation

Assume that the peristaltic transport of Bingham plastic of an incompressible and electrically conduction fluid through two dimensional channel non-uniform of width ($a_1 + a_2$). The upper wall has T_0 temperature while the lower wall retains at temperature T_1, see Figure (1). The flow is created by sinusoidal wave trains reproduction with speed constant c along channel walls. We choose the two dimension system, X is along centerline of channel while Y is transverse to it. Uniform constant magnetic field on the external transverse H_0, H(h_x(X,Y, t), $H_0 + h_y$(X,Y, t),0) is the induced magnetic field and H^+(h_x(X,Y,t), $H_0 +$ h_y(X,Y, t),0) total magnetic field are taken into account.

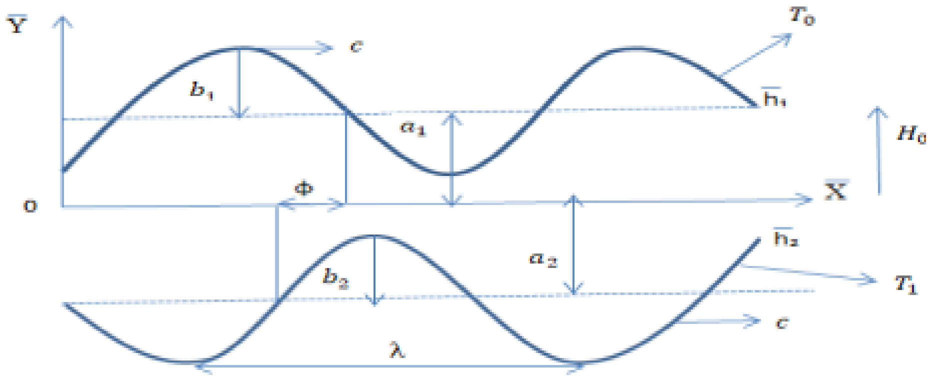


Figure--Geometric of problem

The geometry for wall surface is defined:

$$\bar{h}_1 = a_1 + b_1 Cos[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})] \quad \text{Upper wall} \qquad \ldots(1)$$

$$\bar{h}_2 = -a_2 - b_2 Cos[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi] \quad \text{Lower wall} \qquad \ldots(2)$$

Where b_1 and b_2 are amplitudes wave, c is velocity, λ is the wavelength, t is the time, and X is the direction of wave reproduction. The ϕ is the phase difference lies $0 \le \phi \le \pi$, when ϕ=0 is the symmetric channel with wave out of phase while ϕ =π waves in phase. When a_1, a_2, b_1, b_2 and ϕ satisfy the condition

$$b_1^{\,2} + b_1^{\,2} + 2\, b_1 b_2 \cos\phi \le (a_1 + a_2)^2 \qquad \ldots(3)$$

3- Fundamental Equations Governing

We denote that the velocity component $\bar{U}$ along the $\bar{X}$ and $\bar{V}$ along $\bar{Y}$ the velocity field **V**=$[\bar{U}(\bar{X},\bar{Y},\bar{t}), \bar{V}(\bar{X},\bar{Y},\bar{t}), 0]$ in the fixed frame. The basic equations which are govern the present flow:

(i) Maxwell's equations

$$\nabla.\mathbf{E} = 0, \quad \nabla.\boldsymbol{H} = 0 \qquad \ldots(4)$$

$$\nabla\times\mathbf{E} = -\mu_e \frac{\partial \mathbf{H}}{\partial t}, \quad \nabla\times\mathbf{H} = \mathbf{J} \qquad \ldots(5)$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mu_e(\mathbf{V}\times\mathbf{H})) \qquad \ldots(6)$$

Where H, E, J, σ and μ_e indicate the magnetic field, electric field, current density, electrical conductivity and magnetic permeability, respectively.

(ii) Continuity equation

$$\nabla.\mathbf{V} = 0 \qquad \ldots(7)$$

(iii) Equation of Motion

$$\rho\frac{d\mathbf{V}}{dt} - \rho\Omega\left(\Omega\mathbf{V} + 2\frac{d\mathbf{V}}{dt}\right) = div\,\mathbf{T} + \mu_e(\nabla\times H^+)\times\mathrm{H}^+ + \rho g \sigma_T (T - T_0)$$

$$= div\,\mathbf{T} + \mu_e(\nabla\times H^+)\mathrm{H}^+ - \frac{\nabla {H^+}^2}{2}) + \rho g \sigma_T (T - T_0) \qquad \ldots(8)$$

(iv) Energy equation

$$\rho C_p \frac{d\mathbf{T}}{dt} = k_t\, \nabla^2\mathbf{T} + \sigma_{cs}(\nabla\mathbf{V}) \qquad \ldots(9)$$

Combine equations (4-6) we obtain the:

(v) Induction equation

$$\frac{d\mathbf{H}^+}{dt} = \nabla\times(\mathrm{V}\times\mathrm{H}^+) + \frac{1}{\mathfrak{z}}\nabla^2\mathrm{H}^+ \qquad \ldots(10)$$

Where $\mathfrak{z} = \sigma\,\mu_e$ is magnetic diffusivity, C_p is specific heat, T is temperature, k_t is thermal conductivity, $\nabla = \left[\frac{\partial}{\partial\bar{X}}, \frac{\partial}{\partial\bar{Y}}\right]$, ρ is the density, Ω is the rotation, g is the acceleration due to gravity, σ_T is the coefficient of thermal expansion, and σ_{cs} is the Cauchy stress tensor which is defined for the Bingham plastic fluid by [22]:

$$\sigma_{cs} = -\overline{\mathrm{P}}\mathrm{I} + \overline{\mathrm{S}} \quad \ldots\ldots(11)$$

Where, $\bar{S} = 2\mu\tau + 2\tau_0\hat{\tau}$ ……(12)

In equation (12) the product stress is τ_0, while the tensor is $\hat{\tau}$,and rate of deformation tensor τ is defined by:

$$\hat{\tau} = \tau/\sqrt{2tra\,\tau^2}, \quad \tau = \frac{1}{2}(\nabla\overline{\mathrm{V}} + (\nabla\overline{\mathrm{V}})^T) \quad \ldots\ldots(13)$$

The components of the extra stress tensor $\bar{S}$ are $\bar{S}_{\bar{X}\bar{X}}$, $\bar{S}_{\bar{X}\bar{Y}}$,and $\bar{S}_{\bar{Y}\bar{Y}}$.We introduce the transformations to wave frame , this is given by :

$$\bar{x} = \bar{X} - c\bar{t}, \bar{y} = \bar{Y}, \bar{u} = \bar{U} - c, \bar{v} = \bar{V},\ \bar{p}(\bar{x},\bar{y}) = \bar{P}(\bar{X},\bar{Y},\bar{t}) \quad \ldots\ldots(14)$$

Where $\bar{u}$,and $\overline{v\ are}$ the components of velocity. $\bar{p}$ is the pressure in frame wave, then the equations(7-13) with help of (14) become to:

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \qquad \text{Continuity equation} \quad \ldots\ldots(15)$$

$$\rho\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) - \rho\Omega\left(\Omega\bar{u} + 2\frac{\partial\bar{v}}{\partial\bar{t}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} + \frac{\partial\bar{S}_{\bar{x}\bar{x}}}{\partial\bar{x}} + \frac{\partial\bar{S}_{\bar{x}\bar{y}}}{\partial\bar{y}} + \mu_e\left(\bar{h}_{\bar{x}}\frac{\partial\bar{h}_{\bar{x}}}{\partial\bar{x}} + \bar{h}_{\bar{y}}\frac{\partial\bar{h}_{\bar{x}}}{\partial\bar{y}} + H_0\frac{\partial\bar{h}_{\bar{x}}}{\partial\bar{y}}\right) - \frac{\mu_e}{2}\left(\frac{\partial {H^+}^2}{\partial\bar{x}}\right) + \rho g\sigma_T(T - T_0) \quad \text{Motion equation on x-axes} \quad \ldots\ldots(16)$$

$$\rho\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) - \rho\Omega\left(\Omega\bar{v} - 2\frac{\partial\bar{u}}{\partial\bar{t}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}} + \frac{\partial\bar{S}_{\bar{y}\bar{y}}}{\partial\bar{y}} + \frac{\partial\bar{S}_{\bar{x}\bar{y}}}{\partial\bar{x}} + \mu_e\left(\bar{h}_{\bar{x}}\frac{\partial\bar{h}_{\bar{y}}}{\partial\bar{x}} + \bar{h}_{\bar{y}}\frac{\partial\bar{h}_{\bar{y}}}{\partial\bar{y}} + H_0\frac{\partial\bar{h}_{\bar{y}}}{\partial\bar{y}}\right) - \frac{\mu_e}{2}\left(\frac{\partial {H^+}^2}{\partial\bar{y}}\right) \quad \text{Motion equation on y-axes} \quad \ldots\ldots(17)$$

$$\rho C_p\left(\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial T}{\partial\bar{y}}\right) = k_t\left(\frac{\partial^2 T}{\partial\bar{x}^2} + \frac{\partial^2 T}{\partial\bar{y}^2}\right) + \bar{S}_{\bar{x}\bar{x}}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{S}_{\bar{x}\bar{y}}\left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\right) + \bar{S}_{\bar{y}\bar{y}}\frac{\partial\bar{v}}{\partial\bar{y}} \quad \ldots\ldots(18)$$

