



The Real and Imaginary Volume Integrals in Spherical-Statistical Optical Potential of Neutron scattering from ^{56}Fe nuclei

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Abstract

A spherical-statistical optical model (SOM) has been used to calculate and evaluate the neutron interaction with medium nuclei ($40 \leq A \leq 141$). Empirical formulae of the optical potentials parameters are predicted with minimize accuracy compared with experimental bench work data. With these optical formulae an evaluation of the shape and compound elastic scattering cross-section of interaction neutrons with ^{56}Fe nuclei at different energy range (1-20) MeV has been calculated and compared with experimental results. Also, volume integrals for real and imaginary potential energies have been evaluated and matched with the standard ABAREX code. Good agreements with have been achieved with the available experimental data.

Keywords: spherical-statistical optical model, shape and compound elastic scattering cross-section, volume integrals, neutron, ^{56}Fe nuclei, ABAREX code,

حجم التكامل الحقيقي والخيالي في الجهد البصري الكروي الاحصائي لتشتت النيوترونات من نوى الحديد

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الخلاصة

تم استخدام نموذج بصري الكروي الاحصائي لحساب وتقييم التفاعل النيوتروني مع نوى المتوسطة ($141 \geq A \geq 40$) وتثبيت توقع المعادلات التجريبية لمعاملات الجهد البصري مع تقليل الدقة عند مقارنة بيانات التجارب العملية الاخرى باستخدام هذه الصيغ البصرية. تم قيم الشكل والمقطع العرضي المركب لمرور التفاعل من النيوترونات مع نوى ^{56}Fe في نطاق طاقات مختلفة (20-1MeV) وتم المقارنة مع النتائج العملية. أيضا ، تم حساب تكاملات الحجم للطاقات المحتملة الحقيقية والخيالية ومقارنتها مع الشفرة القياسية ABAREX. كانت نتائج المقارنة مع النتائج العملية المتوفرة جيدة .

1. General Introduction:

The nuclear optical model has been implemented to study and analyze the fast neutron scattered elastically by different target nuclei. The investigations have shown that neutron elastic scattering cross section can be well fitted by this model with complex potential and suitably adjusted parameters that have been used and gave information around the energy and isospin dependence of the optical model [1]. This is analogy between scattering and absorption of particle by nucleus, as well as, the scattering and absorption of light nuclei by cloudy the crystal ball, that the reason it is called the optical model [2]. The former may be treated mathematically using a complex potential just as the latter may be treated using a complex refractive index. The model was first proposed by Serber [3] and

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used by Fernbach et al [4] to calculate the scattering and absorption of 90 MeV neutrons by a range of nuclei. For phenomenological analysis based on the optical model, the central term in the optical model described by $V(r) = Uf(r) + iw g(r)$, where the U and w are real and imaginary potential depths and the factors, $f(r)$ and $g(r)$, are the Saxon-Woods forms for volume and surface, where the real and imaginary potentials are represented the elastic scattering and absorption respectively. The elastic channel of the optical model characterize the target nuclei is to be spherical symmetric and the model is called the spherical model[5, 6]. It was possible to find a single optical model that gives the scattering probability for medium and heavy nuclei for neutron energy[2]. They used a non-local optical potential in general that gives the best results than those obtained with local potential.

A significant contribution to the optical model theory through the previous years can be considered the work of Mahaux and Co-workers on dispersion optical model analysis [7, 8]. The dispersion in optical potentials can be described the nuclear mean field between the negative energy (bound state) and positive energy (scatter state). Then, it is able to fit the experimental nuclear probabilities more accurately than the simple optical model.

2. Theoretical background of SOM

The nucleon – nucleus optical model potential is written in the form

$$V_{op} = U_R(r) + iU_I(r) + U_{so}(r) \quad (1)$$

where $U_R(r)$, $U_I(r)$ and $U_{so}(r)$ are the real, imaginary and spin orbit potentials, respectively.

