Effects of the Rotation on the Mixed Convection Heat Transfer Analysis for the Peristaltic Transport of Viscoplastic Fluid in Asymmetric Channel

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Abstract
In this paper, we study the impact of the variable rotation and different variable on mixed convection peristaltic flow of incompressible viscoplastic fluid. This is investigated in two dimensional asymmetric channel, such as the density, viscosity, rate flow, Grashof number, Bingham number, Brinkman number and tapered, on the mixed convection heat transfer analysis for the peristaltic transport of viscoplastic fluid with consideration small Reynolds number and long wavelength, peristaltic transport in asymmetric channel tapered horizontal channel and non-uniform boundary walls to possess different amplitude wave and phases. Perturbation technique is used to get series solutions. The effects of different values of these parameters for the axial velocity, stream function, stress, temperature and pressure gradient have been obtained. The impact of these parameters is discussed and illustrated graphically through the set of figures. Numerical results have been computed by using MATHEMATICA software.

Keywords: peristaltic transport, heat transfer, rotation, velocity, stream function, stress, viscoplastic.
1- Introduction

The peristaltic pumping is known with special type pumping when it can easily be transported the variety of complex rheological fluids from one place to another place. This pumping precept is called peristaltic. The mechanism includes involuntary periodic contraction, which is followed by the expansion of the ducts that the fluid moves through. This leads to rising in a pressure gradient that eventually pushes the fluid forward. The movements of the small intestine are due to peristaltic, these movements of the small intestine are essential for the normal processes of digestion to take place, since they are responsible for mixing food with digestive juices that expose chime to the surface for absorption of nutrients and propelling material along the gastrointestinal tract. Besides this, urine transports from kidney to the bladder through ureter, semen which moves through the vas deferens, vasomotion of small blood vessels such as arterioles, lymphatic fluid moves through lymphatic vessels, and bile flows from the gall bladder into the duodenum. Peristaltic has been greatly exploited for industrial and engineering purposes e.g. peristaltic can be used in mining slurries, waste water slurries, sodium bromide and slurry pumping. Applications of peristaltic in industrial fluid mechanics like aggressive chemical, high solid slurries, noxious (nuclear industries) and other material that are transported by peristaltic pumps, roller pumps, hose pumps, tube pumps, finger pumps, heart-lung machines, blood pump machines, and dialysis machines are engineering on the basis of peristalsis. The study of the peristaltic transport of the fluid in the presence of an external magnetic field and rotation is of great importance with regard to certain problems involving the movement of conductive physiological fluid, e.g. blood and saline water. The various applications of peristalsis have been attracting the interests of researchers after the seminal work of Latham[1]. Heat transfer is an important principle in biological systems and industrial fluid transport. One of the most important functions of the cardiovascular system is that maintains the temperature of the body. Air enters the lungs must be also warmed (or cooled) to body temperature. This is accomplished through all blood vessels. There are three mechanisms of heat transfer, but the convection is the most applicable heat transfer modality within the circulation of fluid in human body. Convection heat transfer is used by human and animal bodies to lose the heat that generated by metabolic processes to the environment. The industrial applications include the thermal insulation, cooling of nuclear reactor, oil extraction, and thermal energy storage. In the recently years, the mixed of the effects the heat and mass, the effect of variable viscosity, and temperature variable are studied in [2-9]. In [10], authors investigated the effect the rotation and initial stress on the peristaltic flow of an incompressible fluid. The mechanism of peristaltic transport has attracted the attention of many researchers since it is investigated by Ahmed M. and Tamara S. [11], Hina and Muhammed and Nadeem [12], Safia and Nadeem [13], Abd-Alla and Abo-Dahab[14], Zaheer and Nasir [15], Salman and Abdulhadi [16], Murad and Abdulhadi [17], Kareem and Abdulhadi [18]. Impress of rotation and an inclined MHD on waveform motion of the non-Newtonian fluid through porous canal are discussed in [19], K. Kalyani, N. Seshagiri Rao and Sudha Ran [20].

