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# On Soft LC-Spaces and Weak Forms of Soft LC-Spaces

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**Abstract**
The main purpose of this article is to study the soft LC-spaces as soft spaces in which every soft Lindelöf subset of $\tilde{U}$ is soft closed. Also, we study the weak forms of soft LC-spaces and we discussed their relationships with soft LC-spaces as well as among themselves.

**Keywords:** Soft $F_\sigma$ -closed set, soft compact space, soft Lindelöf space, soft LC-space, soft $L_i$ -space, $i = 1,2,3,4$ , soft KC-space, and soft P-space.

# حول فضاءات –LC الميسرة والصيغ الضعيفة لفضاءات –LC الميسرة

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**الخلاصة**
الهدف من هذة المقالة هو دراسة فضاءات –LC الميسرة وهي الفضاءات الميسرة التي فيها كل مجموعة جزئية لندلوف ميسرة مغلقة ميسرة. كذلك درسنا الصيغ الضعيفة لفضاءات–LC الميسرة وناقشنا علاقاتهم مع فضاءات –LC الميسرة كذلك علاقاتهم مع بعضهم.

## Introduction

Molodtsov [1] in 1999 introduced and studied soft set theory as a new mathematical tool for dealing with uncertainty while modeling problems in medical sciences, economics, computer science, engineering physics and social sciences. Shabir and Naz [2] in 2011 investigated the notion of soft topological spaces over an initial universe set with a fixed set of parameters. Molodtsov and et. al. [3] in 2006 and Rong [4] in 2012 introduced and studied soft compact spaces and soft Lindelöf spaces respectively. The main purpose of this paper is to introduce and study a new type of soft spaces called soft LC-spaces and we show that a soft topological space $(U,\tilde{\tau},P)$ is a soft LC-space if and only if each soft point in $\tilde{U}$ has a soft closed neighborhood that is a soft LC-space. Moreover we discussed weak forms of soft LC-spaces such as soft $L_1$-spaces, soft $L_2$-spaces, soft $L_3$-spaces and soft $L_4$-spaces. The characteristics of these soft spaces and the relation among them also have been studied.

## 1. Preliminaries:

In this paper P is the set of parameters, U is an initial universe set, $P(U)$ is the power set of U, and $A \subseteq P$.

**Definition (1.1) [1]:** A soft set over U is a pair $(H, A)$, where H is a function defined by $H: A \to P(U)$ and A is a non-empty subset of P.

**Definition (1.2)[5]:** A soft set $(H, A)$ over U is called a soft point if there is $e \in A$ such that $H(e) = \{u\}$ for some $u \in U$ and $H(e') = \varphi, \forall\ e' \in A \setminus \{e\}$ and is denoted by $\tilde{u} = (e,\{u\})$.

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**Definition (1.3)[5]:** A soft point $\tilde{u}=(e,\{u\})$ is called belongs to a soft set $(H,A)$ if $e\in A$ and $u\in H(e)$, and is denoted by $\tilde{u}\,\tilde{\in}\,(H,A)$.

**Definition (1.4)[2]:** A soft topology on U is a family $\tilde{\tau}$ of soft subsets of $\tilde{U}$ having the following properties:

**(i)** $\tilde{U}\,\tilde{\in}\,\tilde{\tau}$ and $\tilde{\varphi}\,\tilde{\in}\,\tilde{\tau}$.

**(ii)** If $(H_1,P),(H_2,P)\,\tilde{\in}\,\tilde{\tau} \;\Rightarrow\; (H_1,P)\,\tilde{\bigcap}\,(H_2,P)\,\tilde{\in}\,\tilde{\tau}$.

**(iii)** If $(H_j,P)\,\tilde{\in}\,\tilde{\tau},\forall j\in\mathbf{\Omega} \;\Rightarrow\; \bigcup_{j\in\mathbf{\Omega}}(H_j,P)\,\tilde{\in}\,\tilde{\tau}$.

The triple $(U,\tilde{\tau},P)$ is called a soft topological space. The members of $\tilde{\tau}$ are called soft open sets over U. The complement of a soft open set is called soft closed.

**Definition (1.5) [6]:** Let $(U,\tilde{\tau},P)$ be a soft topological space and $(H,P)\,\tilde{\subseteq}\,\tilde{U}$. Then the soft closure of $(H,P)$, denoted by $cl((H,P))$ is the intersection of all soft closed sets in $\tilde{U}$ which contains $(H,P)$.

**Definition (1.6)[2]:** If $(U,\tilde{\tau},P)$ is a soft topological space and $\tilde{\varphi}\neq(Y,P)\,\tilde{\subseteq}\,\tilde{U}$. The family $\tilde{\tau}_{(Y,P)}=\{(V,P)\,\tilde{\bigcap}\,(Y,P):(V,P)\,\tilde{\in}\,\tilde{\tau}\}$ is called the relative soft topology on $(Y,P)$ and $((Y,P),\tilde{\tau}_{(Y,P)},P)$ is called a soft subspace of $(U,\tilde{\tau},P)$.

**Definition (1.7)[7]:** A soft topological space $(U,\tilde{\tau},P)$ is called a soft $\tilde{T}_1$-space if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there exists a soft open set in $\tilde{U}$ containing $\tilde{x}$ but not $\tilde{y}$ and a soft open set in $\tilde{U}$ containing $\tilde{y}$ but not $\tilde{x}$.

**Theorem (1.8)[7]:** A soft topological space $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space if and only if each soft point in $\tilde{U}$ is soft closed.

**Definition (1.9)[7]:** A soft topological space $(U,\tilde{\tau},P)$ is called a soft $\tilde{T}_2$-space if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{U}$, there are two soft open sets $(H,P)$ and $(K,P)$ in $\tilde{U}$ such that $\tilde{x}\,\tilde{\in}\,(H,P)$, $\tilde{y}\,\tilde{\in}\,(K,P)$, and $(H,P)\,\tilde{\bigcap}\,(K,P)=\tilde{\varphi}$.