The real central potential is generated by Woods-Saxon well, and effect for shape elastic scattering cross- section. The imaginary potential has combination of volume and surface terms. The volume term was considered and taken to have the same Woods –Saxon shape and geometric parameter as the real potential [9,10]. The absorption that occurred within the nuclear volume is represented by the derivative of Woods-Saxon form. The surface part dominant the low energy but volume part becomes important at high energy (more penetration).The spin orbit potential has Thomas form with the radial variation and the primary effect in the polarization of the scattered particle.

This potential can be written as follows [2]

$$V_{op}(r) = -V_R f(x) + \left(\frac{\hbar}{m\pi c}\right)^2 V_{SO}(\vec{l} \cdot \vec{\sigma}) \frac{1}{r} \frac{df(x_{so})}{dr} - i \left[W_v f(x_{IV}) - 4W_D \frac{df(x_{ID})}{dx_{ID}} \right] \quad (2)$$

where V_R , W_v , W_D and V_{SO} are the real, imaginary volume, imaginary surface and spin-orbit potentials, respectively, and $\left[\frac{\hbar}{m_0 c}\right]^2$ is the square of pion-Compton wavelength $\cong 2\text{fm}$, the quantity $(\vec{l} \cdot \vec{\sigma})$ is the scalar product of the orbital and intrinsic angular momentum operators and given by [11], $\vec{l} \cdot \vec{\sigma} = 1$ for $j = l + 1/2$ is parallel, $\vec{l} \cdot \vec{\sigma} = -(l + 1)$ For $j = l - 1/2$ is anti-parallel, and $f(x)$, $f(x_{so})$, $f(x_{IV})$ and $f(x_{ID})$ are the radial dependent form factor for the real, spin-orbit, volume and surface terms respectively. These form factors can be defined [2]:

$$f(r, R, a) = \left[1 + e^{\frac{r-R}{a}} \right]^{-1}, \quad (3)$$

where R is the nuclear radius, $R = r_0 A^{1/3}$ and a the surface diffuseness parameter.

The Hauser-Feshbach theory, which is an extension of Wolfenstein's works or splitting the compound cross section into the elastic channel and other channel, where the nucleus is primarily left in an excited state and the neutron, is emitted with reduced energy[12,13].

The absorption cross-section can be written as:

$$\sigma_{abs} = \frac{1}{2} \pi \lambda^2 \sum_{j, \bar{l}} (2j + 1) T_{j, \bar{l}} \quad (4)$$

The transmission coefficient, $T_{j, \bar{l}} = 1 - \left\{ (n_{j, \bar{l}}^R)^2 + (n_{j, \bar{l}}^I)^2 \right\}$ for the l^{th} partial wave, and, $n_{j, \bar{l}}^R$ and $n_{j, \bar{l}}^I$ are the Bessel functions of the second type or is called Neumann function.

It can see from equation (4) the probability of capturing (absorbing) a neutron with energy E , angular momentum j and parity l is proportional to the transmission coefficient $T_{j, l}(E)$. By reciprocity, the probability of emission of the neutron with \tilde{E} , \tilde{j} and \tilde{l} will be proportional to $T_{\tilde{j}, \tilde{l}}(\tilde{E})$. Consequently the probability of emission into the channel $(\tilde{j}, \tilde{l}, \tilde{E})$ when the neutron is capture in the channel (j, l, E) is proportional to the product of these two transmission coefficient, provided the total angular momentum and parity of the nucleus plus neutron are the same in the initial and final states.

The volume integrals are relatively invariant functions of the OMP parameters and give insight on the behavior of the optical potentials as function of mass, energy and nuclear asymmetry. They are particularly useful in the sense that contributions from the well depth and the geometry parameters are included [14, 15]. With phenomenological determined OMPs, the energy or mass dependence of the potential depths may be compensated by that of the geometry parameters, thereby masking particular structure effects [16, 17].

