 Iraqi Journal of Science, 2022, Vol. 63, No. 3, pp: 1240-1257
DOI: 10.24996/ijs.2022.63.3.29

ISSN: 0067-2904

Effects of the Rotation on the Mixed Convection Heat Transfer Analysis for the Peristaltic Transport of Viscoplastic Fluid in Asymmetric Channel

Hatem Nahi Mohaisen[1], Ahmed M.Abedulhadi[2]

[1]*Department of Mathematics, ministers office, Ministry of High Education and Scientific Research, Baghdad, Iraq*

[2]*Department of Mathematics, College of Science, University of Baghdad, Iraq*

Received: 6/6/2021 Accepted: 30/7/2021

Abstract

In this paper, we study the impact of the variable rotation and different variable on mixed convection peristaltic flow of incompressible viscoplastic fluid. This is investigated in two dimensional asymmetric channel, such as the density, viscosity, rate flow, Grashof number, Bingham number, Brinkman number and tapered, on the mixed convection heat transfer analysis for the peristaltic transport of viscoplastic fluid with consideration small Reynolds number and long wavelength, peristaltic transport in asymmetric channel tapered horizontal channel and non-uniform boundary walls to possess different amplitude wave and phases. Perturbation technique is used to get series solutions. The effects of different values of these parameters for the axial velocity, stream function, stress, temperature and pressure gradient have been obtained. The impact of these parameters is discussed and illustrated graphically through the set of figures. Numerical results have been computed by using MATHEMATICA software.

Keywords: peristaltic transport, heat transfer, rotation, velocity, stream function , stress, viscoplastic.

تأثير الدوران على تحليل انتقال الحرارة بالحمل المختلط على النقل التمعجي للسائل اللزج في القناة غير المتماثلة

حاتم ناهي محيسن, أحمد مولود عبدالهادي

[1]قسم الرياضيات ، مكتب الوزير ، وزارة التعليم العالي والبحث العلمي ، بغداد، العراق

[2]قسم الرياضيات ، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة

في هذا البحث تمت دراسة تأثير الدوران المتغير والمتغير المختلف على التدفق التمعجي الحراري المختلط لسائل اللزوجة غير القابلة للضغط في قناة ثنائية الأبعاد غير متماثلة ، مثل الكثافة ، اللزوجة ، معدل التدفق ، رقم جراشوف ، رقم بينغهام ، رقم برينكمان. وقناة مدببة على تحليل نقل الحرارة بالحمل الحراري المختلط للانتقال التمعجي للمائع البلاستيكي اللزج ، مع مراعاة عدد رينولدز القصير والطول الموجي الطويل ، والنقل التمعجي في القناة غير المتماثلة القناة الأفقية المستدقة والجدران الحدودية غير المنتظمة لتمتلك موجة ومراحل مختلفة السعة. تم الحصول على حلول متسلسلة باستخدام تقنية الاضطراب ، وكذلك تمن دراسة تأثيرات القيم المختلفة لهذه المعلمات للسرعة المحورية ، ووظيفة التدفق ، والضغط ، ودرجة الحرارة وتدرج

*Email: ha19652010@yahoo.com

الضغط. مناقشة تأثير هذه المعايير وتوضيحها بيانيا من خلال مجموعة من الأرقام. تم حساب النتائج العددية باستخدام برنامج ماثيماتيكا.

1- Introduction

The peristaltic pumping is known with special type pumping when it can easily be transported the variety of complex rheological fluids from one place to another place. This pumping precept is called peristaltic. The mechanism includes involuntary periodic contraction, which is followed by the expansion of the ducts that the fluid moves through. This leads to rising in a pressure gradient that eventually pushes the fluid forward. The movements of the small intestine are due to peristaltic, these movements of the small intestine are essential for the normal processes of digestion to take place, since they are responsible for mixing food with digestive juices that expose chime to the surface for absorption of nutrients and propelling material along the gastrointestinal tract. Besides this, urine transports from kidney to the bladder through ureter, semen which moves through the vas deferens, vasomotion of small blood vessels such as arterioles, lymphatic fluid moves through lymphatic vessels, and bile flows from the gall bladder into the duodenum. Peristaltic has been greatly exploited for industrial and engineering purposes e.g. peristaltic can be used in mining slurries, waste water slurries, sodium bromide and slurry pumping. Applications of peristaltic in industrial fluid mechanics like aggressive chemical, high solid slurries, noxious (nuclear industries) and other material that are transported by peristaltic pumps, roller pumps, hose pumps, tube pumps, finger pumps, heart-lung machines, blood pump machines, and dialysis machines are engineering on the basis of peristalsis. The study of the peristaltic transport of the fluid in the presence of an external magnetic field and rotation is of great importance with regard to certain problems involving the movement of conductive physiological fluid, e.g. blood and saline water. The various applications of peristalsis have been attracting the interests of researchers after the seminal work of Latham[1]. Heat transfer is an important principle in biological systems and industrial fluid transport. One of the most important functions of the cardiovascular system is that maintains the temperature of the body. Air enters the lungs must be also warmed (or cooled) to body temperature. This is accomplished through all blood vessels. There are three mechanisms of heat transfer, but the convection is the most applicable heat transfer modality within the circulation of fluid in human body. Convection heat transfer is used by human and animal bodies to lose the heat that generated by metabolic processes to the environment. The industrial applications include the thermal insulation, cooling of nuclear reactor, oil extraction, and thermal energy storage. In the recently years, the mixed of the effected the heat and mass, the effect of variable viscosity, and temperature variable are studied in [2-9]. In [10], authors investigated the effect the rotation and initial stress on the peristaltic flow of an incompressible fluid. The mechanism of peristaltic transport has attracted the attention of many researchers since it is investigated by Ahmed M. and Tamara S. [11], Hina and Muhammed and Nadeem [12], Safia and Nadeem [13]. Abd-Alla and Abo-Dahab[14], Zaheer and Nasir [15], Salman and Abdulhadi [16], Murad and Abdulhadi [17], Kareem and Abdulhadi [18]. Impress of rotation and an inclined MHD on waveform motion of the non-Newtonian fluid through porous canal are discussed in [19], K. Kalyani, N. Seshagiri Rao and Sudha Ran [20].

In this paper, we will study the rotation effects of the mixed of heat transfer for the peristaltic transport in an asymmetric channel, we use different values of the parameters of rotation, density, amplitude wave and tapered of channel, as well as we also use different value of the Bingham number, Prandtl number, Brinkman number and Grashof number by study from through change of the velocity, pressure gradient, stream function, variation of stress and temperature.

2- Formulation of the problem

Let the peristaltic transport of an incomperssiable viscoplastic fluid in an asymmetric channel of width $c_1 + c_2$ in two-dimensional coordinate. The flow is generated by continously moving sinusoidal waves with speed (c) along the walls of the channel. The wall surface at the upper wall is defined by:

$$\bar{H}_1(\bar{X},\bar{t})=c_1+\bar{M}\bar{X}+\bar{\alpha}\ \cos[\frac{2\pi}{\lambda}(\bar{X}\text{-c}\bar{t})], \quad (1)$$

whlie at the lower wall is given by .

$$\bar{H}_2(\bar{X},\bar{t})=-c_2\text{-}\ \bar{M}\bar{X}-\bar{\beta}\cos[\frac{2\pi}{\lambda}(\bar{X}\text{-c}\bar{t})+\phi], \quad (2)$$

Where (c_1, c_2) indicate the channel width, $(\bar{\alpha},\bar{\beta})$ are the amplitudes of the wave, (λ) wavelength, and (c) is the speed wave. ϕ varies in the range ($0 \le \phi \le \pi$).
$\bar{M}$ is the non-uniform parametares(trapping), when the value of ϕ = 0 is the symmetric channel with waves out of phase and ϕ = π waves are in phase, $(\bar{X}\ ,\bar{Y})$ are the rectangular coordinates in fixed frams of refernce, $\bar{t}$ is the time, further c_1, c_2, $\bar{\alpha}$, $\bar{\beta}$ and ϕ satisfy the following condition.

$$\bar{\alpha}^2+\bar{\beta}^2+2\bar{\alpha}^2\bar{\beta}^2\cos\phi \le (c_1+c_2)^2 \quad (3)$$

Here T_0, and T_1 prescribe temperature at the walls. The governed equations of the flow by three coupled non-linear partial differential of continuity, momentum and energy, which in frame $(\bar{X}\ ,\bar{Y})$ are expressed as:

$$\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\partial\bar{V}}{\partial\bar{Y}}=0, \quad (4)$$

$$\rho(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}})\bar{U}-\rho\Omega(\Omega\bar{U}+2\frac{\partial\bar{V}}{\partial\bar{t}})=-\frac{\partial\bar{P}}{\partial\bar{X}}+\frac{\partial\bar{S}_{\bar{X}\bar{X}}}{\partial\bar{X}}+\frac{\partial\bar{S}_{\bar{X}\bar{Y}}}{\partial\bar{Y}}+\rho g\gamma(T-T_0) \quad (5)$$

$$\rho\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right)\bar{V}-\rho\Omega(\Omega\bar{V}-2\frac{\partial\bar{U}}{\partial\bar{t}})=-\frac{\partial\bar{P}}{\partial\bar{X}}+\frac{\partial\bar{S}_{\bar{Y}\bar{X}}}{\partial\bar{X}}+\frac{\partial\bar{S}_{\bar{Y}\bar{Y}}}{\partial\bar{Y}} \quad (6)$$

$$\rho\ C_p\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right)T=\text{Z}\ (\frac{\partial^2T}{\partial\bar{X}^2}+\frac{\partial^2T}{\partial\bar{Y}^2})+\sigma\cdot(\nabla\bar{V}) \quad (7)$$