**Definition (1.10)[7]:** A soft topological space $(U,\tilde{\tau},P)$ is called a soft regular space if for any soft closed set $(F,P)$ in $\tilde{U}$ and any soft point $\tilde{x}$ in $\tilde{U}$ such that $\tilde{x}\,\tilde{\notin}\,(F,P)$ there exists two soft open sets $(H,P)$ and $(K,P)$ in $\tilde{U}$ such that $\tilde{x}\,\tilde{\in}\,(H,P)$, $(F,P)\,\tilde{\subseteq}\,(K,P)$ and $(H,P)\,\tilde{\bigcap}\,(K,P)=\tilde{\varphi}$.

**Definition (1.11)[3]:** A soft topological space $(U,\tilde{\tau},P)$ is called soft compact if every soft open cover of $\tilde{U}$ has a finite soft subcover.

**Theorem (1.12)[8]:** A soft closed subset of a soft compact space is soft compact.

**Theorem (1.13)[9]:** A soft compact set in a soft $\tilde{T}_2$-space is soft closed.

**Definition (1.14)[4]:** A soft topological space $(U,\tilde{\tau},P)$ is called soft Lindelöf if every soft open cover of $\tilde{U}$ has a countable soft subcover.

**Theorem (1.15)[8]:** A soft closed subset of a soft Lindelöf space is soft Lindelöf.

**2. Soft LC-Spaces and Weak Forms of Soft LC-Spaces**

Now, we introduce and study new types of soft spaces called soft LC-spaces also, we study weak forms of soft LC-spaces such as soft $L_1$-spaces, soft $L_2$-spaces, soft $L_3$-spaces and soft $L_4$-spaces. Further we discussed the equivalent definitions of these soft spaces and the relation among them.

**Definition (2.1):** A soft topological space $(U,\tilde{\tau},P)$ is called a soft LC-space if every soft Lindelöf subset of $\tilde{U}$ is soft closed.

**Definition (2.2):** A soft subset $(F,P)$ of a soft topological space $(U,\tilde{\tau},P)$ is called soft $F_\sigma$-closed if it is the soft union of a countable soft closed sets.

**Definition (2.3):** A soft topological space $(U,\tilde{\tau},P)$ is called a soft P-space if every soft $F_\sigma$-closed set in $\tilde{U}$ is soft closed.

**Definition (2.4):** A soft topological space $(U,\tilde{\tau},P)$ is called:

**(i)** A soft $L_1$-space if every soft Lindelöf $F_\sigma$-closed set in $\tilde{U}$ is a soft closed set.

**(ii)** A soft $L_2$-space if $cl((L,P))$ is soft Lindelöf whenever $(L,P)$ is a soft Lindelöf set in $\tilde{U}$.

**(iii)** A soft $L_3$-space if every soft Lindelöf set in $\tilde{U}$ is a soft $F_\sigma$-closed set.

**(iv)** A soft $L_4$-space if whenever $(L,P)$ is a soft Lindelöf set in $\tilde{U}$, then there is a soft Lindelöf $F_\sigma$-closed set $(F,P)$ in $\tilde{U}$ such that $(L,P)\tilde{\subseteq}(F,P)\tilde{\subseteq}cl((L,P))$.

**Theorem (2.5):**

**(i)** If $(U,\tilde{\tau},P)$ is a soft LC -space, then $(U,\tilde{\tau},P)$ is a soft $L_i$-space, $i=1,2,3,4$.

**(ii)** If $(U,\tilde{\tau},P)$ is a soft $L_1$-space and a soft $L_3$-space, then $(U,\tilde{\tau},P)$ is a soft LC-space.

**(iii)** If $(U,\tilde{\tau},P)$ is a soft $L_1$-space and a soft $L_4$-space, then $(U,\tilde{\tau},P)$ is a soft $L_2$-space.

**(iv)** Every soft $L_2$-space is a soft $L_4$-space and every soft $L_3$-space is a soft $L_4$-space.

**(v)** Every soft $L_3$-space is a soft $\tilde{T}_1$-space.

**(vi)** Every soft Lindelöf space is a soft $L_2$-space and every soft $L_2$-space having a soft dense Lindelöf set is soft Lindelöf.

**(vii)** The property soft $L_3$ is soft hereditary and the properties $L_1, L_2$ and $L_4$ are soft hereditary on a soft $F_\sigma$-closed set.

**(viii)** The property soft LC-space is soft hereditary.

**(ix)** Every soft P-space is a soft $L_1$-space.

**Proof: (i)** It is obvious.

**(ii)** Let $(L,P)$ be a soft Lindelöf set in $\tilde{U}$, since $(U,\tilde{\tau},P)$ is a soft $L_3$-space, then $(L,P)$ is soft $F_\sigma$-closed, but $(U,\tilde{\tau},P)$ is a soft $L_1$-space, then $(L,P)$ is a soft closed set in $\tilde{U}$. Thus $(U,\tilde{\tau},P)$ is a soft LC-space.

**(iii)** Let $(L,P)$ be a soft Lindelöf set in $\tilde{U}$, since $(U,\tilde{\tau},P)$ is a soft $L_4$-space, then there is a soft Lindelöf $F_\sigma$-closed set $(F,P)$ in $\tilde{U}$ such that $(L,P)\tilde{\subseteq}(F,P)\subseteq cl((L,P))$. Since $(U,\tilde{\tau},P)$ is a soft $L_1$-space, then $(F,P)$ is soft closed. Hence $cl((L,P))\tilde{\subseteq}(F,P)\subseteq cl((L,P))$, thus $cl((L,P))=(F,P)$ is a soft Lindelöf set in $\tilde{U}$. Therefore $(U,\tilde{\tau},P)$ is a soft $L_2$-space.

**(iv)** Let $(L,P)$ be a soft Lindelöf set in $\tilde{U}$, since $(U,\tilde{\tau},P)$ is a soft $L_2$-space, then $cl((L,P))$ is soft Lindelöf. Hence $(L,P)\tilde{\subseteq}cl((L,P))\tilde{\subseteq}cl((L,P))$. Since $cl((L,P))$ is soft closed, then there is $(F,P)=cl((L,P))$ is a soft Lindelöf $F_\sigma$-closed set in $\tilde{U}$ such that $(L,P)\tilde{\subseteq}(F,P)\tilde{\subseteq}cl((L,P))$. Therefore $(U,\tilde{\tau},P)$ is a soft $L_4$-space. Similarly, we can prove $(U,\tilde{\tau},P)$ is a soft $L_4$-space, if $(U,\tilde{\tau},P)$ is a soft $L_3$-space.