In this paper, we will study the rotation effects of the mixed of heat transfer for the peristaltic transport in an asymmetric channel, we use different values of the parameters of rotation, density, amplitude wave and tapered of channel, as well as we also use different value of the Bingham number, Prandtl number, Brinkman number and Grashof number by study from through change of the velocity, pressure gradient, stream function, variation of stress and temperature.
2- Formulation of the problem

Let the peristaltic transport of an incompressible viscoplastic fluid in an asymmetric channel of width \( c_1 + c_2 \) in two-dimensional coordinate. The flow is generated by continuously moving sinusoidal waves with speed \( c \) along the walls of the channel. The wall surface at the upper wall is defined by:

\[
\bar{H}_1(\bar{X}, \bar{t}) = c_1 + \bar{M} \bar{X} + \bar{\alpha} \cos\left(\frac{2\pi}{\lambda}(\bar{X} - c \bar{t})\right), \tag{1}
\]

while at the lower wall is given by:

\[
\bar{H}_2(\bar{X}, \bar{t}) = -c_2 - \bar{M} \bar{X} - \bar{\beta} \cos\left(\frac{2\pi}{\lambda}(\bar{X} - c \bar{t})+ \phi\right), \tag{2}
\]

Where \( (c_1, c_2) \) indicate the channel width, \( (\bar{\alpha}, \bar{\beta}) \) are the amplitudes of the wave, \( (\lambda) \) wavelength, and \( (c) \) is the speed wave. \( \bar{M} \) is the non-uniform parameter (trapping), when the value of \( \phi = 0 \) is the symmetric channel with waves out of phase and \( \phi = \pi \) waves are in phase, \( (\bar{X}, \bar{Y}) \) are the rectangular coordinates in fixed frame of reference, \( \bar{t} \) is the time, further, \( \bar{X}, \bar{Y}, \bar{t}, \bar{\alpha}, \bar{\beta} \) and \( \phi \) satisfy the following condition.

\[
\bar{\alpha}^2 + \bar{\beta}^2 + 2\bar{\alpha}\bar{\beta} \cos \phi \leq (c_1 + c_2)^2 \tag{3}
\]

Here \( T_0 \) and \( T_1 \) prescribe temperature at the walls. The governed equations of the flow by three coupled non-linear partial differential of continuity, momentum and energy, which in frame \((\bar{X}, \bar{Y})\) are expressed as:

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{4}
\]

\[
\rho \left( \frac{\partial \bar{u}}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{u}}{\partial \bar{y}} \right) - \rho \Omega (\bar{U} \bar{u} + 2 \frac{\partial \bar{v}}{\partial \bar{t}}) = - \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{y}} + \rho \gamma (T - T_0) \tag{5}
\]

\[
\rho \left( \frac{\partial \bar{v}}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{v}}{\partial \bar{y}} \right) - \rho \Omega (\bar{U} \bar{v} - 2 \frac{\partial \bar{u}}{\partial \bar{t}}) = - \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial \bar{S}_{\bar{y}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{y}\bar{y}}}{\partial \bar{y}} \tag{6}
\]

\[
\rho C_p \left( \frac{\partial \bar{T}}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = Z \left( \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right) + \sigma \cdot (\nabla \bar{V}) \tag{7}
\]

Where \( \rho \) is fluid density, \( \bar{V} = [\bar{U}, \bar{V}] \) is the velocity vector, \( \bar{P} \) is the hydrodynamic pressure, \( \gamma \) is acceleration due to gravity, \( \gamma \) is the coefficient of thermal expansion. \( \bar{S}_{\bar{x}\bar{x}}, \bar{S}_{\bar{y}\bar{y}} \) and \( \bar{S}_{\bar{x}\bar{y}} \) are the component of extra stress tensor \( \bar{S} \), \( C_p \) is specific heat, while \( Z \) is the thermal conductivity, \( \Omega \) is the rotation, \( T \) is a temperature, \( \nabla = [\frac{\partial \bar{u}}{\partial \bar{x}}, \frac{\partial \bar{v}}{\partial \bar{y}}] \), \( \sigma \) is the cauchy stress tensor which for the Bingham plastic fluid is defined [21]:

\[
\sigma = -\bar{P} \bar{I} + \bar{S} \tag{8}
\]

where,

\[
\bar{S} = 2\mu \omega + 2\tau_0 \bar{\omega} \tag{9}
\]

In equation (9) \( \tau_0 \) is the yield stress while the rate of deformation tensor \( \omega \) and \( \bar{\omega} \) is the tensor are defined:

\[
\omega = \frac{1}{2} \left( (\nabla \bar{V}) + (\nabla \bar{V})^T \right), \quad \bar{\omega} = \frac{\omega}{\sqrt{\epsilon_0 + 2 \tau_0 \omega}} \tag{10}
\]

the presence \( \epsilon_0 \) is necessary event prevents from unboundedness when \( \omega \rightarrow 0 \). The definition of dot product of arbitrary tensors D and E, i.e., \( D \cdot E = tra(D)E \) enables us to write energy equation (5) as

\[
\rho C_p \left( \frac{\partial \bar{T}}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = Z \left( \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right) + \bar{S}_{\bar{x}\bar{x}} \left( \frac{\partial \bar{U}}{\partial \bar{x}} \right) + \bar{S}_{\bar{y}\bar{y}} \left( \frac{\partial \bar{V}}{\partial \bar{y}} \right) + \bar{S}_{\bar{x}\bar{y}} \left( \frac{\partial \bar{U}}{\partial \bar{y}} \right) \tag{11}
\]

