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Cubic arcs in the projective plane over a finite field of order 16

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Abstract
The main aims purpose of this study is to find the stabilizer groups of a cubic curves over a finite field of order 16, also studying the properties of their groups, and then constructing all different cubic curves, and known which one of them is complete or not. The arcs of degree 2 which are embedding into a cubic curves of even size have been constructed.

Key words: stabilizer groups, arcs, cubic curves.

اقواس مكعبة في المستوي الاسقاطي حول الحقل المنتهي من الرتبة السادسة عشر

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الخلاصة
الاهداف الرئيسية لغرض دراسة هذا البحث هو لإيجاد الزمر المثبتة للمنحنيات المكعبة حول الحقل المنتهي من الرتبة ١٦ ، ودراسة الخواص لهذه الزمر ، وكذلك تشكيل كل المنحنيات المكعبة المختلفة ، ومعرفة اي واحده منها هو كامل او لا . الاقواس من الدرجة الثانية والتي غمرت في منحنيات مكعبة ذات حجم زوجي تم تشكيلها.

1. Introduction

The subject of this research depends on themes of

- Projective geometry over a finite field.
- Group theory.
- Linear algebra.
- Field theory.

The strategy of this research is to construct the stabilizer groups and finding the linear transformations groups in $PGL(3,q)$ of $PG(2,q)$, where $q = 16$ which its element are considering the non-singular matrices $A_n = [a_{ij}]$,a_{ij} in F_q, i, $j = 1,2,3$ for some n in $\mathbb{N}$ satisfying $K(tA_n) = K$ for all t in $F_q \setminus \{0\}$ and K be any arc.The set of all matrices A_n, which construct the group, and according to the number of A_n, and its order and then make comparison with the groups in [6], so we can find which one of them similar than it. on the other hand, we have found the arcs which are embedding cubic curves which are splitting into two sets, one of them contains the inflection points and the other does not, the set which does not contain the inflection points is considering the arc of degree two.

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The brief history of this theme is shown as follows :

- All theorems and definitions of the research are taken from James Hirshfeld [1].
- In 2010, Najm AL-Seraji [2] studied the cubic curves over finite field of order 17.
- In 2011, Emad AL-Zangana [3] showed the cubic curves over finite field of order 19.
- In 2013, Emad AL-Zangana [4] described the cubic curves over finite field of orders 2,3,5,7.
- In 2013, Emad AL-Zangana [5] classified the cubic curves over finite field of orders 11,13.

Definition 1.1 [1]:- " Denote by S and S^* two subspaces of $P(n,K)$, A projectivity $\beta: S \rightarrow S^*$ is a bijection given by a matrix T, necessarily non-singular, where $P(X) = P(X)\beta$ if $tX^* = XT$, with $t \in K$. Write $\beta = M(T)$; then $\beta = M(\lambda T)$ for any λ in K. The group of projectivities of $PG(n,K)$ is denoted by $PG(n+1,K)$".

Definition 1.2 [1]:-"The stabilizer of x in Λ in under the action of G is the group $G_x = \{g \in G | xg = x\}$".

Definition 1.3 [1] :- "An $(n;r)$ arc K or arc of degree r in $PG(k,q)$ with $n \geq r+1$ is a set of points with property that every hyperplane meets K in at most r points of K and there is some hyperplane meeting K in exactly r points. An $(n;3)$ -arc is also called an -arc. A $(n;r)$-arc is called complete if it is contained within $(n+1;r)$-arc"

Theorem 1.4 [1]:- A non-singular plane cubic curve with form and nine rational inflexions exists over F_q if and only if $q \equiv 1\ (mod\ 3)$, and $\mathcal{F}$ then has canonical form $\mathcal{F} = x^3 + y^3 + z^3 - 3cxyz$.

Theorem 1.5 [1]:- A non-singular plane cubic curve with form $\mathcal{F}$ and three rational inflexions exists over F_q for all . The inflexions are necessary collinear.

i - If the inflexional tangent are concurrent, the canonical forms are as follows:

$$q \equiv 1\ (mod\ 3),$$
$$\mathcal{F} = xy(x+y) + z^3;$$
$$\mathcal{F} = xy(x+y) + cz^3;$$
$$\mathcal{F} = xy(x+y) + c^2z^3;$$

Where c is a primitive of F_q .

ii - If the in flexional tangent are not concurrent, the canonical form is as follows:

$$\mathcal{F} = xyz + e(x+y+z)^3, \quad e \neq 0,1\,/\,27.$$

Theorem 1.6 [1]:- A non-singular plane cubic curve with form $\mathcal{F}$ defined over F_q , $q = p^h$ and at least one rational inflexion has one of following canonical forms.

$p = 2$,

(a) $\mathcal{F} = yz^2 + xyz + x^3 + bx^2y + cxy^2$, where $b = 0$ or a fixed element of trace 1 and $c \neq 0$;

(b) $\mathcal{F} = z^2y + zy^2 + ex^3 + cxy^2 + dy^3$, where $e = 1$ when $(q-1,3) = 1$ and $e = 1, \alpha, \alpha^2$ when $(q-1,3) = 3$, with α a primitive element of F_q; also $d = 0$ or a particular element of trace 1 .

Theorem 1.7 [1]:- A non-singular plane cubic curve with form $\mathcal{F}$ defined over F_q , $q = p^h$, with no rational inflexion has one of following canonical forms.

$q \equiv 1\ (mod\ 3)$,

(a) $\mathcal{F} = x^3 + \alpha y^3 + \alpha^2 z^3 - 3cxyz$, with α a primitive element of F_q.