Where ρ is fluid density, $\bar{V}=[\bar{U},\bar{V}]$ represents the velocity vector, $\bar{P}$ is the hydrodynamic pressure, g is acceleration due to gravity, γ is the coefficient of thermal expansion. $\bar{S}_{\bar{X}\bar{X}}$, $\bar{S}_{\bar{X}\bar{Y}}$ and $\bar{S}_{\bar{Y}\bar{Y}}$ are the compenent of extra stress tensor $\bar{S}$, C_p is specific heat,while Z is the thermal conductivity, Ω is the rotation, T is a temperature, $\nabla=[\frac{\partial}{\partial\bar{X}},\frac{\partial}{\partial\bar{Y}}]$, σ is the cauchy stress tensor which for the Bingham plastic fluid is defined [21]:

$$\sigma=-\bar{P}\,\bar{I}+\bar{S} \quad (8)$$

where,

$$\bar{S}=2\mu\omega+2\tau_0\,\widehat{\omega} \quad (9)$$

In equation (9) τ_0 is the yield stress while the rate of deformation tensor ω and $\widehat{\omega}$ is the tensor are defined:

$$\omega=\frac{1}{2}\ (\nabla\bar{V}+(\nabla\bar{V})^T)\ ),\quad \widehat{\omega}=\frac{\omega}{\sqrt{\epsilon_0+2\,tr\,\omega^2}} \quad (10)$$

the persence ϵ_0 is necessary event prevents from unboundedness when $\omega\rightarrow 0$. The definition of dot product of arbitrary tensors D and E, i.e., $D\cdot E=tra(DE)$ enables us to write energy equation (5) as

$$\rho\ C_p\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right)T=\text{Z}\ (\frac{\partial^2T}{\partial\bar{X}^2}+\frac{\partial^2T}{\partial\bar{Y}^2})+\bar{S}_{\bar{X}\bar{X}}\left(\frac{\partial\bar{U}}{\partial\bar{X}}\right)+\bar{S}_{\bar{X}\bar{Y}}\left(\frac{\partial\bar{V}}{\partial\bar{X}}\right)+\bar{S}_{\bar{X}\bar{Y}}\left(\frac{\partial\bar{U}}{\partial\bar{Y}}\right)+\bar{S}_{\bar{Y}\bar{Y}}\left(\frac{\partial\bar{V}}{\partial\bar{Y}}\right) \quad (11)$$

The compenents of extra stress tensor in laboratory frame in equation (9), and (10) become

$$\bar{S}_{\bar{X}\bar{X}}=2\,\mu\,\omega_{\bar{X}\bar{X}}+\frac{2\tau_0\omega_{\bar{X}\bar{X}}}{\sqrt{\epsilon_0+2tr\omega^2}},\ \bar{S}_{\bar{X}\bar{Y}}=2\,\mu\,\omega_{\bar{X}\bar{Y}}+\frac{2\tau_0\omega_{\bar{X}\bar{Y}}}{\sqrt{\epsilon_0+2tr\omega^2}},\bar{S}_{\bar{Y}\bar{Y}}=2\,\mu\,\omega_{\bar{Y}\bar{Y}}+\frac{2\tau_0\omega_{\bar{Y}\bar{Y}}}{\sqrt{\epsilon_0+2tr\omega^2}} \quad (12)$$

Peristaltic motion in natural is unsteady phenomenon, however it can be assumed steady by using the transformation from laboratory frame(fixed frame) $(\bar{X},\bar{Y})$ to wave frame(move frame) $(\bar{x}, \bar{y})$. The relationship between coordinates, velocities and pressure in laboratory frame $(\bar{X}, \bar{Y})$ and wave frame $(\bar{x}, \bar{y})$ is provided by the following transformations

$$\bar{x} = \bar{X} + c\bar{t},\ \bar{y} = \bar{Y},\ \bar{u} = \bar{U} - c,\ \bar{v} = \bar{V},\ \bar{p}(\bar{x}, \bar{y}) = \bar{P}(\bar{X}, \bar{Y}, \bar{t}) \tag{13}$$

Where $\bar{u}, \bar{v}, and\ \bar{p}$ are velocity compenents and pressure in wave frame. Now,we transform equations (1,2,4,5,6,11,12) in wave frame with helping of equation (13) and normalize the resulting equation by using the following dimensionless quantities:

$$x = \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{c_1},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c\delta},\ \bar{t} = \frac{\lambda}{c}\, t,\ \bar{p} = \frac{c\mu}{{c_1}^2}\lambda p,\ R_e = \frac{\rho c_1 c}{\mu},\ \alpha = \frac{\bar{\alpha}}{c_1},\ \beta = \frac{\bar{\beta}}{c_1},\ \delta = \frac{c_1}{\lambda},$$

$$h_1 = \frac{\bar{H}_1}{c_1},\ h_2 = \frac{\bar{H}_2}{c_1},\ S = \frac{c_1}{c\mu}\bar{S},\ \theta = \frac{T - T_0}{T_1 - T_0},\ \bar{M} = \frac{c_1}{\lambda}m,\ a = \frac{c_2}{c_1},\ E_c = \frac{c^2}{C_p T_0},\ G_r = \frac{\rho g \gamma {c_1}^2}{\mu c}(T_1 - T_0),$$

$$B_r = P_r E_c,\ B_n = \frac{c_1 \tau_0}{\mu c},\ P_r = \frac{\mu C_p}{k},\ \varepsilon = \frac{\epsilon_0 {c_1}^2}{c^2}\ (0 < \varepsilon \ll 1)$$

To obtain,

$$h_1 = 1 + mx + \alpha\cos(2\pi x) \tag{14}$$

$$h_2 = -a - mx - \beta\cos(2\pi x + \phi) \tag{15}$$

The equation (3) in dimensionless frames is aslo given by :

$$\alpha^2 + \beta^2 + 2\alpha^2\beta^2\cos\phi \le (1 + a)^2 \tag{16}$$

$$\delta\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{17}$$

$$R_e\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - \frac{\rho {c_1}^2}{\mu}\Omega^2 u = -\frac{\partial p}{\partial x} + \delta\frac{S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} + G_r\,\theta \tag{18}$$

$$R_e\delta^2\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - \delta\frac{\rho\Omega^2 {c_1}^2 v}{\mu} - 2\Omega\, R_e\delta^2 u\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial S_{yx}}{\partial x} + \delta\frac{\partial S_{yy}}{\partial y} \tag{19}$$

$$R_e\delta\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right) = \frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + \delta E_c S_{xx}\left(\frac{\partial u}{\partial x}\right) + E_c S_{xy}\left(\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x}\right) + E_c S_{yy}\left(\frac{\partial v}{\partial y}\right) \tag{20}$$

$$S_{xx} = 2\delta\frac{\partial u}{\partial x} + \frac{2\delta B_n(\frac{\partial u}{\partial x})}{\sqrt{\varepsilon + 2\delta^2(\frac{\partial u}{\partial x})^2 + 2(\frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x})^2}} \tag{21}$$

$$S_{xy} = \left(\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x}\right) + B_n\frac{\left(\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x}\right)}{\sqrt{\varepsilon + 2\delta^2(\frac{\partial u}{\partial x})^2 + 2(\frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x})^2}} \tag{22}$$

$$S_{yy} = 2\frac{\boldsymbol{\partial v}}{\boldsymbol{\partial y}} + \frac{2\delta B_n(\frac{\partial v}{\partial y})}{\sqrt{\varepsilon + 2\delta^2(\frac{\partial u}{\partial x})^2 + 2(\frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x})^2}} \tag{23}$$

In previous equations, T_1, and T_0 are the temperature at the upper and lower walls, respectively. The dimensionless number δ is the wave number, B_n is Bingham number, R_e Reynolds number, P_r Prandtl number, E_c Eckert number, G_r Grashof number, $\varepsilon \ll 1$, B_r Brinkman number and θ is the temperature in dimensionless form. Introduction to the stream function (ψ) by relation:

$u = \psi_y,\ \ v = -\delta\psi_x$,

From equations(17-23) show that the continuity equation (17) satisfies identically while other equations will take the following form:

$$R_e\delta\left(\psi_y\frac{\partial\psi_y}{\partial x} - \psi_x\frac{\partial\psi_y}{\partial y}\right) - \frac{\Omega^2 {c_1}^2\rho}{\mu}\psi_y + 2R_e\delta^2\frac{\partial\psi_x}{\partial t} = -\frac{\partial p}{\partial x} + \delta\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} + G_r\theta \tag{24}$$

$$-R_e\delta^3\left(\psi_y\frac{\partial\psi_x}{\partial x} - \delta\,\psi_x\frac{\partial\psi_x}{\partial y}\right) - \frac{\Omega^2 {c_1}^2\rho}{\mu}\delta^2\psi_x + 2\Omega R_e\delta^2\psi_y\frac{\partial\psi_y}{\partial t} = -\frac{\partial p}{\partial y} + \delta\frac{\partial S_{xx}}{\partial y} + \delta^2\frac{\partial S_{xy}}{\partial x} \tag{25}$$

$$R_e P_r\delta\left(\psi_y\frac{\partial\theta}{\partial x} - \delta\psi_x\frac{\partial\theta}{\partial y}\right) = \left(\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + B_r\delta S_{xx}\psi_{yy} + B_r S_{xy}\left(\psi_{yy} - \delta^2\psi_{xx}\right) - \delta B_r S_{yy}\psi_{xy} \tag{26}$$

$$S_{xx} = 2\delta\psi_{xy} + \frac{2\delta B_n\psi_{xy}}{\sqrt{\varepsilon + 2\delta^2(\psi_{xy})^2 + 2\delta^2(\psi_{xy})^2 + (\psi_{yy} - \delta^2\psi_{xx})^2}} \tag{27}$$

$$S_{xy} = (\psi_{yy} - \delta^2\psi_{xx}) + \frac{B_n(\psi_{yy}-\delta^2\psi_{xx})}{\sqrt{\varepsilon+2\delta^2(\psi_{xy})^2+2\delta^2(\psi_{xy})^2+(\psi_{yy}-\delta^2\psi_{xx})^2}} \tag{28}$$

$$S_{yy} = -2\delta\psi_{xy} - \frac{2\delta B_n\psi_{xy}}{\sqrt{\varepsilon+2\delta^2(\psi_{xy})^2+2\delta^2(\psi_{xy})^2+(\psi_{yy}-\delta^2\psi_{xx})^2}} \tag{29}$$

The equations from(24) to (29) when (R_e and $\delta \ll 1$) become in the form:

$$-\frac{\Omega^2 {c_1}^2\rho}{\mu}\psi_y = -\frac{\partial p}{\partial x} + \frac{\partial S_{xy}}{\partial y} + G_r\theta \tag{30}$$

$$-\frac{\partial p}{\partial y}=0 \tag{31}$$

$$\frac{\partial^2\theta}{\partial y^2} + B_r S_{xy}\psi_{yy} = 0 \tag{32}$$

Whereas the component of extra stess tensor become in the form:

$$S_{xy} = (\psi_{yy}) + \frac{B_n\psi_{yy}}{\sqrt{\varepsilon+(\psi_{yy})^2}}\ ,\ S_{yy} = 0, \qquad S_{xx} = 0. \tag{33}$$

