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ON A Topological Γ - Rings

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Abstract
In this study we introduce the concepts of topological Γ-ring as generalization of Topological ring. We also present and study the concepts of topological Γ-ring, Homomorphism of topological Γ-ring, and compact of topological Γ-ring. The results have confirmed that: if (G, +, . ,$\mathcal{J}$) is a compact topological Γ-ring and f : (G,+ ,. ,$\mathcal{J}$)→(G^*,+',.', T^*) epimorphism topological Γ-ring Then G^* is compact.

Keywords: topological Γ-ring, norm, homomorphism and compact.

حول الحلقات التبولوجية من النمط Γ −

بشرى جارالله توفيق
قسم الرياضيات, كلية التربية, الجامعة المستنصرية, بغداد, العراق

الخلاصه
في البحث قدمنا مفهوم تبولوجيه الحلقات من النمط كاما كتعميم لمفهوم تبولوجيا الحلقات حيث قدمنا ودرسنا مفهوم تبولوجيا الحلقات من النمط كاما وتشاكل تبولوجيا الحلقات من النمط كاما وكذلك تراص تبولوجيا الحلقات من النمط كاما كذلك داله f:G⟶G* تشاكل تبولوجي حلقي من النمط كاما وكانت G متراصه فان G*متراصه.

1. INTRODUCTION

The concepts of topological ring is one of the most important topics in topological algebra. The objective of the concept of topological Γ-ring is to generalize the definition of topological ring.

The concept of Γ-ring was presented by Nobusawa **1964 in [1],** and it was generalized by Barnes **1966 [2]** as below:

Let M and Γ be two additive abelian groups. Suppose that there is a mapping from M×Γ×M→M, the image of (a,α,b) is denoted by aαb, a,b M and α Γ, that satisfies the following for all a,b,c M, α,β Γ:

i) (a+b)αc= aαc+ bαc
a(α +β)c = aαc + aβc
aα(b +c) = aα b + aα c
ii) (aα b)βc = aα(bβc)

Then M is called a Γ-ring., where every ring is a Γ-ring. M is said to be 2-torsion free if 2a = 0 implies a=0 for all a∈ M. Besides, M is called a prime Γ-ring if for all a,b M, aMΓMb = (0) implies either a=0 or b=0, and M is called a semiprime if aMΓMa=(0) with a∈M implies a=0. Note that every prime Γ-ring is obviously a semiprime [3], [4] ,[5-10].

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In this paper we present the concept of topological Γ-ring as generalization of topological ring where we get every topological ring is topological Γ-ring. In general the converse is not true .

In this paper we introduce and study the concepts of topological Γ-ring, norm, homomorphism of topological Γ-ring some type of homomorphism kernel of homomorphism, and natural mapping as well as we present the compact of topological Γ-rings. We also prove that: If f:(G,+, . , T) → ($G_1, +', .', T^*$) is homomorphism of topological Γ-ring G onto topological Γ-ring G_1,then

1)If I is an ideal of topological Γ-ring of G then f(I) is an ideal of topological Γ- ring G_1.

2) If J is an ideal of topological Γ-ring of G_1 then f^{-1} (J) is an ideal of topological Γ-ring G.

The results have confirmed that: If f:(G,+, . , $\mathcal{J}$)→ (G^*,+',.', T^*) is isomorphism topological Γ-ring from G into compact topological Γ-ring G^* then G is compact topological Γ-ring.

2. Topological Γ- Rings:

Basic concepts related to the talked topic are given as follows with few important properties has been presented in the next section.

Definition (2-1): A topology τ on a Γ-ring G is a topological Γ-ring, which is denoted by (G, +, . , τ), iff:

i)(m,n)→m+n is continuous from G×G into G

ii)(m,y)$_\alpha$ → m α n is continuous from G×G into G

iii)m →- m is continuous from G×G into G

where G is given topology τ and G×G the Cartesian product topology determined by τ.

Example (2-2) : Let G be any Γ-ring andτ be discrete topology on G then (G,+, . ,τ) is topology Γ-ring, which is called discrete topological Γ-ring

Example (2-3) : Let G be any Γ-ring andτ be indiscrete topology on G then (G,+, . ,τ) is topology Γ-ring,which is called indiscrete topological Γ-ring

Example (2-4) : Let R be the set of all real numbers and Q be the set of all rational numbers then R is Q-ring if τ is usual topology on R ,and then (R,+, . , τ) is topological Q–ring, which is called usual topological Γ-ring. By Definition (2.1), G=R and Γ= Q, also the operations +, · and – are continuous from $R \times R$ into R.