**(v)** Since $\{\tilde{x}\}$ is a soft Lindelöf set and $(U,\tilde{\tau},P)$ is a soft $L_3$-space, then $\{\tilde{x}\}$ is a soft $F_\sigma$-closed set. Therefore $\{\tilde{x}\}$ is soft closed. Thus $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space.

**(vi)** Let $(L,P)$ be a soft Lindelöf set in $\tilde{U}$, since $cl((L,P))$ is soft closed in $(U,\tilde{\tau},P)$ which is a soft Lindelöf space, then by theorem (1.15), $cl((L,P))$ is soft Lindelöf in $\tilde{U}$. Thus $(U,\tilde{\tau},P)$ is a soft $L_2$-space. Also, if $(U,\tilde{\tau},P)$ is a soft $L_2$-space having a soft dense Lindelöf set $(L,P)$, then $cl((L,P)) = \tilde{U}$. Since $(U,\tilde{\tau},P)$ is a soft $L_2$-space, then $(U,\tilde{\tau},P)$ is soft Lindelöf.

**(vii)** Let $(U,\tilde{\tau},P)$ be a soft $L_1$-space and $(Y,\tilde{\tau}_Y,P)$ be a soft $F_\sigma$-closed subspace of $(U,\tilde{\tau},P)$. To prove that $(Y,\tilde{\tau}_Y,P)$ is a soft $L_1$-space. Let $(A,P)$ be a soft Lindelöf $F_\sigma$-closed set in $\tilde{Y}$. Since $\tilde{Y} \tilde{\subseteq} \tilde{U}$, then $(A,P)$ is soft Lindelöf in $\tilde{U}$ and $(A,P) = \bigcup_{n\in N}(F'_n,P)$, where $(F'_n,P)$ is soft closed in $\tilde{Y}$, $\forall\ n \in N$, thus $(A,P) = \bigcup_{n\in N}(\tilde{Y} \tilde{\bigcap} (F_n,P)) = \tilde{Y} \tilde{\bigcap} (\bigcup_{n\in N}(F_n,P))$, Where $(F_n,P)$ is soft closed in $\tilde{U}$, $\forall\ n \in N$. Since $\tilde{Y}$ is a soft $F_\sigma$-closed set in $\tilde{U}$, then $\tilde{Y} = \bigcup_{m\in N}(G_m,P)$, where $(G_m,P)$ is soft closed in $\tilde{U}$, $\forall\ m \in N$. Hence $(A,P) = (\bigcup_{m\in N}(G_m,P)) \tilde{\bigcap} (\bigcup_{n\in N}(F_n,P)) = \bigcup_{n,m\in N}((G_m,P) \tilde{\bigcap} (F_n,P))$, but $(G_m,P) \tilde{\bigcap} (F_n,P)$ is soft closed in $\tilde{U}$, thus $(A,P)$ is a soft union of a countable soft closed sets in $\tilde{U}$, hence $(A,P)$ is a soft $F_\sigma$-closed set in $\tilde{U}$. Since $(U,\tilde{\tau},P)$ is a soft $L_1$-space, then $(A,P)$ is soft closed in $\tilde{U} \Rightarrow (A,P) = \tilde{Y} \tilde{\bigcap} (A,P) = \tilde{Y} \tilde{\bigcap} (\bigcup_{n,m\in N}((G_m,P) \tilde{\bigcap} (F_n,P))) = \bigcup_{n,m\in N}[\tilde{Y} \tilde{\bigcap} ((G_m,P) \tilde{\bigcap} (F_n,P))]$ is soft closed in $\tilde{Y}$. Therefore $(Y,\tilde{\tau}_Y,P)$ is a soft $L_1$-space. Similarly, we can prove other cases.

**(viii)** It is obvious.

**(ix)** It is obvious.

**Theorem (2.6):** A soft topological space $(U,\tilde{\tau},P)$ is a soft LC-space if and only if each soft point in $\tilde{U}$ has a soft closed neighborhood that is a soft LC-subspace.

**Proof:** If $(U,\tilde{\tau},P)$ is a soft LC-space, then for each $\tilde{x} \tilde{\in} \tilde{U}$, $\tilde{U}$ itself is a soft closed neighborhood that is a soft LC-space. Conversely, let $(L,P)$ be a soft Lindelöf set in $\tilde{U}$ and let $\tilde{x} \tilde{\notin} (L,P)$. Choose a soft closed neighborhood $(W,P)_{\tilde{x}}$ of $\tilde{x}$ such that $((W,P)_{\tilde{x}}, \tilde{\tau}_{(W,p)_{\tilde{x}}}, P)$ is a soft LC-subspace. Then $(W,P)_{\tilde{x}} \tilde{\bigcap} (L,P)$ is soft Lindelöf in the soft subspace $((W,P)_{\tilde{x}}, \tilde{\tau}_{(W,p)_{\tilde{x}}}, P)$. Since $((W,P)_{\tilde{x}}, \tilde{\tau}_{(W,p)_{\tilde{x}}}, P)$ is a soft LC-space, therefore $(W,P)_{\tilde{x}} \tilde{\bigcap} (L,P)$ is soft closed in $((W,P)_{\tilde{x}}, \tilde{\tau}_{(W,p)_{\tilde{x}}}, P)$ and so also soft closed in $(U,\tilde{\tau},P)$. Hence $(W,P)_{\tilde{x}} - (W,P)_{\tilde{x}} \tilde{\bigcap} (L,P) = (W,P)_{\tilde{x}} - (L,P)$ is soft open neighborhood of $\tilde{x}$ in $(W,P)_{\tilde{x}}$ soft disjoint from $(L,P)$, that is $(L,P)$ is soft closed in $(W,P)_{\tilde{x}}$. Thus $(L,P)$ is soft closed in $(U,\tilde{\tau},P)$.