(b) $\mathcal{F} = xy^2 + x^2z + eyz^2 - c(x^3 + ey^3 + e^2z^3 - 3exyz)$, with α a primitive element of F_q and $e = \alpha, \alpha^2$.

2. The classification of cubic curves over a finite field of order 16

Let the polynomial $f_1(x) = x^4 - 16x + 4$ and $F_{16} = \frac{F_2[x]}{\langle f_1(x) \rangle}$ which has 16 elements namely $0,1,\sigma,\sigma^2,\sigma^3,\sigma^4,\sigma^5,\sigma^6,\sigma^7,\sigma^8,\sigma^9,\sigma^{10},\sigma^{11},\sigma^{12},\sigma^{13},\sigma^{14}$ where σ be x plus the ideal $\langle f_1(x) \rangle$ which generated by polynomial of degree 4 with coefficients in $F_2 = \{0,1\}$. The polynomial $f_2(x) = x^3 + x + \sigma^7$ is primitive over F_{16} , since $f_2(0) = \sigma^7$, $f_2(1) = \alpha^7$, $f_2(\sigma) = 1$, $f_2(\sigma^2) = \sigma^4$, $f_2(\sigma^3) = \sigma^{14}$, $f_2(\sigma^4) = \sigma^{10}$, $f_2(\sigma^5) = \sigma^6$, $f_2(\sigma^6) = \sigma^{12}$, $f_2(\sigma^7) = \sigma^6$ $f_2(\sigma^8) = \sigma^2$, $f_2(\sigma^9) = \sigma^{11}$, $f_2(\sigma^{10}) = \sigma^{13}$, $f_2(\sigma^{11}) = \sigma^{13}$, $f_2(\sigma^{12}) = \sigma^3$, $f_2(\sigma^{13}) = \sigma^6$ and $f_2(\sigma^{13}) = \sigma^{13}$, this means f_2 is irreducible over F_{16} , also $f_2(\tau^{187}) = f_2(\tau^{2992}) = f_2(\tau^{2827}) = 0$, where $\tau^{187}, \tau^{2992}, \tau^{2827}$ in F_{16^3} , this means f_2 is reducible over F_{4096}. The companion matrix of $f_2(x) = x^3 + x + \sigma^7$ generated the points and lines of $PG(2,16)$ as follows:

$$P(k) = [1,0,0]C(f)^k = [1,0,0]\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \sigma^7 & 1 & 0 \end{pmatrix}^k, k = 0,1,\dots,272$$

With select the points in $PG(2,16)$ which are the third coordinate equal to zero, this means belong to $L_0 = v(z)$ such that $v(z) = tz = z$ for all t in $F_{16}\backslash\{0\}$, therefore with $P(k) = k$, $k = 0,1,\dots,272$,we obtain

$$L_0 = \{0,1,3,7,15,31,63,90,116,127,136,181,194,204,233,238,255\}$$

Moreover,

$$L_k = L_0C(f)^k = L_0\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \sigma^7 & 1 & 0 \end{pmatrix}^k, k = 0,\dots,272$$

By substituting the points of $PG(2,16)$ in theorem (1.4), we obtain

$\mathcal{F}_1 = x^3 + y^3 + z^3$ $|\mathcal{F}_1| = 9$
$\mathcal{F}_2 = x^3 + y^3 + z^3 + \sigma xyz$ $|\mathcal{F}_2| = 18$
$\mathcal{F}_3 = x^3 + y^3 + z^3 + \sigma^2 xyz$ $|\mathcal{F}_3| = 18$
$\mathcal{F}_4 = x^3 + y^3 + z^3 + \sigma^3 xyz$ $|\mathcal{F}_4| = 18$
$\mathcal{F}_5 = x^3 + y^3 + z^3 + \sigma^4 xyz$ $|\mathcal{F}_5| = 18$
$\mathcal{F}_6 = x^3 + y^3 + z^3 + \sigma^6 xyz$ $|\mathcal{F}_6| = 18$
$\mathcal{F}_7 = x^3 + y^3 + z^3 + \sigma^7 xyz$ $|\mathcal{F}_7| = 18$
$\mathcal{F}_8 = x^3 + y^3 + z^3 + \sigma^8 xyz$ $|\mathcal{F}_8| = 18$
$\mathcal{F}_9 = x^3 + y^3 + z^3 + \sigma^9 xyz$ $|\mathcal{F}_9| = 18$
$\mathcal{F}_{10} = x^3 + y^3 + z^3 + \sigma^{11} xyz$ $|\mathcal{F}_{10}| = 18$
$\mathcal{F}_{11} = x^3 + y^3 + z^3 + \sigma^{12} xyz$ $|\mathcal{F}_{11}| = 18$
$\mathcal{F}_{12} = x^3 + y^3 + z^3 + \sigma^{13} xyz$ $|\mathcal{F}_{12}| = 18$
$\mathcal{F}_{13} = x^3 + y^3 + z^3 + \sigma^{14} xyz$ $|\mathcal{F}_{13}| = 18$

$\mathcal{F}_2$ is equivalent to $\mathcal{F}_3$, this means $\mathcal{F}_2 \equiv \mathcal{F}_3, \mathcal{F}_4, \mathcal{F}_5, \mathcal{F}_6, \mathcal{F}_7, \mathcal{F}_8, \mathcal{F}_9, \mathcal{F}_{10}, \mathcal{F}_{11}, \mathcal{F}_{12}, \mathcal{F}_{13}$.
By the same method for $\mathcal{F}_{18}, \dots, \mathcal{F}_{262}$

Therefore, the number of different cubic curves is shown in following theorem as follows :

Theorem 2. 1:- On $PG(2,16)$ there are precisely 54 distinct cubic curves which given in Table-2, where n represent the number inflexion point.

