DOI: 10.24996/ijs.2022.63.2.13

ISSN: 0067-2904

A Comprehensive Study and Analysis of the Chaotic Chua Circuit

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Received: 16/5/2021 Accepted: 25/7/2021

Abstract
was studied by taking several different values for the constant α and fixing the other three variables β, c and d with the values 25.58, -0.7142857, and -1.142, respectively. The purpose of this paper is to know the values by which the system transforms from a steady state to a chaotic state under the initial conditions x, y, and z that equal -1.6, 0 and 1.6 respectively. It was found that when the value of α is equal to 0, the Chua system is in a steady state, and when the value of α is equal to 9.5 and the wave is sinusoidal, the system is in oscillation, and when α is equal 13.4 the system is in a Quasi-chaotic state, and finally the system turns to the chaotic state when the value of α equals 15.05.

Keywords: Chaos, Chua, time series, attractor, bifurcation, FFT.

دراسة وتحليل شامل لدائرة تشوا الفوضوية

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الخلاصة
في هذا البحث تم دراسة سلوك نظام تشوا بأخذ عدة قيم مختلفة للمتغير α وتثبيت المتغيرات الثلاثة الأخرى β و c و d بالقيم 25.58 و −0.7142857 و −1.142 على التوالي. الغرض من هذا البحث هو معرفة القيم التي يتحول بها النظام من حالة مستقرة إلى حالة فوضوية في ظل الظروف الأولية x و y و z التي تساوي −1.6 و 0 و 1.6 على التوالي. لقد وجد أنه عندما تكون قيمة α تساوي 0 ، يكون نظام تشوا في حالة المستقرة ، وعندما تكون قيمة α تساوي 9.5 تكون الموجة جيبية ويكون النظام في حالة تذبذب ، وعندما تكون قيمة α تساوي 13.4 يصبح النظام في حالة شبه فوضوية ، وأخيراً يتحول النظام إلى حالة الفوضى عندما تكون قيمة α تساوي 15.05.

1. Introduction

In 1983, Leon Chua invented his electrical circuit, in response to two unfulfilled endeavors by many researchers on chaos, and that was about the required aspects of Lorenz's equations [1]. Chua decided to create or design a laboratory system that could realistically simulate Lorenz's equations(which are known to be Chaotic) in order to demonstrate that chaos is a strong physical phenomenon and not just an imaginary fantasy resulting from the errors of computer approximation[2][3]. So Chua wanted to prove that chaotic behavior occurs in all systems, even simple ones, which exist in all of our daily lives, such as radio and television. His second endeavor was to prove that the Lorenz's attractor obtained from these equations was a chaotic attractor [4]. Thus, Chua designed his circuit, known as the "Chua's Circuit." It is a simple

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electronic circuit that displays classic chaotic behavior, which means that it is an irregular oscillator. It produces an oscillating wave but unlike a normal electronic oscillator, the oscillating wave is never repeated[5]. The ease of building the circuit made it a realistic example of chaotic systems, leading some scientists to admit that it is a standard paradigm of studying chaos. Moreover, he invented it to produce chaos and show many dynamic phenomena such as complexity, synchronization, and bifurcation [6].
There are many previous studies that have used chaos theory to solve and develop many security problems [7-8], but the most important of these studies that have been conducted now is an attempt to hide important and confidential documents using chaotic technology, which are currently in the process of publication [9].

2. Chua system

The Chua circuit is a simple circuit that contains all the necessary elements as well as all the measurement components needed to control chaotic attractors. The circuit is easily designed, at a little cost, using standard electronic components [10] (as shown in Figure 1). The components of the Chua electronic circuit include the following: a linear inductor L, two linear capacitors C_1 and C_2, a linear dispersive resistor R, and a nonlinear voltage regulating resistor NR known as a Chua diode [6]. Table (1) shows the values of Chua circuit components. The Chua circuit has the following four dimensionless equations:

$$x' = \alpha(y - x - g(x))$$

$$y' = x - y + z \quad \text{--------------(1)}$$

$$z' = -\beta y$$

Where:

$$g(x) = cx + (\tfrac{1}{2})(d - c)(|x + 1| - |x - 1|) \text{---------------(2)}$$

Here α and β are bifurcation constants parameters for Chua's system (real number), and $g(x)$ is the nonlinear function, b , c, and d are stochastic parameters.
The major component of Chua's circuit is a nonlinear resistor (also known as Chua's diode) with an electric characteristic defined by function g(x). Since Chau's signals are usually broadband, noisy, and challenging to predict, they may be used in a variety of applications, including secure communication. Chau's system has many significant features, including strong sensitivity to the initial condition [11], pseudo-randomness, no periodicity, and system parameter dependence. These properties are related to Shannon's permutation and diffusion criteria for cryptosystem design [12]. So, for a given set of parameters, the circuit exhibits a chaotic strange attractor called the double scroll strange attractor [13-15]. But, generally, in order for an circuit to appear that chaos, it must fulfill some basic conditions which are absolutely necessary for chaos to appear in electrical circuits.
The conditions are:
1- The electrical circuit must contain at least three elements to store energy. In the case of the Chua's circuit, it contains an inductor and two capacitors, which are elements for electrical energy storage.
2- The electrical circuit must also contain at least one nonlinear element, for oscillation to be possible. In the case of the Chua's circuit, that element is represented by the Chua's diode.
3- The circuit must contain a locally active resistance element, the Chua's diode.
So, Chua's diode satisfies the last two points above, it represents the non-linear locally active resistor in Chua's circuit. It is active and has a negative resistance, and it supplies power to the circuit to generate an oscillating current.

Using Matlab in Bifurcations and Chaos allowed us to explore some important mathematical operations and how to integrate mathematical concepts into the programming environment. Many studies have been conducted in the field of chaos science [16-19] and one of these studies combined two different systems to produce a new type of chaotic behavior [20].

3. Results and discussion

In this paper, the behavior of the Chua circuit was studied experimentally and numerically by programming the four differential equations of Chua (Equation 1) using the Matlab program, where the differential equations were solved using Runga-Kutta integration of the fourth degree. The values of β, c and d were fixed at 25.58, -0.7142857, and -1.142, respectively but the value of the constant α was varied, noting that the values of the initial conditions of the system $x, y, and\ z$ were -1.6, 0, and 1.6, respectively

Modeling results

At α = zero, the system was stable and there was no oscillation (no peaks appeared), this case is called steady state as shown in Figure 2. Figure 3 represents the corresponding attractor for the steady state at a fixed point, which means there are no spikes in time series. At α = 9.5 , the system started to work as an oscillating system and a sine wave began to appear (as shown in Figure 4). This state is called the periodic state. and the time series is called the periodic state (that is, it has an almost uniform amplitude height) so the system moved from a steady state to a periodic state. Figure 5 represents the attractor corresponding to this state where a limit cycle was obtained where the internal orbits of the attractor occurred due to divergence in the amplitude chaotic signal. At α = 13.4 , this case is called quasi-chaotic, as shown in Figure 6. In this case, the appearance of many peaks in the time series of different amplitudes was noticed, but the system still did not enter the state of excessive chaotic system. The shape of the attractant, as shown in Figure 7, changed from the concentric circles and started to exit from the binary plane. Numerical solutions to the chaotic differential equations, when implementing normal computers, will exhibit quasi-chaotic behavior. The term "quasi-chaotic signal" refers to a periodic waveform with an unusually long duration that appears chaotic at short time intervals. Finally, when the value of α was increased to 15.05, the system became a hyper-chaotic system, as shown in Figure 8, which is represented by the time series of the dynamics of (x), (y) and (z). Figure 9 shows a double scroll attractor which indicates that although the system might not be sensitive to certain system parameters, it was highly responsive to others. The multi-structural geometry of the double scroll attractor renders it a more complex entity than any other 3D flow attractor.