**Definition (2.7):** A soft topological space $(U,\tilde{\tau},P)$ is called a soft Q-set space if each soft subset of $\tilde{U}$ is a soft $F_\sigma$-closed set.

**Definition (2.8):** A soft topological space $(U,\tilde{\tau},P)$ is called a soft hereditarily Lindelöf if each soft subspace of $\tilde{U}$ is soft Lindelöf.

**Proposition (2.9):(i)** Every soft Q-set space is a soft $L_3$-space.

**(ii)** Every soft hereditarily Lindelöf $L_3$-space is a soft Q-set space.

**Proof: (i)** Let $(L,E)$ be a soft Lindelöf set in $\tilde{U}$, since $(U,\tilde{\tau},P)$ is a soft Q-set space, then $(L,P)$ is a soft $F_\sigma$-closed set. Thus $(U,\tilde{\tau},P)$ is a soft $L_3$-space.

**(ii)** Let $(L,P)$ be a soft subset of $\tilde{U}$, since $(U,\tilde{\tau},P)$ is a soft hereditarily Lindelöf, then $(L,P)$ is a soft Lindelöf set in $(U,\tilde{\tau},P)$ which is a soft $L_3$-space, then $(L,P)$ is a soft $F_\sigma$-closed set in $\tilde{U}$. Hence

$(U,\tilde{\tau},P)$ is a soft Q-set space.

**Corollary (2.10):(i)** Every soft Q-set space is a soft $\tilde{T}_1$-space.

**(ii)** Every soft hereditarily Lindelöf LC-space is a soft Q-set space.

**(iii)** Every soft $L_1$ Q-set space is a soft LC-space.

**(iv)** Every soft $L_3$ P-space is a soft LC-space.

**(v)** Every soft P Q-set space is a soft LC-space.

**Proof: (i)** If $(U,\tilde{\tau},P)$ is a soft Q-set space, then by proposition ((2.9),(i)), $(U,\tilde{\tau},P)$ is a soft $L_3$-space, hence $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space by theorem ((2.5), (v)).

**(ii)** Since $(U,\tilde{\tau},P)$ is a soft LC-space, then by theorem (2.5),(i)), $(U,\tilde{\tau},P)$ is a soft $L_3$-space, but $(U,\tilde{\tau},P)$ is a soft hereditarily Lindelöf, then by proposition ((2.9),(ii)), $(U,\tilde{\tau},P)$ is a soft Q-set space.

**(iii)** Since $(U,\tilde{\tau},P)$ is a soft Q-set space, then by proposition ((2.9),(i)), $(U,\tilde{\tau},P)$ is a soft $L_3$-space, since $(U,\tilde{\tau},P)$ is a soft $L_1$-space, then $(U,\tilde{\tau},P)$ is a soft LC-space by theorem (2.5),(ii)).

**(iv)** It is obvious.

**(v)** If $(L,P)$ is a soft Lindelöf set in $(U,\tilde{\tau},P)$ which is a soft Q-set space, then $(L,P)$ is a soft $F_\sigma$-closed set, but $(U,\tilde{\tau},P)$ is a soft P-space, so $(L,P)$ is a soft closed set. Hence $(U,\tilde{\tau},P)$ is a soft LC-space.

**Corollary (2.11):** Every soft $L_i$ Q-set space is soft hereditary, $i = 1,2,4$.

**Proof:** This is obvious by theorem ((2.5), (vii)) and definition (2.7).

**Corollary (2.12):** For a soft hereditarily Lindelöf P-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft Q-set space.

**Proof:** $(i) \to (ii)$: This is obvious by proposition ((2.9),(ii)).

$(ii) \to (i)$: This is obvious by corollary ((2.10),v).

**Definition (2.13):** A soft topological space $(U,\tilde{\tau},P)$ is called a soft KC-space if every soft compact subset of $\tilde{U}$ is soft closed.

**Proposition (2.14):**

**(i)** Every soft $\tilde{T}_2$-space is a soft KC-space.

**(ii)** Every soft KC-space is a soft $\tilde{T}_1$-space.

**Proof:** It is obvious.

**Proposition (2.15):**

**(i)** Every soft LC-space is a soft KC-space.

**(ii)** Every soft LC-space is a soft $\tilde{T}_1$-space.

**Proof:** It is obvious.

**Theorem (2.16):** Every soft $\tilde{T}_2$ P-space $(U,\tilde{\tau},P)$ is a soft LC-space.

**Proof:** Let $(L,P)$ be a soft Lindelöf set in $\tilde{U}$. To prove that $(L,P)$ is soft closed in $\tilde{U}$. Let $\tilde{x} \tilde{\in} (L,P)^c \Rightarrow \forall\, \tilde{y} \tilde{\in} (L,P)$, we get $\tilde{x} \neq \tilde{y}$, since $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space, then $\exists$ $(H,P)_{\tilde{x}}$ and $(K,P)_{\tilde{y}}$ are soft open sets in $\tilde{U}$ such that $\tilde{x} \tilde{\in} (H,P)_{\tilde{x}}$, $\tilde{y} \tilde{\in} (K,P)_{\tilde{y}}$ and $(H,P)_{\tilde{x}} \tilde{\bigcap} (K,P)_{\tilde{y}} = \tilde{\varphi}$. Hence $(L,P) \tilde{\subseteq} \bigcup_{\tilde{y} \tilde{\in} (L,P)} (K,P)_{\tilde{y}}$, thus $\{(K,P)_{\tilde{y}} : \tilde{y} \tilde{\in} (L,P)\}$ is a soft open cover of $(L,P)$. Since $(L,P)$ is soft Lindelöf $\Rightarrow$ $\exists \{(K,P)_{\tilde{y}_n}\}_{n \in N}$ is a countable soft subcover of $(L,P)$. Let $(W,P) = \bigcup_{n \in N} (K,P)_{\tilde{y}_n}$ and $(V,P) = \bigcap_{n \in N} (H,P)_{\tilde{x}_n} \Rightarrow (W,P)$ is soft open, since it is a soft union of soft open sets and $(V,P)$ is also soft open, since $(U,\tilde{\tau},P)$ is a soft P-space and soft intersection of a countable soft open sets is soft open. Hence $\tilde{x} \in (V,P)$ and $(L,P) \tilde{\subseteq} (W,P)$. To prove that $(V,P) \tilde{\cap} (W,P) = \tilde{\varphi}$. Since $(H,P)_{\tilde{x}_n} \tilde{\cap} (K,P)_{\tilde{y}_n} = \tilde{\varphi}$ , $\forall\, n \in N$ $\Rightarrow$ $(V,P) \tilde{\cap} (K,P)_{\tilde{y}_n} = \tilde{\varphi}$, $\forall\, n \in N$ $\Rightarrow$ $(V,P) \tilde{\cap} (W,P) = \tilde{\varphi}$ $\Rightarrow$ $(V,P) \tilde{\cap} (L,P) = \tilde{\varphi}$ $\Rightarrow$ $\tilde{x} \in (V,P) \tilde{\subseteq} (L,P)^c$ $\Rightarrow$ $(L,P)^c$ is soft open $\Rightarrow$ $(L,P)$ is soft closed. Thus $(U,\tilde{\tau},P)$ is a soft LC-space.