The volume integrals for present real volume per nucleon (J_R/A), imaginary volume ($J_{W_{V,D}}/A$) and imaginary surface ($J_{W_{SO}}/A$) potential parts are defined and when $V_R(r)$ is the Woods-Saxon potential given by equation (3):

$$\frac{J_{V_R}}{A} = \frac{4\pi}{A} \int_0^\infty V_R(r) r^2 dr = \frac{4\pi}{3A} r_R^3 \left[1 + \left(\frac{\pi a_R}{r_R} \right)^2 \right] \quad (5)$$

$$\frac{J_{W_{V,D}}}{A} = \frac{4\pi}{A} \int_0^\infty w_{V,D}(r) r^2 dr = \frac{4\pi}{3A} r_{wv}^3 \left[1 + \left(\frac{\pi a_{V,D}}{r_{V,D}} \right)^2 \right] \quad (6)$$

$$\frac{J_{W_{SO}}}{A} = \frac{4\pi}{A} \int_0^\infty w_{SO}(r) r^2 dr = \frac{16\pi}{A} a_{SO} R_{SO}^2 \left[1 + \frac{1}{3} \left(\frac{\pi a_{SO}}{R_{SO}} \right)^2 \right] \quad (7)$$

3. Results, discussion and conclusion:

The SOM have been used to analyze the optical model parameters (OMPs) for neutron energy range 1-20 MeV. The theoretical differential elastic cross-section for ^{56}Fe nuclei has been calculated at different neutron energy rang using ABAREK code [18]. As shown in Figures-(1 and 2), the relation between the differential elastic scattering cross-sections at different scattering angles. At backward scattering angles, $> 90^\circ$, the neutron energy reduced to the energy level value during the collision in with nucleus, this caused the nucleon at higher excited state can share absorption and its energy effects the differential elastic cross-section at these scattering angles. As shown in the Figures-1 and 2 around ($30^\circ - 70^\circ$) scattering angle the theoretical calculate of differential elastic cross-section for shape elastic on ^{56}Fe at energy (6.96, 11.93, 13.92, 20) MeV indicates large compared with experimental data difference in value due the selected potential parameters (real, imaginary and spin orbit) terms and one can noticed the forwards direction a good agree with experimental results, while the compound elastic cross section still constant not change. Also, it shows that both direction and compound nucleus processes can contribute to any reaction. As the neutron energy increased, the compound elastic cross section rapidly becomes negligible compared with the direct or shapes elastic cross section, so in many cases it is sufficient to ignore the compound elastic contribution.

The calculate potentials and geometric parameters for ^{56}Fe nuclei and neutron energy range to evaluate and compare the present result with else the present produce started with a subject examination of the optical model parameter of Becchetti and Greenless [19].

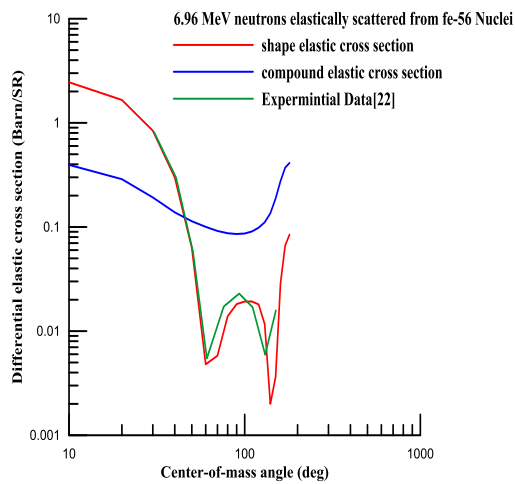
The predicted OMPs are found nuclear asymmetry, $\xi = (N - Z/A)$, and neutron energy (E) dependence, while the other optical parameters are fixed as shown in Table-1. These parameters differ from [20, 21]. Also, from the analysis of the OMPs, it concluded that the imaginary potential W_1 value, which has the Saxon-Woods derivative forms, is a nuclear asymmetry and neutron energy dependence rather than neutron energy dependence mentioned by (Hodgson) [2].

This conclusion also withdraws for the dependence of W_{SO} parameters on neutron energy and nuclear asymmetry for unaffected analysis.