Table 2-The distinct cubic curves in $PG(2,16)$

n	No	Canonical form	Size	Description	Maximum
9	1	$\mathcal{F}_1 = x^3 + y^3 + z^3$	9	incomplete	22
	2	$\mathcal{F}_2 = x^3 + y^3 + z^3 + \sigma xyz$	18	incomplete	26
3	3	$\mathcal{F}_{18} = xy(x + y) + \sigma z^3$	21	complete	–
	4	$\mathcal{F}_{19} = xy(x + y) + \sigma^2 z^3$	21	complete	–
	5	$\mathcal{F}_{26} = xyz + \sigma(x + y + z)^3$	12	incomplete	21
	6	$\mathcal{F}_{27} = xyz + \sigma^2(x + y + z)^3$	12	incomplete	21
	7	$\mathcal{F}_{28} = xyz + \sigma^3(x + y + z)^3$	18	incomplete	21

	8	$\mathcal{F}_{29} = xyz + \sigma^4(x+y+z)^3$	12	incomplete	21
	9	$\mathcal{F}_{30} = xyz + \sigma^5(x+y+z)^3$	24	complete	–
	10	$\mathcal{F}_{31} = xyz + \sigma^6(x+y+z)^3$	18	incomplete	21
	11	$\mathcal{F}_{32} = xyz + \sigma^7(x+y+z)^3$	18	incomplete	26
	12	$\mathcal{F}_{33} = xyz + \sigma^8(x+y+z)^3$	12	incomplete	19
	13	$\mathcal{F}_{34} = xyz + \sigma^9(x+y+z)^3$	18	incomplete	21
	14	$\mathcal{F}_{35} = xyz + \sigma^{10}(x+y+z)^3$	24	complete	–
	15	$\mathcal{F}_{37} = xyz + \sigma^{12}(x+y+z)^3$	18	incomplete	23
1	16	$\mathcal{F}_{40} = yz^2 + xyz + x^3 + xy^2$	16	incomplete	17
	17	$\mathcal{F}_{41} = yz^2 + xyz + x^3 + \sigma xy^2$	16	incomplete	22
	18	$\mathcal{F}_{42} = yz^2 + xyz + x^3 + \sigma^2 xy^2$	16	incomplete	19
	19	$\mathcal{F}_{43} = yz^2 + xyz + x^3 + \sigma^3 xy^2$	20	complete	–
	20	$\mathcal{F}_{44} = yz^2 + xyz + x^3 + \sigma^4 xy^2$	16	incomplete	19
	21	$\mathcal{F}_{45} = yz^2 + xyz + x^3 + \sigma^5 xy^2$	24	complete	–
	22	$\mathcal{F}_{46} = yz^2 + xyz + x^3 + \sigma^6 xy^2$	20	complete	–
	23	$\mathcal{F}_{47} = yz^2 + xyz + x^3 + \sigma^7 xy^2$	12	incomplete	20
	24	$\mathcal{F}_{48} = yz^2 + xyz + x^3 + \sigma^8 xy^2$	16	incomplete	17
	25	$\mathcal{F}_{49} = yz^2 + xyz + x^3 + \sigma^9 xy^2$	20	complete	–
	26	$\mathcal{F}_{50} = yz^2 + xyz + x^3 + \sigma^{10} xy^2$	24	complete	–
	27	$\mathcal{F}_{51} = yz^2 + xyz + x^3 + \sigma^{11} xy^2$	12	incomplete	21
	28	$\mathcal{F}_{52} = yz^2 + xyz + x^3 + \sigma^{12} xy^2$	20	complete	–
	29	$\mathcal{F}_{53} = yz^2 + xyz + x^3 + \sigma^{13} xy^2$	12	incomplete	21
	30	$\mathcal{F}_{54} = yz^2 + xyz + x^3 + \sigma^{14} xy^2$	12	incomplete	20
	31	$\mathcal{F}_{55} = z^2 y + zy^2 + x^3$	9	incomplete	21
	32	$\mathcal{F}_{56} = z^2 y + zy^2 + x^3 + xy^2$	25	complete	–
	33	$\mathcal{F}_{57} = z^2 y + zy^2 + x^3 + \sigma xy^2$	17	complete	–