Equation (18) represents the energy equation. The stress becomes as follows :

$$\bar{S}_{\bar{x}\bar{x}} = 2\mu\tau_{\bar{x}\bar{x}} + \frac{2\tau_0\tau_{\bar{x}\bar{x}}}{\sqrt{2\,tra(\tau^2)}}, \bar{S}_{\bar{x}\bar{y}} = 2\mu\tau_{\bar{x}\bar{y}} + \frac{2\tau_0\tau_{\bar{x}\bar{y}}}{\sqrt{2\,tra(\tau^2)}}, \bar{S}_{\bar{y}\bar{y}} = 2\mu\tau_{\bar{y}\bar{y}} + \frac{2\tau_0\tau_{\bar{y}\bar{y}}}{\sqrt{2\,tra(\tau^2)}} \quad \ldots\ldots(19)$$

Where $(\bar{u},\bar{v})$ are the components of velocity along the $(\bar{x},\bar{y})$ directions, respectively.
We normalize the equations (15-19) by using the following dimensionless variables:

$$x = \frac{\bar{x}}{\lambda}, y = \frac{\bar{y}}{a_1}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c}, \delta = \frac{\lambda}{a_1}, d = \frac{a_2}{a_1}, p = \frac{{a_1}^2\bar{p}}{\mu c\lambda}, t = \frac{c\bar{t}}{\lambda}, h_1 = \frac{\bar{h}_1}{a_1}, h_2 = \frac{\bar{h}_2}{a_1}, \alpha = \frac{b_1}{a_1}, \beta = \frac{b_2}{a_1}, R_e = \frac{c\rho a_1}{\mu}, P_r = \frac{\mu C_\rho}{k_t}, B_n = \frac{a_1\tau_0}{\mu c}, E_c = \frac{c^2}{C_\rho T_0}, B_r = P_r E_c, G_r = \frac{\rho g\sigma_T {a_1}^2}{\mu c}, \theta = \frac{T - T_0}{T_1 - T_0}, S = \frac{a_1\bar{S}}{\mu c}, S_1 = \frac{H_0}{c}\sqrt{\frac{\mu_e}{\rho}}, M^2 = R_e {S_1}^2 R_m, R_m = \sigma\mu_e a_1 c, p_m = p + \frac{1}{2}R_e\delta\frac{\mu_e(H^+)^2}{\rho c^2}, \varphi = \frac{\bar{\varphi}}{H_0 a_1}, \bar{h}_{\bar{x}} = \bar{\varphi}_{\bar{y}}, \bar{h}_{\bar{y}} = \bar{\varphi}_{\bar{x}}, E = \frac{-\bar{E}}{cH_0\mu_e} \quad \ldots\ldots(20)$$

$$h_1 = 1 + \alpha\cos(2\pi x),\ h_2 = -d - \beta\cos(2\pi x + \phi) \quad \ldots\ldots(21)$$

$$u = \frac{\partial\psi}{\partial y},\ v = -\delta\frac{\partial\psi}{\partial x},\ h_x = \frac{\partial\varphi}{\partial y},\ h_y = -\delta\frac{\partial\varphi}{\partial x} \quad \ldots\ldots(22)$$

Where δ, α, β, R_e, P_r, B_n, E_c, B_r, G_r, θ, S_1, M, R_m, p_m, φ, T_0 ,T_1, ψ and E are as follows , the wave number, upper amplitude wave, lower amplitude wave, Reynolds number, Prandtl number, Bingham number, Eckert number, Brinkman number, temperature, Grashof number, Stormmer number, Hartman number, magnetic Reynolds number, total pressure, magnetic force, temperature at h_1, temperature at h_2, stream function and electric field strength, respectively. If we substitute equations (20,22) into equations (15-19) we obtain that:
The continuity equation (15) satisfies identically whereas other equations take the following formulas:

$$R_e\delta\left(\psi_y\psi_{xy} - \psi_x\psi_{yy}\right) - \frac{\Omega^2\rho {a_1}^2}{\mu}\psi_y = -\frac{\partial p_m}{\partial x} + \delta\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} + R_e {S_1}^2\varphi_{yy} + \delta R_e {S_1}^2\left(\varphi_y\varphi_{xy} - \varphi_x\varphi_{yy}\right) + G_r\theta \quad \ldots\ldots(23)$$

$-R_e\delta^2\left(\psi_y\psi_{xx}-\psi_x\psi_{yx}\right)=-\frac{\partial p_m}{\partial y}-\delta^2\frac{\Omega^2\rho a_1{}^2}{\mu}\psi_x+\delta^2\frac{\partial S_{xx}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y}-\delta^3 R_e S_1{}^2\left(\varphi_y\varphi_{xx}-\varphi_x\varphi_{yx}\right)-R_e\delta^2 S_1{}^2\varphi_{xy}$
……………………………………………….(24)

$\delta P_r R_e\left(\psi_y\theta_x-\psi_x\theta_y\right)=\delta^2\theta_{xx}+\theta_{yy}+B_r[\delta\left(S_{xx}-S_{yy}\right)\psi_{xy}+(\psi_{yy}-\delta^2\psi_{xx})S_{xy}]$
…(25)

E=$E=\psi_y-\delta\left(\psi_y\varphi_x-\psi_x\varphi_y\right)+\frac{1}{R_m}(\delta^2\varphi_{xx}+\varphi_{yy})$
………………………………(26)

$S_{xx}=2\delta\psi_{xy}+\frac{2\delta B_n\psi_{xy}}{\sqrt{2\delta^2(\psi_{xy})^2+2\delta^2(\psi_{xy})^2+(\psi_{yy}-\delta^2\psi_{xx})^2}}$
………………………………(27)

$S_{xy}=(\psi_{yy}-\delta^2\psi_{xx})+\frac{\mathbf{B}_n(\psi_{yy}-\delta^2\psi_{xx})}{\sqrt{2\delta^2(\psi_{xy})^2+2\delta^2(\psi_{xy})^2+(\psi_{yy}-\delta^2\psi_{xx})^2}}$
……………………………(28)

$S_{yy}=-2\delta\psi_{xy}-\frac{2\delta B_n\psi_{xy}}{\sqrt{2\delta^2(\psi_{xy})^2+2\delta^2(\psi_{xy})^2+(\psi_{yy}-\delta^2\psi_{xx})^2}}$
……………………………..(29)

Under the assumption of long wavelength, low Reynolds number and $\delta\ll 1$ the equations (23-29) become as follows:

$\frac{\partial p}{\partial x}=\frac{\Omega^2 a_1{}^2\rho}{\mu}\psi_y+\frac{\partial S_{xy}}{\partial y}+R_e S_1{}^2\varphi_{yy}+G_r\theta$ ……………..………………(30)

$\frac{\partial p}{\partial y}=0$ ………………………………(31)

$\frac{\partial^2\theta}{\partial y^2}+B_r\psi_{yy}S_{xy}=0$ ...……………………………(32)

$E=\psi_y+\frac{1}{R_m}\varphi_{yy}$ …………………………(33)