The components of extra stress tensor in laboratory frame in equation (9), and (10) become

\[
\bar{S}_{\bar{x}\bar{x}} = 2 \mu \omega_{\bar{x}\bar{x}} + \frac{2\tau_0 \omega_{\bar{x}\bar{y}}}{\sqrt{\epsilon_0 + 2 \tau_0 \omega}}, \quad \bar{S}_{\bar{y}\bar{y}} = 2 \mu \omega_{\bar{y}\bar{y}} + \frac{2\tau_0 \omega_{\bar{x}\bar{y}}}{\sqrt{\epsilon_0 + 2 \tau_0 \omega}} \tag{12}
\]

Peristaltic motion in natural is unsteady phenomenon, however it can be assumed steady by using the transformation from laboratory frame(fixed frame) \((\bar{X}, \bar{Y})\) to wave frame(move...
Mohaisen and Abedulhadi  

The relationship between coordinates, velocities and pressure in laboratory frame \( (\tilde{x}, \tilde{y}) \) and wave frame \((\tilde{x}, \tilde{y})\) is provided by the following transformations

\[
\tilde{x} = \frac{x}{\lambda}, \quad \tilde{y} = \frac{y}{c}, \quad \tilde{u} = \frac{u}{c}, \quad \tilde{v} = \frac{v}{c}, \quad \tilde{t} = \frac{t}{\lambda}, \quad \tilde{p} = \frac{p}{c^2_1}.
\] (13)

Where \( \tilde{u}, \tilde{v}, \tilde{p} \) are velocity components and pressure in wave frame. Now, we transform equations (1,2,4,5,6,11,12) in wave frame with helping of equation (13) and normalize the resulting equation by using the following dimensionless quantities:

\[
h_1 = \frac{1}{c_1}, \quad h_2 = \frac{\tilde{h}_2}{c_1}, \quad S = \frac{\tilde{S}}{c_1}, \quad \theta = \frac{T_T - T_0}{T_1 - T_0}, \quad M = \frac{c_1^2 \tilde{m}}{c^2}, \quad a = \frac{c^2_1}{\tilde{c}_1}, \quad E_c = \frac{c^2_1}{c_1 \rho r}, \quad G_r = \frac{\rho g c_1^2}{\mu c} (T_1 - T_0),
\]

\[
B_r = P_r E_c, \quad B_n = \frac{c_1 \tau_0}{\mu c}, \quad \varepsilon = \frac{e \alpha c_1^2}{c^2} (0 < \varepsilon \ll 1)
\]

To obtain,

\[
h_1 = 1 + mx + \alpha \cos(2\pi x) \quad \text{(14)}
\]

\[
h_2 = -a - mx - \beta \cos(2\pi x + \phi) \quad \text{(15)}
\]

The equation (3) in dimensionless frames is also given by:

\[
\alpha^2 + 2\alpha^2 \beta^2 \cos \phi \leq (1 + \alpha)^2
\]

\[
\delta \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \quad \text{(16)}
\]

\[
R_e \delta \left( \frac{\partial \tilde{u}}{\partial x} + \gamma \frac{\partial \tilde{v}}{\partial y} \right) - \frac{\rho c_1^2}{\mu} \Omega^2 u = - \frac{\partial p}{\partial x} + \delta \frac{\partial \tilde{S}_{xx}}{\partial x} + \frac{\partial \tilde{S}_{xy}}{\partial y} + G_r \theta \quad \text{(17)}
\]

\[
R_e \delta^2 \left( \frac{\partial \tilde{u}}{\partial x} + \gamma \frac{\partial \tilde{v}}{\partial y} \right) - \delta \frac{\partial \tilde{S}_{xy}}{\partial y} - 2\Omega R_e \delta^2 \frac{\partial \tilde{u}}{\partial x} = - \frac{\partial p}{\partial x} + \delta^2 \frac{\partial \tilde{S}_{yy}}{\partial x} + \delta \frac{\partial \tilde{S}_{xy}}{\partial y} \quad \text{(18)}
\]

\[
R_e \delta \left( \frac{\partial \tilde{u}}{\partial x} + \gamma \frac{\partial \tilde{v}}{\partial y} \right) = \frac{1}{\nu_r} \left( \delta^2 \frac{\partial \tilde{\theta}}{\partial x^2} + \frac{\partial \tilde{\theta}}{\partial y} \right) + \delta E_c S_{xy} \frac{\partial \tilde{u}}{\partial x} + E_c S_{xy} \frac{\partial \tilde{v}}{\partial y} \quad \text{(19)}
\]