	34	$\mathcal{F}_{71} = z^2y + zy^2 + \sigma x^3$	21	complete	–
	35	$\mathcal{F}_{73} = z^2y + zy^2 + \sigma x^3 + \sigma xy^2$	13	incomplete	20
	36	$\mathcal{F}_{87} = z^2y + zy^2 + \sigma^2 x^3$	21	complete	–
	37	$\mathcal{F}_{89} = z^2y + zy^2 + \sigma^2 x^3 + \sigma xy^2$	13	incomplete	21
0	38	$\mathcal{F}_{103} = x^3 + \sigma y^3 + \sigma^2 z^3$	9	incomplete	23
	39	$\mathcal{F}_{104} = x^3 + \sigma y^3 + \sigma^2 z^3 + xyz$	18	incomplete	21
	40	$\mathcal{F}_{106} = x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma^2 xyz$	18	incomplete	21
	41	$\mathcal{F}_{107} = x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma^3 xyz$	18	incomplete	21
	42	$\mathcal{F}_{108} = x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma^4 xyz$	18	incomplete	21
	43	$\mathcal{F}_{231} = xy^2 + x^2z + \sigma yz^2$	21	complete	–
	44	$\mathcal{F}_{232} = xy^2 + x^2z + \sigma yz^2 + (x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$	18	complete	–
	45	$\mathcal{F}_{233} = xy^2 + x^2z + \sigma yz^2 + \sigma(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$	24	complete	–
	46	$\mathcal{F}_{234} = xy^2 + x^2z + \sigma yz^2 + \sigma^2(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$	18	complete	–
	47	$\mathcal{F}_{235} = xy^2 + x^2z + \sigma yz^2 + \sigma^3(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$	12	incomplete	20
	48	$\mathcal{F}_{236} = xy^2 + x^2z + \sigma yz^2 + \sigma^4(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$	12	incomplete	21
	49	$\mathcal{F}_{247} = xy^2 + x^2z + \sigma^2 yz^2$	21	complete	–
	50	$\mathcal{F}_{248} = xy^2 + x^2z + \sigma^2 yz^2 + (x^3 + \sigma^2 y^3 + \sigma^4 z^3 + \sigma^2 xyz)$	18	complete	–
	51	$\mathcal{F}_{249} = xy^2 + x^2z + \sigma^2 yz^2 + \sigma(x^3 + \sigma^2 y^3 + \sigma^4 z^3 + \sigma^2 xyz)$	12	incomplete	22
	52	$\mathcal{F}_{250} = xy^2 + x^2z + \sigma^2 yz^2 + \sigma^2(x^3 + \sigma^2 y^3 + \sigma^4 z^3 + \sigma^2 xyz)$	24	complete	–
	53	$\mathcal{F}_{251} = xy^2 + x^2z + \sigma^2 yz^2 + \sigma^3(x^3 + \sigma^2 y^3 + \sigma^4 z^3 + \sigma^2 xyz)$	12	incomplete	20
	54	$\mathcal{F}_{252} = xy^2 + x^2z + \sigma^2 yz^2 + \sigma^4(x^3 + \sigma^2 y^3 + \sigma^4 z^3 + \sigma^2 xyz)$	18	complete	–

The number distinct cubic curves is 54 see [table 2], one of them is given as following : $\mathcal{F}_1 = x^3 + y^3 + z^3$. The points of $PG(2,16)$ on $\mathcal{F}_1$ are $[\sigma^5,1,0]$,$[0,\sigma^5,1]$,$[\sigma^{10},0,1]$,$[1,1,0]$, $[0,1,1]$,$[\sigma^5 0,1]$,$[\sigma^{10},1,0]$,$[0,\sigma^{10},1]$,$[1,0,1]$.After calculations with computer help, we are note that the number of matrices which are stabilizing of $\mathcal{F}_1$ and their orders is 216, and we can not write them, because they are too much.

Therefore , the stabilizer group of $\mathcal{F}_1$ which is denoted by $G_{\mathcal{F}_1}$ contains

- 9 matrices of order 2.
- 80 matrix of order 3.
- 54 matrix of order 4.
- 72 matrix of order 6.
- The identity matrix.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_2 = x^3 + y^3 + z^3 + \sigma xyz$ .The points of $PG(2,16)$ on $\mathcal{F}_2$ are $[1,\sigma^8,1]$,$[\sigma^5,1,0]$,$[0,\sigma^5,1]$,$[\sigma^{10},0,1]$,$[\sigma^8,1,1]$,$[\sigma^{13},\sigma^{10},1]$, $[\sigma^2,\sigma^{12},1]$,$[\sigma^5,\sigma^3,1]$,$[\sigma^3,\sigma^5,1]$,$[1,1,0]$,$[0,1,1]$,$[\sigma^{12},\sigma^2,1]$,$[\sigma^7,\sigma^7,1]$,$[\sigma^5,0,1]$,$[\sigma^{10},1,0]$, $[0,\sigma^{10},1]$,$[\sigma^{10},\sigma^{13},1]$,$[1,0,1]$.To find the stabilizer group of $\mathcal{F}_2$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_2$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&0&\sigma^{12}\\0&\sigma^{12}&0\\\sigma^{12}&0&0\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{2}\\\sigma^{2}&0&0\\0&\sigma^{2}&0\end{pmatrix}:3,\begin{pmatrix}0&\sigma^{10}&0\\\sigma^{10}&0&0\\0&0&\sigma^{10}\end{pmatrix}:2$$

$$\begin{pmatrix}0&\sigma^{10}&0\\\sigma^{5}&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{7}\\\sigma^{12}&0&0\\0&\sigma^{2}&0\end{pmatrix}:3,\begin{pmatrix}0&\sigma^{7}&0\\0&0&\sigma^{2}\\\sigma^{12}&0&0\end{pmatrix}:3,\begin{pmatrix}\sigma^{12}&0&0\\0&0&\sigma^{2}\\0&\sigma^{7}&0\end{pmatrix}:2$$

$$\begin{pmatrix}0&0&\sigma^{2}\\0&\sigma^{12}&0\\\sigma^{7}&0&0\end{pmatrix}:2,\begin{pmatrix}\sigma^{10}&0&0\\0&1&0\\0&0&\sigma^{5}\end{pmatrix}:3,\begin{pmatrix}0&\sigma^{10}&0\\1&0&0\\0&0&\sigma^{5}\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{12}\\\sigma^{7}&0&0\\0&\sigma^{2}&0\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{7}&0&0\\0&0&\sigma^{2}\\0&\sigma^{12}&0\end{pmatrix}:2,\begin{pmatrix}0&\sigma^{12}&0\\0&0&\sigma^{2}\\\sigma^{7}&0&0\end{pmatrix}:3,\begin{pmatrix}0&\sigma^{2}&0\\0&0&\sigma^{2}\\\sigma^{2}&0&0\end{pmatrix}:3,\begin{pmatrix}\sigma^{2}&0&0\\0&0&\sigma^{2}\\0&\sigma^{2}&0\end{pmatrix}:2$$

$$\begin{pmatrix}0&0&\sigma^{7}\\0&\sigma^{12}&0\\\sigma^{2}&0&0\end{pmatrix}:2,\begin{pmatrix}\sigma^{5}&0&0\\0&1&0\\0&0&\sigma^{10}\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_2$ which is denoted by $G_{\mathcal{F}_2}$ contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- The identity matrix.