If we substitut equation (33) into (30) and take the derivative with respect of y, we obtain the following equation:

$$-\frac{\Omega^2 {c_1}^2\rho}{\mu}\psi_{yy} = \frac{\partial^2 S_{xy}}{\partial y^2} + G_r\frac{\partial\theta}{\partial y} \tag{34}$$

while if we substitut equation (33) into equations (34,32) we get couple and high nonlinear differential equations:

$$\frac{\partial^2}{\partial y^2}(\psi_{yy} + \frac{B_n\psi_{yy}}{\sqrt{\varepsilon+(\psi_{yy})^2}}) + G_r\frac{\partial\theta}{\partial y} + \frac{\Omega^2 {c_1}^2\rho}{\mu}\psi_{yy}=0 \tag{35}$$

$$\frac{\partial^2\theta}{\partial y^2} + B_r\ ((\psi_{yy})^2 + \frac{B_n(\psi_{yy})^2}{\sqrt{\varepsilon+(\psi_{yy})^2}}) \tag{36}$$

the dimensionless volume flow rate and boundary condition in the wave frams are [22]:

$$\psi = -\ F/2,\ \ \frac{\partial\psi}{\partial y} = -1,\ \ \theta = 0,\ \ at\ y = h_2 \tag{37}$$

$$\psi = \ F/2,\ \ \frac{\partial\psi}{\partial y} = -1,\ \ \theta = 0,\ \ at\ y = h_1 \tag{38}$$

$$q\text{-}2= F =\int_{h_2}^{h_1}\frac{\partial\psi}{\partial y}dy = \psi(h_1) - \psi\ (h_2) \tag{39}$$

F and q are the dimensionless mean flow rate in fixed and wave frams. The dimensionless expression of the pressure rise over one cycle of the wave is:

$$\Delta P_\lambda = \int_0^1\frac{\partial p}{\partial x}\ dx \tag{40}$$

3- Solution of the Problem

The perturbation method is used to get the solution of previous non-linear partial differential equation system . We start with inserting the following expressions:

$$\Psi= \sum_{i=0}^{\infty}\psi_{0i}\,(G_r)^i + B_n\sum_{i=0}^{\infty}\psi_{1i}\,(G_r)^i + \mathrm{O}(\mathrm{B}n^2) \tag{41}$$

$$\theta = \sum_{i=0}^{\infty}\theta_{0i}\,(G_r)^i + B_n\sum_{i=0}^{\infty}\theta_{1i}\,(G_r)^i + \mathrm{O}(\mathrm{B}n^2) \tag{42}$$

from equation (35) to the (39) we get the following system:

3.1- Zeroth Order system

$$\frac{\partial^4\psi_{00}}{\partial y^4} + \mathrm{k} = 0, \qquad \frac{\partial^2\theta_{00}}{\partial^2} + B_r(\frac{\partial^2\psi_{00}}{\partial y^2})^2 =0 \tag{43}$$

$$\frac{\partial^4\psi_{01}}{\partial y^4} + \frac{\partial\theta_{00}}{\partial y} + k = 0,\ \ \frac{\partial^2\theta_{01}}{\partial y^2} + 2B_r\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)\left(\frac{\partial^2\psi_{01}}{\partial y^2}\right) = 0 \tag{44}$$

Where the boundary conditions are:

$$\psi_{00}(h_1)=q/2,\ \psi_{00}(h_2)= -q/2,\ \ \psi_{00y}(h_1)= -1,\ \ \psi_{00y}(h_{12})= -1,\ \ \psi_{01}(h_1)=0,\ \psi_{01}(h_2)=0, \tag{45}$$

$$\psi_{01y}(h_1)=0,\ \ \psi_{01y}(h_2)=0,\ \theta_{00}(h_1) = 0,\ ,\ \theta_{00}(h_2) = 0,\ ,\ \theta_{01}(h_1) = 0,\ ,\ \theta_{01}(h_2) = 0, \tag{46}$$

3.2- First Order System

$$\psi_{10yyyy} + \frac{3(\psi_{00yy})^3(\psi_{00yyy})^2}{(\varepsilon+ \left(\psi_{00yy}\right)^2)^{5/2}} - \frac{3\,(\psi_{00yy})(\psi_{00yyy})^2}{(\varepsilon+ \left(\psi_{00yy}\right)^2)^{3/2}} + \mathrm{k}=0 \quad (47)$$

$$\theta_{10yy} + \frac{B_r(\psi_{00yy})^2}{\sqrt{\varepsilon+ (\psi_{00yy})^2}} + 2B_r\left(\psi_{00yy}\right)\left(\psi_{10yy}\right) = 0 \quad (48)$$

Where the boundary conditions are:

$$\psi_{10}(h_1)\text{=0, } \psi_{10}(h_2)\text{= 0, } \psi_{10y}(h_1)\text{= 0, } \psi_{10y}(h_{12})\text{= 0,} \quad (49)$$

$$\theta_{10}(h_1) = 0,\, ,\, \theta_{10}(h_2) = 0. \quad (50)$$

Where k= $\frac{\Omega^2 {c_1}^2 \rho}{\mu}\psi_{yy}$.

For example of of the solution of zeroth order when BR= B_r:

$$\psi_{00} = \frac{k\mathrm{t3}+k\mathrm{t4}y-\mathrm{t1Cos}[\sqrt{k}y]-\mathrm{t2Sin}[\sqrt{k}y]}{k};$$

$$\theta_{00} = \mathrm{t5} + y*\mathrm{t6} - \frac{\mathrm{BR}(2k(\mathrm{t1}^2+\mathrm{t2}^2)y^2+(-\mathrm{t1}^2+\mathrm{t2}^2)\mathrm{Cos}[2\sqrt{k}y]-2\mathrm{t1t2Sin}[2\sqrt{k}y])}{8k};$$

$$\psi01 = \mathrm{t9} + y*\mathrm{t10} + \frac{-12k\mathrm{t6}y^2 + 2\mathrm{BR}k\mathrm{t1}^2y^3 + 2\mathrm{BR}k\mathrm{t2}^2y^3 - 24k*\mathrm{t7}*\mathrm{Cos}\left[\sqrt{k}y\right] - \frac{\mathrm{BRt1t2Cos}\left[2\sqrt{k}y\right]}{\sqrt{k}} - 24k*\mathrm{t8}*\mathrm{Sin}\left[\sqrt{k}y\right] + \frac{\mathrm{BR}(\mathrm{t1}^2-\mathrm{t2}^2)\mathrm{Sin}\left[2\sqrt{k}y\right]}{2\sqrt{k}}}{24k^2};$$

$$\begin{aligned}\theta_{01} = \mathrm{t11} + \mathrm{t12}y - \frac{1}{108k^{5/2}}\mathrm{BR}(&54k^{5/2}(\mathrm{t1t7}+\mathrm{t2t8})y^2 - 108\mathrm{BRt2}(\mathrm{t1}^2+\mathrm{t2}^2)\mathrm{Cos}[\sqrt{k}\\&- 9(-24\sqrt{k}\mathrm{t1t6} + \mathrm{BR}(\mathrm{t1}^2+\mathrm{t2}^2)(13\mathrm{t2}+12\sqrt{k}\mathrm{t1}y))\mathrm{Cos}[\sqrt{k}y]\\&- 27k^{3/2}(\mathrm{t1t7}-\mathrm{t2t8})\mathrm{Cos}[2\sqrt{k}y] + \mathrm{BRt2}(-3\mathrm{t1}^2+\mathrm{t2}^2)\mathrm{Cos}[3\sqrt{k}y]\\&+ 108\mathrm{BRt1}(\mathrm{t1}^2+\mathrm{t2}^2)\mathrm{Sin}[\sqrt{k}y] + 9(24\sqrt{k}\mathrm{t2t6} + \mathrm{BR}(\mathrm{t1}^2+\mathrm{t2}^2)(13\mathrm{t1}\\&- 12\sqrt{k}\mathrm{t2}y))\mathrm{Sin}[\sqrt{k}y] - 27k^{3/2}(\mathrm{t2t7}+\mathrm{t1t8})\mathrm{Sin}[2\sqrt{k}y] + \mathrm{BRt1}(\mathrm{t1}^2\\&- 3\mathrm{t2}^2)\mathrm{Sin}[3\sqrt{k}y]);\end{aligned}$$

4- Results and discusions

In this section we will discuss the effect of different parameters on the axioal velocity Figure 1, pressure gradient Figure 5, stress distribution Figure 3 and temperature distribution Figure 4. The result described by the graphical clarification. The trapping phenomenon also studied for the different parameters through contoure graphs Figure 2.

4.1- Velocity profile

At the x=0.5 cross-section the velocity axial is calculated.The camparism is done in Figure 1 besed on different physical parametares such as the non-uniform parameter (m), the amplitudes wave (α, β), varies in the range(ϕ), Bingham number (BN), Brinkman number (BR), flow rate (q), rotation (Ω), viscosity (μ), density (ρ),and channel width (c_1).

The effect of (m) on the velocity is shown in Figure(1.a).The velocity distribution decreases in the middle of channel, but it increases near the walls channel when (m) is increasing. When the impact of amplitudes wave(β) decreases this leads to increase the velocity distrbution near the walls see Figure (1.b). Varies in the range(ϕ) is shown in Figure (1.c), the value of velocity in the middle decreases and it expands near the walls if the value is increasing. Flow rate (q) is shown in Figure (1.d) a slight drop in the middle and increasing at near the walls of channel when the value decreases. Bingham number (BN) is shown in Figure (1.e) velocity a slight drop in the middle but continiuty increases when its value is deacreasing. The rotation (Ω) is shown in Figure (1.f) the velocity decreases in the middle, while it continues to expand near the walls of the channel when its value is decreasing. Channel width (c_1) is shown in Figure (1.g) to observe the velocity a slight drop in the middle, and expands near the walls of channel when its value is increasing. The viscosity (μ) is the convers of (c_1). Figure (1.h) shows that with a slight drop in middel of channel. The density (ρ), the graph is nearly with other and a slight drop in middle channel that is shown in Figure(1.i). Brinkman number (BR) does not affect by change of value but a slight drop in the middle of channel in all change. This is clearly seen in Figure (1.j).

4.2- Trapping Phenomenon

The process of the formulation of internally circulation of fluid bolus by closed stream line in the fluid flow is called trapping phenomenon, the behavior of stream function is shown in Figure (2). In Figure(2) camparism is done besed on different physical parametares such that, the non-uniform parameter (m), the amplitudes wave (α, β), varies in the range(ϕ), Bingham number (BN), rotation (Ω), viscosity (μ), density (ρ), and channel width (c_1). In the case of increasing the value of Bingham the value of bolus at near of wall is increasing ,that is shown in Figures (2.a,2.b,2.c). Figures (2.d, 2.e, 2.f, 2.m, 2.n, 2.o) show that if the (α, β) is increasing then the value of bolus is in middle and nearly to the entering to center. Figures (2.g, 2.h, 2.i) illustrate that if the non-uniform parameter (m) is increasing, then the value of m is obseved , and the wave is centered at the center of the walls . When the value of (ϕ) increases this leads to an increasing in the number of boluses. This is clearly seen in Figures (2.j, 2.k, 2.l). The increasing value of density (ρ) leads to few change value for every bolus this is shown in Figures (2.p, 2.q, 2.r). An increasing value of the viscosity (μ) leads to increase values of bolus in direction of center of channel , this is shown Figues (2.s, 2.t, 2.u). The increasing value of channel width (c_1) leads to an increasing in number of bolus in upper wall and gather inward of channel this is obseved in Figures (2.v, 2.w, 2.x). An increasing value of rotation (Ω) leads to an increasing in number of bolus in upper wall but decreasing in number of bolus in lower wall and gather inward of channel this is shown in Figure (2.y, 2.z, 2.a1).