\[
S_{xx} = 2 \frac{\partial \tilde{u}}{\partial x} + \frac{2\beta_n \frac{\partial \tilde{u}}{\partial x}}{\sqrt{\varepsilon + 2\beta_n^2 \frac{\partial \tilde{u}}{\partial x} + 2 \left( \frac{\partial \tilde{u}}{\partial y} \right)^2}}, \quad \text{(20)}
\]

\[
S_{xy} = \frac{\left( \frac{\partial \tilde{u}}{\partial y} + \delta \frac{\partial \tilde{v}}{\partial x} \right)}{\sqrt{\varepsilon + 2\beta_n^2 \frac{\partial \tilde{u}}{\partial x} + 2 \left( \frac{\partial \tilde{u}}{\partial y} \right)^2}} + B_n \frac{\left( \frac{\partial \tilde{u}}{\partial y} + \delta \frac{\partial \tilde{v}}{\partial x} \right)}{\sqrt{\varepsilon + 2\beta_n^2 \frac{\partial \tilde{u}}{\partial x} + 2 \left( \frac{\partial \tilde{u}}{\partial y} \right)^2}}, \quad \text{(21)}
\]

\[
S_{xy} = 2 \frac{\partial \tilde{v}}{\partial y} + \frac{2\beta_n \frac{\partial \tilde{v}}{\partial y}}{\sqrt{\varepsilon + 2\beta_n^2 \frac{\partial \tilde{u}}{\partial x} + 2 \left( \frac{\partial \tilde{u}}{\partial y} \right)^2}}, \quad \text{(22)}
\]

In previous equations, \( T_1 \) and \( T_0 \) are the temperature at the upper and lower walls, respectively. The dimensionless number \( \delta \) is the wave number, \( B_n \) is Bingham number, \( R_e \) Reynolds number, \( P_r \) Prandtl number, \( E_c \) Eckert number, \( G_r \) Grashof number, \( \varepsilon \ll 1 \), \( B_r \) Brinkman number and \( \theta \) is the temperature in dimensionless form. Introduction to the stream function \( (\psi) \) by relation:

\[
u = \psi_y, \quad v = -\delta \psi_x.
\]

From equations (17-23) show that the continuity equation (17) satisfies identically while other equations will take the following form:

\[
R_e \delta \frac{\partial \tilde{\psi}_y}{\partial x} - \frac{\partial \tilde{\psi}_x}{\partial y} \frac{\partial \tilde{\psi}_y}{\partial x} = \frac{\mu^2 c_1^2 \rho}{2} \frac{\partial \tilde{\psi}_x}{\partial t} = - \frac{\partial p}{\partial x} + \delta \frac{\partial \tilde{S}_{xx}}{\partial x} + \frac{\partial \tilde{S}_{xy}}{\partial y} + G_r \theta \quad \text{(24)}
\]

\[-R_e \delta^3 \left( \psi_y \frac{\partial \psi_x}{\partial x} - \delta \psi_x \frac{\partial \psi_x}{\partial y} \right) = \frac{\mu^2 c_1^2 \rho}{2} \frac{\partial^2 \psi_x}{\partial t^2} = - \frac{\partial p}{\partial x} + \delta^2 \frac{\partial \tilde{S}_{xx}}{\partial x} + \delta^2 \frac{\partial \tilde{S}_{xy}}{\partial y} \quad \text{(25)}
\]

\[
R_e P_r \delta \left( \frac{\partial \tilde{\theta}}{\partial y} - \frac{\partial \tilde{\psi}_x}{\partial t} \right) = \left( \delta^2 \frac{\partial \tilde{\theta}}{\partial x^2} + \frac{\partial \tilde{\theta}}{\partial y^2} \right) + B_r \delta S_{xx} \psi_{yy} + B_r \delta S_{xy} \left( \psi_{yy} - \delta^2 \psi_{xx} \right) - \delta B_r S_{yy} \psi_{xy} \quad \text{(26)}
\]

\[
S_{xx} = 2 \delta \psi_x + \frac{2\beta_n \psi_{xy}}{\sqrt{\varepsilon + 2\beta_n^2 (\psi_{xy})^2 + 2 \left( \psi_{yy} - \delta^2 \psi_{xx} \right)^2}}, \quad \text{(27)}
\]
\[
S_{xy} = (\psi_{yy} - \delta^2 \psi_{xx}) + \frac{B_n(\psi_{yy} - \delta^2 \psi_{xx})}{\sqrt{\varepsilon + 2\delta^2(\psi_{xx})^2 + 2\delta^2(\psi_{yy})^2 + (\psi_{yy} - \delta^2 \psi_{xx})^2}} \\
S_{yy} = -2\delta \psi_{xy} - \frac{2\delta B_n \psi_{xy}}{\sqrt{\varepsilon + 2\delta^2(\psi_{xx})^2 + 2\delta^2(\psi_{yy})^2 + (\psi_{yy} - \delta^2 \psi_{xx})^2}}
\]