**Corollary (2.17):** Every soft $\tilde{T}_1$-regular P-space is a soft LC -space.

**Proof:** Let $\tilde{x}, \tilde{y} \tilde{\in} \tilde{U}$ such that $\tilde{x} \neq \tilde{y}$. Since $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space, then by theorem (1.8), $\{\tilde{x}\}$ is soft closed in $\tilde{U}$ and $\tilde{y} \tilde{\notin} \{\tilde{x}\}$. Since $(U,\tilde{\tau},P)$ is a soft regular space, then by definition (1.10), $\exists$ $(H,P)$ and $(K,P)$ are soft open sets in $\tilde{U}$ such that $\{\tilde{x}\} \tilde{\subseteq} (H,P)$, $\tilde{y} \tilde{\in} (K,P)$ and $(H,P) \tilde{\cap} (K,P) = \tilde{\varphi}$. Hence $\exists$ $(H,P)$ and $(K,P)$ are soft open sets in $\tilde{U}$ such that $\tilde{x} \tilde{\in} (H,P)$, $\tilde{y} \tilde{\in} (K,P)$ and $(H,P) \tilde{\cap} (K,P) = \tilde{\varphi}$ $\Rightarrow$ $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space, since $(U,\tilde{\tau},P)$ is a soft P-space, then by theorem (2.16), $(U,\tilde{\tau},P)$ is a soft LC-space.

**Proposition (2.18):** Countable soft union of soft Lindelöf sets is soft Lindelöf.

**Proof:** Let $\{(A_n,P)\}_{n\in N}$ be a countable family of soft Lindelöf sets in $\tilde{U}$. To prove that $\bigcup_{n\in N}(A_n,P)$ is soft Lindelöf. Let $\{(V_\alpha,P)\}_{\alpha\in\Lambda}$ be any soft open cover of $\bigcup_{n\in N}(A_n,P)$ $\Rightarrow \{(V_\alpha,P)\}_{\alpha\in\Lambda}$ is soft open cover of $(A_n,P), \forall n \in N$. Since $(A_n,P)$ is soft Lindelöf $\forall n \in N \Rightarrow \exists$ $\{(V_{\alpha_{nm}},P)\}_{m\in N}$ is a countable soft subcover $\forall n \in N$. That is $(A_n,P) \tilde{\subseteq} \bigcup_{m\in N}(V_{\alpha_{nm}},P), \forall n \in N$ $\Rightarrow$ $\bigcup_{n\in N}(A_n,P) \tilde{\subseteq}$ $\bigcup_{n\in N}(\bigcup_{m\in N}(V_{\alpha_{nm}},P)) = \bigcup_{n,m\in N}(V_{\alpha_{nm}},P)$. Since union of countable family of countable set is countable

$\Rightarrow \{(V_{\alpha_{nm}},P)\}_{n,m\in N}$ is a countable soft subcover of $\bigcup_{n\in N}(A_n,P)$ $\Rightarrow$ $\bigcup_{n\in N}(A_n,P)$ is soft Lindelöf.

**Proposition (2.19):** For a soft Lindelöf $\tilde{T}_2$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft P-space.

**Proof:** $(i) \rightarrow (ii)$: Let $(A,P)$ be a soft $F_\sigma$-closed set in $\tilde{U}$ $\Rightarrow$ $(A,P) = \bigcup_{n\in N}(F_n,P)$, where $(F_n,P)$ is soft closed in $\tilde{U}, \forall\, n \in N$. Since $(U,\tilde{\tau},P)$ is soft Lindelöf, then by theorem (1.15), $(F_n,P)$ is soft Lindelöf in $\tilde{U}, \forall\, n \in N$, hence by proposition (2.18), $(A,P) = \bigcup_{n\in N}(F_n,P)$ is soft Lindelöf in $\tilde{U}$, but $(U,\tilde{\tau},P)$ is a soft LC-space, then $(A,P)$ is soft closed in $\tilde{U}$. Thus $(U,\tilde{\tau},P)$ is a soft P-space.

$(ii) \rightarrow (i)$: This is obvious by theorem (2.16).

**Proposition (2.20):** For a soft Lindelöf Q-set space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft P-space.

**Proof:** $(i) \rightarrow (ii)$: This is obvious by proposition (2.19).

$(ii) \rightarrow (i)$: This is obvious by corollary ((2.10),v).

**Proposition (2.21):** For a soft regular P-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft KC-space.

**(iii)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space.

**Proof:** $(i)\rightarrow(ii)$: This is obvious by proposition ( 2.15),(i)).

$(ii)\rightarrow(i)$: Let $(U,\tilde{\tau},P)$ be a soft KC-space, then by proposition ((2.14), (ii)), $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space, but $(U,\tilde{\tau},P)$ is soft regular, then $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space. Since $(U,\tilde{\tau},P)$ is a soft P-space, then by theorem (2.16), $(U,\tilde{\tau},P)$ is a soft LC-space.