4.3- Stress Distribution

It is shown in the Figure(3) from (3.a-3.r) the variation of the stress and comparism between the upper and lower channel using differents parameters. For the upper channel it is represented by Figures (3.a, 3.c, 3.e, 3.g, 3.i, 3.k, 3.m, 3.o, 3.q), and the lower channel which is represented by Figures (3.b, 3.d, 3.f, 3.h, 3.j, 3.l, 3.n, 3.p, 3.r). where the parameters are used for comparison, non-uniform parameter (m), the amplitudes wave (α, β), varies in the range(ϕ), Bingham number (BN), rotation (Ω), viscosity (μ), density (ρ), and channel width (c_1).

4.4- Temperature profile

In Figures(4) it is shown the varition of temperature distribution, we look at that from Figure (4.a-4.h), with different parameters with respect to the density (ρ), viscosity (μ), channel width (c_1), rotation (Ω), amplitudes wave (α, β), varies in the range(ϕ) and flow rate (q). To observe that if an decreasing value of (ρ) fluid leads to an increasing in temperature this is shown in Figure (4.a), and an increasing value of(μ) starts to increase the value of temperature and starts to low befer to arrive to the middle of channel,however when increasing the value of (μ) starts to high after the meddle of channel this is shown in Figure (4.b). The increasing the value of (c_1) with decreasing it is balance at the middle of channel but for increasing of channel width the temperature continuous to high at near the walls see Figure (4.c). The discussion of (Ω) is specially very sensitive if the value is increasing and continous to high at near of walls see Figure (4.d). In the case of upper wall the decreasing of (α) leads to an increasing of temperature this is shown in Figure (4.e), however in case of lower the increasing value of (β) starts by high when it passed of the middle channel see Figure (4.f). In case of varies of rang (ϕ) the increasing value leads to increase the of temperture and an decreasing leads to an increasing at near of walls see Figure (4.g), with respect flow rate (q) choosing the perfect value leads to the increasing of the temperature see Figure (4.h).

4.5- Pressure Gradient Distribution

Figures from (5.a-5.l) are shown the change behavior of pressure gradient with respect of axial coordinate x for the various parameters. We obsereve that the increasing value of (BN), (β), and (m) leads to decrease value of the pressure gradient see Figure (5.a), (5.g) and (5.d), respectively.However the increasing magnitud of the (BR), (GR), (q), (α), (ϕ), (c_1) and (Ω) leads to increasing value of pressure gradient this is shown in Figures (5.b), (5.c), (5.e), (5.f), (5.h), (5.i) and (5.j) respectivily, as for the (ρ) and (μ), their effect is minimal of incresing or decreasing values on the pressure gradient distribution see Figures (5.k), and (5.l), respectivily.

5- Conclusion

In this reseach, we study the effect of the rotation on mixed convection heat transfer analysis peristaltic flow of the viscoplastic through the tapered horizontal asymmetric channel. The channel asymmetric is produced by choosing the peristaltic waves on the non-uniform walls to have different amplutude and phases low Renolds number and along wavelength. We adopt the perturbation method to obtain the expression of the axial velocity, stream function, stress, temperature and pressure gradient. A parametric analysis is performed through various graphs.

A- When an increasing value of (m), (c_1), (ϕ) and (ρ) leads to increase the value of velocity, but an decreasing of values of (β), (q), (BN), (μ) and (Ω) lead to increase the value of velocity , as for the (BR) when it values change not impact the value of velocity.

B- In case of trapping, when the value of (BN) increases, the bolus expands and moves from the lower wall towards the upper wall. When an increasing in (α, β) we note that the bolus value increases in the middle and recedes in the center of the channel. When the value of (m) increases, it observes that the wave is directed towards the center of the channel, when the increasing value of (ϕ) leads to an increasing in the number of boluses, an increasing value of (ρ) leads to few change in the value for every bolus, an increasing value of the (μ) leads to that values of bolus are increasing in direction of center of channel, the increasing value of (c_1) leads to an increasing in number of bolus in upper wall and gather inward of channel, if the value of rotation (Ω) increasing leads to an increasing in number of bolus in upper wall but decreasing in number of bolus in lower wall and gather inward of channel.

C- The stress distribution, in case of the upper wall, when (BN) and (μ) are decreasing the value of stress is increasing at the upper wall, but an increseaing values of (ϕ), (β), (Ω), (c_1),and (ρ) leads to an increasing in stress at lower walls, but for (m) and (α) at left of wall. In lower wall, the stress increasing when (BN), (ϕ) and (μ) are decreasing, increasing value of (m), (Ω), (α), (β), (c_1) and (ρ) implies to an increasing of stress value.

D- Temperature profile, an decreasing value of (ρ), (α) and (β) leads to increasing of temperture, but an increasing of (μ), (c_1), (Ω) and (ϕ) implies to increasing of temperature.

E- In case of the pressure gradient, when an decreasing of (BN), (m) and (β) high product of pressure gradient, but an increasing of (BN), (GR), (q), (α),), (c_1), (Ω) and (ϕ) high product of pressure gradient.

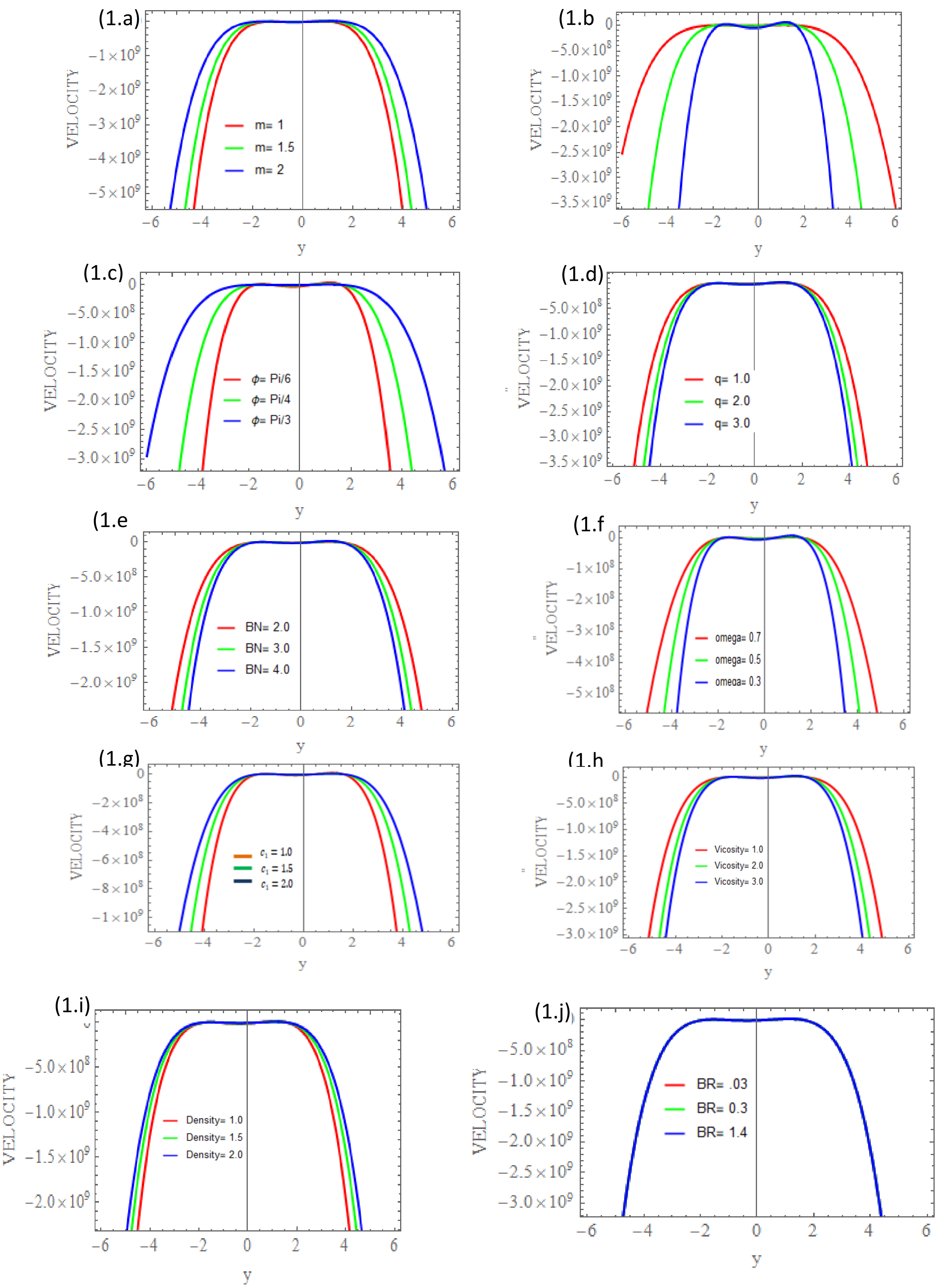
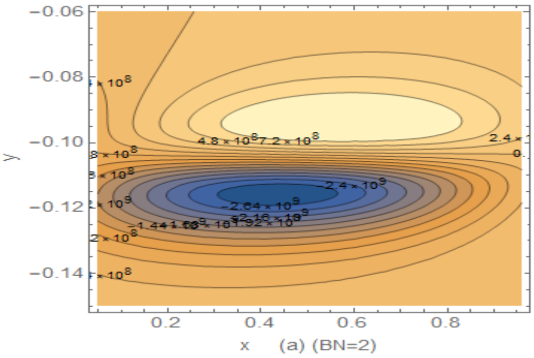
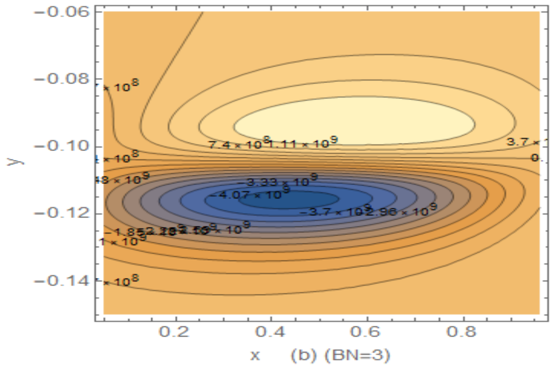


Figure-1 Change velocity with different parameters{ GR= 0.2,BR= 0.4,BN= 4,x=0.5,ϵ=0.01,α=0.4,m=2.0,ϕ=Pi/4,q=1.6,Ω=0.2, c_1=0.8,μ=2.0,ρ=0.9,a=1}