$S_{xx}=0,\quad S_{xy}=\psi_{yy}+B_n$ and $S_{yy}=0$ …………………………(34)

Elimination pressure between (30) and (31) by derivative with respect to y we obtain:

$\frac{\Omega^2 a_1{}^2\rho}{\mu}\psi_{yy}+\frac{\partial^2 S_{xy}}{\partial y^2}+R_e S_1{}^2\varphi_{yyy}+G_r\theta_y=0$ …………………………(35)

$\frac{\partial^2\theta}{\partial y^2}+B_r\psi_{yy}S_{xy}=0$ …………………………(36)

$\frac{\partial^2\varphi}{\partial y^2}=R_m\left(E-\psi_y\right)==\gg\frac{\partial^3\varphi}{\partial y^3}=-R_m\psi_{yy}$ …………………………(37)

Now substitute the equation (34) into equations (35,36) and equation (37) into equation (35) then we get :

$\frac{\partial^4\psi}{\partial y^4}+\frac{\Omega^2 a_1{}^2\rho}{\mu}\psi_{yy}-M^2\psi_{yy}+G_r\theta_y=0$ ………………………………(38)

$\frac{\partial^2\theta}{\partial y^2}+B_r(B_n\psi_{yy}+\left(\psi_{yy}\right)^2)=0$ ………………………………(39)

$\frac{\partial^2\varphi}{\partial y^2}=R_m\left(E-\psi_y\right)$ ………………………………(40)

The boundary conditions in wave frame and dimensionless volume flow rate are [21]:

$\varphi=0$, $\psi=\frac{q}{2}$, $\frac{\partial\psi}{\partial y}=-1$, $\theta=0$, at y= h_1 ……………...……………..(41)

$\varphi=0$, $\psi=-\frac{q}{2}$, $\frac{\partial\psi}{\partial y}=-1$, $\theta=0$, at y= $-h_2$ ………………………………(42)

In equations (41,42) the q is the flux in the wave frame. α,β,ϕ and d satisfy the relation

$\alpha^2+\beta^2+2\alpha\beta\cos\phi\le(1+d)^2$ ……………………………(43)

4- Method of the solution

We use the perturbation method and Mathematica software inserting the following expressions:

$$\psi = \psi_{00} + G_r\psi_{01} + O(G_r)^2 \qquad ....(44)$$

$$\theta = \theta_{00} + B_n\theta_{01} + O(B_n)^2 \qquad ....(45)$$

$$\varphi = \varphi_{00} + B_n\varphi_{01} + O(B_n)^2 \qquad ....(46)$$

4.1- zeroth order

$$\frac{\partial^4\psi_{00}}{\partial y^4} - \mathrm{Mg}\frac{\partial^2\psi_{00}}{\partial y^2} + \mathrm{Ro}\frac{\partial^2\psi_{00}}{\partial y^2} = 0 \qquad ....(47)$$

$$\frac{\partial^2\theta_{00}}{\partial y^2} + B_r\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)^2 = 0 \qquad ....(48)$$

$$\frac{\partial^2\varphi_{00}}{\partial y^2} - R_m E + R_m\frac{\partial\psi_{00}}{\partial y} = 0 \qquad ....(49)$$

For example of the zeroth order:

$$\psi_{00} = \frac{e^{-\sqrt{\mathrm{Mg-Ro}}y}\left(e^{2\sqrt{\mathrm{Mg-Ro}}y}*\mathrm{t1}+\mathrm{t2}\right)}{\mathrm{Mg-Ro}} + \mathrm{t3} + y*\mathrm{t4};$$

$$\vartheta_{00} = -\frac{\mathrm{BR}(e^{2\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t1}^2+e^{-2\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t2}^2+4(\mathrm{Mg-Ro})\mathrm{t1t2}y^2)}{4(\mathrm{Mg-Ro})} + \mathrm{t5} + y*\mathrm{t6};$$

$$\varphi_{00} = \mathrm{t7} + y*\mathrm{t8} + \left(-\frac{e^{\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t1}}{(\mathrm{Mg-Ro})^{3/2}} + \frac{e^{-\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t2}}{(\mathrm{Mg-Ro})^{3/2}} + \frac{E*y^2}{2} - \frac{\mathrm{t4}y^2}{2}\right)R_M;$$

4.2- first order

$$\frac{\partial^4\psi_{01}}{\partial y^4} - \mathrm{Mg}\frac{\partial^2\psi_{01}}{\partial y^2} + \mathrm{Ro}\frac{\partial^2\psi_{01}}{\partial y^2} + \frac{\partial\theta_{00}}{\partial y} = 0 \qquad ....(50)$$

$$\frac{\partial^2\theta_{01}}{\partial y^2} + 2B_r\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)\left(\frac{\partial^2\psi_{01}}{\partial y^2}\right) = 0 \qquad ....(51)$$

$$\frac{\partial^2\varphi_{00}}{\partial y^2} + R_m\frac{\partial\psi_{01}}{\partial y} = 0 \qquad ....(52)$$

Where $\mathrm{Ro}=\frac{\Omega^2 a_1{}^2\rho}{\mu}$ and $\mathrm{Mg} = \mathrm{M}^2$.

For example of the first order:

$$\psi_{01} = \frac{\mathrm{t6}y^2}{2\mathrm{Mg}-2\mathrm{Ro}} + \frac{\mathrm{BR}(e^{2\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t1}^2-e^{-2\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t2}^2-8(\mathrm{Mg-Ro})^{3/2}\mathrm{t1t2}y^3)}{24(\mathrm{Mg-Ro})^{5/2}} + \frac{e^{\sqrt{\mathrm{Mg-Ro}}y}*\mathrm{t9}}{\mathrm{Mg-Ro}} + \frac{e^{-\sqrt{\mathrm{Mg-Ro}}y}*\mathrm{t10}}{\mathrm{Mg-Ro}} + \mathrm{t11} + y*\mathrm{t12};$$

$$\theta_{01} = -\frac{1}{54(\mathrm{Mg-Ro})^{5/2}}\mathrm{BR}(2\mathrm{BR}e^{3\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t1}^3 + 27e^{-2\sqrt{\mathrm{Mg-Ro}}y}(\mathrm{Mg-Ro})^{3/2}\mathrm{t10t2} - 2\mathrm{BR}e^{-3\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t2}^3 + 27e^{2\sqrt{\mathrm{Mg-Ro}}y}(\mathrm{Mg-Ro})^{3/2}\mathrm{t1t9} + 54(\mathrm{Mg-Ro})^{5/2}(\mathrm{t1t10} + \mathrm{t2t9})y^2 + 18e^{\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t1}(6\sqrt{\mathrm{Mg-Ro}}\mathrm{t6} + \mathrm{BRt1t2}(25 - 12\sqrt{\mathrm{Mg-Ro}}y)) - 18e^{-\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t2}(-6\sqrt{\mathrm{Mg-Ro}}\mathrm{t6} + \mathrm{BRt1t2}(25 + 12\sqrt{\mathrm{Mg-Ro}}y))) + \mathrm{t13} + y*\mathrm{t14};$$

$$\varphi_{01} = \mathrm{t15} + y*\mathrm{t16} + \frac{e^{-2\sqrt{\mathrm{Mg-Ro}}y}(-\mathrm{BR}(e^{4\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t1}^2+\mathrm{t2}^2-4e^{2\sqrt{\mathrm{Mg-Ro}}y}(\mathrm{Mg-Ro})^2\mathrm{t1t2}y^4)+8e^{\sqrt{\mathrm{Mg-Ro}}y}(\mathrm{Mg-Ro})^{3/2}(6\mathrm{t10}-e^{\sqrt{\mathrm{Mg-Ro}}y}(6e^{\sqrt{\mathrm{Mg-Ro}}y}\mathrm{t9}+\sqrt{\mathrm{Mg-Ro}}y^2(3\mathrm{Mgt12}-3\mathrm{Rot12}+\mathrm{t6}y))))R_M}{48(\mathrm{Mg-Ro})^3};$$

Where from(t1-t16) are the constant of integration and $h_1 = 1 + \alpha\cos(2\pi x)$ and $h_2 = -d - \beta\cos(2\pi x + \phi)$.