$(ii)\rightarrow(iii)$: This is obvious by proposition ((2.14),(ii)).

$(iii)\rightarrow(ii)$: Let $(U,\tilde{\tau},P)$ be a soft $\tilde{T}_1$-space, since $(U,\tilde{\tau},P)$ is soft regular, then $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space. Hence $(U,\tilde{\tau},P)$ is a soft KC-space by proposition ((2.14), (i)).

**Definition (2.22):** A soft topological space $(U,\tilde{\tau},P)$ is called a soft $R_1$-space if $\tilde{x}$ and $\tilde{y}$ have disjoint soft neighborhoods whenever $cl(\{\tilde{x}\})\neq cl(\{\tilde{y}\})$. Clearly a soft space is soft $\tilde{T}_2$ if and only if its soft $\tilde{T}_1$ and soft $R_1$.

**Theorem (2.23):** For a soft $R_1$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft KC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space.

**(iii)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space.

**Proof:** $(i)\rightarrow(ii)$**:** This is obvious by proposition ((2.14),(ii)).

$(ii)\rightarrow(i)$: Let $(U,\tilde{\tau},P)$ be a soft $\tilde{T}_1$-space, since $(U,\tilde{\tau},P)$ is a soft $R_1$-space, then $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space by definition (2.22). So $(U,\tilde{\tau},P)$ is a soft KC-space by proposition ((2.14),(i)).

$(ii)\rightarrow(iii)$**:** This is obvious by definition (2.22).

$(iii)\rightarrow(ii)$: It is obvious.

**Corollary (2.24):(i)** Every soft $R_1$KC-space is a soft $\tilde{T}_2$-space.

**(ii)** Every soft $R_1$Q-set space is a soft $\tilde{T}_2$-space.

**(iii)** Every soft $R_1$Q-set space is a soft KC-space.

**(iv)** Every soft $L_1$ $L_3$-space is a soft KC-space.

**(v)** Every soft $R_1$ $L_3$-space is a soft $\tilde{T}_2$-space.

**(vi)** Every soft $R_1$ $L_3$-space is a soft KC-space.

**Proof:** It is obvious.

**Corollary (2.25):** For a soft regular space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft KC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space.

**Proof:** $(i)\rightarrow(ii)$**:** This is obvious by proposition ((2.14),(ii)).

$(ii)\rightarrow(i)$: Let $(U,\tilde{\tau},P)$ be a soft $\tilde{T}_1$-space, since $(U,\tilde{\tau},P)$ is soft regular, then $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space. Hence by proposition ((2.14),(i)), $(U,\tilde{\tau},P)$ is a soft KC-space.

**Theorem (2.26):** For a soft $\tilde{T}_2$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft $L_1$-space and a soft $L_2$-space.

**Proof:** $(i)\rightarrow(ii)$**:** This is obvious by theorem ((2.5),(i)).

$(ii)\rightarrow(i)$**:** Let $(L,P)$ be a soft Lindelöf set in $\tilde{U}$ and $\tilde{x}\,\tilde{\notin}\,(L,P)$. To prove that $\tilde{x}\,\tilde{\notin}\,cl((L,P))$. Since $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space, then $\forall\ \tilde{y}\,\tilde{\in}\,(L,P)$, $\exists$ $(V,P)_{\tilde{y}}\,\tilde{\in}\,\tilde{\tau}$ such that $\tilde{y}\,\tilde{\in}\,(V,P)_{\tilde{y}}$ and

$\tilde{x} \tilde{\notin} cl((V,P)_{\tilde{y}})$. Hence $\{(V,P)_{\tilde{y}} : \tilde{y} \tilde{\in} (L,P)\}$ is a soft open cover of $(L,P)$. Since $(L,P)$ is soft Lindelöf $\Rightarrow \exists \{(V,P)_{\tilde{y}_n}\}_{n\in N}$ is a countable soft subcover of $(L,P)$. Thus $(L,P) \tilde{\subseteq} \bigcup_{n\in N}(V,P)_{\tilde{y}_n} \tilde{\subseteq} \bigcup_{n\in N} cl((V,P)_{\tilde{y}_n})$. For each $n \in N$, $(L,P) \tilde{\bigcap} cl((V,P)_{\tilde{y}_n})$ is soft Lindelöf. Since $(U,\tilde{\tau},P)$ is a soft $L_2$-space, then $cl[(L,P) \tilde{\bigcap} cl((V,P)_{\tilde{y}_n})]$ is soft Lindelöf. If $(W,P) = \bigcup_{n\in N} cl[(L,P) \tilde{\bigcap} cl((V,P)_{\tilde{y}_n})]$, then $(W,P)$ is soft Lindelöf $F_\sigma$-closed set in $\tilde{U}$, but $(U,\tilde{\tau},P)$ is a soft $L_1$-space, then $(W,P)$ is soft closed and $\tilde{x} \tilde{\notin} (W,P)$, hence $\tilde{x} \tilde{\notin} cl((L,P))$. Thus $(L,P)$ is a soft closed set in $\tilde{U}$. Therefore $(U,\tilde{\tau},P)$ is a soft LC-space.

**Corollary (2.27):** For a soft Lindelöf $\tilde{T}_2$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:
**(i)** $(U,\tilde{\tau},P)$ is a soft P-space .
**(ii)** $(U,\tilde{\tau},P)$ is a soft LC-space.
**(iii)** $(U,\tilde{\tau},P)$ is a soft $L_1$-space and a soft $L_2$-space.
**Proof:** This is obvious by proposition (2.19) and theorem (2.26).

**Corollary (2.28):** For a soft regular $L_1$ $L_2$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:
**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.
**(ii)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space.
**Proof:** $(i) \to (ii)$: This is obvious by proposition ((2.15),(ii)).
$(ii) \to (i)$: This is obvious by theorem (2.26).

**Corollary (2.29):** For a soft $R_1$ $L_1$ $L_2$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:
**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.
**(ii)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space.
**Proof:** $(i) \to (ii)$: This is obvious by proposition ((2.15), (ii)).
$(ii) \to (i)$: This is obvious by theorem (2.26).