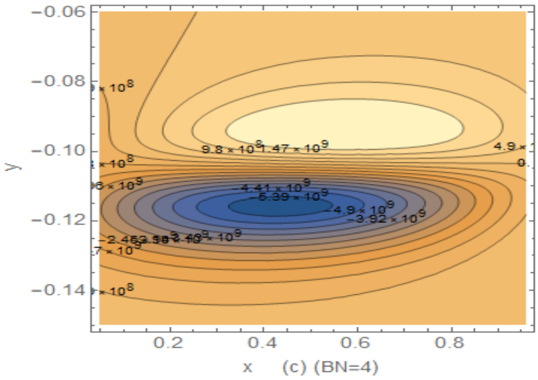
(2.a



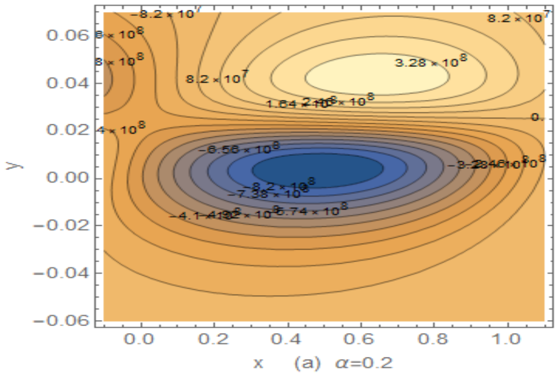
(2.b)



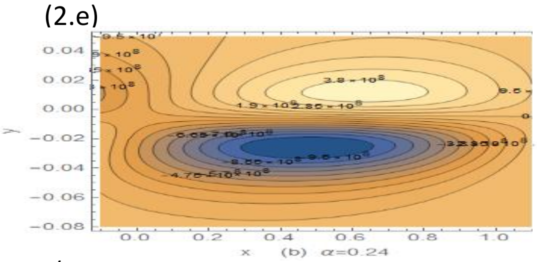
(2.c)



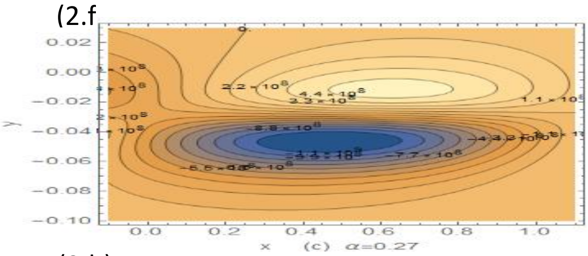
(2.d



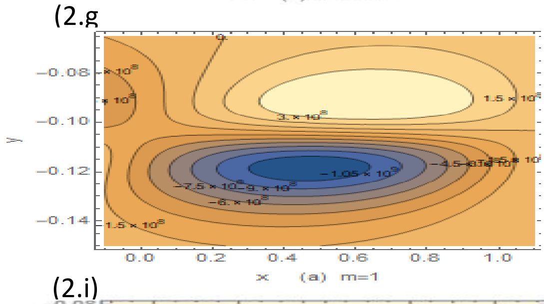
(2.e)



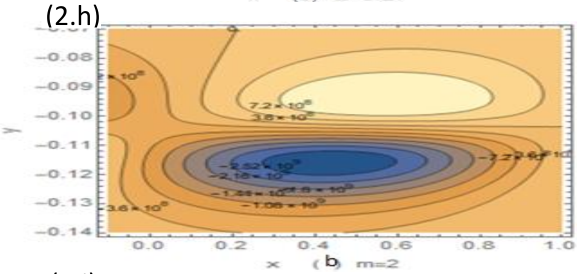
(2.f



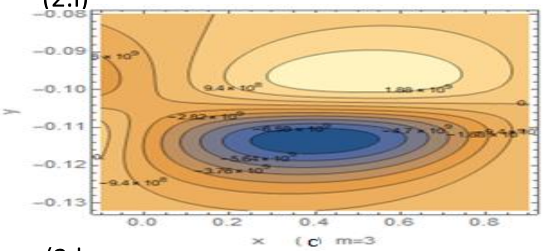
(2.g



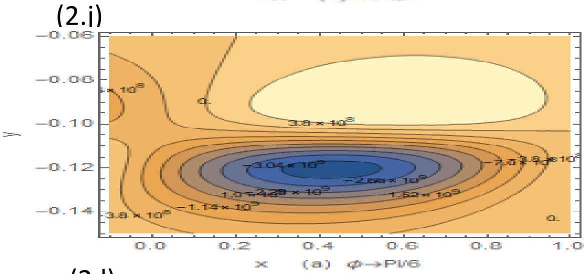
(2.h)



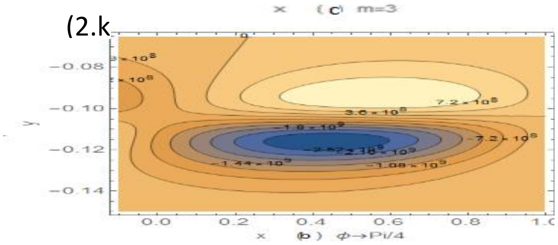
(2.i)



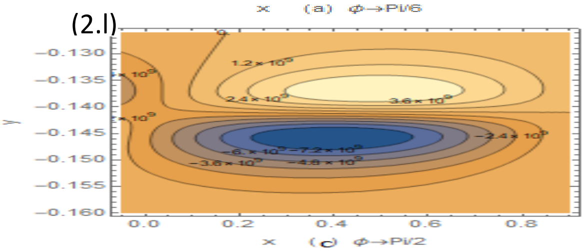
(2.j)



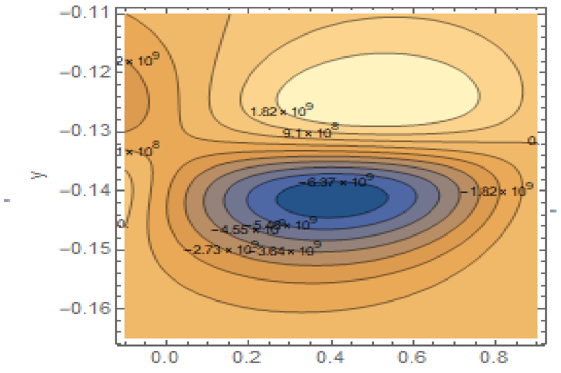
(2.k



(2.l)

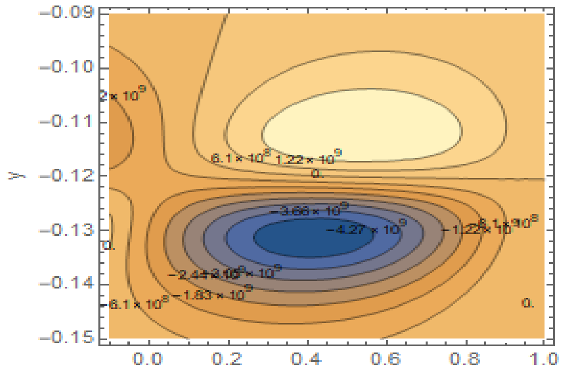


(2.m)



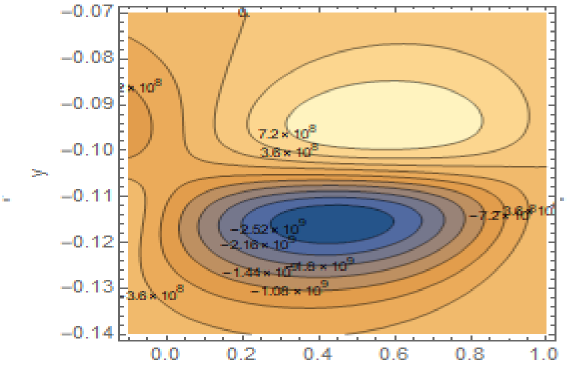
x (a) β=0.25

(2.n)



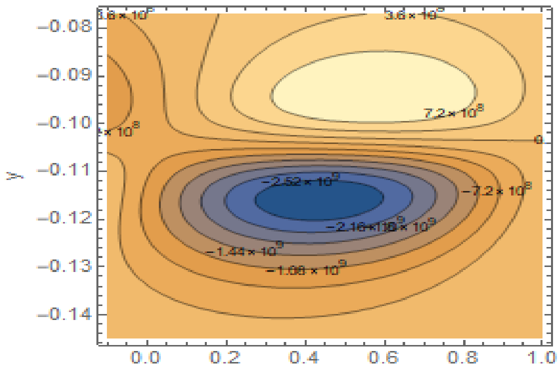
x (b) β=0.27

(2.o



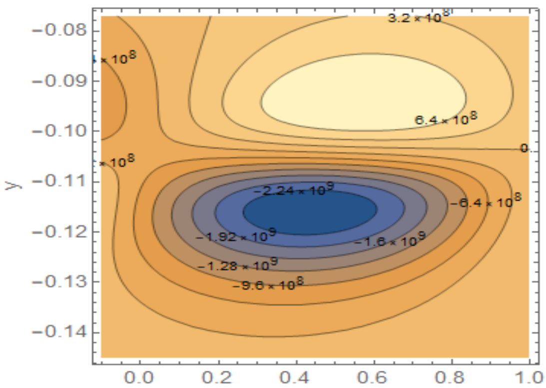
x (c) β=0.3

(2.p)