Form [6] , $G_{\mathcal{F}_2}$ is isomorphic to $(\boldsymbol{Z_3} \times \boldsymbol{Z_3}) \rtimes \boldsymbol{Z_2}$, that is $G_{\mathcal{F}_2} \cong (\boldsymbol{Z_3} \times \boldsymbol{Z_3}) \rtimes \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{18} = xy(x+y) + \sigma z^3$ .The points of $PG(2,16)$ on $\mathcal{F}_{18}$ are $[1,0,0]$,$[0,1,0]$,$[\sigma^2,\sigma^{10},1]$,$[\sigma^{14},\sigma^5,1]$,$[\sigma^{14},\sigma^{12},1]$,$[\sigma^{10},\sigma^2,1]$, $[1,\sigma^9,1]$,$[\sigma^{12},\sigma^{14},1]$,$[1,1,0]$,$[\sigma^7,1,1]$,$[\sigma^7,\sigma^9,1]$,$[\sigma^{10},\sigma^4,1]$,$[\sigma^9,1,1]$,$[\sigma^4,\sigma^2,1]$,$[\sigma^5,\sigma^{14},1]$, $[\sigma^4,\sigma^{10},1]$,$[\sigma^{12},\sigma^5,1]$,$[\sigma^5,\sigma^{12},1]$,$[\sigma^9,\sigma^7,1]$,$[\sigma^2,\sigma^4,1]$,$[1,\sigma^7,1]$. To find the stabilizer group of $\mathcal{F}_{18}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{18}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^{10}&0\\0&0&1\end{pmatrix}:3,\begin{pmatrix}\sigma^{5}&0&0\\0&\sigma^{5}&0\\0&0&1\end{pmatrix}:3,\begin{pmatrix}\sigma^{10}&0&0\\\sigma^{10}&\sigma^{10}&0\\0&0&1\end{pmatrix}:6$$

$$\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}\sigma^{5}&0&0\\\sigma^{5}&\sigma^{5}&0\\0&0&1\end{pmatrix}:6,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&\sigma^{5}&0\\\sigma^{5}&0&0\\0&0&1\end{pmatrix}:6$$

$$\begin{pmatrix}0&\sigma^{10}&0\\\sigma^{10}&0&0\\0&0&1\end{pmatrix}:6,\begin{pmatrix}0&\sigma^{10}&0\\\sigma^{10}&\sigma^{10}&0\\0&0&1\end{pmatrix}:3,\begin{pmatrix}0&\sigma^{5}&0\\\sigma^{5}&\sigma^{5}&0\\0&0&1\end{pmatrix}:3,\begin{pmatrix}0&1&0\\1&1&0\\0&0&1\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{10} & \sigma^{10} & 0\\ \sigma^{10} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}\sigma^{5} & \sigma^{5} & 0\\ \sigma^{5} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}1 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}\sigma^{5} & \sigma^{5} & 0\\ 0 & \sigma^{5} & 0\\ 0 & 0 & 1\end{pmatrix}:6$$

$$\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:2,\begin{pmatrix}\sigma^{10} & \sigma^{10} & 0\\ 0 & \sigma^{10} & 0\\ 0 & 0 & 1\end{pmatrix}:6$$

Therefore, the stabilizer groups of $\mathcal{F}_{18}$ which is denoted by $G_{\mathcal{F}_{18}}$ contains

- 3 matrices of order 2.
- 8 matrices of order 3.
- 6 matrix of order 6.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{18}}$ is isomorphic to $\boldsymbol{S_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{18}} \cong \boldsymbol{S_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{19} = xy(x+y) + \sigma^2 z^3$.The points of $PG(2,16)$ on $\mathcal{F}_{19}$ are $[1,0,0]$,$[0,1,0]$,$[1,\sigma^{14},1]$,$[\sigma^9,\sigma^{13},1]$,$[\sigma^8,\sigma^4,1]$,$[\sigma^4,\sigma^8,1]$, $[\sigma^{14},\sigma^3,1]$,$[\sigma^9,\sigma^{10},1]$,$[\sigma^3,\sigma^{14},1]$,$[1,\sigma^3,1]$,$[\sigma^{13},\sigma^{10},1]$,$[\sigma^8,\sigma^5,1]$,$[1,1,0]$,$[\sigma^{13},\sigma^9,1]$,$[\sigma^{10},\sigma^9,1]$, $[\sigma^{14},1,1]$,$[\sigma^5,\sigma^4,1]$,$[\sigma^3,1,1]$,$[\sigma^5,1,0]$,$[\sigma^{10},\sigma^{13},1]$,$[\sigma^4,\sigma^5,1]$. To find the stabilizer group of $\mathcal{F}_{19}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{19}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1,\begin{pmatrix}\sigma^{10} & 0 & 0\\ 0 & \sigma^{10} & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}\sigma^{5} & 0 & 0\\ 0 & \sigma^{5} & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}\sigma^{5} & 0 & 0\\ \sigma^{5} & \sigma^{5} & 0\\ 0 & 0 & 1\end{pmatrix}:6$$

$$\begin{pmatrix}\sigma^{10} & 0 & 0\\ \sigma^{10} & \sigma^{10} & 0\\ 0 & 0 & 1\end{pmatrix}:6,\begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:2,\begin{pmatrix}0 & \sigma^{10} & 0\\ \sigma^{10} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:6,\begin{pmatrix}0 & \sigma^{5} & 0\\ \sigma^{5} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:6$$

$$\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:2,\begin{pmatrix}0 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}0 & \sigma^{10} & 0\\ \sigma^{10} & \sigma^{10} & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}0 & \sigma^{5} & 0\\ \sigma^{5} & \sigma^{5} & 0\\ 0 & 0 & 1\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{5} & \sigma^{5} & 0\\ \sigma^{5} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}1 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}\sigma^{10} & \sigma^{10} & 0\\ \sigma^{10} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:3,\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma^{5} & \sigma^{5} & 0\\ 0 & \sigma^{5} & 0\\ 0 & 0 & 1\end{pmatrix}:6,\begin{pmatrix}\sigma^{10} & \sigma^{10} & 0\\ 0 & \sigma^{10} & 0\\ 0 & 0 & 1\end{pmatrix}:6$$