5- Graphical results and discussions

In this problem, we discuss the effects of the rotation and induction on the velocity, temperature, magnetic force, current density, induced magnetic field, pressure gradient, stream function and the lower, upper walls stress by using different parameters.

5.1- Velocity distribution

We use different parameters to show the profile of velocity. The change of the velocity for different values of BR, GR, q , ϕ, β, α, d, Ω, Mg are clearly in figure (2) from (A-I) we notes that the profile of velocity is parabolic distribution and it mostly increases at the center of channel. The effect of (BR,GR, Ω) the Brinkman, Grashof numbers and rotation on the velocity for increasing value lead to an increasing of velocity near the channel walls and they are merged with other at the center, while an increasing in value of (q) leads to an increasing in value of velocity it is away from the walls and descends inward, as well as in cases of (ϕ, β, α). In the event of a decrease in the value of (d), the velocity increases and it expands to the walls when its value increases. When the value of (Mg) decreases, the velocity increases near the walls and is identical at the center.

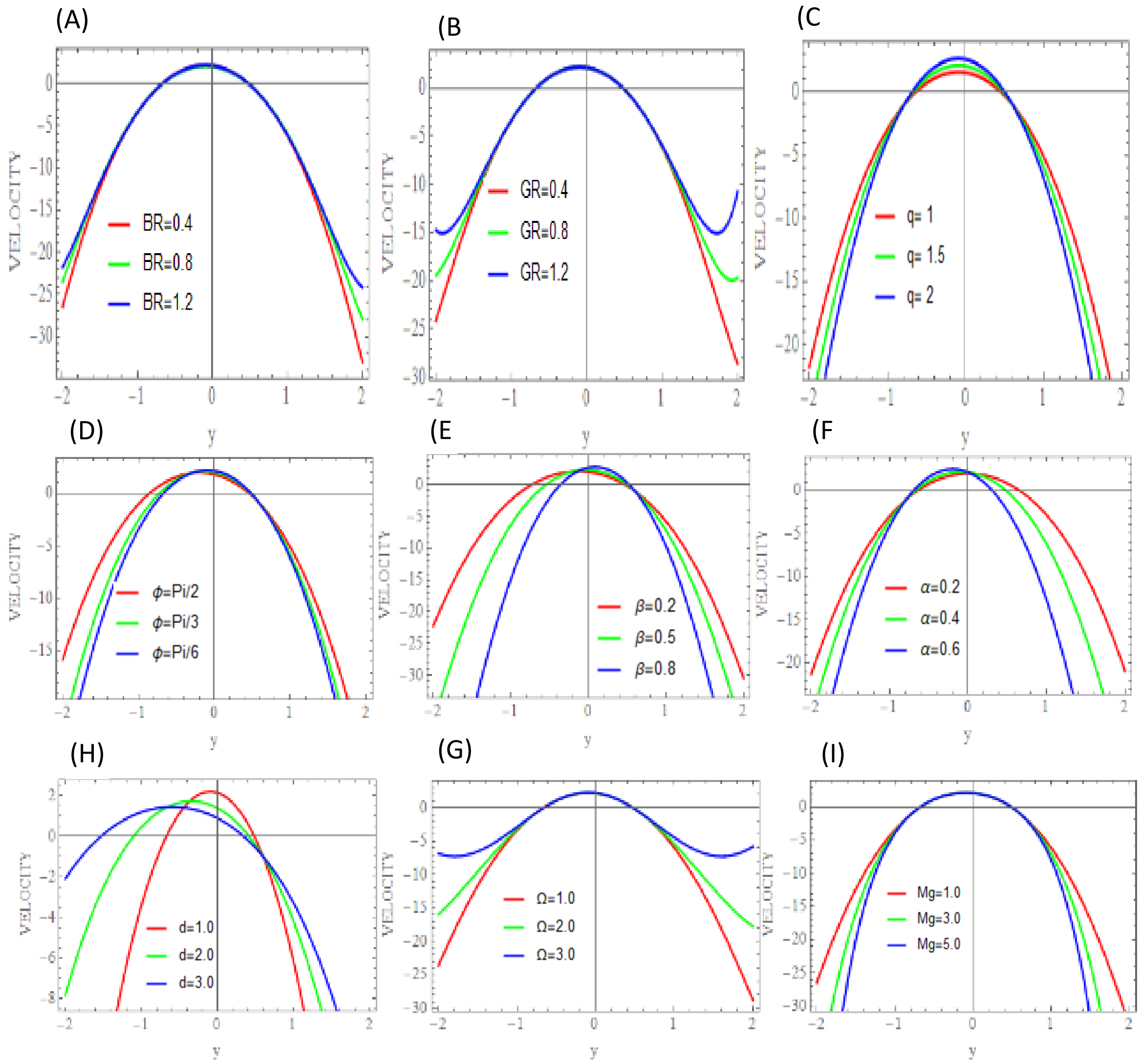


Figure-2 Represented velocity with different values of parameters$\{G_r = 0.2, q = 1.6, x = 0.5, \phi = Pi/4, B_r = 0.4, \alpha = 0.4, \beta = 0.3, \rho = 0.9, Mg = 1.0, \Omega = 2.0, \mu = 2.0, a_1 = 1.0\}$.

5.2- temperature distribution

To show the distribution of the temperature using different parameters such as (BR,BN,q, ϕ, β, α, Ω,d,Mg). It can be seen from the figure (3) from (A-I), where we note that (BR,q) when an increasing in these values leads to increase temperature values and any decreasing in these values the temperature expands near of walls, whereas (BN) in case this increasing leads to expand near walls and identical at the center of channel. In cases of (ϕ, β, α, Mg) ,when the value of these paramreters are decreasing, the temperature will continue to increase at the areas near the walls, while the increasing value (Ω) implies that continue increasing near of tha walls channel. When any decreasing in (d) leads to increase the value of temperature.