Figure 10 shows the Fourier spectrum (FFT) corresponding to all the previous cases. Figure (10a) shows the steady state of the system as the system does not display any frequency (i.e. there are no spikes). Figure (10b) shows the Fourier spectrum in the periodic state of the system, which displays a single frequency. Figure (10c) shows the Fourier spectrum of the qusi-chaotic system, which displays many frequencies distinct from each another. Figure (10d) shows the Fourier spectrum of chaotic behavior of the system, which is characterized by the behavior of exponential decay, and this is in contrast to noise systems that display Gaussian behavior.

Studying the bifurcation of the Chua system, it was observed that there were specific values of α , where the system changed from a steady state to a chaotic state and these values are summarized as follows: -

- At α =0 → The system is a steady state.
- At α =9.5 → The system is a periodic state.
- At α =13.4 → The system is a quasi-chaotic.
- At α =15.05 → The system is a chaotic.

. Experimental results

Figure 1 shows Chua circuit, in the (V_{C2}, i_L)-plane, the parallel attachment of C_2 and L acts as a single simple oscillatory system, while the resistor R provides interaction between the

(V_{C2}, i_L)-oscillatory portion and the active resistor NR in conjunction with C_1. Since the active resistor NR continues supplying power to the circuit, the R resistor must be dissipative in order to stimulate the turbulent trajectories' attracting existence. Figure 11 shows the experimental set up of the circuit. When a voltage of ±9 V is supplied to the circuit the circuit exhibited a chaotic behavior when the variable resistance R was 0.980 kΩ. , Figure 12 shows the time series of the *x* and *y* dynamics where chaotic behavior is very clear. Figure 13 represents the (*x*,*y*) attractor of Chua's circuit with the characteristic double-scroll attractor. Figure 14 represents the FFT spectrum of *x* dynamics of Chua circuit. It shows an exponential decay pattern, which suggests a turbulent environment.

4. Conclusions

From this work, it was concluded that the system showed different behaviors of being stable, periodic, double parodic, quasi- chaotic, and chaotic at specific values of the constant α Moreover, it is not necessarily that the high value of α results in more chaos, but it is possible for the system to revert to a periodic or double cyclic state, which means that the behavior of the system may return to the initial state. Since the third equation of Chua's method has only one nonlinear term, experimental design of the electronic circuit to analyze Chua's attractor is simpler than Lorenz circuit. Chua's oscillator is easily assembled from low-cost basic electronic parts, as seen in the circuit schematic. Furthermore, the Chua circuit exhibits a wide range of bifurcations and chaos. Through this work, it was found that it is possible to control chaos and that the four variables have a tendency to reach equilibrium points [0, 0, 0, 0] at finite time values.

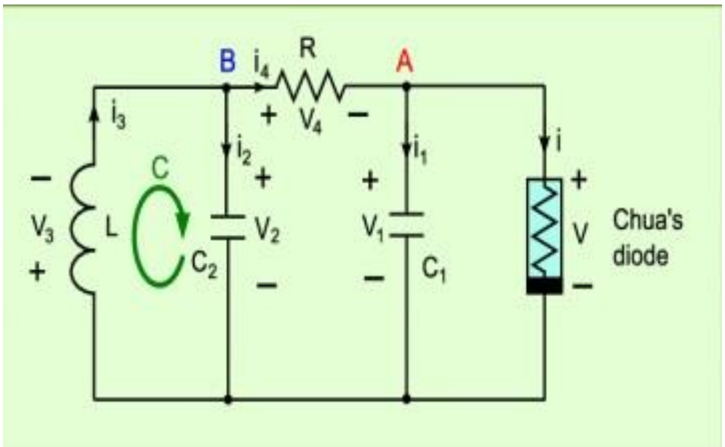
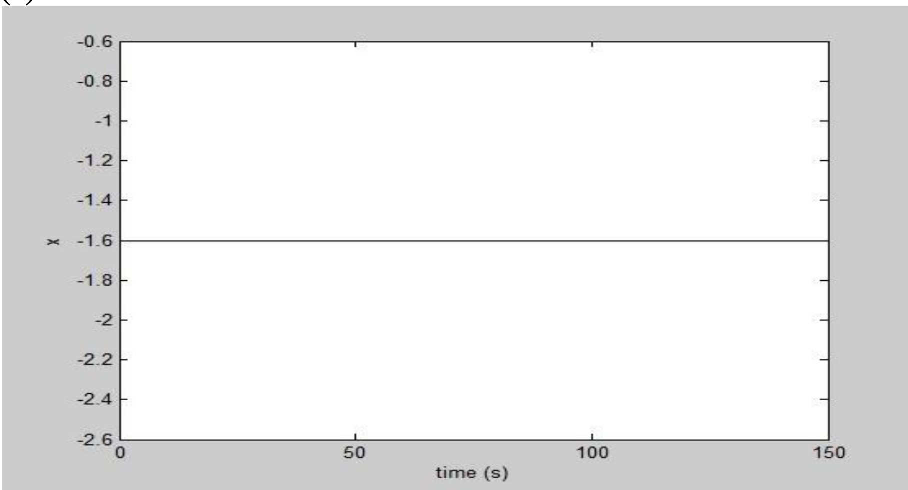


Figure 1-Simple Chua circuit [16]

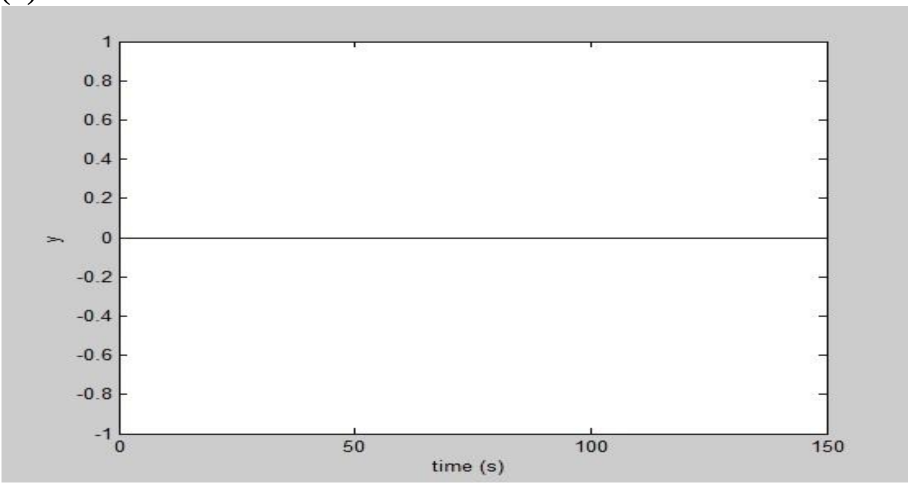
Table 1-The values of Chua circuit components