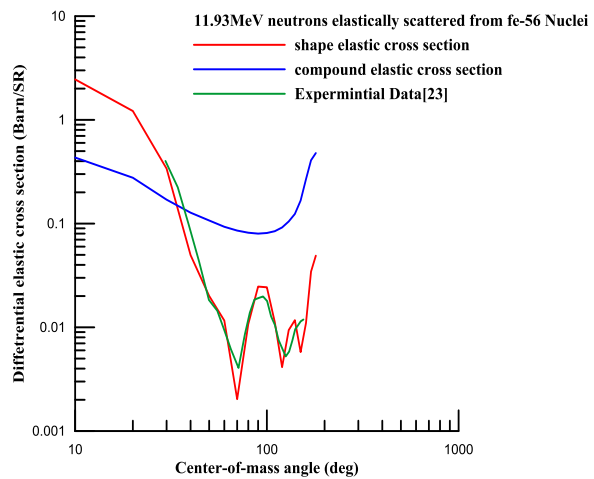
As shows in Figure-3 the relationship between the volumes integral predicted compared with calculated result that obtained from SOM at different neutron energy range, below 20MeV for ^{56}Fe nuclei, and an acceptable agreement in the case of the real potential volume integral in [18].

Table 1-The neutron energy and nuclear asymmetry dependence for empirical form parameters in calculated in terms of SOM, for target masses $40 \leq A \leq 141$ amu [21].

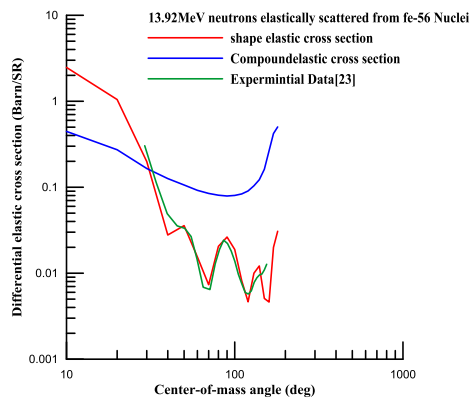
Empirical formula	F-Ratio	R^2
$V_R = 51.523 - (0.32 \pm 0.133) E_n - (42.581 \pm 22.335) \xi + (0.0581 \pm 0.032) A$ $r_R = 1.17 \text{ fm}, a_R = 0.75 \text{ fm}$	1.90	2.0
$W_I = 6.672 + (0.173 \pm 0.069) E_n + (33.209 \pm 11.518) \xi - (0.059 \pm 0.016) A$ $r_I = 1.26 \text{ fm}, a_I = 0.58 \text{ fm}$	5.94	1.0
$V_{SO} = 6.2 \text{ MeV}, r_I = 1.1 \text{ fm}, a_R = 0.75 \text{ fm}$		
$J_R = 476.107 - (2.125 \pm 0.708) E_n - (132.004 \pm 175.065) \xi - (0.362 \pm 0.284) A$ $(\text{MeV} \cdot \text{fm})^3$	13.99	20.0
$J_I = 100.078 + (1.963 \pm 0.463) E_n + (303.853 \pm 101.63) \xi - (0.855 \pm 0.162) A$ $(\text{MeV} \cdot \text{fm})^3$	27.78	12.0



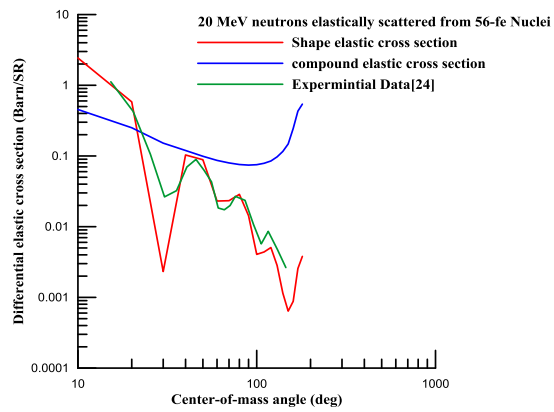
A



B



C



D

Figure 1-The calculated differential elastic scattering cross-sections of A-6.96MeV, B-11.93MeV, C-13.92MeV and D-20 MeV neutron by ^{56}Fe nuclei using the present predicted optical potential

parameters in SOM compared with experimental results , 6.96MeV [22], 11.93MeV,13.92MeV[23] and 20MeV[24].