Since every ring is Γ-ring, then we can get the following lemma

Lemma (2-5): Every topological ring is topological Γ-ring

The next example shows that the converse of Lemma (2-5) is not true in general.

Example (2-6):Let G =$\left\{\begin{pmatrix} a & b \\ c & d \\ s & t \end{pmatrix} : a, b, c, d, s, t \in Z\right\}$ and Γ= $\left\{\begin{pmatrix} x & y & z \\ w & u & v \end{pmatrix} : x.y.z.w.u.v \in Z\right\}$

then G is Γ-ring, if we define the discrete topology τ on G then (G, +, . , τ) is topological Γ-ring. However (G, +, . , τ) is not topological ring .

Definition (2-7): A function N from a Γ-ring G into $R^+ \cup \{0\}$ is a norm if the following conditions hold for all p, hϵG and $\alpha \epsilon$Γ:

1)N(0)=0

2) $N(p+h)$ ≤N(P)+N (h)

3) N(-p)=N(p)

4) N(pαh) ≤N(p)αN(h)

5) N(p) =0 if and only if p=0.

Remark (2-8): If N is a norm on a Γ- ring G, then d defined by d(p,h) = N(p-h) for all p,$h\epsilon$G and αϵΓ, is a metric. Indeed (1) and (5) imply that d(p,h)=0 iff p=h, (3) implies that d(p,h)= d(h,p) , and (2) yields the triangle inequality.

Theorem (2.8): Let N be a norm on a Γ-ring. The topology is given by the metric d which is defined by N is a Γ- ring topology

Proof: Let a, b, p, h ∈G and α ∈Γ.

d(p+h, a+b)= N((p+h)-(a+b))
= N((p-a)+(h-b))
$\leq$ N(p-a) + N(h-b)
=d(p,a) + d(h-b)
Hence the definition (2-1) (1) holds, for all a , b ϵG
d(-p,-a) = N(-p+a)
= N(p-a)
=d(p,a)
By (3) of definition 2-6-hence the definition 2-1 (2) holds. Finally for all p, aϵG
d(pαh,aαb)= N((p-a) α (h-b)+aα(h-b)+(p-a)ααb)
$\leq N$(p-a)αN(h-b)+N(a)αN(h-b)+N(p-a)αN(b)
Hence the definition 2-1 (3) holds, and the proof is finished
Let f be a partial function from the carrier of S to R, we recall that f is said to be uniformly continuous on X if and only if the following conditions are satisfied (i) X ⊆ dom f, and (ii) for every r such that 0 < r there exists s such that 0 < s and for all x_1, x_2 such that x_1, x_2 ∈ X and $\|x1 - x2\|$ < s holds $|f(x_1) - f(x_2)| < r$.) from G into $R^+ \cup \{0\}$.

Theorem (2-9): Let N be a norm on a Γ-ring G. for all p,hϵG, then$|N(p) - N(h)| \leq N(p-h)$,and hence N is a uniformly continuous function .

Proof: N(p) = N((p-h)+h)
$\leq$N(p-h)+N(h)
Then $N(h) - N(p) \leq N(h-p) = N(p-h)$
Therefore $|N(p) - N(h)| \leq N(p-h)$
Definition(2-10):Let (G, +, . , τ) be topological Γ-ring. A basis of τ is a collection $\mathcal{B}$ of τ U = ∪{B:B∈$\mathcal{B}_1$}.

Theorem(2-11): The Cartesian product of topological Γ-rings is a topological Γ-ring .

Proof: `Let (G, +, . , τ) and (G_1, +, . , τ_1) are two topological spaces and $\mathcal{B}$ be a basis of τ and $\mathcal{C}$ be a basis for $\tau_1$then
$\mathcal{H}$={B×C: B∈$\mathcal{B}$ and C∈$\mathcal{C}$} is a basis for the product topology (G×G_1 ,+,., $\mathcal{W}$)
Let (x,y) be any point of G×$G_1$and $\mathcal{N}$be a neighborhood of (x,y)
Since E={X×Y: X∈ τ and Y∈ τ_1 } is a basis for $\mathcal{W}$ there exists a member
X×Y of E such that (x,y)∈X×Y⊆$\mathcal{N}$ ....(1)
Since X is τ-open and $\mathcal{B}$ is a basis for τ there exists some B∈$\mathcal{B}$ such that
x∈B⊆X. Similarly there exists some C∈$\mathcal{C}$ such that y∈C⊆Y. It follows that
(x,y)∈ B×C ⊆ X×Y …..(2)
Hence from (1) and (2) , we get (x,y) ∈ B×C⊆$\mathcal{N}$ .This implies that $\mathcal{H}$ is basis for G×G_1