L(mH)	C1(nf)	C2(nf)	R(kΩ)	voltage	Operational amplifier
15	10	100	2.5	±9	TL082

(a)



(b)



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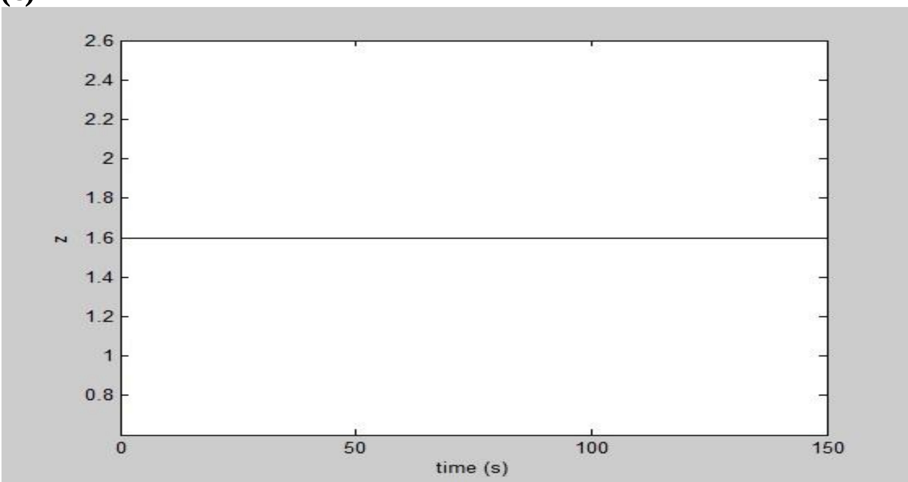


Figure 2-Numerical simulation results of time series; (a) in x-dynamic, (b) in y-dynamic, (c) in z-dynamic, at initial condition x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where $\alpha = 0$, (steady state).

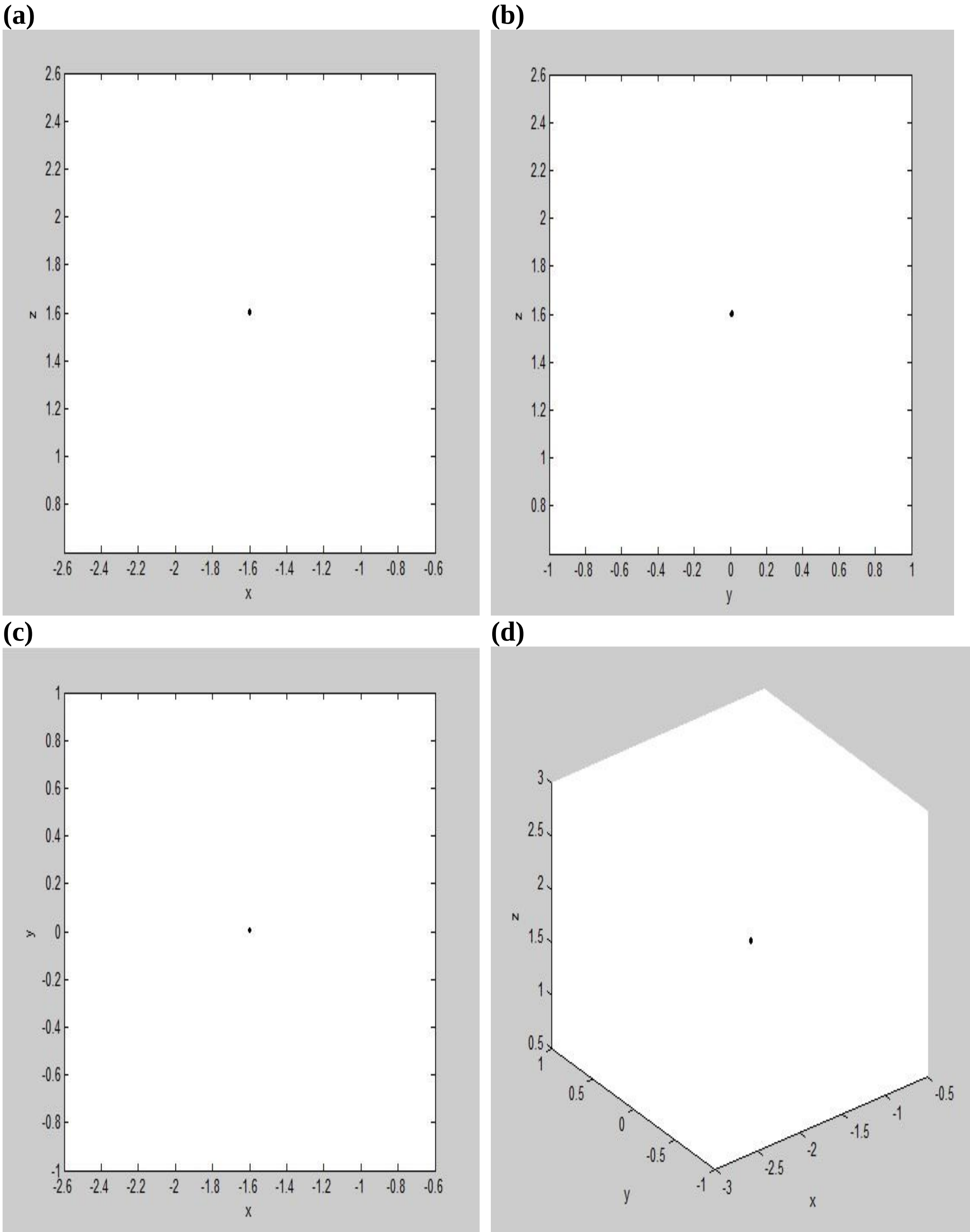
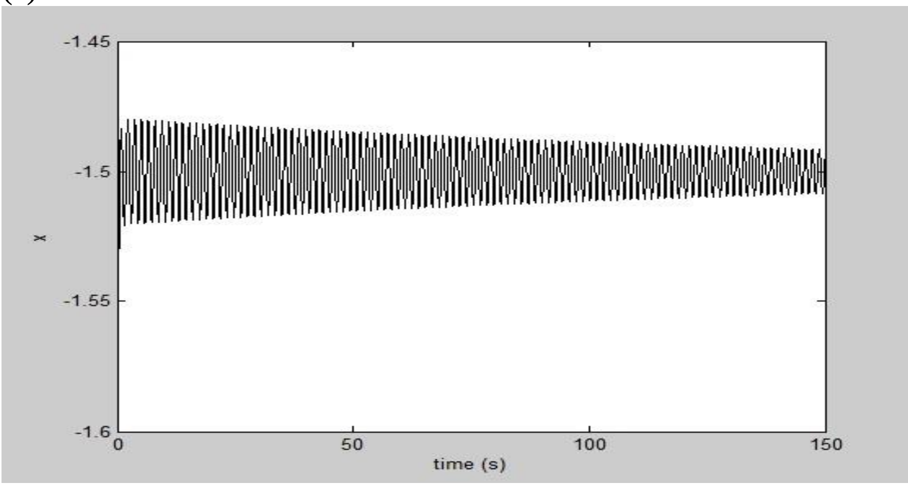


Figure 3- (a) (z-x) attractor, (b) (z-y) attractor, (c) (y-x) attractor, and (d) 3D (x-y-z) attractor, at initial conditions x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where $\alpha = 0$, (steady state).