**Corollary (2.30):** For a soft discrete space $(U,\tilde{\tau},P)$ the following statements are equivalent:
**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.
**(ii)** $(U,\tilde{\tau},P)$ is a soft $L_2$-space.
**Proof:** It is obvious.

**Theorem (2.31):** If $(U,\tilde{\tau},P)$ is a soft topological space and $\tilde{Y} \tilde{\subseteq} \tilde{U}$, $\tilde{Y} = \bigcup_{i=1}^{n} \tilde{Y}_i$, where $\tilde{Y}_i$, $i = 1,2,...,n$ are soft closed LC-subspaces of $\tilde{U}$, then $\tilde{Y}$ is a soft LC-subspace.
**Proof:** Let $(L,P)$ be a soft Lindelöf subset of $\tilde{Y}$, then $\tilde{Y}_i \tilde{\bigcap} (L,P)$, $i = 1,2,...,n$ are soft closed in $(L,P)$ which is soft Lindelöf so $\tilde{Y}_i \tilde{\bigcap} (L,P)$, $i = 1,2,...,n$ are soft Lindelöf subset of $\tilde{Y}_i$ $i = 1,2,...,n$. Since $\tilde{Y}_i$, $i = 1,2,...,n$ is a soft LC-subspace, then $\tilde{Y}_i \tilde{\bigcap} (L,P)$ is a soft closed in $\tilde{Y}_i$, $i = 1,2,...,n$. Since $\tilde{Y}_i$, $i = 1,2,...,n$ is soft closed in $\tilde{U}$, then $\tilde{Y}_i \tilde{\bigcap} (L,P)$, $i = 1,2,...,n$ is soft closed in $\tilde{U}$. But $(L,P) = \bigcup_{i=1}^{n} (\tilde{Y}_i \tilde{\bigcap} (L,P))$, so $(L,P)$ is soft closed in $\tilde{U}$ and also in $\tilde{Y}$. Hence $\tilde{Y}$ is a soft LC-subspace.

**Proposition (2.32):** Every soft Lindelof $L_1$-space is a soft P-space.
**Proof:** Let $(A,P)$ be a soft $F_\sigma$-closed set in $\tilde{U}$ $\Rightarrow$ $(A,P) = \bigcup_{n\in N}(F_n,P)$, where $(F_n,P)$ is soft closed in $\tilde{U}$, $\forall\, n \in N$. Since $(U,\tilde{\tau},P)$ is soft Lindelöf, then by theorem (1.15), $(F_n,P)$ is soft Lindelöf in

$\tilde{U}$, $\forall\ n \in N$. Hence $(A,P)=\bigcup_{n\in N}(F_n,P)$ is soft Lindelöf in $\tilde{U}$ by proposition (2.18). Since $(U,\tilde{\tau},P)$ is a soft $L_1$- space, then $(A,P)$ is soft closed in $\tilde{U}$. Thus $(U,\tilde{\tau},P)$ is a soft P-space.

**Proposition (2.33):** For a soft Lindelöf $\tilde{T}_2$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft $L_1$-space.

**Proof**: $(i)\rightarrow(ii)$: This is obvious by theorem ((2.5), (i)).

$(ii)\rightarrow(i)$: Let $(U,\tilde{\tau},P)$ be a soft $L_1$-space, since $(U,\tilde{\tau},P)$ is a soft Lindelöf space, then by proposition (2.32), $(U,\tilde{\tau},P)$ is a soft P-space. Since $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space, then $(U,\tilde{\tau},P)$ is a soft LC-space by theorem (2.16).

**Proposition (2.34):** For a soft Q-set space $(U,\tilde{\tau},P)$ having a soft dense Lindelöf subset the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft P-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft Lindelöf and a soft $L_1$-space.

**Proof:** $(i)\rightarrow(ii)$**:** If $(U,\tilde{\tau},P)$ is a soft P-space, then $(U,\tilde{\tau},P)$ is a soft $L_1$-space. Since $(U,\tilde{\tau},P)$ is a soft Q-set space, then by corollary ((2.10),(iii)), $(U,\tilde{\tau},P)$ is a soft LC-space, hence $(U,\tilde{\tau},P)$ is a soft $L_2$-space. Since $(U,\tilde{\tau},P)$ having a soft dense Lindelöf subset, then by theorem ((2.5),(vi)), $(U,\tilde{\tau},P)$ is a soft Lindelöf space.

$(ii)\rightarrow(i)$: This is obvious by proposition (2.32).

**Proposition (2.35):** For a soft $\tilde{T}_2$ $L_1$-space $(U,\tilde{\tau},P)$ having a soft dense Lindelöf subset the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft Lindelöf space.

**Proof:** $(i)\rightarrow(ii)$: If $(U,\tilde{\tau},P)$ is a soft LC-space, then $(U,\tilde{\tau},P)$ is a soft $L_2$-space. Since $(U,\tilde{\tau},P)$ having a soft dense Lindelöf subset, then by theorem (2.5),(vi)), $(U,\tilde{\tau},P)$ is a soft Lindelöf space.

$(ii)\rightarrow(i)$: Let $(U,\tilde{\tau},P)$ be a soft Lindelöf space, then by theorem ((2.5),(vi)), $(U,\tilde{\tau},P)$ is a soft $L_2$-space. Since $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$ $L_1$-space, then by theorem (2.26), $(U,\tilde{\tau},P)$ is a soft LC-space.

**Proposition (2.36):** For a soft Lindelöf Q-set space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft $L_1$-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft $L_2$-space and a soft P-space.

**Proof**: $(i)\rightarrow(ii)$: Let $(U,\tilde{\tau},P)$ be a soft $L_1$-space, since $(U,\tilde{\tau},P)$ is a soft Lindelöf space, then by proposition (2.32), $(U,\tilde{\tau},P)$ is a soft P-space. Since $(U,\tilde{\tau},P)$ is a soft Q-set space, then by corollary ((2.10),v), $(U,\tilde{\tau},P)$ is a soft $L_2$-space.