3. HOMOMORPHISM of TOPOLOGICAL Γ- RINGS:

We introduce and study in this section the following concepts subtopological Γ-ring, ideal of topological Γ-ring, and homomorphism of topological Γ-ring as well as some properties of them are given.

Definition (3-1) : Let (G,+,. , T) be topological Γ-ring a subring S of topological Γ- ring is called topological Γ-subring (or called subtopological Γ-ring) denoted by (S,+,. , T_S) where $A_S \in T_S$ $A_S = A \cap S$, for all $A \in T$.
Every topological Γ-ring (G,+,. , T) have two topologica Γ-subrings (0) and G itself which is called trivial topologica Γ-subrings. Any topological Γ-subring except trivial is called proper topological Γ-subring.

Definition (3-2): A topological Γ-subring (I , +, ., T) of topological Γ-ring (G,+, . ,T) is called an ideal of (G,+, . ,T) if IΓG⊆I and GΓI⊆I.

Definition (3-3): Let (G,+,. , T) and (G_1, $+'$, $.'$, T^*)be two topological Γ- rings.
A map f: (G,+,. , T) → (G_1, $+'$, $.'$, T^*) is called homomorphism of topological Γ-ring , if

f:(G, T)→ (G_1, T^*) is continuous and f satisfies:
$f(p+h) = f(p) +' f(h)$,and $f(p.\alpha.h) = f(p).'\alpha.'f(h)$, for all p,h ϵ G and αϵΓ.
An epimorphism topological Γ-ring if f is continuous map and epimorphism Γ-ring, a monomorphism topological Γ- ring is a continuous mapping and monomorphism Γ-ring. An isomorphism topological Γ-ring is a homeomorphism topological map (A mapping f:(X,T_Y)⟶(Y,T_Y) is called homeomorphism if f is 1-1 and onto also f and f^{-1} are continuous) and isomorphic Γ- rings, an automrphism topological Γ-ring is a continuous mapping and automorphism Γ-ring.

Lemma (3-4): Let f: (G,+,. , T) →($G_1, +́, .́, T^*$)be isomomorphism of topological Γ -ring then**:**
1) If S is a subtopological Γ- ring of , then$f$(S) is a subtopological Γ-ring of G_1.
2) If E is a subtopologicalΓ-ring of G_1,then f^{-1}(E) is a subtopologicalΓ-ring of G.
proof : We prove (2) and (1) by using the same technical
2) By assumption we get
(E,+,., $T^*{}_E$) is subtopologicalΓ-ring of ($G_1, +́, .́, T^*$)
Since f is homomorphism
Therefore for all E∈ $T^*{}_E$ thenf^{-1}(E)∈ $T_{f^{-1}(s_E)}$
Now, let p,h ∈ f^{-1}(E) then f(p), 22222a f(h)∈E and αϵΓ.
i)f(p-h)= f(p) – f(h)
Since f(p) , f(h)ϵE and E is subtopologicalΓ-ring then f (p) – f(h)∈E, and hence p-h∈ f^{-1}(E)
ii)f(pα h) = f(p)αf(h)
Since f(p), f(h)∈E, αϵΓ and E is subtopological Γ- ring then f(p)αf(h)∈E hence pαh ϵ f^{-1}(E) . Therefore f^{-1}(E) is subtopological Γ- ring.