(a)



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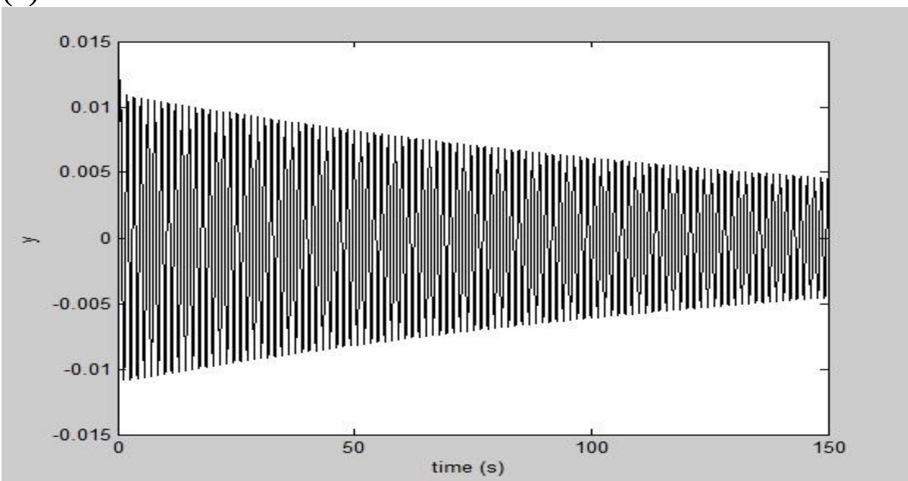

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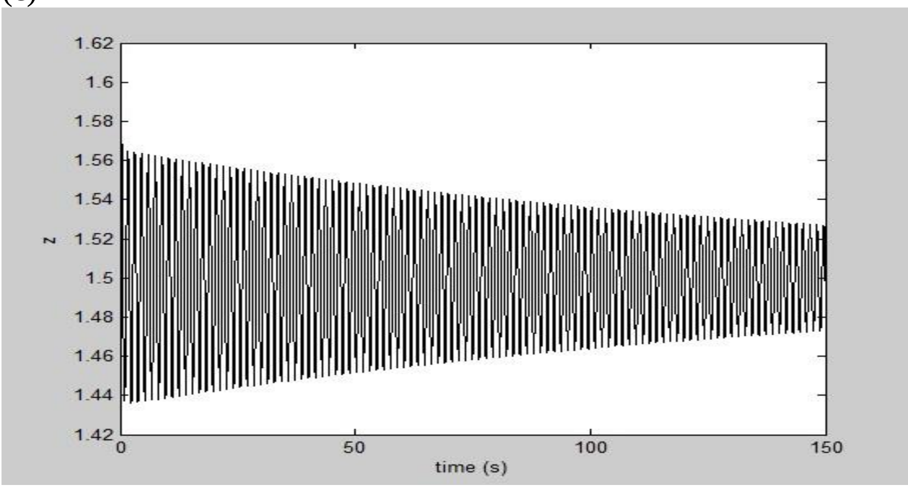


Figure 4-Numerical simulation results of time series; (a) in x-dynamic, (b) in y-dynamic, (c) in z-dynamic, at initial condition x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where α = 9.5, (periodic state).

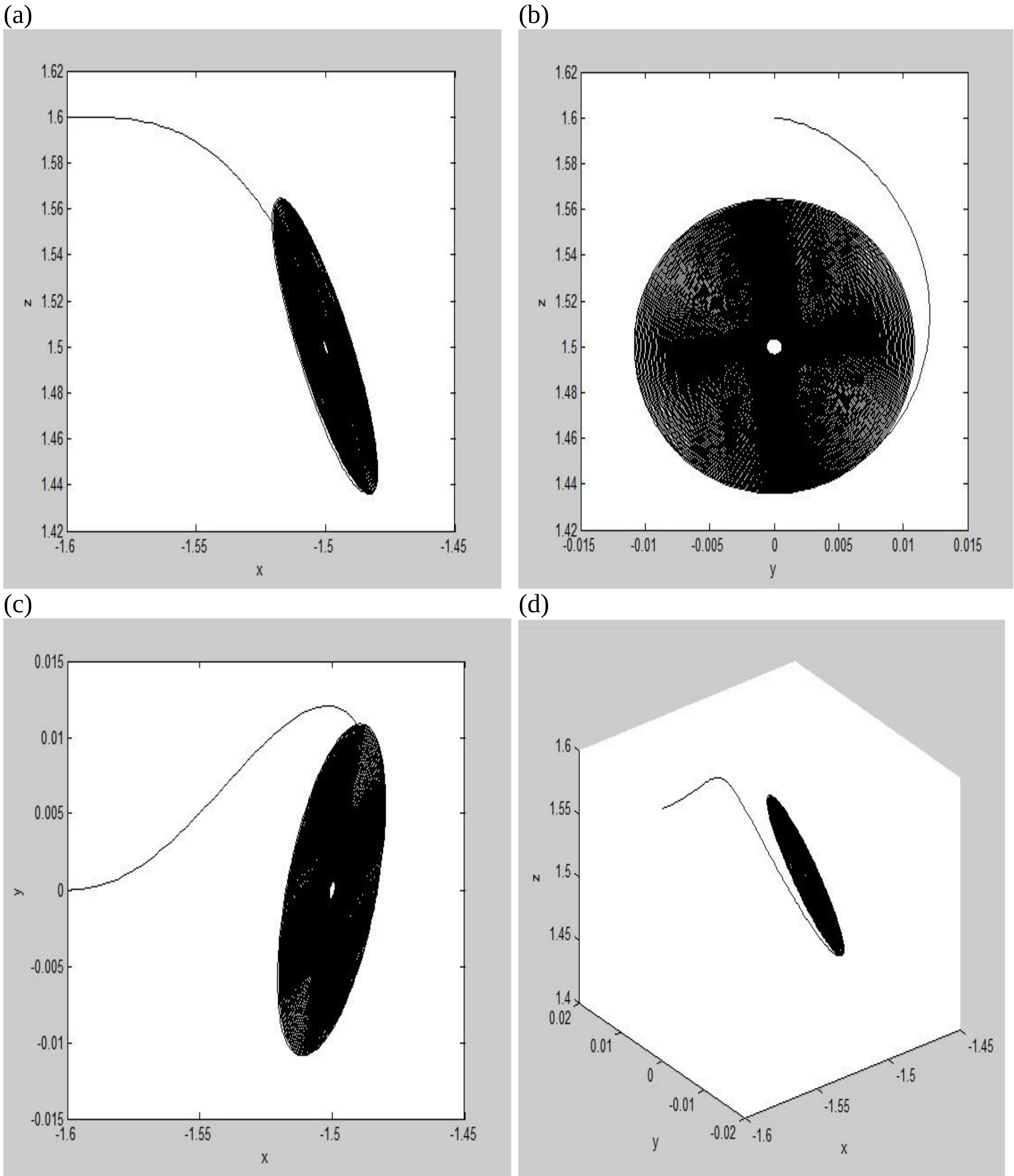
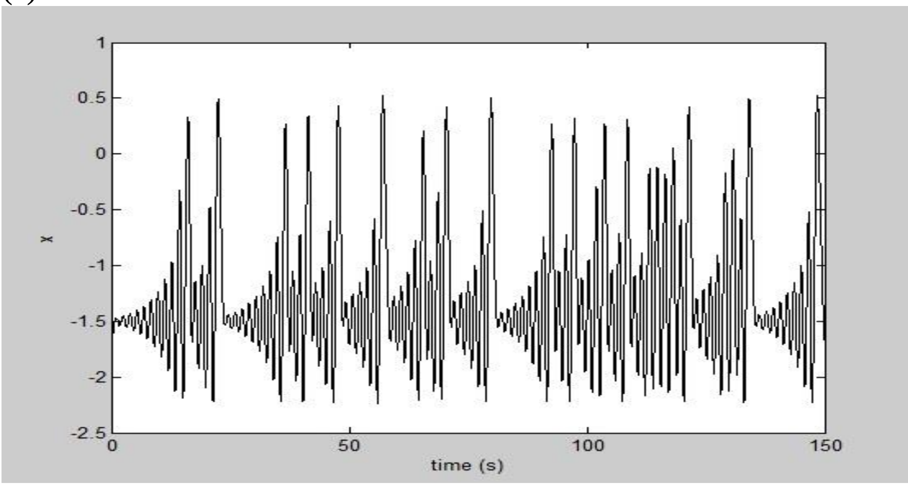