The equations from (24) to (29) when \((R_e \text{ and } \delta \ll 1)\) become in the form:
\[
-\frac{\alpha^2 c_1^2 \rho}{\mu} \psi_y = -\left( \frac{\partial \rho}{\partial x} + \frac{\partial S_{xy}}{\partial y} + G_r \theta \right) \\
-\frac{\partial \rho}{\partial y} = 0
\]
\[
\frac{\partial^2 \theta}{\partial y^2} + B_r S_{xy} \psi_{yy} = 0
\]

Whereas the component of extra stress tensor become in the form:
\[
S_{xy} = (\psi_{yy}) + \frac{B_n \psi_{yy}}{\sqrt{\varepsilon + (\psi_{yy})^2}} \quad S_{yy} = 0, \quad S_{xx} = 0.
\]

If we substitute equation (33) into (30) and take the derivative with respect of \(y\), we obtain the following equation:
\[
-\frac{\alpha^2 c_1^2 \rho}{\mu} \psi_{yy} = \frac{\partial^2 S_{xy}}{\partial y^2} + G_r \frac{\partial \theta}{\partial y}
\]

while if we substitute equation (33) into equations (34,32) we get a coupled and high nonlinear differential equations:
\[
\frac{\partial^2}{\partial y^2} ((\psi_{yy}) + \frac{B_n \psi_{yy}}{\sqrt{\varepsilon + (\psi_{yy})^2}}) + G_r \frac{\partial \theta}{\partial y} + \frac{\alpha^2 c_1^2 \rho}{\mu} \psi_{yy} = 0
\]
\[
\frac{\partial^2 \theta}{\partial y^2} + B_r ((\psi_{yy})^2 + \frac{B_n (\psi_{yy})^2}{\sqrt{\varepsilon + (\psi_{yy})^2}})
\]

the dimensionless volume flow rate and boundary condition in the wave frames are [22]:
\[
\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 0, \quad \text{at } y = h_2
\]
\[
\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 0, \quad \text{at } y = h_1
\]
\[
q - 2 = F \int_{h_1}^{h_2} \frac{\partial \psi}{\partial y} dy = \psi(h_1) - \psi(h_2)
\]

F and q are the dimensionless mean flow rate in fixed and wave frames. The dimensionless expression of the pressure rise over one cycle of the wave is:
\[
\Delta P_h = \int_0^1 \frac{\partial \rho}{\partial x} dx
\]

3- Solution of the Problem

The perturbation method is used to get the solution of previous non-linear partial differential equation system. We start with inserting the following expressions:
\[
\Psi = \sum_{i=0}^{\infty} \theta_{0i} (G_r)^i + B_n \sum_{i=0}^{\infty} \theta_{1i} (G_r)^i + O(Bn^2)
\]
\[
\theta = \sum_{i=0}^{\infty} \theta_{0i} (G_r)^i + B_n \sum_{i=0}^{\infty} \theta_{1i} (G_r)^i + O(Bn^2)
\]

from equation (35) to the (39) we get the following system:
3.1-Zeroth Order system
\[
\frac{\partial^4 \psi_{00}}{\partial y^4} + k = 0, \quad \frac{\partial^2 \theta_{00}}{\partial y^2} + B_r \left( \frac{\partial^2 \psi_{00}}{\partial y^2} \right)^2 = 0
\]
\[
\frac{\partial^4 \psi_{01}}{\partial y^4} + \frac{\partial \theta_{00}}{\partial y} + k = 0, \quad \frac{\partial^2 \theta_{01}}{\partial y^2} + 2B_r \left( \frac{\partial^2 \psi_{00}}{\partial y^2} \right) \left( \frac{\partial^2 \psi_{01}}{\partial y^2} \right) = 0
\]