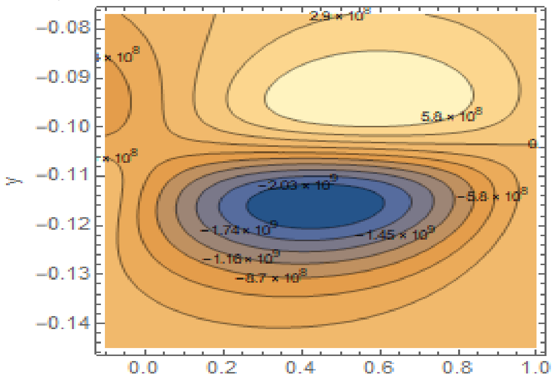
x (a) density=0.9

(2.q



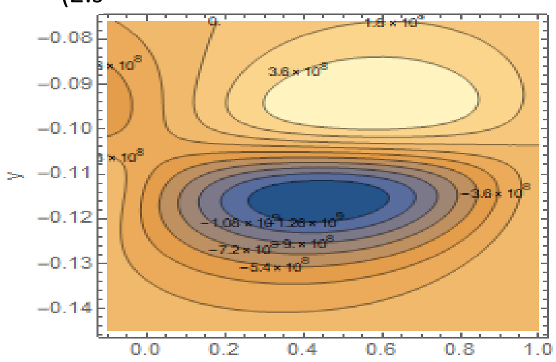
x (b) density=1.0

(2.r



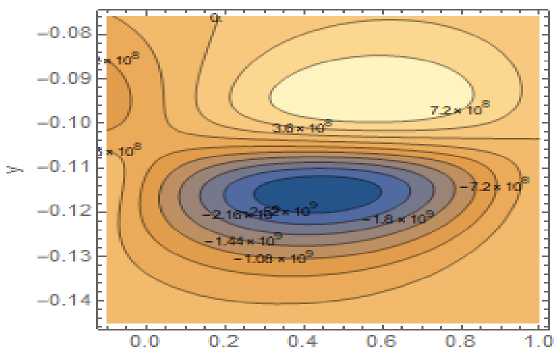
x (c) density=1.1

(2.s



x (a) viscosity=1.0

(2.t



x (b) viscosity=2.0

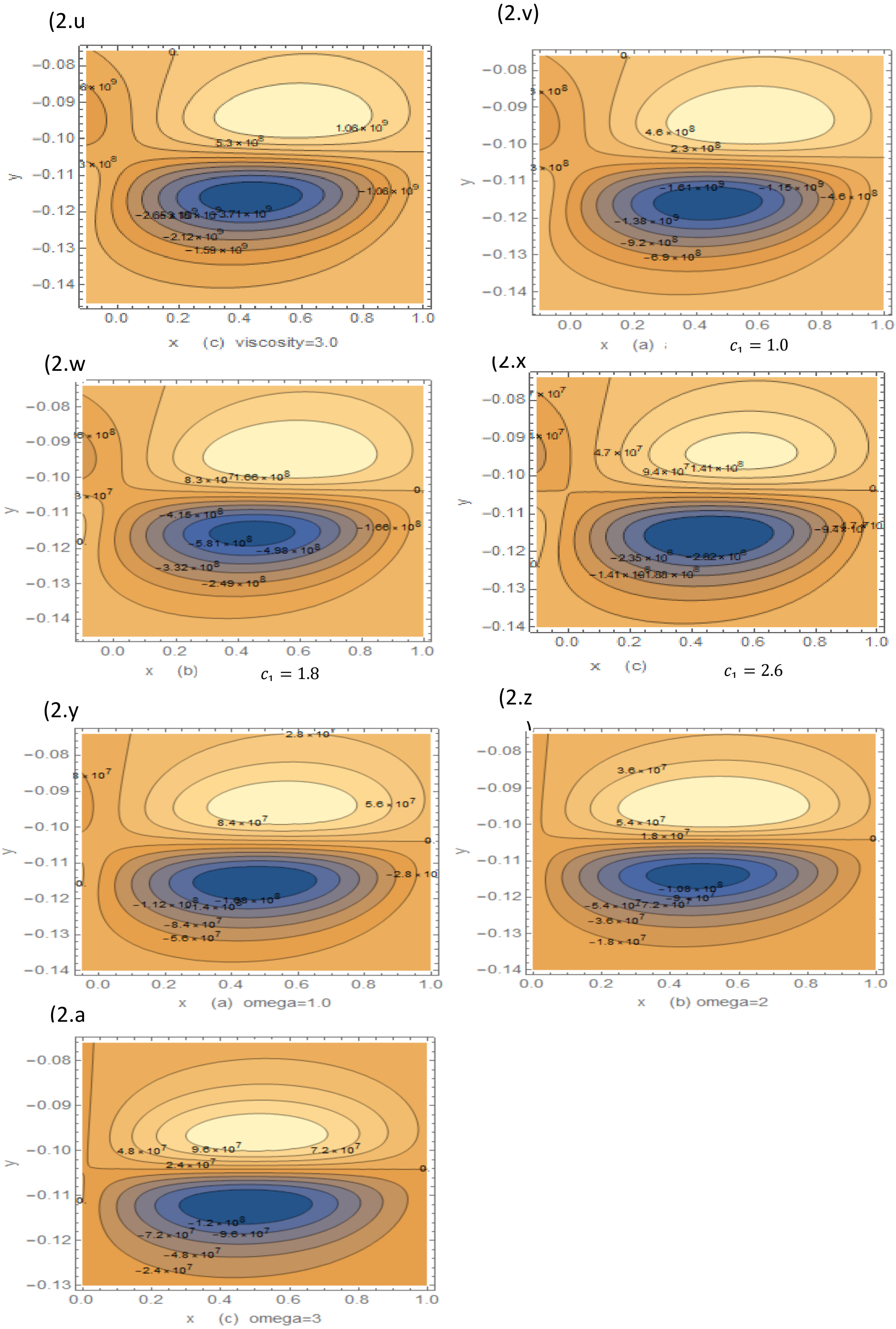
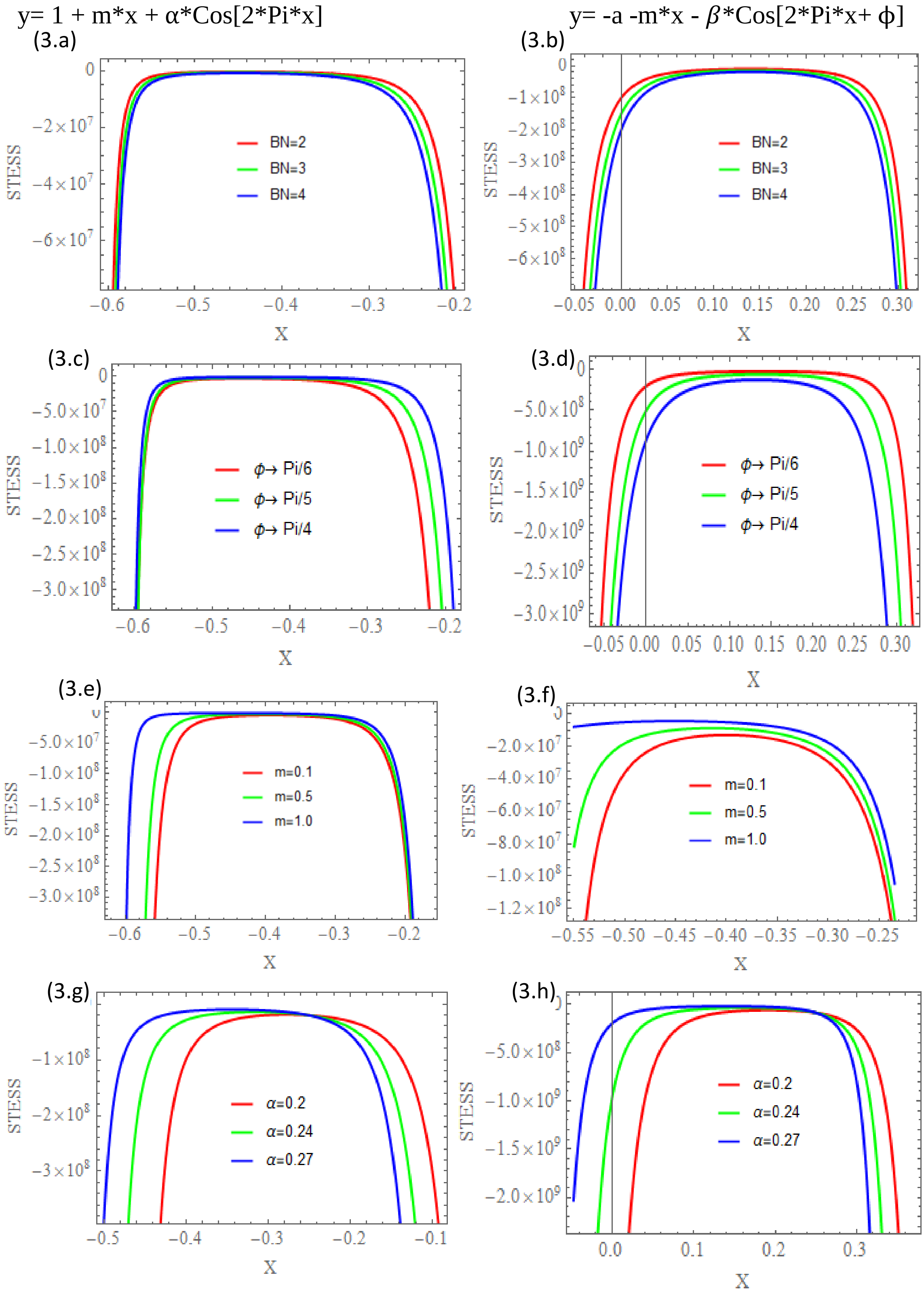
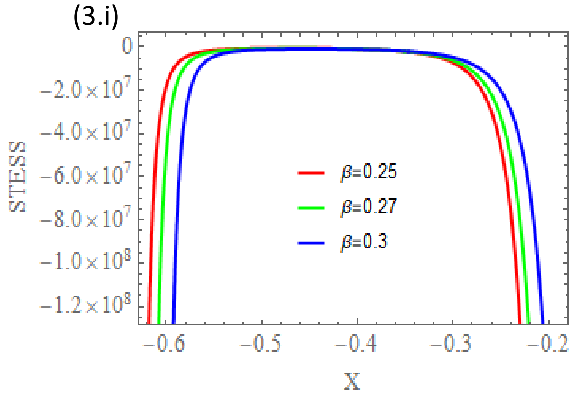


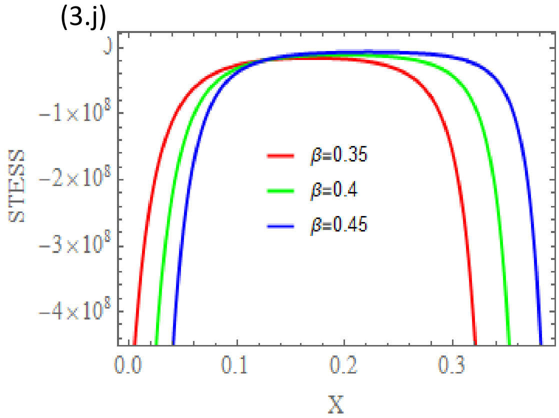
Figure-2 Change stream function with different parameters{ GR= 0.2,BR= 0.4,BN= 4,x=0.5,ϵ=0.01,α=0.4,m=2.0,ϕ=Pi/4,q=1.6,Ω=0.2, c_1=0.8,μ=2.0,ρ=0.9,a=1}


y= 1 + m*x + α*Cos[2*Pi*x]
y= -a -m*x - β*Cos[2*Pi*x+ ϕ]
(3.a)
(3.b)
(3.c)
(3.d)
(3.e)
(3.f)
(3.g)
(3.h)
STESS
X
BN=2
BN=3
BN=4
ϕ→ Pi/6
ϕ→ Pi/5
ϕ→ Pi/4
m=0.1
m=0.5
m=1.0
α=0.2
α=0.24
α=0.27

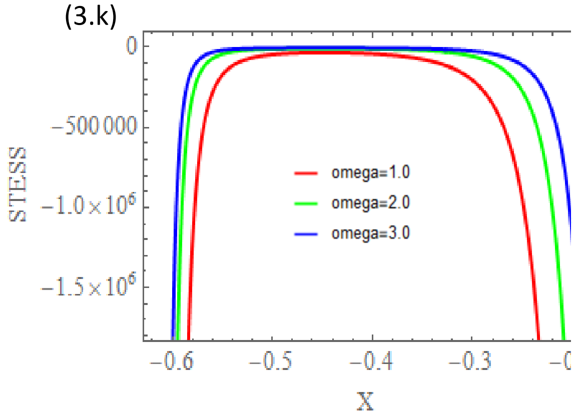
(3.i)