Figure 5-(a) (z-x) attractor, (b) (z-y) attractor, (c) (y-x) attractor, and (d) 3D (x-y-z) attractor, at initial conditions x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where α = 9.5, (periodic state).

(a)



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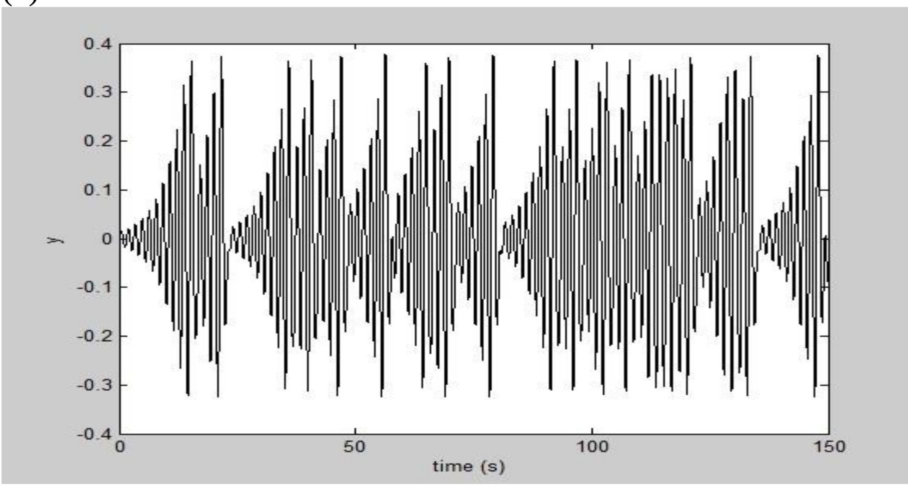

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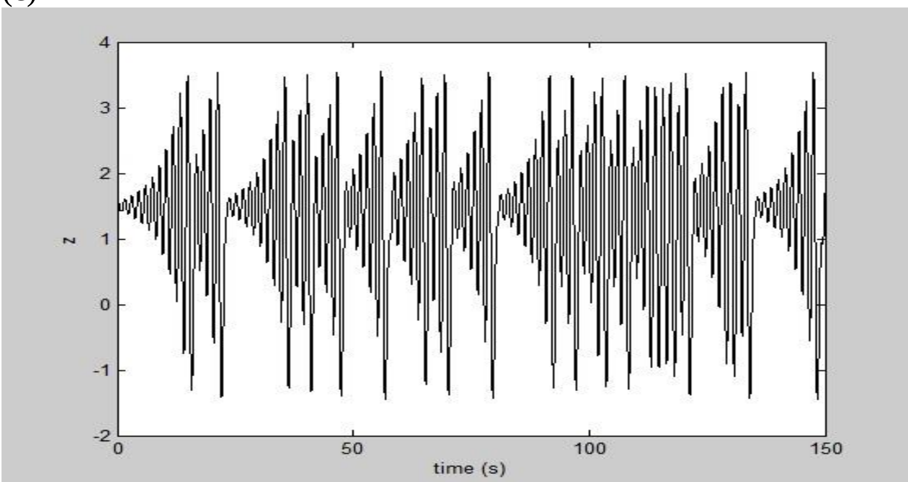


Figure 6-Numerical simulation results of time series; (a) in x-dynamic, (b) in y-dynamic, (c) in z-dynamic, at initial condition x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where α = 13.4, (quasi-chaotic system).