Therefore, the stabilizer groups of $\mathcal{F}_{19}$ which is denoted by $G_{\mathcal{F}_{19}}$ contains

- 3 matrices of order 2.
- 8 matrix of order 3.
- 6 matrix of order 6.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{19}}$ is isomorphic to $\boldsymbol{S_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{19}} \cong \boldsymbol{S_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{26} = xyz + \sigma(x+y+z)^3$.The points of $PG(2,16)$ on $\mathcal{F}_{26}$ are $[\sigma^{14},\sigma^5,1]$,$[\sigma^9,\sigma^{10},1]$,$[1,1,0]$,$[0,1,1]$,$[\sigma^7,1,1]$,$[\sigma^8,\sigma^8,1]$, $[\sigma^{10},\sigma^9,1]$,$[\sigma^6,\sigma,1]$,$[\sigma^5,\sigma^{14},1]$,$[\sigma,\sigma^6,1]$,$[1,\sigma^7,1]$,$[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{26}$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_{26}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^7&0&0\\0&0&\sigma^7\\0&\sigma^7&0\end{pmatrix}:2,\begin{pmatrix}0&\sigma^7&0\\0&0&\sigma^7\\\sigma^7&0&0\end{pmatrix}:3,\begin{pmatrix}0&0&\sigma^7\\\sigma^7&0&0\\0&\sigma^7&0\end{pmatrix}:3$$
$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^7\\0&\sigma^7&0\\\sigma^7&0&0\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{26}$ which is denoted by $G_{\mathcal{F}_{26}}$ contains

- 3 matrices of order 2;
- 2 matrix of order 3;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{26}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{26}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{26}^* = \{[\sigma^9, \sigma^{10}, 1], [1,1,0], [0,1,1], [\sigma^7, 1,1], [\sigma^8, \sigma^8, 1], [\sigma^6, \sigma, 1]\}$ be a subset of $\mathcal{F}_{26}$ which is forming by partition $\mathcal{F}_{26}$ into two sets such that $\mathcal{F}_{26}^*$ dose not contains the inflexion points of $\mathcal{F}_{26}$, so we note that $\mathcal{F}_{26}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{26}^*$, by some calculation .we get that the matrix which is stabilizing of $\mathcal{F}_{26}^*$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&0&\sigma^{14}\\0&\sigma^{14}&0\\\sigma^{14}&0&0\end{pmatrix}:2$$

Therefore , the stabilizer group of $\mathcal{F}_{26}^*$ which is denoted by $G_{\mathcal{F}_{26}^*}$ which contains

- One matrix of order 2;
- The identity matrix.

Thus, $G_{\mathcal{F}_{26}^*}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{26}^*} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{27} = xyz + \sigma^2(x+y+z)^3$.The points of $PG(2,16)$ on $\mathcal{F}_{27}$ are $[1,\sigma^{14},1]$,$[\sigma^{13},\sigma^{10},1]$,$[\sigma^2,\sigma^{12},1]$,$[\sigma^5,\sigma^3,1]$,$[\sigma^3,\sigma^5,1]$, [1,1,0],[0,1,1],$[\sigma^{12},\sigma^2,1]$,$[\sigma^{14},1,1]$,$[\sigma,\sigma,1]$,$[\sigma^{10},\sigma^{13},1]$,[1,0,1]. To find the stabilizer group of $\mathcal{F}_{27}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{27}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&0&\sigma^{14}\\0&\sigma^{14}&0\\\sigma^{14}&0&0\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{14}\\\sigma^{14}&0&0\\0&\sigma^{14}&0\end{pmatrix}:3,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2$$
$$\begin{pmatrix}0&\sigma&0\\0&0&\sigma\\\sigma&0&0\end{pmatrix}:3,\begin{pmatrix}\sigma&0&0\\0&0&\sigma\\0&\sigma&0\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{27}$ which is denoted by $G_{\mathcal{F}_{27}}$ contains

- 3 matrices of order 2;
- 2 matrix of order 3;
- The identity matrix.

Form [6] , $G_{\mathcal{F}_{27}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{27}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{27}^* = \{[1,\sigma^{14},1], [\sigma^3,\sigma^5,1], [1,1,0], [\sigma,\sigma.1], [\sigma^{10},\sigma^{13},1], [1,0,1]\}$ be a subset of $\mathcal{F}_{27}$ which is forming by partition $\mathcal{F}_{27}$ into two sets such that $\mathcal{F}_{27}^*$ dose not contains the inflexion points of $\mathcal{F}_{27}$, so we note that $\mathcal{F}_{27}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{27}^*$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}_{27}^*$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1$$

Therefore, the stabilizer group of $\mathcal{F}_{27}^*$ which is denoted by $G_{\mathcal{F}_{27}^*}$ which contains

- The identity matrix .