$(ii)\rightarrow(i)$: This is obvious by theorem ((2.5),(ix)).

**Theorem (2.37):** If $(U,\tilde{\tau},P)$ is a soft topological space and $\tilde{Y}\tilde{\subseteq}\tilde{U}$, $\tilde{Y}=\bigcup_{i=}^{n}\tilde{Y}_i$, where $\tilde{Y}_i$, $i=1,2,...,n$ are soft closed $L_2$-subspaces of $\tilde{U}$, then $\tilde{Y}$ is a soft $L_2$-subspace.

**Proof:** Let $(L,P)$ be a soft Lindelöf subset of $\tilde{Y}$, then $\tilde{Y}_i\tilde{\cap}(L,P)$, $i=1,2,...,n$ are soft closed in $(L,P)$ which is soft Lindelöf, so $\tilde{Y}_i\tilde{\cap}(L,P)$, $i=1,2,...,n$ are soft Lindelöf subset of $\tilde{Y}_i$ $i=1,2,...,n$. Since $\tilde{Y}_i$, $i=1,2,...,n$ is a soft $L_2$-subspace, then $cl(\tilde{Y}_i\tilde{\cap}(L,P))$ is a soft Lindelöf in $\tilde{Y}_i$, $i=1,2,...,n$. Hence $cl(\tilde{Y}_i\tilde{\cap}(L,P))$, $i=1,2,...,n$ is soft Lindelöf in $\tilde{Y}$. But

$cl((L,P)) = cl(\bigcup_{i=1}^{n}(\tilde{Y}_i \tilde{\cap} (L,P))) = \bigcup_{i=1}^{n} cl((\tilde{Y}_i \tilde{\cap} (L,P)))$, so $cl((L,P))$ is soft Lindelöf in $\tilde{Y}$. Hence $\tilde{Y}$ is a soft $L_2$-subspace.

**Theorem (2.38):** For a soft $R_1$P-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft KC-space.

**(iii)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space.

**(iv)** $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space.

**(v)** $(U,\tilde{\tau},P)$ is a soft $L_3$-space.

**Proof:** (i) $\rightarrow$ (ii): This is obvious by proposition ((2.15),(i)).

(ii) $\rightarrow$ (i): Let $(U,\tilde{\tau},P)$ be a soft KC-space, since $(U,\tilde{\tau},P)$ is a soft $R_1$-space, then by proposition ((2.14),(ii)) and definition (2.22), $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space, since $(U,\tilde{\tau},P)$ is a soft P-space, then by theorem (2.16), $(U,\tilde{\tau},P)$ is a soft LC-space.

(ii) $\rightarrow$ (iii): This is obvious by proposition ((2.14),(ii)).

(iii) $\rightarrow$ (ii): Let $(U,\tilde{\tau},P)$ be a soft $\tilde{T}_1$-space, since $(U,\tilde{\tau},P)$ is a soft $R_1$-space, then $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space by definition (2.22). So $(U,\tilde{\tau},P)$ is a soft KC-space by proposition ((2.14),(i)).

(iii) $\rightarrow$ (iv): Let $(U,\tilde{\tau},P)$ be a soft $\tilde{T}_1$-space, since $(U,\tilde{\tau},P)$ is a soft $R_1$-space, then $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space by definition (2.22).

(iv) $\rightarrow$ (iii): It is obvious.

(iv) $\rightarrow$ (v): Let $(U,\tilde{\tau},P)$ be a soft $\tilde{T}_2$-space, since $(U,\tilde{\tau},P)$ is a soft P-space, then $(U,\tilde{\tau},P)$ is a soft LC-space by theorem (2.16), so $(U,\tilde{\tau},P)$ is a soft $L_3$-space.

(v) $\rightarrow$ (iv)**:** Let $(U,\tilde{\tau},P)$ be a soft $L_3$-space, then $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_1$-space. Since $(U,\tilde{\tau},P)$ is a soft $R_1$-space, then by definition (2.22), $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space.

**Theorem (2.39):** For a soft $\tilde{T}_2$ $L_1$-space $(U,\tilde{\tau},P)$ the following statements are equivalent:

**(i)** $(U,\tilde{\tau},P)$ is a soft LC-space.

**(ii)** $(U,\tilde{\tau},P)$ is a soft $L_4$-space.

**(iii)** $(U,\tilde{\tau},P)$ is a soft $L_3$-space.

**(iv)** $(U,\tilde{\tau},P)$ is a soft $L_2$-space.

**Proof:** (i) $\rightarrow$ (ii): This is obvious by theorem ((2.5),(i)).

(ii) $\rightarrow$ (i): This is obvious by theorem ((2.5),(iii)) and theorem (2.26).

(ii) $\rightarrow$ (iii): Let $(U,\tilde{\tau},P)$ be a soft $L_4$-space, since $(U,\tilde{\tau},P)$ is a soft $L_1$-space, then $(U,\tilde{\tau},P)$ is a soft $L_2$-space by theorem ((2.5),(iii)). Since $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$-space, then by theorem (2.26), $(U,\tilde{\tau},P)$ is a soft LC-space. Hence $(U,\tilde{\tau},P)$ is a soft $L_3$-space by theorem ((2.5),(i)).

(iii) $\rightarrow$ (ii): This is obvious by theorem ((2.5),(iv)).

(iii) $\rightarrow$ (iv): Let $(U,\tilde{\tau},P)$ be a soft $L_3$-space, since $(U,\tilde{\tau},P)$ is a soft $L_1$-space, then by theorem ((2.5),(ii)), $(U,\tilde{\tau},P)$ is a soft LC-space. Hence $(U,\tilde{\tau},P)$ is a soft $L_2$-space by theorem ((2.5),(i)).

(iv) $\rightarrow$ (iii): Let $(U,\tilde{\tau},P)$ be a soft $L_2$-space, since $(U,\tilde{\tau},P)$ is a soft $\tilde{T}_2$ $L_1$-space, then $(U,\tilde{\tau},P)$ is a soft LC-space by theorem (2.26). Hence $(U,\tilde{\tau},P)$ is a soft $L_3$-space by theorem ((2.5),(i)).

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