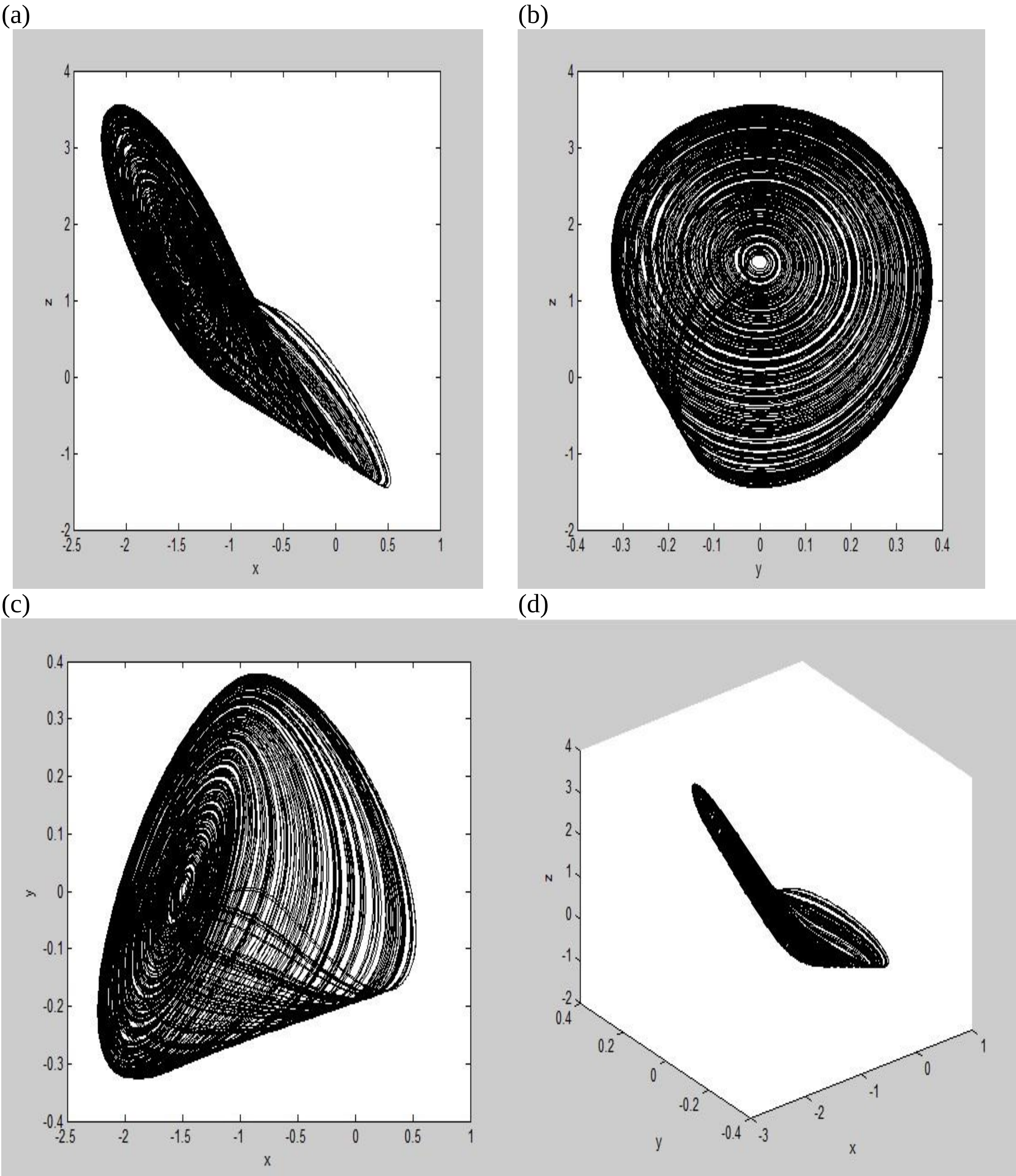
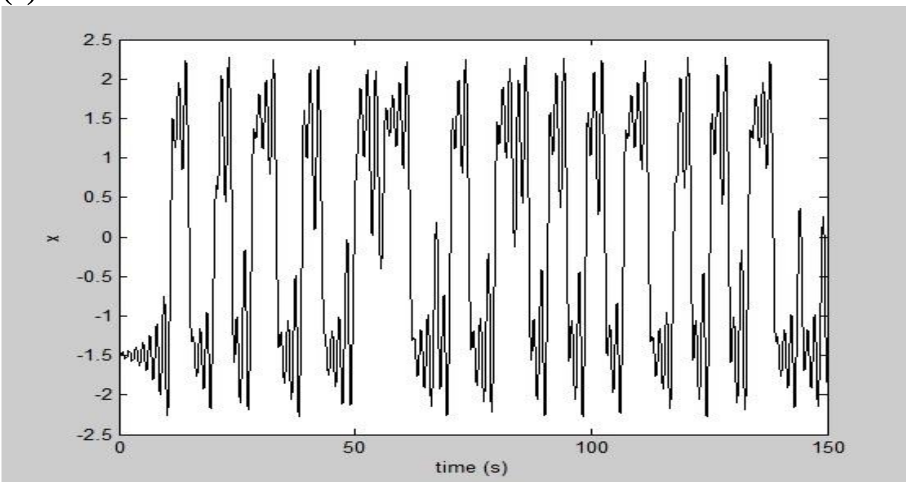


Figure 7- (a) (z-x) attractor, (b) (z-y) attractor, (c) (y-x) attractor, and (d) 3D (x-y-z) attractor, at initial conditions x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where α = 13.4, (quasi-chaotic system).

(a)



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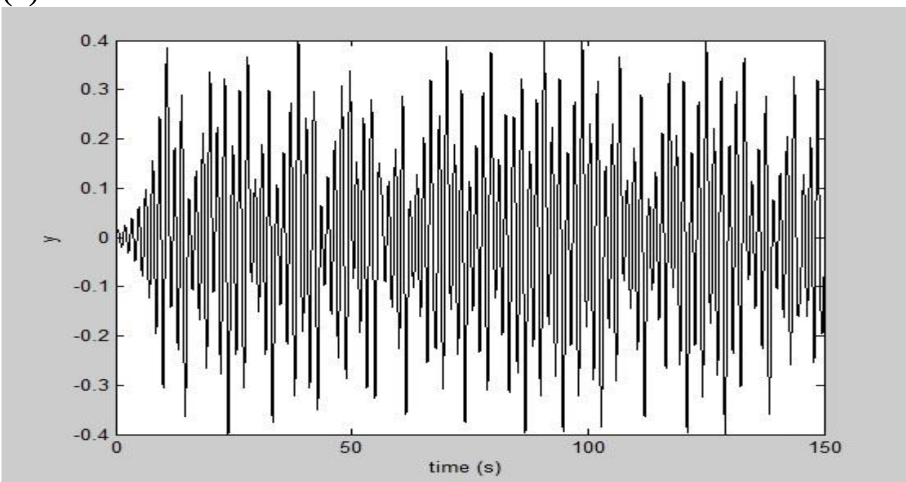

(c)

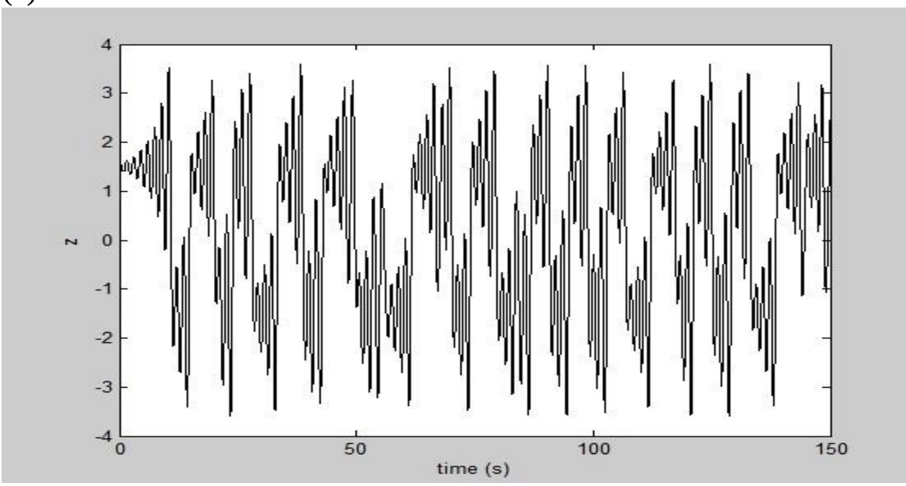

Figure 8-Numerical simulation results of time series; (a) in x-dynamic, (b) in y-dynamic, (c) in z-dynamic, at initial condition x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where α = 15.05, (chaotic system).