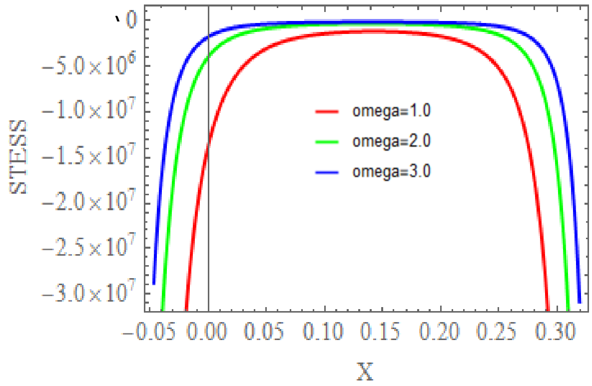
(3.j)



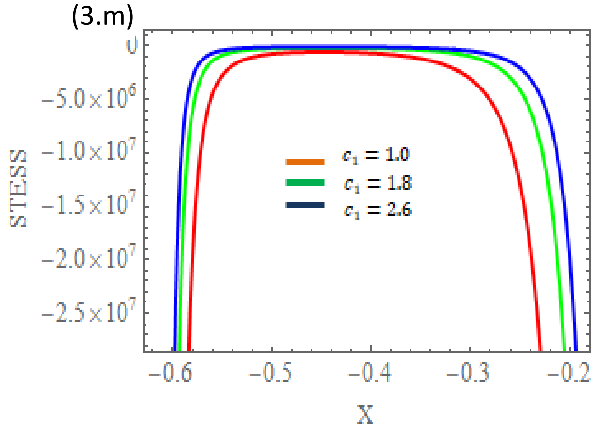
(3.k)



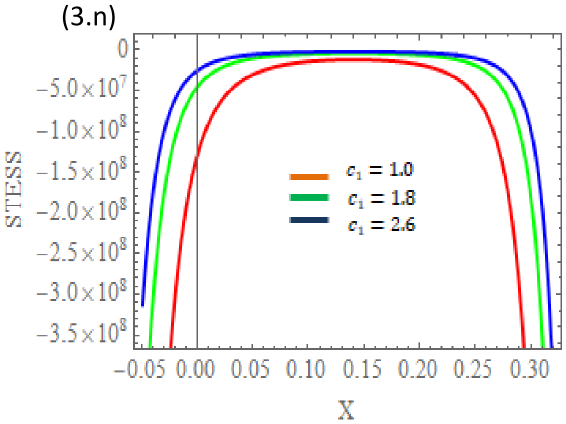
(3.l

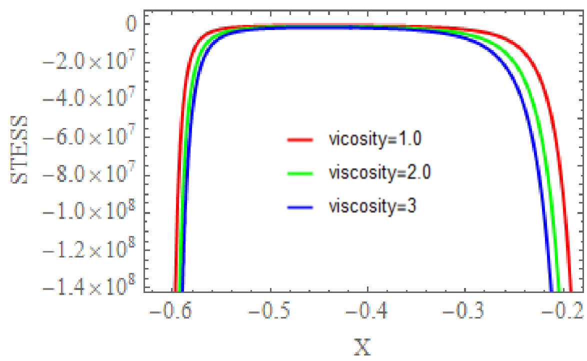


(3.m)

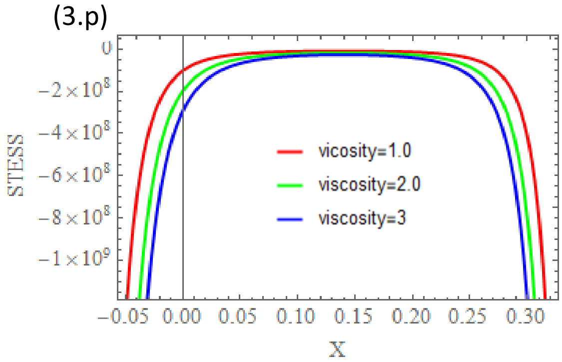


(3.n)





(3.p)



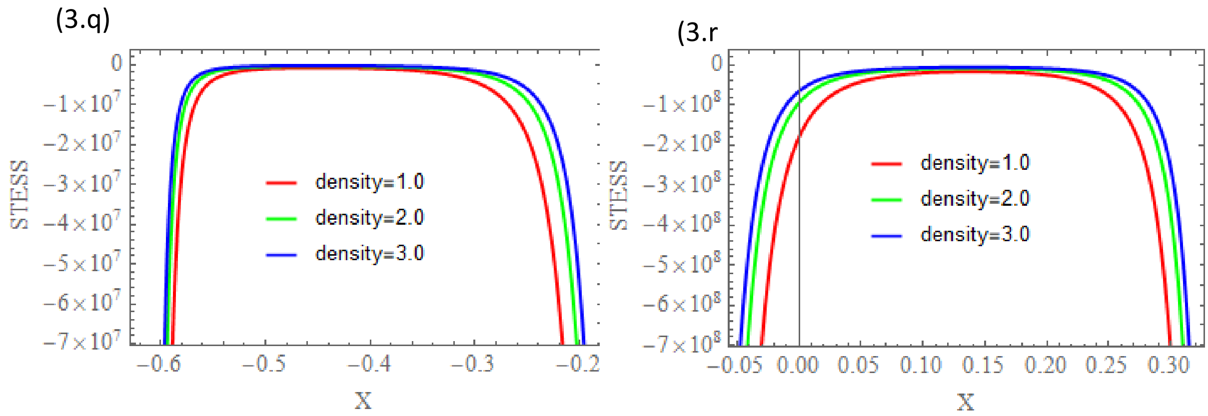


Figure 3- Change stress of two sides channel with different parameters{ GR= 0.2,BR= 0.4,BN= 4,y=0.5,ϵ=0.01,α=0.4,m=2.0,ϕ=Pi/4,q=1.6,Ω=0.2, c_1=0.8,μ=2.0,ρ=0.9,a=1}

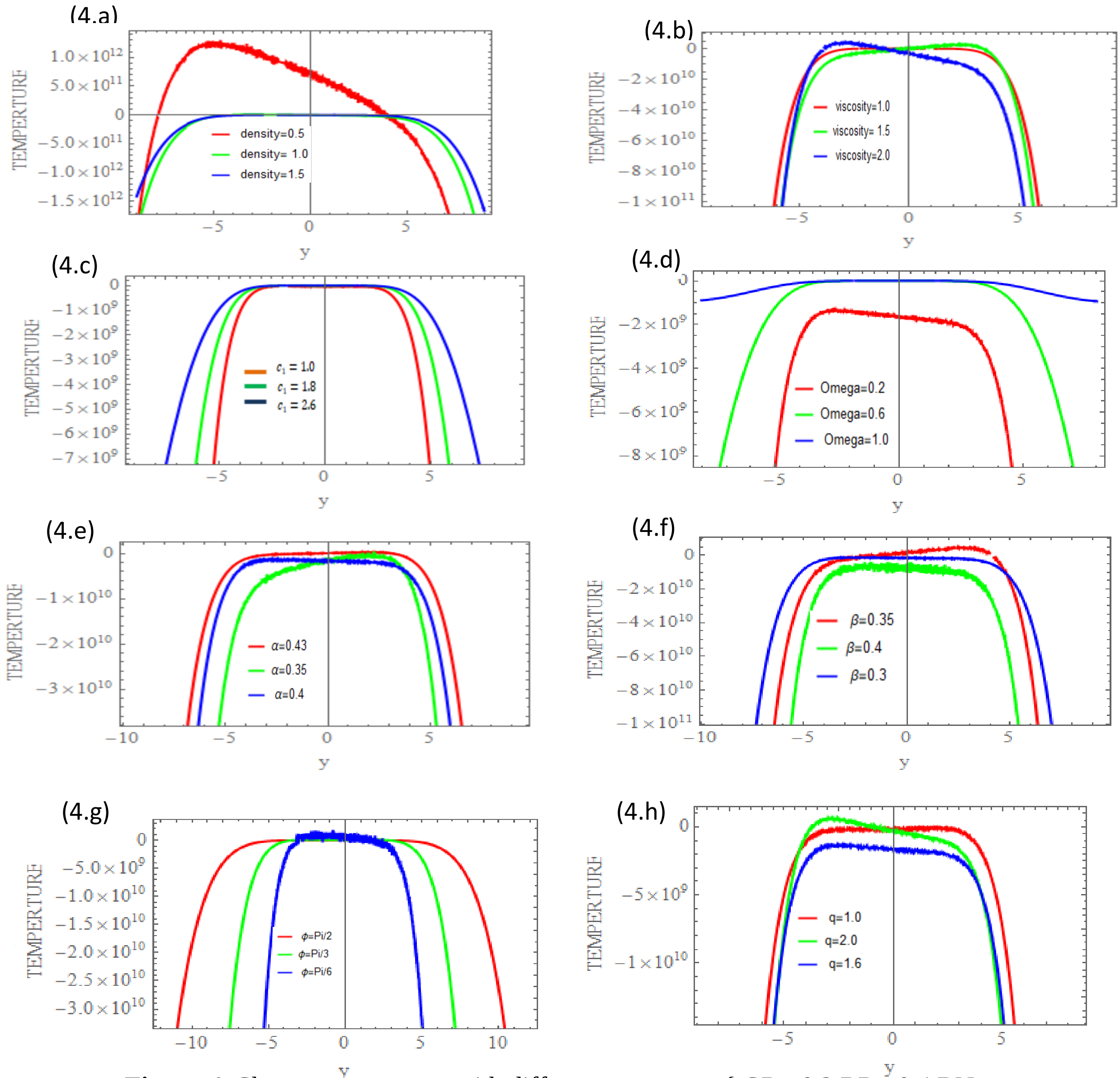
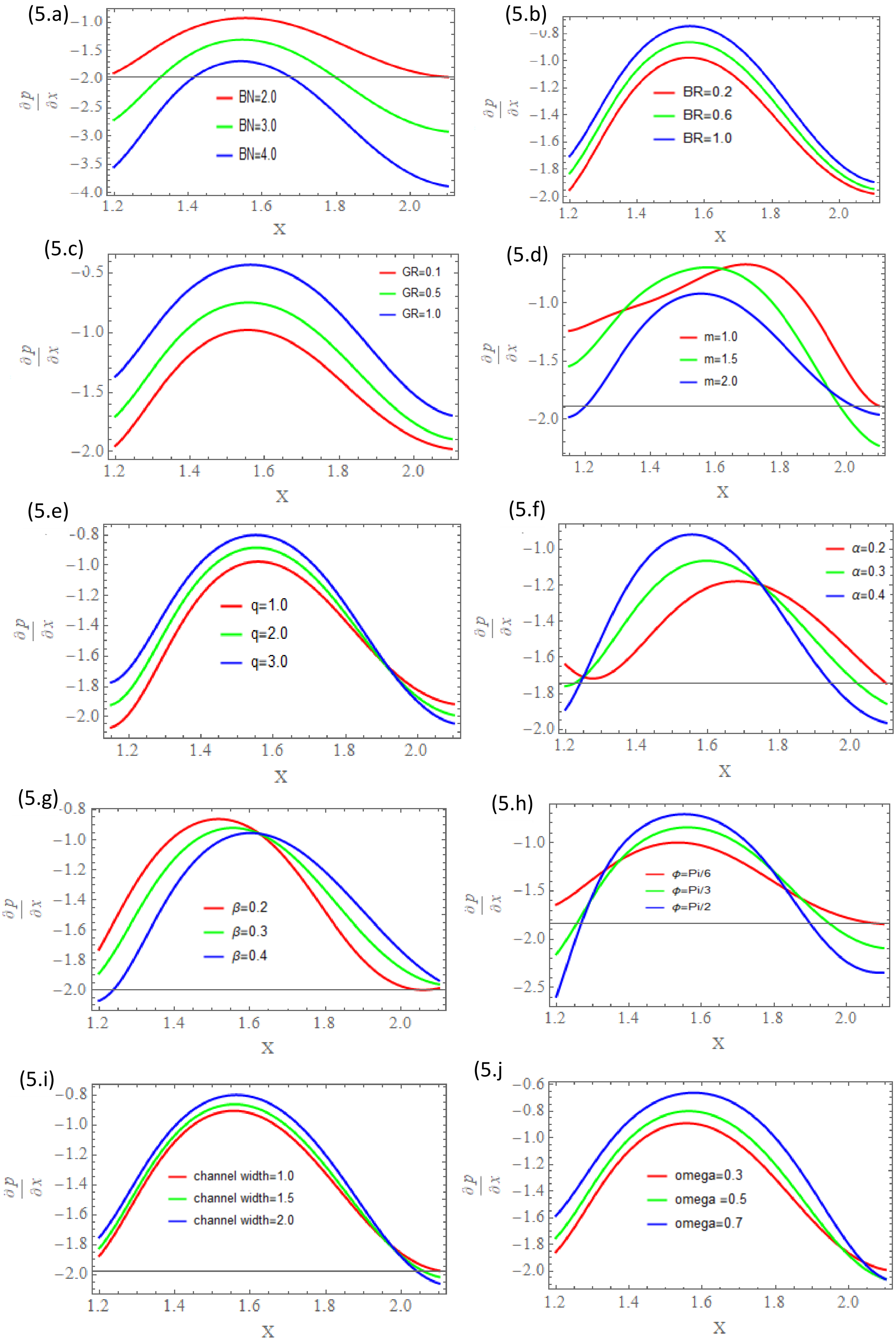