Thus, $G_{\mathcal{F}_{27}^*}$ is isomorphic to $\boldsymbol{Z_1}$, that is $G_{\mathcal{F}_{27}^*} \cong \boldsymbol{Z_1}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{28} = xyz + \sigma^3(x+y+z)^3$. The points of $PG(2,16)$ on $\mathcal{F}_{28}$ are $[\sigma^9,\sigma^9,1]$,$[\sigma^{14},\sigma^{10},1]$,$[\sigma^{10},\sigma^{14},1]$,$[\sigma,\sigma^{11},1]$,$[\sigma,\sigma^{14},1]$, $[\sigma^{14},\sigma,1]$,$[\sigma^{13},\sigma^{14},1]$,[1,1,0],[0,1,1],$[\sigma^6,1,1]$,$[\sigma^{11},\sigma,1]$,$[\sigma^5,\sigma^4,1]$,$[\sigma^2,\sigma,1]$,$[1,\sigma^6,1]$, $[\sigma^{14},\sigma^{13},1]$, $[\sigma,\sigma^2,1]$,$[\sigma^4,\sigma^5,1]$,[1,0,1]. To find the stabilizer group of $\mathcal{F}_{28}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{28}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^6\\0&\sigma^6&0\\\sigma^6&0&0\end{pmatrix}:2,\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}:3$$
$$\begin{pmatrix}0&0&\sigma^6\\\sigma^6&0&0\\0&\sigma^6&0\end{pmatrix}:3,\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{28}$ which is denoted by $G_{\mathcal{F}_{28}}$ contains

- 3 matrices of order 2;
- 2 matrix of order 3;
- The identity matrix.

Let $\mathcal{F}_{28}^* = \{[\sigma^9,\sigma^9,1], [\sigma^{14},\sigma^{10},1], [\sigma^{10},\sigma^{14},1], [\sigma,\sigma^{11},1], [\sigma^6,1,1], [\sigma^{11},\sigma,1], [\sigma^5,\sigma^4,1], [1,\sigma^6,1], [\sigma^4,\sigma^5,1]\}$ be a subset of $\mathcal{F}_{28}$ which is forming by partition $\mathcal{F}_{28}$ into two sets such that $\mathcal{F}_{28}^*$ dose not contains the inflexion points of $\mathcal{F}_{28}$, so we note that $\mathcal{F}_{28}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{28}^*$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}_{28}^*$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&\sigma&0\\0&0&\sigma\\\sigma&0&0\end{pmatrix}:3,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{11}\\0&\sigma^{11}&0\\\sigma^{11}&0&0\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma&0&0\\0&0&\sigma\\0&\sigma&0\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{11}\\\sigma^{11}&0&0\\0&\sigma^{11}&0\end{pmatrix}:3$$

Therefore, the stabilizer group of $\mathcal{F}_{28}^*$ which is denoted by $G_{\mathcal{F}_{28}^*}$ which contains

- 3 matrices of order 2;
- 2 matrices of order 3;
- The identity matrix.

Thus, $G_{\mathcal{F}_{28}^*}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{28}^*} \cong \boldsymbol{S_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{29} = xyz + \sigma^4(x+y+z)^3$.The points of $PG(2,16)$ on $\mathcal{F}_{29}$ are $[\sigma^4,\sigma^9,1]$,$[\sigma^6,\sigma^{10},1]$,$[\sigma^{10},\sigma^6,1]$,$[\sigma^5,\sigma^{11},1]$,$[\sigma^9,\sigma^4,1]$, $[\sigma^{11},\sigma^5,1]$,$[1,\sigma^{13},1]$,[1,1,0],[0,1,1],$[\sigma^{13},1,1]$,$[\sigma^2,\sigma^2,1]$,[1,0,1]. To find the stabilizer group of $\mathcal{F}_{29}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{29}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&\sigma^2&0\\0&0&\sigma^2\\\sigma^2&0&0\end{pmatrix}:3,\begin{pmatrix}\sigma^2&0&0\\0&0&\sigma^2\\0&\sigma^2&0\end{pmatrix}:2,\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{29}$ which is denoted by $G_{\mathcal{F}_{29}}$ contains

- 3 matrices of order 2;
- 2 matrix of order 3;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{29}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{29}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{29}^{*} = \{[\sigma^6,\sigma^{10},1],[\sigma^5,\sigma^{11},1],[\sigma^9,\sigma^4,1],[\sigma^{11},\sigma^5,1],[1,\sigma^{13},1],[\sigma^2,\sigma^2,1]\}$ be a subset of $\mathcal{F}_{29}$ which is forming by partition $\mathcal{F}_{29}$ into two sets such that $\mathcal{F}_{29}^{*}$ dose not contains the inflexion points of $\mathcal{F}_{29}$, so we note that $\mathcal{F}_{29}^{*}$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{29}^{*}$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}_{29}^{*}$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}:2$$

Therefore, the stabilizer group of $\mathcal{F}_{29}^{*}$ which is denoted by $G_{\mathcal{F}_{29}^{*}}$ which contains

- One matrix of order 2;
- The identity matrix.