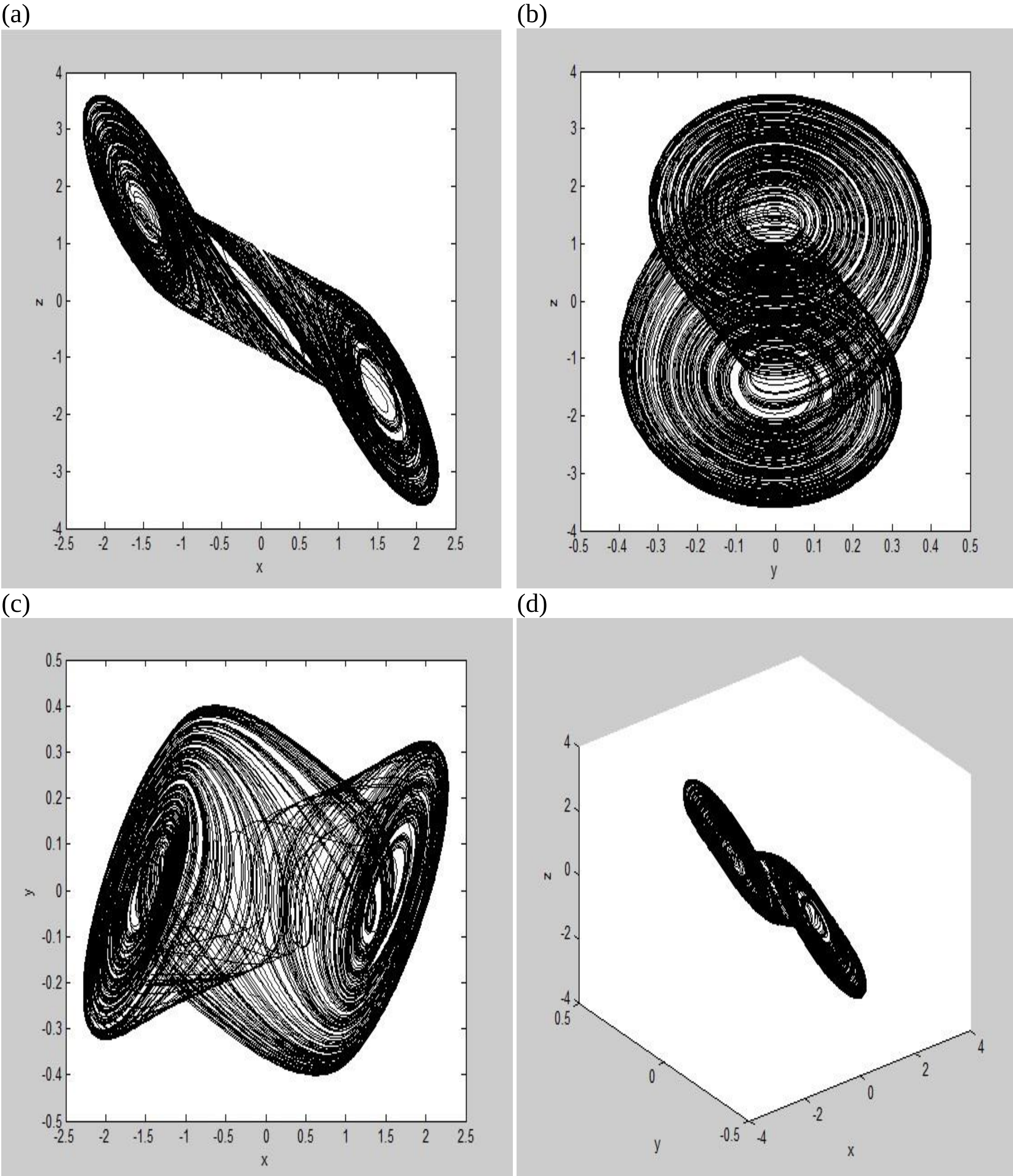

Figure 9-(a) (z-x) attractor, (b) (z-y) attractor, (c) (y-x) attractor, and (d) 3D (x-y-z) attractor, at initial conditions x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where α = 15.05, (Chaotic system).