Where the boundary conditions are:
\[
\psi_{00}(h_1) = q/2, \quad \psi_{00}(h_2) = -q/2, \quad \psi_{00}(h_1) = -1, \quad \psi_{00}(h_2) = -1, \quad \psi_{01}(h_1) = 0, \quad \psi_{01}(h_2) = 0,
\]
\[
\psi_{01y}(h_1) = 0, \quad \psi_{01y}(h_2) = 0, \quad \theta_{00}(h_1) = 0, \quad \theta_{00}(h_2) = 0, \quad \theta_{01}(h_1) = 0, \quad \theta_{01}(h_2) = 0.
\]
### 3.2 - First Order System

\[
\psi_{10yy} + \frac{3(\psi_{0yy})^2(\psi_{0yy})^2}{(e+\psi_{0yy})^{3/2}} - \frac{3(\psi_{0yy})}{(e+\psi_{0yy})^{3/2}} + k=0 \tag{47}
\]

\[
\theta_{10yy} + \frac{B_T(\psi_{0yy})^2}{(e+\psi_{0yy})^{3/2}} + 2B_T(\psi_{0yy})(\psi_{10yy}) = 0 \tag{48}
\]

Where the boundary conditions are:

\[
\psi_{10}(h_1) = 0, \quad \psi_{10}(h_2) = 0, \quad \psi_{10y}(h_1) = 0, \quad \psi_{10y}(h_1) = 0, \quad \theta_{10y}(h_1) = 0, \quad \theta_{10y}(h_2) = 0. \tag{49}
\]

Where \( k = \frac{\alpha^2 c_s^2 \rho}{\mu} \).

For example of the solution of zeroth order when \( B_R = B_T \):

\[
\psi_{00} = \frac{kt^3 + k t \cos(\sqrt{k}y) - t \sin(\sqrt{k}y)}{k};
\]

\[
\theta_{00} = t^5 + y \ast t^6 - \frac{k(2k(t^2 + t^2)y^2 + (-t^2 + t^2)\cos[\sqrt{k}y] - 2t^2 \sin[\sqrt{k}y])}{8k},
\]

\[
\theta_{01} = t^11 + t^12y - \frac{1}{108k^{5/2}}(54k^{5/2}(t^1t + t^2t)y^2 - 108B RT^2(t^1 + t^2)\cos[\sqrt{k}y] - 9(-24\sqrt{k}t^1t + BR(t^1 + t^2)(13t^2 + 12\sqrt{k}t^1y))\cos[\sqrt{k}y] - 27k^{3/2}(t^1 - t^2)\cos[\sqrt{k}y] + BRT^2(-3t^1 + t^2)^2\cos[3\sqrt{k}y] + 108BRT^2(t^1 + t^2)\sin[\sqrt{k}y] + 9(24\sqrt{k}t^2t + BR(t^1 + t^2)(13t^1 - 12\sqrt{k}t^2y))\sin[\sqrt{k}y] - 27k^{3/2}(t^2t + t^1t^2)\sin[\sqrt{k}y] + BRT^1(t^1 - 3t^2)^2\sin[3\sqrt{k}y]).
\]

### 4- Results and discussions

In this section we will discuss the effect of different parameters on the axial velocity. Figure 1, pressure gradient Figure 5, stress distribution Figure 3 and temperature distribution Figure 4. The result described by the graphical clarification. The trapping phenomenon also studied for the different parameters through contour graphs Figure 2.

#### 4.1 - Velocity profile

At the \( x=0.5 \) cross-section the velocity axial is calculated. The comparison is done in Figure 1 based on different physical parameters such as the non-uniform parameter \( (m) \), the amplitudes wave \( (\alpha, \beta) \), varies in the range \( (\phi) \), Bingham number \( (B_R) \), Brinkman number \( (B_T) \), flow rate \( (q) \), rotation \( (\Omega) \), viscosity \( (\mu) \), density \( (\rho) \), and channel width \( (c_1) \).

The effect of \( (m) \) on the velocity is shown in Figure (1.a). The velocity distribution decreases in the middle of channel, but it increases near the walls channel when \( (m) \) is increasing. When the impact of amplitudes wave \( (\beta) \) decreases this leads to increase the velocity distribution near the walls see Figure (1.b). Varies in the range \( (\phi) \) is shown in Figure (1.c), the value of velocity in the middle decreases and it expands near the walls if the value is increasing. Flow rate \( (q) \) is shown in Figure (1.d) a slight drop in the middle and increasing at near the walls of channel when the value decreases. Bingham number \( (B_R) \) is shown in Figure (1.e) velocity a slight drop in the middle but continuity increases when its value is decreasing. The rotation \( (\Omega) \) is shown in Figure (1.f) the velocity decreases in the middle, while it continues to expand near the walls of the channel when its value is decreasing. Channel width \( (c_1) \) is shown in Figure (1.g) to observe the velocity a slight drop in the middle, and expands near the walls of channel when its value is increasing. The viscosity \( (\mu) \) is the conjugate of \( (c_1) \). Figure (1.h) shows that with a slight drop in middle of channel. The
density (ρ), the graph is nearly with other and a slight drop in middle channel that is shown in Figure(1.i). Brinkman number (BR) does not affect by change of value but a slight drop in the middle of channel in all change. This is clearly seen in Figure (1.j).