Lemma (3-5) : If f:(G,+, . , T) → ($G_1, +́, .́, T^*$) is homomorphism of topological Γ-ring G onto topological Γ-ring G_1,then
1)If I is an ideal of topological Γ-ring of G then f(I) is an ideal of topological Γ- ring G_1.
2) If J is an ideal of topological Γ-ring of G_1 then f^{-1} (J) is an ideal of topological Γ-ring G.
Proof:1) (i) By theorem (3-4) f(I) is a subtopological Γ-ring
(i.e. f(I) ≠∅ and a-b∈f(I), ∀a,b∈f(I)**)**
(ii) Let ŕ ∈ G_1 and for all a ∈ f(I), and α∈Γ.
T.P. $a .́ \alpha' .́ r'$ ∈ f(I) and ŕ .́ **α**. .́ a ∈ f(I)
∵ f is onto , then ∃ r ∈G s.t. , ŕ = f(r) ⋯ ⋯ ∗
and ∃ x ∈ I . s. t. a = f(x) ⋯ ⋯ ∗∗
since, r ∈ R , x ∈ I and since I is an ideal of a ring
⟹ x . α.r ∈ I and r . α.x ∈ I
⟹ f(x .α. r) ∈ f(I) and f(r .α. x) ∈ f(I)
⟹ f(x) .́α .́ f(r) ∈ f(I) and f(r) .́ α .́ f(x) ∈ f(I)
⟹ a .́ α .́ ŕ ∈ f(I) and ŕ .́ α .́ a ∈ f(I)
Therefore, f(I) is an ideal of topological Γ-ring G_1.
2)By using the same technic of (1) we get the require result.

Theorem (3-6): If f:(M,+, . ,T) → ($M_1, +́, .́, T_1$) is homomorphism of topological Γ-ring M onto topological Γ-ring M_1 then every homomorphism image of
1) If M is commutative topological Γ-ring then M_1 is a commutative topological Γ-ring.
2) If M is topological Γ-ring with identity 1 then M_1 is a topological Γ-ring with identity 1′.
Proof: Since f is a homomorphism from a topological Γ-ring (M,+,.,T) into a topological Γ-ring (M_1,+', .' ,T_1).
1) Suppose that (M,+, . ,T) be a commutative topological Γ-ring
Let a , b ∈ M_1 , αϵΓ and since f is onto ,

Then , $\exists$ x , y $\in$ M , s .t. , a = f(x) and b = f(y)
$\therefore$ a α b = f(x)αf(y) = f (x α y) (since f is homo.)
= f(y αx) (since M is a comm. topological Γ-ring)
= f(y) .، f(x) (since f is homo.)
= b α a
Hence , M_1 is a commutative topological Γ-ring
2) Suppose that M be a topological Γ-ring with identity
Let a $\in M_1$ since f is onto therefore $\exists$ x $\in$M s.t. a=f(x)
x α1 = 1α x =x , $\forall$x $\in$ M, αϵΓ.
$\Longrightarrow$ f(x α1)= f(1αx) =f(x)
$\Longrightarrow$ f(x) α f(1) = f(1) α f(x) = f(x) [f is homomorphism]
$\Longrightarrow$ a α $1'$ = $1'$α a = a , for all a $\in M_1$.
Thus, M_1 is a topological Γ-ring with identity $1'$.
Definition (3-7): Let f : (M,+,.,T) $\rightarrow$ $(M_1, \acute{+}, \acute{.}, T_1)$ is homomorphism of topological Γ-ring s ,then the kernel of f is denoted by ker. (f), and it is defined by
ker. (f) = { $m\epsilon$M: f(m) = 0_{M_1}}
Lemma (3-8): If f: (G,+, . , T) → $(G^*, \acute{+}, \acute{.}, T^*)$ is homomorphism of topological Γ-ring G into topological Γ-ring G^*, then the ker.(f) is an ideal of topological Γ-ring G.
Proof: 1)Let p, h$\in$ ker. (f) and f is homomorphism of topological Γ-ring .,
we get f(p − h) = f(p) −f(h) = 0_{G^*}
then p-h$\in$ ker. (f)
2)Let p, h$\in$ ker. (f),αϵΓ and f is homomorphism of topological Γ-ring
we get f(pαh) = f(p)αf(h) = 0_{G^*}
then pαh$\in$ ker. (f)
Hence ker. (f) is an ideal of topological Γ-ring (G,+, . , T).
Theorem (3-9): If f is homomorphism of topological Γ-ring (G,+, . ,T) into topological Γ-ring $(G^*, \acute{+}, \acute{.}, T^*)$ then f is one-one iff ker.(f)={0}.
Proof : Suppose that ker.(f)= {0}
Let r, e $\in$ G s.t. , f(r) =f(e) $\Longrightarrow$ f(r) − f(e) = $\grave{0}$
f(r − e) = $\grave{0}$ (f is homo.)
r − e $\in$ Ker. f
But , Ker. f = {0} $\Longrightarrow$ r − e =0 $\Longrightarrow$ r = e
Suppose that f is one to one
Let r $\in$ Ker(f) $\Longrightarrow$ f(r) = $\grave{0}$ and since f(0)=$\grave{0}$, therefore f(r) = f(0)
Since , f is one to one $\Longrightarrow$ r = 0 $\Longrightarrow$ r $\in$ {0} $\Longrightarrow$ Ker(f) $\subseteq$ {0}.
Since f(0)=$\grave{0}$
$\Longrightarrow$ 0 $\in$ Ker. f $\Longrightarrow$ {0} $\subseteq$ Ker(f)
Ker(f)= {0} .
Theorem (3-10): If (I,+, .) is an ideal of the topological Γ-ring (M,+,.,T) then the natural map is a homomorphism from a topological Γ-ring M to the topological quotient Γ-ring (M/I , + , .,T) with kernel equal to I.
Proof : *Let* m ,n $\in M, \alpha \in \Gamma$
1- $nat_I(m + \text{n}) = (m + \text{n}) + \text{I}$ = $(m + I)$+ (E + I)= $nat_I(\text{m}) + nat_I(\text{n})$
2- $nat_I(m\alpha\text{n}) = (m\alpha\text{n}) + I$ = $(m + I)\ \alpha$ (E + I)= $nat_I(m)\alpha nat_I(\text{n})$
nat_I: (M,T)→(M/I,T) is continuous map since every open set V in M/I then $natI.^{-1}$(V) is open in M.
Hence nat.$_I$ is homomorphism
Now, to prove nat_I *is on to*
$\forall$ m+I $\in$*R/I* $\Longrightarrow$ $\exists$ m $\in$*R s.t.* $nat_I(m)$ = m+*I*