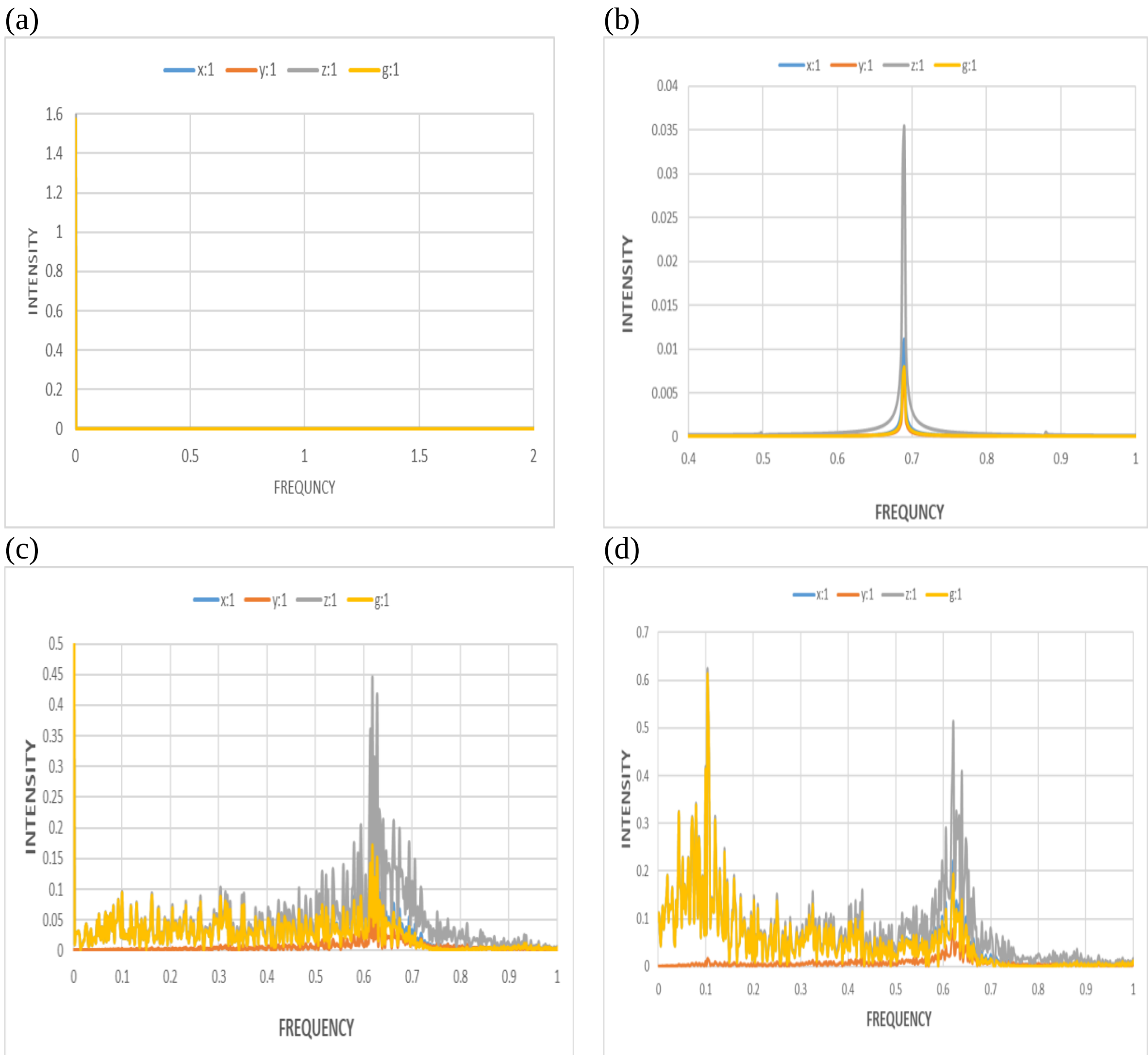

Figure 10-FFT spectra of x, y, and z-dynamics at x_i , y_i and z_i equal -1.6, 0, and 1.6 respectively, where (a) α=0, (b) α=9.5, (c) α=13.4, and (d) α = 15.05.

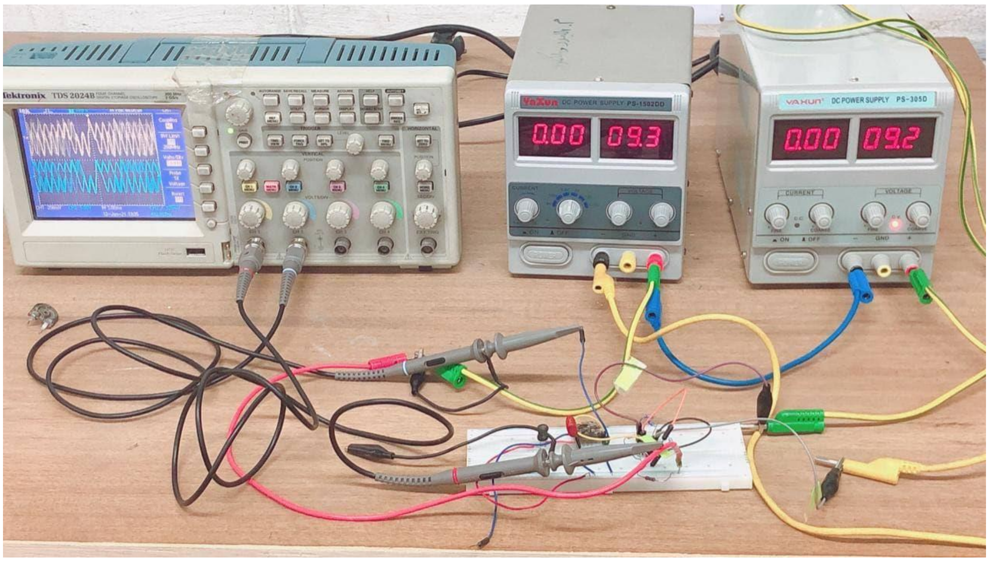

Figure 11-The experimental setup of Chua circuit.

(a) (b)

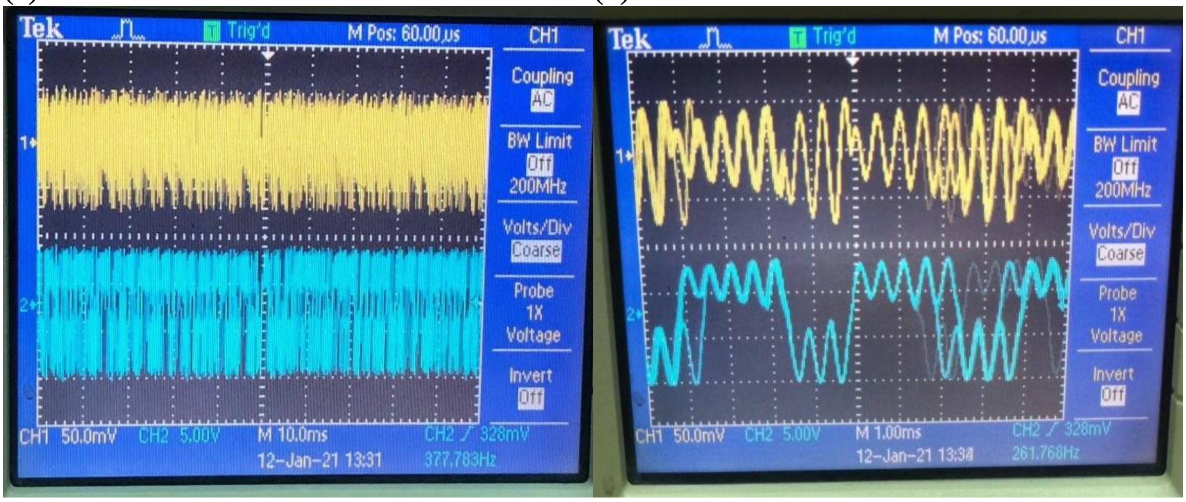

Figure 12-(a) Time series of Chua circuit where the blue line is (y-dynamic) and green line is (x-dynamic) respectively, (b) re-scale of (a).

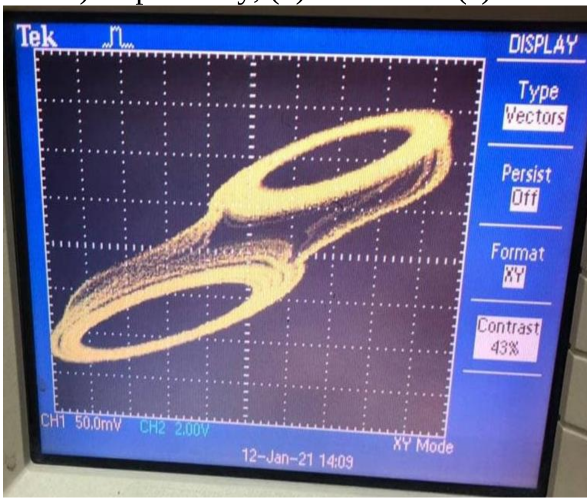

Figure 13-(*x*-*y*) attractor of Chua's circuit. **Figure 14-**FFT spectrum of *x* dynamics of Chua circuit.

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