4.2 - Trapping Phenomenon

The process of the formulation of internally circulation of fluid bolus by closed stream line in the fluid flow is called trapping phenomenon, the behavior of stream function is shown in Figure (2). In Figure(2) comparison is done based on different physical parameters such that, the non-uniform parameter (m), the amplitudes wave (α, β), varies in the range (φ), Bingham number (BN), rotation (Ω), viscosity (μ), density (ρ), and channel width (c₁). In the case of increasing the value of Bingham the value of bolus at near of wall is increasing, that is shown in Figures (2.a,2.b,2.c). Figures (2.d, 2.e, 2.f, 2.m, 2.n, 2.o) show that if the (α, β) is increasing then the value of bolus is in middle and nearly to the entering to center. Figures (2.g, 2.h, 2.i) illustrate that if the non-uniform parameter (m) is increasing, then the value of m is observed, and the wave is centered at the center of the walls. When the value of (φ) increases this leads to an increasing in the number of boluses. This is clearly seen in Figures (2.j, 2.k, 2.l). The increasing value of density (ρ) leads to few change value for every bolus this is shown in Figures (2.p, 2.q, 2.r). An increasing value of viscosity (μ) leads to increase values of bolus in direction of center of channel , this is shown Figures (2.s, 2.t, 2.u). The increasing value of channel width (c₁) leads to an increasing in number of bolus in upper wall and gather inward of channel this is observed in Figures (2.v, 2.w, 2.x). An increasing value of rotation (Ω) leads to an increasing in number of bolus in upper wall but decreasing in number of bolus in lower wall and gather inward of channel this is shown in Figure (2.y, 2.z, 2.a1).

4.3 - Stress Distribution

It is shown in the Figure(3) from (3.a-3.r) the variation of the stress and comparison between the upper and lower channel using different parameters. For the upper channel it is represented by Figures (3.a, 3.c, 3.e, 3.g, 3.i, 3.k, 3.m, 3.o, 3.q), and the lower channel which is represented by Figures (3.b, 3.d, 3.f, 3.h, 3.j, 3.l, 3.n, 3.p, 3.r). where the parameters are used for comparison, non-uniform parameter (m), the amplitudes wave (α, β), varies in the range (φ), Bingham number (BN), rotation (Ω), viscosity (μ), density (ρ), and channel width (c₁).

4.4 - Temperature profile

In Figures(4) it is shown the variation of temperature distribution, we look at that from Figure (4.a-4.h), with different parameters with respect to the density (ρ), viscosity (μ), channel width (c₁), rotation (Ω), amplitudes wave (α, β), varies in the range (φ) and flow rate (q). To observe that if an decreasing value of (ρ) fluid leads to an increasing in temperature this is shown in Figure (4.a), and an increasing value of (μ) starts to increase the value of temperature and starts to low before to arrive to the middle of channel, however when increasing the value of (μ) starts to high after the middle of channel this is shown in Figure (4.b). The increasing the value of (c₁) with decreasing it is balance at the middle of channel but for increasing of channel width the temperature continuous to high at near the walls see Figure (4.c). The discussion of (Ω) is specially very sensitive if the value is increasing and continuous to high at near of walls see Figure (4.d). In the case of upper wall the decreasing of (α) leads to an increasing of temperature this is shown in Figure (4.e), however in case of lower the increasing value of (β) starts by high when it passed of the middle channel see Figure (4.f). In case of varies of range (φ) the increasing value leads to increase the of temperature and an increasing leads to an increasing at near of walls see Figure (4.g), with respect flow rate (q) choosing the perfect value leads to the increasing of the temperature see Figure (4.h).

4.5 - Pressure Gradient Distribution
Figures from (5.a-5.l) are shown the change behavior of pressure gradient with respect of axial coordinate x for the various parameters. We observe that the increasing value of (BN), (β), and (m) leads to decrease value of the pressure gradient see Figure (5.a), (5.g) and (5.d), respectively. However, the increasing magnitude of the (BR), (GR), (q), (α), (ϕ), (c₁) and (Ω) leads to increasing value of pressure gradient this is shown in Figures (5.b), (5.c), (5.e), (5.f), (5.h), (5.i) and (5.j) respectively, as for the (ρ) and (μ), their effect is minimal of increasing or decreasing values on the pressure gradient distribution see Figures (5.k), and (5.l), respectively.