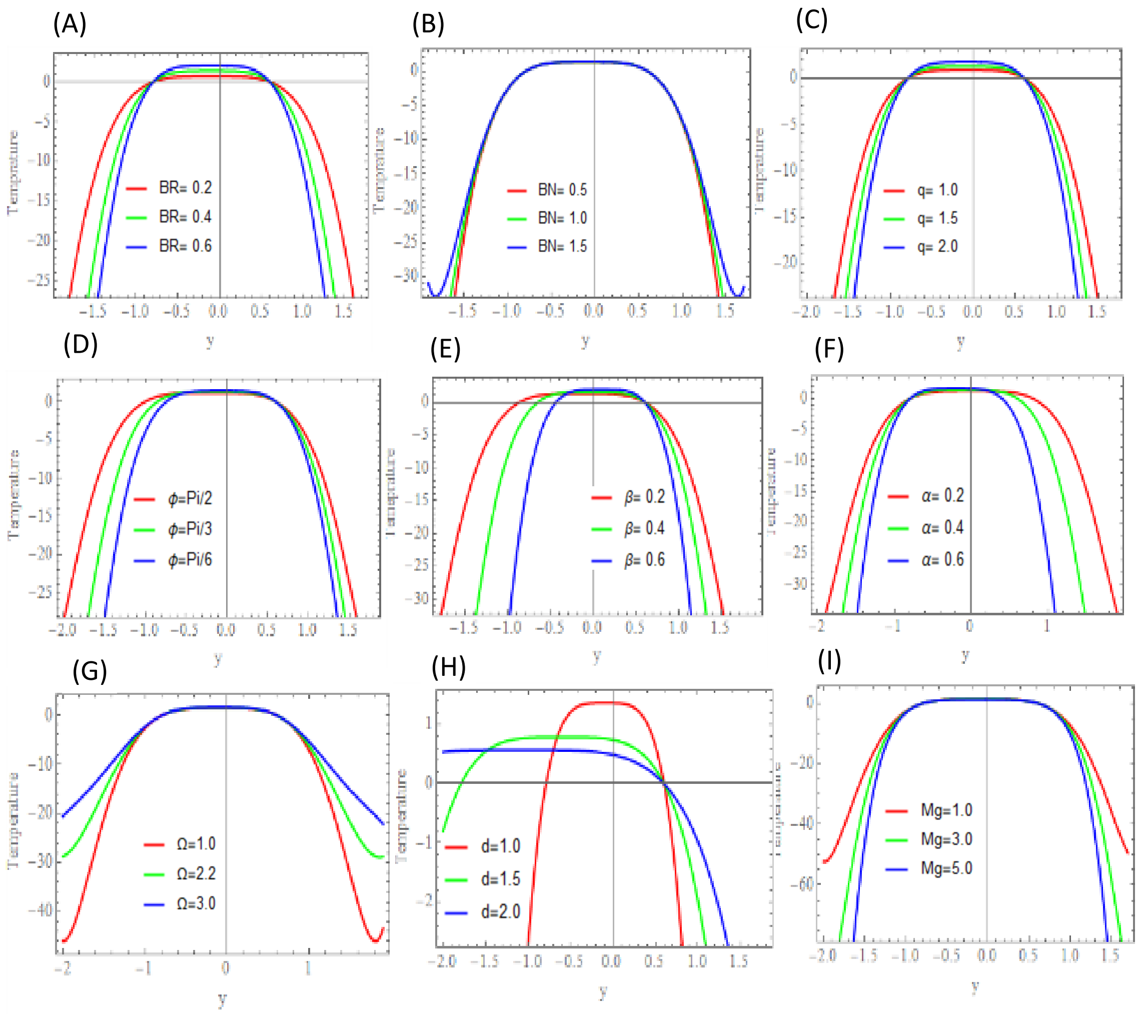


Figure-3 This figure represents the temperature with different values of parameters $\{\alpha = 0.7, E = 0.7, R_M = 4.0, q = 1.6, BN = 1.0, x = 0.5, Mg = 2.0, BR = 0.4, \beta = 1.2, \phi = Pi/2, \Omega = 0.2, a1 = 1.0, d = 2.0, \mu = 2.0, \rho = 0.9\}$

5.3- magnetic force

We use different parameters to study the impact on the behavior of magnetic force as (R_M, E, q, BN, BR, Mg, d, α, Ω) see figure (4) from (A-I), we observed that when any decreasing in values of (R_M, E, Mg) leads to an increasing in value of magnetic force. We also see that when an increasing in values of (q, BN, BR, α, Ω) leads to an increasing in value of magnetic force, in case of (α) this increasing is more direct to upper wall. If we decrease the value of (d) then its direction tends to the upper wall of the channel, and its value will decrease when the value of (d) is increasing.

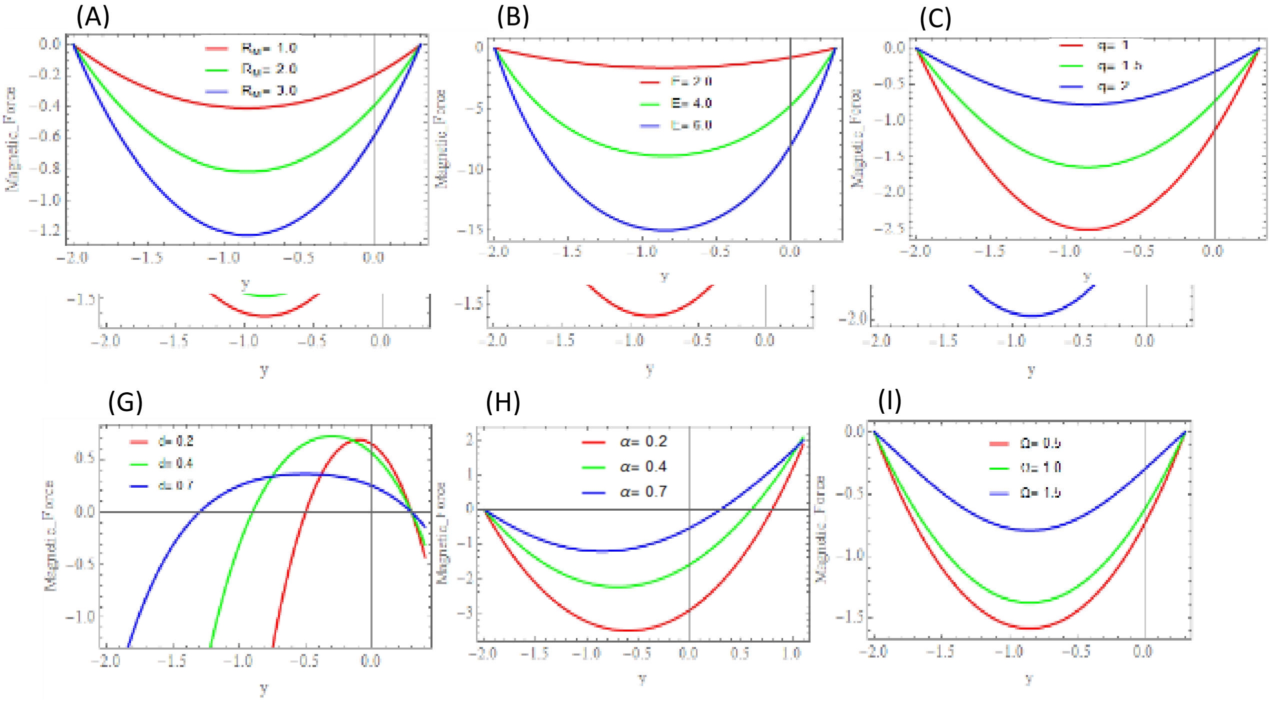
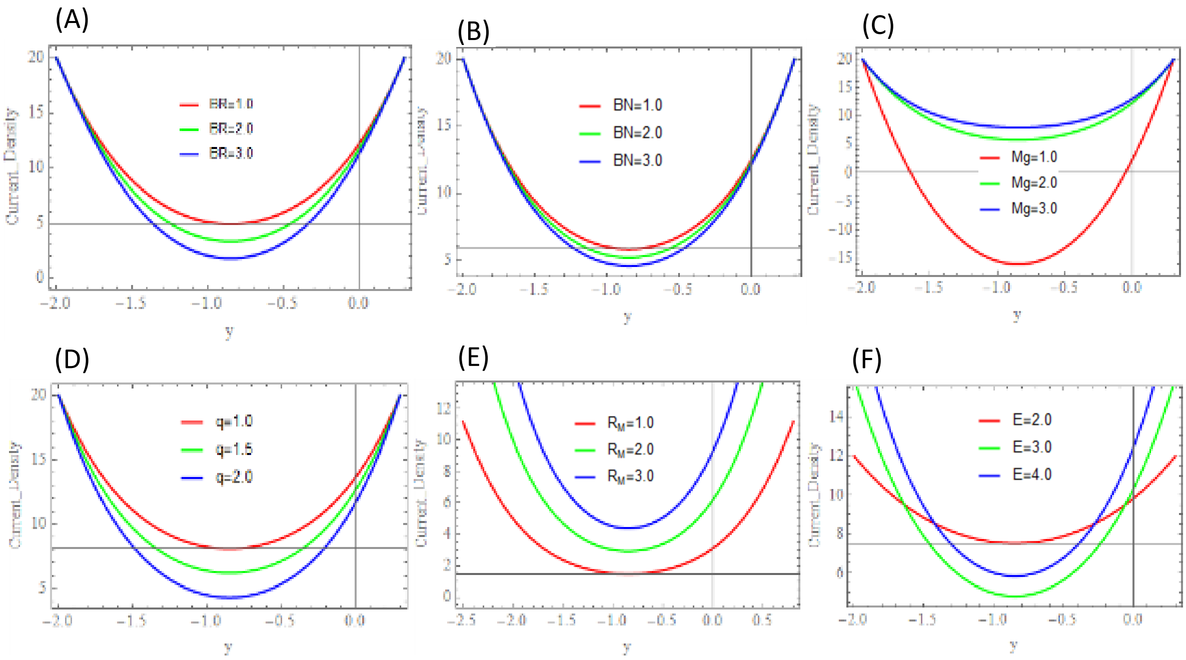


Figure 4-This figure represents the magnetic force with different values of parameters { $\alpha = 0.7, E = 0.7, R_M = 4.0, q = 1.6, BN = 1.0, x = 0.5, Mg = 2.0, BR = 0.4, \beta = 1.2, \phi =$

5.4- Current density

We study the effect of several different parameters on the current density this is shown in figure (5) from (A-I), we observed that any decreasing values of these parameters (BR, BN, q, Ω) leads to increase the value of current density, as well as any decreasing in values of (E, α) implies that increasing current density and continues to be high near the walls, while an increasing in values of (Mg, R_M, d) leads to increase the value of current density near the walls channel except the (d) is to inward.