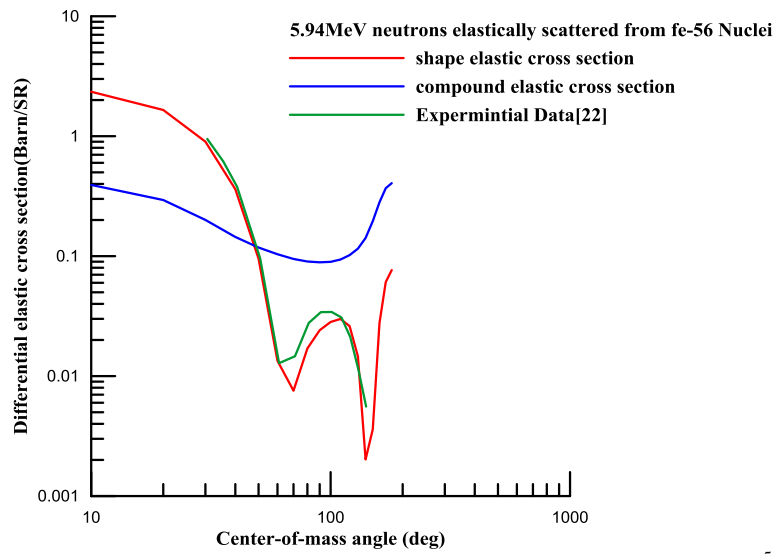


Figure 2-the differential elastic scattering cross-sections of 5.49 MeV neutron on ⁵⁶Fe, compared with present theoretical calculations using neutron SOM.

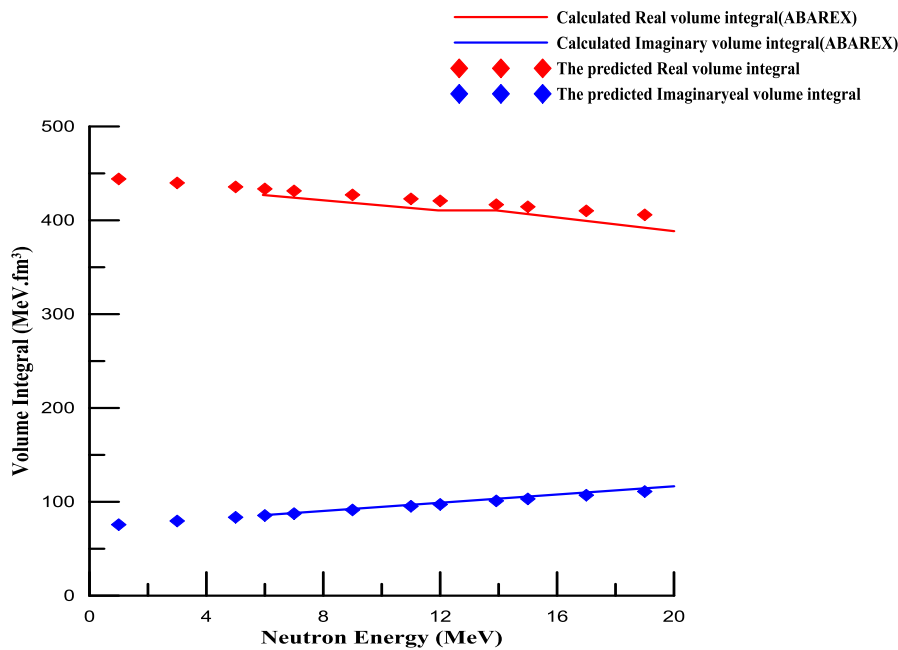


Figure 3-The predicted real and imaginary volume integrals as a function of neutron energy on ⁵⁶Fe nuclei compared with theoretical calculation using the SOM in [19].

References

1. Perey, F. G. **1963**. Optical-Model Analysis of Proton Elastic Scattering in the Range of 9 to 22 MeV, *Phys. Rev.* **131**(2): 745.
2. Hodgson, P.E. **1971**. "Nuclear reaction and nuclear structure", Clarendon press, Oxford.
3. Serber, R.D. **1947**. The production of high energy neutron by stripping, *Phys. Rev.* **72**: 1008.
4. Fernbach, S., Serber, R.D. and Taylor, T.B. **1949**. The scattering of high energy neutrons by nuclei, *Phys.Rev.* **75**: 1352.
5. Kang, A.J. and Delarache, J.P. **2003**. Local and global nucleon optical models from 1 keV to 200 MeV, *Nuclear Physic A*, **713**: 231- 310.
6. Hodgson, P.E. **1963**. *The optical model of elastic scattering*, Clarendon press Oxford.