5- Conclusion

In this research, we study the effect of the rotation on mixed convection heat transfer analysis peristaltic flow of the viscoplastic through the tapered horizontal asymmetric channel. The channel asymmetric is produced by choosing the peristaltic waves on the non-uniform walls to have different amplitude and phases low Reynolds number and along wavelength. We adopt the perturbation method to obtain the expression of the axial velocity, stream function, stress, temperature and pressure gradient. A parametric analysis is performed through various graphs.

A- When an increasing value of (m), (c₁), (ϕ) and (ρ) leads to increase the value of velocity, but an decreasing of values of (β), (q), (BN), (μ) and (Ω) lead to increase the value of velocity, as for the (BR) when it values change not impact the value of velocity.

B- In case of trapping, when the value of (BN) increases, the bolus expands and moves from the lower wall towards the upper wall. When an increasing in (α, β) we note that the bolus value increases in the middle and recedes in the center of the channel. When the value of (m) increases, it observes that the wave is directed towards the center of the channel, when the increasing value of (ϕ) leads to an increasing in the number of boluses, an increasing value of (ρ) leads to few change in the value for every bolus, an increasing value of the (μ) leads to that values of bolus are increasing in direction of center of channel, the increasing value of (c₁) leads to an increasing in number of bolus in upper wall and gather inward of channel, if the value of rotation (Ω) increasing leads to an increasing in number of bolus in upper wall but decreasing in number of bolus in lower wall and gather inward of channel.

C- The stress distribution, in case of the upper wall, when (BN) and (μ) are decreasing the value of stress is increasing at the upper wall, but an increasing value of (ϕ), (β), (Ω), (c₁), and (ρ) leads to an increasing in stress at lower walls, but for (m) and (α) at left of wall. In lower wall, the stress increasing when (BN), (ϕ) and (μ) are decreasing, increasing value of (m), (Ω), (α), (β), (c₁) and (ρ) implies to an increasing of stress value.

D- Temperature profile, an decreasing value of (ρ), (α) and (β) leads to increasing of temperature, but an increasing of (μ), (c₁), (Ω) and (ϕ) implies to increasing of temperature.

E- In case of the pressure gradient, when an decreasing of (BN), (m) and (β) high product of pressure gradient, but an increasing of (BN), (GR), (q), (α), (c₁), (Ω) and (ϕ) high product of pressure gradient.
Figure-1 Change velocity with different parameters {GR = 0.2,BR= 0.4,BN= 4,x=0.5,c=0.01,α=0.4,m=2.0,ϕ=Π/4,q=1.6,Ω=0.2, c₁=0.8,μ=2.0,ρ=0.9,a=1}
Figure-2 Change stream function with different parameters\{ GR= 0.2, BR= 0.4, BN= 4, x=0.5, \( \epsilon \)=0.01, \( \alpha \)=0.4, \( m \)=2.0, \( \phi \)=Pi/4, \( q \)=1.6, \( \Omega \)=0.2, \( c_1 \)=0.8, \( \mu \)=2.0, \( \rho \)=0.9, a=1 \}
\[ y = 1 + m \cdot x + \alpha \cdot \cos(2\pi x) \]  
\[ (3.a) \]

\[ y = -a - m \cdot x - \beta \cdot \cos(2\pi x + \phi) \]  
\[ (3.b) \]

\[ (3.c) \]

\[ (3.d) \]

\[ \phi \rightarrow \pi/6 \]
\[ \phi \rightarrow \pi/5 \]
\[ \phi \rightarrow \pi/4 \]

\[ (3.e) \]

\[ (3.f) \]

\[ m=0.1 \]
\[ m=0.5 \]
\[ m=1.0 \]

\[ (3.g) \]

\[ (3.h) \]

\[ \alpha=0.2 \]
\[ \alpha=0.24 \]
\[ \alpha=0.27 \]
Figure 3: Change stress of two sides channel with different parameters\( \{ GR=0.2, BR=0.4, BN=4, y=0.5, c=0.01, \alpha=0.4, m=2.0, \phi=\pi/4, q=1.6, \Omega=0.2, c_1=0.8, \mu=2.0, \rho=0.9, a=1 \}\)

Figure-4 Change temperature with different parameters\( \{ GR=0.2, BR=0.4, BN=4, x=0.5, c=0.01, \alpha=0.4, m=2.0, \phi=\pi/4, q=1.6, \Omega=0.2, c_1=0.8, \mu=2.0, \rho=0.9, a=1 \}\)
References