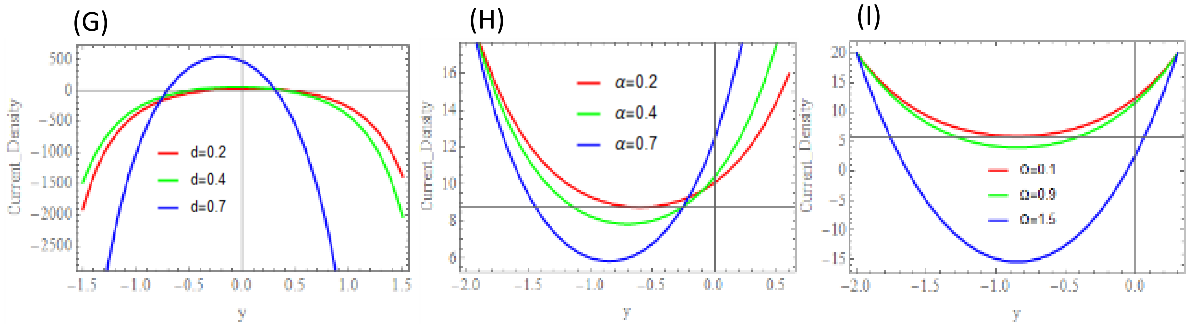


Figure 5-This figure represents the current density with different values of parameters{ α= 0.7, $E = 4.0, BN = 1.0, q = 1.6", x = 0.5, R_M = 4.0, BR = 0.4, \beta = 1.2, \phi = Pi/2, \Omega = 0.2, a1 = 1.0, d = 2.0, \mu = 2.0, \rho = 0.9, Mg = 2.0$}

5.5- Induced Magnetic

By looking at figure (6) from (A-I), we see that the impact of several parameters (BN, R_M, E, α, Ω, BR, q, Mg, d) on magnetic induction as follows : any decreasing in values of (BN, BR, q, Mg, Ω) leads to increase the value of induced magnetic, where the wall starts from the lower wall negative up to the upper side until it reaches close to the upper walls begins to land and merge all values at the center. The impact of (E, d, R_M) can be seen when an increasing in values implies that increases the value of induced magnetic and the magnetic induction motion recedes inward, while any decreasing in value of (α) leads to near of the lower and lower walls of channel.

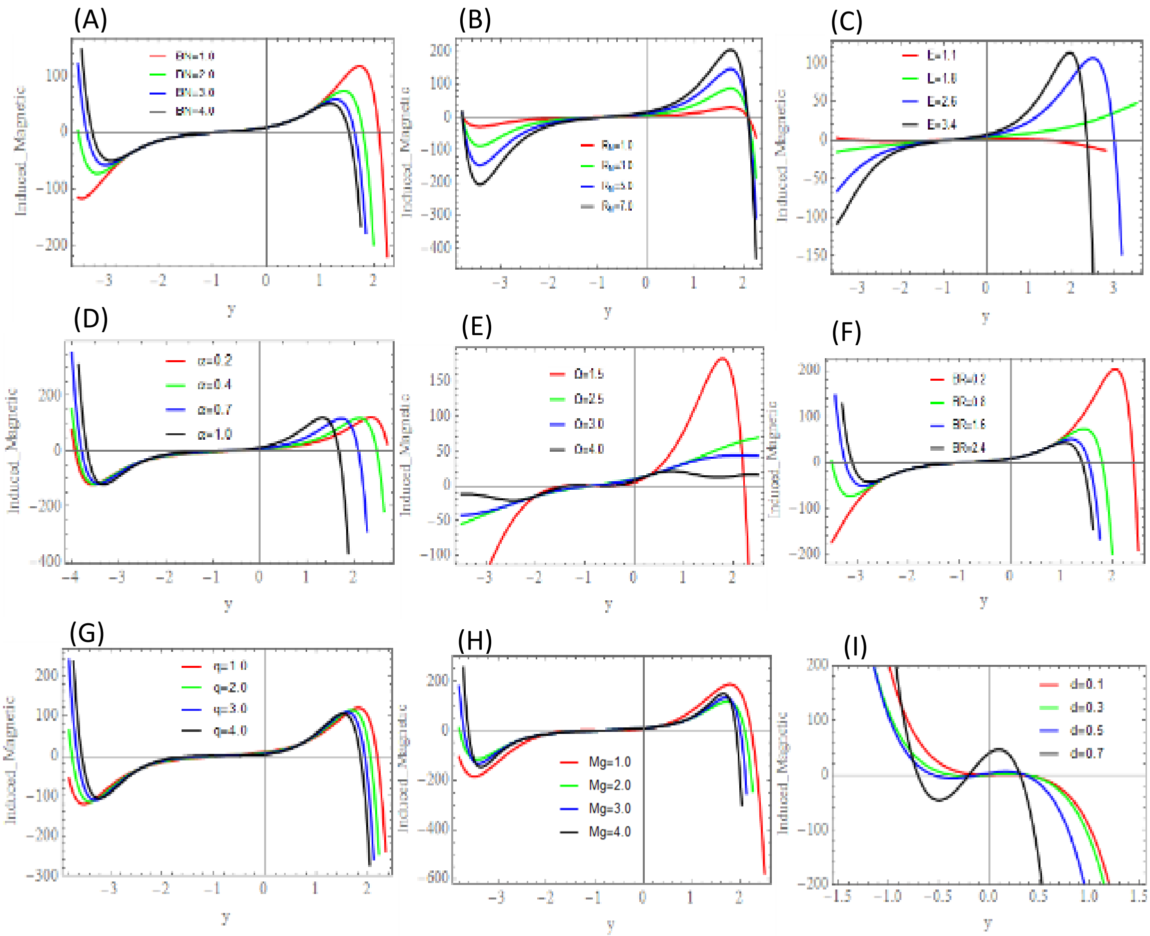


Figure-6 Represented Induced Magnetic With Different Parameters{ $\alpha = 0.7, R_M = 4.0, BN = 1.0, E = 4.0, q = 1.6, x = 0.5, Mg = 5.0, BR = 0.4, \beta = 1.2, \phi = Pi/2, \Omega =$

5.6- Behavior gradient pressure

In the studying the behavior of gradient pressure Figure (7) from (A-I) is parabola by application different parameters (BN, ϕ, E, α, Ω, BR, q, Mg, GR), we observe that, if the value of ϕ decreases the wave of gradient pressure begins at the lower wall and ends away from the upper wall. While if the values of (Mg, E, BN, GR, BR) are increasing lead to increase the value of gradient pressure. We also see any decreasing in value of (q) implies that increases in value of gradient pressure whereas of (Ω) a little change occurs of gradient pressure.