∴ nat_I *is onto.*
Ker. (nat_I) = { $m \in$ R : nat_I (m)= $0 + I$ }
= {$m \in$ R : $m + I$ = 0+I }
= { $\in R$:$m + I$ = I }
= {$m \in R$: $m \in I$ }
= I

Theorem (3-11): If f is homomorphism of topological Γ-ring (G,+,.,T) into topological Γ-ring ($G^*, \acute{+}, \acute{.}, T^*$) then M/ker.($f$) is isomorphic to topological Γ-ring $G^*{}_1$.

Proof: We define a map g from topological Γ-ring (M/ker.(f),+, . , T) into topological Γ-ring ($G^*, \acute{+}, \acute{.}, T^*$) by g(E+ker.(f))= f(E), for all Eϵ G
It's clear that g is will define and one to one.
Now, to show g is homomorphism.
Let p+ ker.(f), h+ ker.(f)ϵ M/ker.(f)
g((p+ker.(f) +(h+ ker.(f))) = f(p+h) = f(p) + f(h) =g(p+ker.(f)) + g(h+ker(f))
g(((p+ ker(f)α(h+ker.(f))) = f(pαħ) = f(p) α f(ħ) =g(p+ker.(f))αg(h+ker.(f))
To prove g is onto.
Let E+ ker.(f)ϵ M /ker.(f) where mϵ G, since f is onto then there exist f(E) ϵ G*such that g (E+ ker.(f))=f(E).hence ğ is onto.
Hence g is isomorphism Γ-ring.
Now, since f is continuous therefore g is continuous.
Let U be an open set in G/ker(f)
Since g(E+ker(f))=f(E) and f is continuous, so it g^{-1} is continuous
g^{-1}: $G^* \rightarrow G$/ker.(f) and f is continuous then $g^{-1}$is continuous .
Hence the topological Γ-ring G/ker.(f) is isomorphic topological Γ-ring to G^*.

Theorem (3-12): If M is a topological Γ-ring let Ş be a subring, and I be an ideal of M. Then:
(1) S + I = { s + a : s ∈ S, a ∈ I } is a topological subring of M,
(2) S ∩ I is an ideal of S.
(3) (S + I)/ I is isomorphic topological Γ-ring to S /(S ∩ I).

Proof : It is clear so that the proof is omitted.