Figure-4 Change temperature with different parameters{ GR= 0.2,BR= 0.4,BN= 4,x=0.5,ϵ=0.01,α=0.4,m=2.0,ϕ=Pi/4,q=1.6,Ω=0.2, c_1=0.8,μ=2.0,ρ=0.9,a=1}


(5.a)
BN=2.0
BN=3.0
BN=4.0
(5.b)
BR=0.2
BR=0.6
BR=1.0
(5.c)
GR=0.1
GR=0.5
GR=1.0
(5.d)
m=1.0
m=1.5
m=2.0
(5.e)
q=1.0
q=2.0
q=3.0
(5.f)
α=0.2
α=0.3
α=0.4
(5.g)
β=0.2
β=0.3
β=0.4
(5.h)
φ=Pi/6
φ=Pi/3
φ=Pi/2
(5.i)
channel width=1.0
channel width=1.5
channel width=2.0
(5.j
omega=0.3
omega =0.5
omega=0.7
∂p/∂x
X

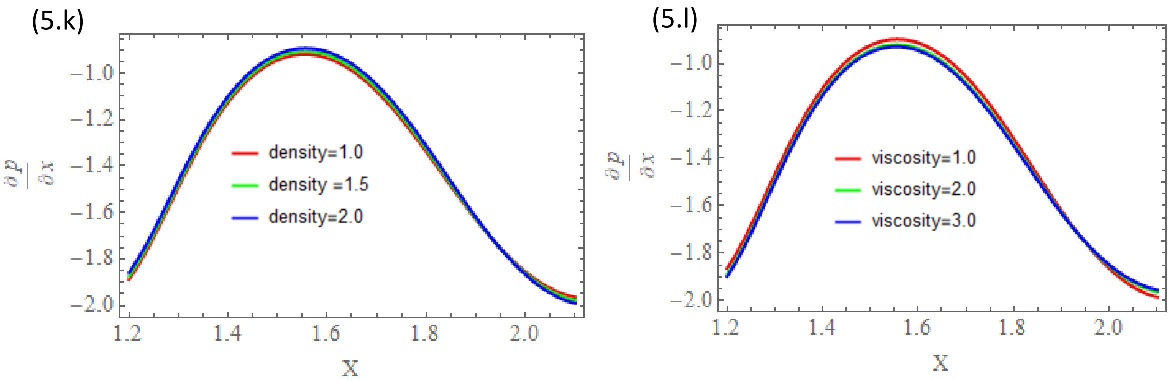


Figure-5 Change pressure gradient with different parameters{ GR= 0.2,BR= 0.4,BN= 4,γ=0.5,ϵ=0.01,α=0.4,m=2.0,ϕ=Pi/4,q=1.6,Ω=0.2, c_1=0.8,μ=2.0,ρ=0.9,a=1}

References

[1] T.W. Latham ,M.S. Thesis , MIT, Cambridge 1966.

[2] Jaffrin, M.Y., Shapiro, A.H. and Weinberg, S.L. "peristaltic pumping with long wavelength at low Reynolds numbers*," Journal of Fluid Mechanic*, 37: 799-825, 1969.

[3] Lew, H.S., Fung Y.C. and Lowenstein, C.B., "Peristaltic carrying and mixing of chime in the small intestine," *Journal of Biomechanics*, 4: 297-315, 1971.

[4] Hayat, T. and Ali, N., "Effect of Variable Viscosity on the Peristaltic Transport of Newtonian Fluid in an Symmetric Channel". *Appl. Math. Modell*. 32: 761-774, 2008.

[5] Nadeem S., Hayat T., Akber N.S. and Malik M.Y., "On the influence of hear transfer in peristaltic with variable viscosity," *Int. J. Heat Mass Transfer*. 52: 4722-4730, 2009.

[6] Kothandapani, M., Prakash, J. and Pushparaj, "Analysis of Heat and Mass Transport of MHD Peristaltic Flow through a Tapered Asymmetric Channel*," Journal of fluid* v.2015: Article ID 561263, 2015.

[7] Farooq, S., Awais, M., Naseem, M., Hayat, T. and Ahmed, B., "Magnetohydrodynamic peristalsis of variable viscosity Jeffrey fluid with heat and mass transfer," *Nuclear Engineering and Technology*, 49: 1396-1404, 2017.

[8] Huda, A.B., Akbar, N. Sh., Beg, O. A., Khan and M. Y., "Dynamics of variable-viscosity nanofluid flow with heat transfer in a flexible vertical tube under propagating waves*," Result in physics*, 7: 413-425, 2017.

[9] Misa, J.C., Mallick, B. and Sinha A., "Heat and mass transfer in asymmetric channels during peristatic transport of an MHD fluid having temperature-dependent properties*," Alexandria Engineering Journal*, 57: 391-406, 2018.

[10] A. M. Abd-All, S.M. Abo-Dahab, H.D. El-Shahrany, "Effect of rotation and initial stress on peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field," *Journal of Magnetism and Magnetic Materials*, 349: 268-280, 2014.

[11] Ahmed M.Abdulhadi, Tamara S.Ahmed, "Effect of magnetic field on peristaltic flow of Walters-B fluid through a porous medium in s tapered asymmetric channel," *Journal of Advance in Mathematics*, Volume 12, Number 12, ISSN 2347-1921, 2017.

[12] Hina Sadaf, Muhammed Akber, S. Nadeem, "Induced magnetic field analysis for the peristaltic transport of non- Newtonian nanofluid in an annulus," *Mathematics and Computers in Simulation*,148(2018) 16-36, 2018.

[13] Safia Akram, S. Nadeem, "Influence of induced magnetic field and heat transfer on the peristaltic motion of a Jeffery fluid in an asymmetric channel: Closed form solution," *Journal of Magnetism and Magnetic Material*, 328, 11-20, 2013.

[14] A.M. Abd-Alla, S.M. Abo-Dahab, "Rotation effect on peristaltic transport of a Jeffrey fluid in an asymmetric channel with gravity field," *Alexandria Engineering Journal* Vol. 55, 2016.

[15] Zaheer Asghar, Nasir Ali, "Mixed convective heat transfer analysis for the peristaltic transport of viscoplastic fluid: perturbation and numerical study, AIP Advances 9, 095001; doi: 10.1063/1.5118846, 2019.

[16] M.R. Salman and A.M. Abdulhadi, "Influence of MHD on peristaltic transport and heat transfer with variable viscosity in porous medium," *International Journal of science and Reseach*,7(2), 391-406, 2018.
[17] M.A. Murad and A.M. Abdulhadi, "Influence of MHD on mixed convective heat and mass transfer analysis for the peristaltic transport of viscoplastic fluid with porous medium in tapered channel," *Journal of Al-Qadisiyah foe computer Science and Mathematics*,Vol.12(4),79-90, 2020.
[18] R. S. Kareem and A.M. Abdulhadi, "Impact of Heat and Mass Transfer on Magneto Hydrodynamic Peristaltic Flow Having Temperature-dependent Properties in an Inclined Channel Through Porous Media," *Iraqi Journal of Science*, Vol. 61, No. 4, pp: 854-869, 2020.
[19] T Sh Alshareef, "Impress of rotation and an inclined MHD on waveform motion of the non-Newtonian fluid through porous canal", *J. Phys.: Conf. Ser.* 1591 012061, 2020.
[20] K. Kalyani, N. Seshagiri Rao, M.V.V.N.L. Sudha Rani," A Numerical Study on Cross Diffusion Cattaneo-Christov Impacts of MHD Micropolar Fluid Across a Paraboloid", *Iraqi Journal of Science*, Vol. 62, No. 4, pp: 1238-1264,,2021.
[21] Y. Wang, *Acta Mechanica* 186, 187 ,2006.
[22] N. Ali, T. Hayat, and Y. Wang," Method Fluid," *International Journal Number*, 64, 992 ,2009.