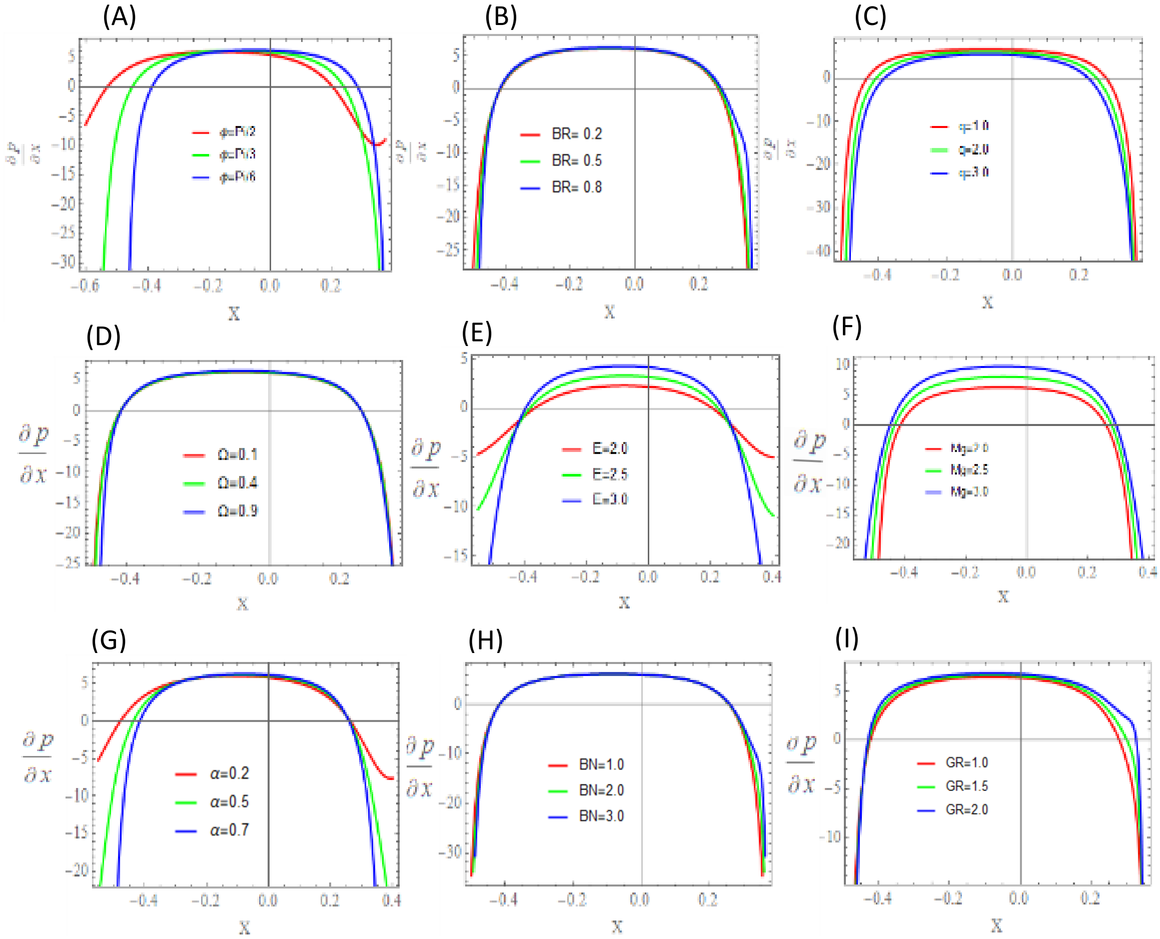


Figure-7 Represented Gradient Pressure With Different Parameters { $\alpha = 0.7, E = 4.0", BN = 1.0, GR = 0.2, y = 0.5, Mg = 2.0, R_M = 4.0, BR = 0.4, \beta = 1.2, \phi = Pi/4, \Omega = 0.2, a1 = 1.0, d = 2.0, q = 1.6, "\mu = 2.0, \rho = 0.9$.

5.7- Stress distribution

In studying the distribution of stress by using different parameters with respect x-axial the behavior obvious at the upper and lower walls of channel in the figure (8) from (A-R) , we observed that at upper wall. Now when the value of ϕ decreases, the wave of gradient pressure begins at the lower wall and ends away from the upper wall at the upper wall, while any decreasing in values of (q, Mg) leads to increase the value of stress whereas an increasing in value of (BN, d) leads to increase the stress.

When an increasing in value of (E, GR, α, Ω) the stress is confined to the inside of the channel away from the walls. In the case of the lower wall, the distribution is in reverse.

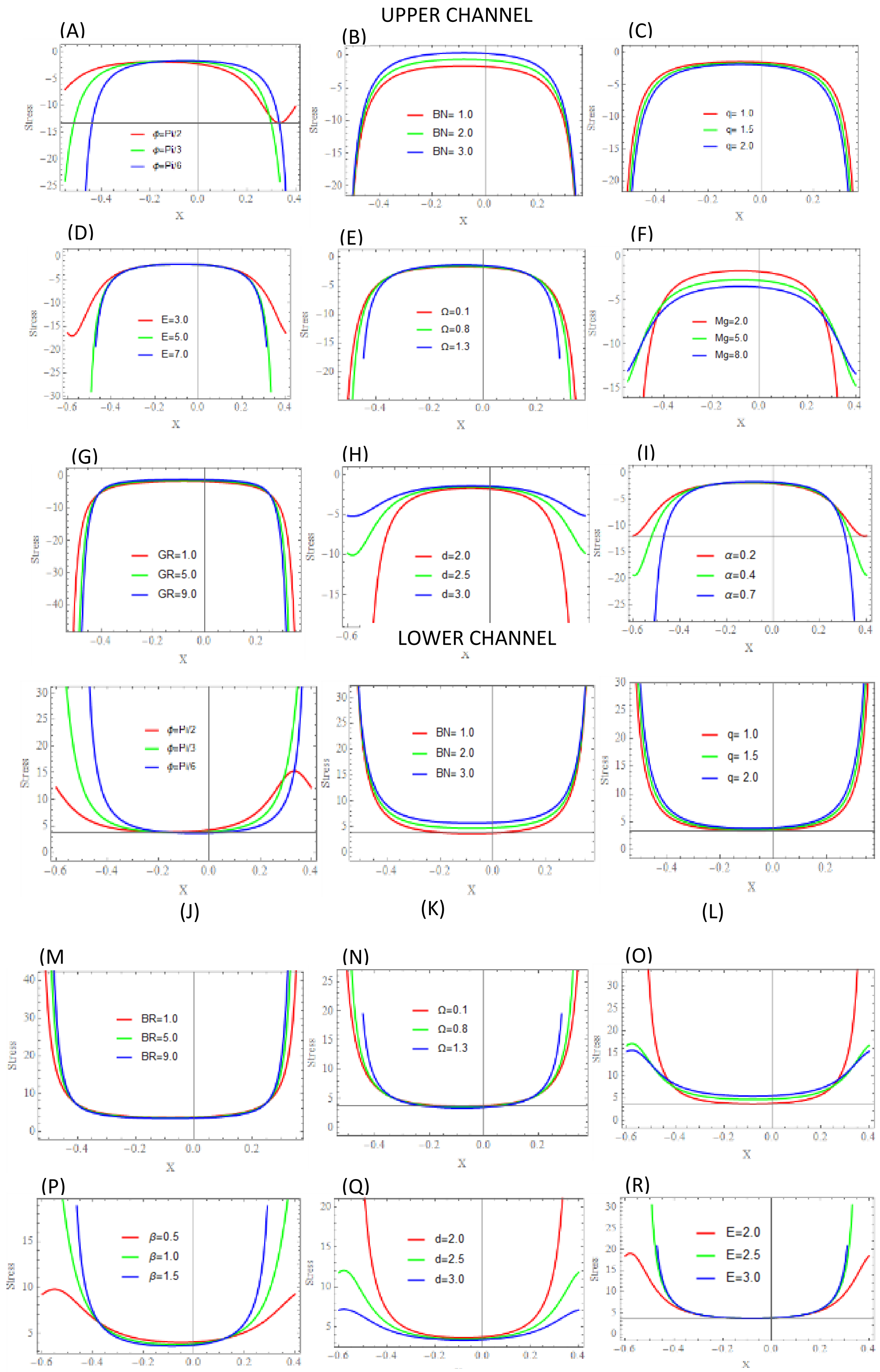
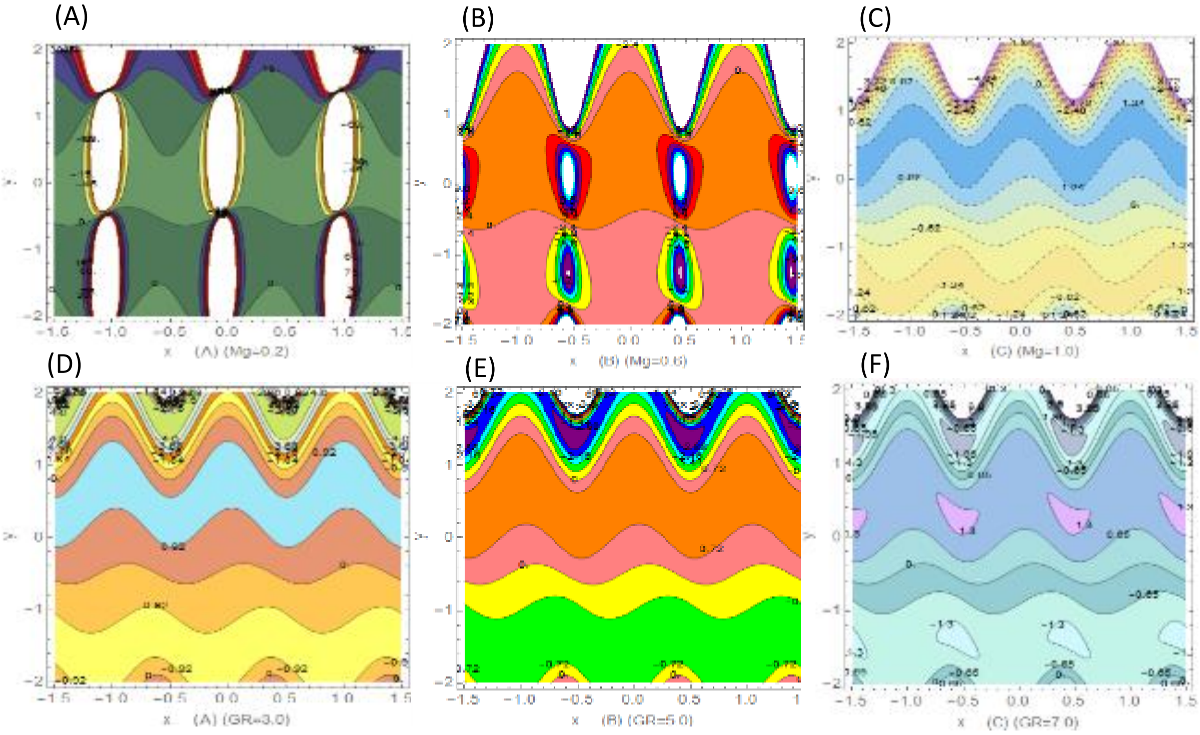