7. Mahaux, C. and Ngo, H. **1984**. Effective masses, occupation probabilities and quasiparticle strengths in ^{208}Pb , *Nucl. Phys. A*, **431**: 486.
8. Negele, J.W. and Vogt, E. **1991**. *Advances in nuclear physics*, Plenum, New York.
9. Hodgson, P. E. **1994**. *The nuclear optical model*, world scientific, Singapore.
10. Mahaux, C., Ngo, H. and Satcher, G.R. **1986**. Causality and the threshold anomaly of the nucleus-nucleus potential, *Nucl. Phys. A*, **449**: 344.
11. Kang, A.J. and Delarache, J.P. **2003**. Local and global nucleon optical models from 1 keV to 200 MeV, *Nuclear physic A*, **713**: 231- 310.
12. Myers, W.F. and Schmidt, K.H. **1983**. "An update on droplet-model charge distributions, *Nuclear Physics, A*, **410**: 61.
13. Hauser, W. and Feshbach, H. **1952**. The Inelastic Scattering of Neutrons, *Phys. Rev.* **87**: 336.
14. Wolfenstein, L. **1951**. Conservation of Angular Momentum in the Statistical Theory of Nuclear Reactions, *Phys. Rev.* **82**: 690.
15. Mahaux, C. and Sartor, R. **1991**. Single-Particle Motion in Nuclei, *Adv. Nucl. Phys.* **20**:1.
16. Zureikat, R.A. and Jahgoub, M.I. **2013**. Surface and volume term non localities in the proton-nucleus elastic scattering process', *Nucl. Phys. A*, **916**: 183.
17. Koning, A.J. and Delaroche, J.P. **2003**. Local and global nucleon optical models from 1 keV to 200 MeV, *Nucl. Phys. A*, **713**: 231–310.
18. Rapaport, J. **1982**. An optical model analysis of neutron scattering, *Phys. Rep.* **87**(2): 27–75.
19. Lawson, R.D. and Smith, A.B. **1999**. A Neutron Spherical Optical-Statistical-Model Code-ABERAX, Argonne National laboratory, June.
20. Becchetti, F.D. and Greenless, G.W. **1969**. Nucleon-Nucleus Optical-Model, $A>40, E<50$ MeV. *Phys. Rev.* **182**: 1190
21. Engelbrecht, C.A. and Fiedeldey, H. **1967**. Nonlocal potentials and the energy dependence of the optical model for neutrons. *Ann. Phys.* **42**(2): 262-295.
22. Jasim, M.H. **2014**. Optimization the Spherical-Statistical Optical Model parameters for fast neutron scattered from ^{90}Zr and ^{138}Ba Nuclei, *Asian J.Appl.Sci.Eng.V*: **3**: 413-420.
23. Ramirez, A.P.D., McEllistern, M.T., Mukhopadhyay, S., Peters, E.E., Yates, S.W., Vanhoy, J.R., Harrison, T.D., Thomas, B.K., Hickks, S.E., Howard, T.J., Jackson, D.T., Lenzen, D., Nguyen, T.D. and Pecha, R.L. **2017**. Neutron cross section measurement for ^{56}Fe , *Phys. Rev. C*, **95**, 064605
24. EL-Kadi, S.M., Nelson, C.E., Purser, F.O., Walter, R.L., Beyerle, A., Gould, C.R., Seagondllar, L.W. **2012**. Elastic and inelastic scattering of neutron from $^{54-56}\text{Fe}$ and $^{63,65}\text{Cu}$, *J,NP/A*,**390**: 509.
25. Mellema, S., Finlay, R.W., Dietrich, F.S. and Petrovich, F. **2008**. Microscopic and conventional optical model analyse of fast neutron scattering from $^{54-56}\text{Fe}$, *J/PR/C* **28**, 2267,198312.