Theorem (3-13): Let (M,+,. , T) be a topological Γ-ring and let J ⊂ I be ideals of M. Then I/J is an ideal of M/J and

$$\frac{M/\mathrm{J}}{\mathrm{I}/\mathrm{J}} \cong M/\mathrm{J}$$

Proof: Since I and J are ideals, they are non-empty and so that I/J = {a + J : a ∈ I} is also non-empty. Let a_1, a_2 ∈ I; m ∈ M, and $\alpha \in \Gamma$. By definition of addition , and multiplication of the cosets , we have (a_1 + J) + (a_2 + J) = (a_1 + a_2) + J, (m + J)α(a_1 + J) = mαa_1 + J, and (a_1 + J)α(m + J) = $a_1\alpha$m + J. Since I is an ideal, a_1 + a_2, mαa_1, and $a_1\alpha$m are contained in I so that I/J is an ideal of M/J. Let φ: M/J → M/I that sends m + J to m+ I. It is clear that this is a well-defined surjective homomorphism with kernel equal to I/J, and φ is also open map. Then (M/J)/(I/J) is isomorphism topological Γ-ring to M/I by the first isomorphism theorem.

4. COPMACT TOPOLOGICAL Γ- RINGS:

Definition (4-1): Let (G,+ , .,$\mathcal{J}$) be a topological Γ-ring ,the family{ G_i ∈J: (G_i, + , . , $\mathcal{J}_i$)}is a proper subrings of G, for all $i \in \wedge$**}** is a cover topological Γ-ring of (G,+ , ., $\mathcal{J}$) if G= $\bigcup_{i\in\wedge}$ Gi.

Definition (4-2): Let (G,+, . , $\mathcal{J}$) be topological Γ-ring then (G,+, . , $\mathcal{J}$) is compact topological Γ-ring if for every cover topological rings of (G,+, . ,$\mathcal{J}$) there is a finite sub cover topological

Example(4-3): Every finite topological Γ-ring is compact topological Γ-ring.

Theorem (4-4): If f:(G,+, . , $\mathcal{J}$)→ (G^*,+',.', T^*) is isomorphism topological Γ-ring from G ito compact topological Γ-ring G^* then G is compact topological Γ-ring.**Proof :** Suppose that S={

$G_i \in \mathcal{J}$: (G_i**, + , .**) is a proper subrings of G, for all $i \in \Lambda$**}** is a cover topological Γ-ring of (G,+ , .,$\mathcal{J}$)**.** That means G= $\bigcup_{i\in\Lambda} G_i$. Therefore f(G)=G* = $\bigcup_{i\in\Lambda}$ f(G^*_i),where f(G_i) ∈ T^*.
Since G* is compact ,it follows that there is a finite subcover $\mathcal{H} \subseteq S$
Such that $G^* \subseteq \mathcal{H}$
Since f is isomorphism we get G= $f^{-1}(\mathcal{H})$.where $f^{-1}(\mathcal{H})$ is a finite subcover of G.
Theorem (4-5): Every closed subset of a compact topological Γ-ring space is compact.
Proof: Let B be a closed subset of the compact topological Γ-ring (G,+,. ,$\mathcal{J}$).
Let D^*={ $G_i \in \mathcal{J}$: (G_i**, + , .**) is a proper subrings of G, for all $i \in \Lambda$**}** be a cover topological Γ-ring of D^*. Since B is closed then G-B is open, and
C=$D^* \cup$ (G-B)is an open cover of G. Since G is compact topological ring, it has a finite subcover, containing only finitely many members $G_{1,\dots,n}$ of D^* and may contain G-B. Since G=(G-B)$\bigcup_{i=1}^{n} G_i$ it follows that B$\subseteq \bigcup_{i=1}^{n} G_i$ and D^* has a finite subcover.
Theorem (4-6): If (G, +, . ,$\mathcal{J}$) is a compact topological Γ-ring, and
f : (G,+ ,. ,$\mathcal{J}$)→(G^*,+',.', T^*) epimorphism topological Γ-ring , then G^* is compact.
Proof: Assume that S^*={$G^*_i \in T^*$: (G^*_i**, +' , .'**) is a proper subrings of G^*, for all $i \in \Lambda$**}** is a cover topological Γ-ring of $\mathcal{S}$ that is G *= $\bigcup_{i\in\Lambda} G^*_i$
Since f is epimorphism topological Γ-ring this implies that
G= $\bigcup_{i\in\Lambda} f^{-1}(G^*_i)$,where$f^{-1}(G^*_i) \in \mathcal{J}$
Since G is compact topological Γ-ring ,this gives G=$\bigcup_{i=1}^{n} f^{-1}(G^*_i)$. Hence f is epimorphism topological Γ-ringG^*=$\bigcup_{i=1}^{n} G^*_i$.
Thus G^* is compact topological Γ-ring

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