Figure-8 This figure represents the stress with different values of Parameters { $\alpha = 0.7, E = 4.0, BN = 1.0, GR = 0.2, y = 0.5, Mg = 2.0, R_M = 4.0, BR = 0.4, \beta = 1.2, \phi = Pi/4, \Omega = 0.2, a1 = 1.0, d = 2.0, q = 1.6, \mu = 2.0, \rho = 0.9$}

5.8- Steam Function

The trapping phenomena for different values (Mg, GR, BR, Ω , ϕ, E, d) are shown in figure (9) form (A-U). It is observed that the volume of the trapped bolus begins to decay when the value of (Mg, d) increasing, whereas it begins to be appear when an increasing in values of (GR), as well as in cases of (BR, Ω, E). when the increasing value of (ϕ) leads to expand of the wave.



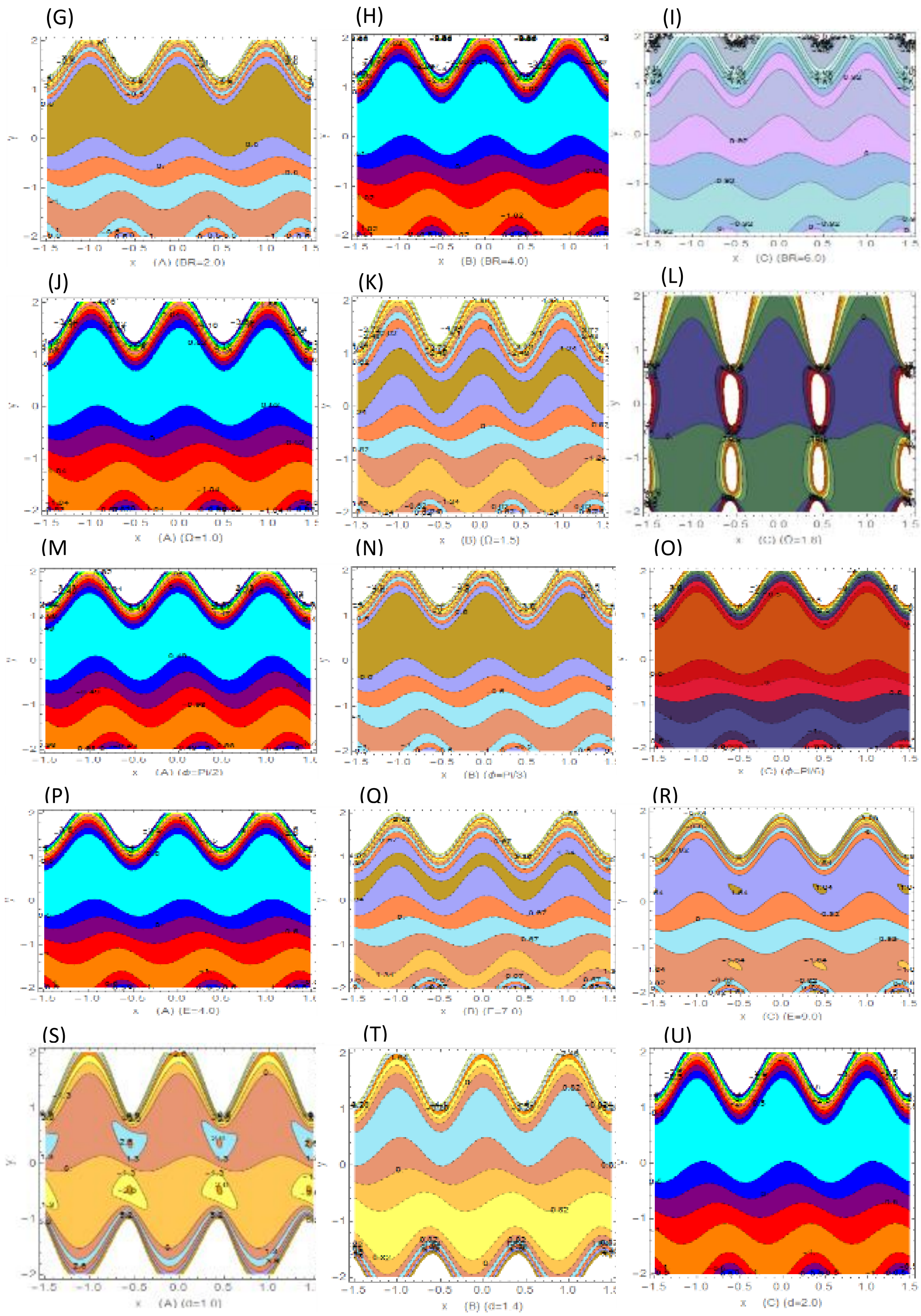


Figure-9 This figure shows the stream function with Different values of Parameters $\{GR = 0.2, BR = 0.4, Mg = 2.0, d = 2.0, E = 4.0, \alpha = 0.4, \beta = 0.3, \phi = Pi/4, q = 1.6, \mu =$

6- Conclusion

In this Paper, The effect of the induced magnetic and rotaion on mixed convection heat transfer peristaltic move of the Binghame plastic fluid trough of the tapered horizontal asymmetric channel have been studied. The channel asymmetric is produced by choosing the peristaltic waves which have different amplutude and phases, low Renolds number and along wavelength. The perturbation method to obtain the expression of the axial velocity, magnetic force, induced magnetic, stress, temperature, stream function, current density and pressure gradient is also adapted. A parametric analysis is performed through various graphs.

A- We note that the profile of velocity is parabolic and it increases at the center of channel mostly when an increasing in the value of parameters .

B- Temperature profile, Generally the temperature value is in the highest at near the center of channel and it increases when the parameters are increasing with exception the value of (d).

C- We note that the magnetic force is t alternates between increasing and decreasing according to the type of parameter that applied. Also with respect to the current density, induced magnetic, gradient pressure and stress.

D- The trapped phenomonent it is observed that the volume of the trapped bolus begins to decay and to increase volume according to which parameters are used.

E- There are several applications such as in engineering and physical applications of peristaltic mobility, in fact such waves have been induced due the expansion and contraction of an extensible tube and propagate along the length of tube, mixing and transporting the fluid in the direction of wave propagation, this process appears in the tubular organs of human body such as ureter, blood transport in the extracorporeal circulation.

F- Another applications including medical, biological sciences,physiology, blood and to generation of electric energy in the industrial field.

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