Thus, $G_{\mathcal{F}_{29}^{*}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{29}^{*}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{30} = xyz + \sigma^5(x+y+z)^3$. The points of $PG(2,16)$ on $\mathcal{F}_{30}$ are $[\sigma^{13},\sigma^7,1]$,$[\sigma^{10},\sigma^{10},1]$,$[\sigma^6,\sigma^8,1]$,$[\sigma^8,\sigma^6,1]$,$[\sigma^5,1,1]$, $[\sigma^2,\sigma^9,1]$,$[1,1,0]$,$[0,1,1]$,$[\sigma^7,\sigma^{13},1]$,$[\sigma^6,\sigma^{11},1]$,$[\sigma^9,\sigma^{14},1]$,$[\sigma^{14},\sigma^9,1]$,$[\sigma^5,\sigma^9,1]$,$[\sigma^6,\sigma^5,1]$, $[\sigma,\sigma^{10},1]$, $[1,\sigma^5,1]$,$[\sigma^{10},\sigma,1]$,$[\sigma^{10},\sigma^4,1]$,$[\sigma^{11},\sigma^6,1]$,$[\sigma^4,\sigma^{10},1]$,$[\sigma^9,\sigma^5,1]$, $[\sigma^5,\sigma^6,1]$,$[\sigma^9,\sigma^2,1]$, $[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{30}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{30}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{13}&0&0\\0&0&\sigma^{13}\\0&\sigma^{13}&0\end{pmatrix}:2,\begin{pmatrix}0&\sigma^{13}&0\\0&0&\sigma^{13}\\\sigma^{13}&0&0\end{pmatrix}:3,\begin{pmatrix}0&0&\sigma^{12}\\0&\sigma^{12}&0\\\sigma^{12}&0&0\end{pmatrix}:2$$
$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{12}\\\sigma^{12}&0&0\\0&\sigma^{12}&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{30}$ which is denoted by $G_{\mathcal{F}_{30}}$ contains

- 3 matrices of order 2;
- 2 matrix of order 3;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{30}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{30}} \cong \boldsymbol{S_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{31} = xyz + \sigma^6(x+y+z)^3$. The points of $PG(2,16)$ on $\mathcal{F}_{31}$ are $[1,\sigma^{12},1]$,$[\sigma^5,\sigma^{13},1]$,$[\sigma^2,\sigma^{13},1]$,$[\sigma^7,\sigma^2,1]$,$[\sigma^{10},\sigma^8,1]$ $[\sigma^3,\sigma^3,1]$,$[\sigma^2,\sigma^7,1]$,$[1,1,0]$,$[0,1,1]$,$[\sigma^{12},1,1]$,$[\sigma^{11},\sigma^{13},1]$,$[\sigma^{13},\sigma^{11},1]$,$[\sigma^{13},\sigma^2,1]$,$[\sigma^{13},\sigma^5,1]$, $[\sigma^4,\sigma^2,1]$, $[\sigma^8,\sigma^{10},1]$,$[\sigma^2,\sigma^5,1]$,$[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{31}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{31}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&0&\sigma^{12}\\0&\sigma^{12}&0\\\sigma^{12}&0&0\end{pmatrix}:2,\begin{pmatrix}\sigma^9&0&0\\0&0&\sigma^9\\0&\sigma^9&0\end{pmatrix}:2,\begin{pmatrix}0&\sigma^9&0\\0&0&\sigma^9\\\sigma^9&0&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{12}\\\sigma^{12}&0&0\\0&\sigma^{12}&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{31}$ which is denoted by $G_{\mathcal{F}_{31}}$ contains

- 3 matrices of order 2.
- 2 matrix of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{31}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{31}} \cong \boldsymbol{S_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{32} = xyz + \sigma^7(x+y+z)^3$. The points of $PG(2,16)$ on $\mathcal{F}_{32}$ are $[\sigma^9,\sigma^{13},1]$,$[\sigma^4,1,1]$,$[\sigma^{11},\sigma^8,1]$,$[1,1,0]$,$[0,1,1]$, $[\sigma^{11},\sigma^{11},1]$, $[\sigma^{11},\sigma^2,1]$,$[\sigma^{13},\sigma^9,1]$,$[\sigma^7,\sigma^3,1]$,$[\sigma^4,\sigma^6,1]$,$[\sigma^6,\sigma^4,1]$,$[\sigma^4,\sigma^{12},1]$,$[\sigma^8,\sigma^{11},1]$,$[1,\sigma^4,1]$, $[\sigma^2,\sigma^{11},1]$, $[\sigma^{12},\sigma^4,1]$,$[\sigma^3,\sigma^7,1]$,$[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{32}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{32}$ and their orders are shown as follows :

$$\begin{pmatrix}\sigma^7 & 1 & \sigma^4\\ \sigma^9 & \sigma^{12} & \sigma^5\\ \sigma^{10} & \sigma^{14} & \sigma^2\end{pmatrix}:3, \begin{pmatrix}\sigma^{14} & \sigma^7 & \sigma^{11}\\ \sigma^2 & \sigma^6 & \sigma^9\\ \sigma & \sigma^4 & \sigma^{12}\end{pmatrix}:2, \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^6 & 0 & 0\\ 0 & 0 & \sigma^6\\ 0 & \sigma^6 & 0\end{pmatrix}:2$$

$$\begin{pmatrix}0 & 0 & \sigma^6\\ 0 & \sigma^6 & 0\\ \sigma^6 & 0 & 0\end{pmatrix}:2, \begin{pmatrix}0 & 0 & \sigma^6\\ \sigma^6 & 0 & 0\\ 0 & \sigma^6 & 0\end{pmatrix}:3, \begin{pmatrix}\sigma^{12} & \sigma^9 & \sigma^5\\ \sigma^{14} & \sigma^{10} & \sigma^2\\ 1 & \sigma^7 & \sigma^4\end{pmatrix}:2, \begin{pmatrix}\sigma & \sigma^{13} & \sigma^9\\ \sigma^4 & \sigma^{11} & \sigma^8\\ \sigma^3 & \sigma^{14} & \sigma^6\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^7 & \sigma^{14} & \sigma^{11}\\ \sigma^6 & \sigma^2 & \sigma^9\\ \sigma^4 & \sigma & \sigma^{12}\end{pmatrix}:3, \begin{pmatrix}1 & \sigma^7 & \sigma^4\\ \sigma^{12} & \sigma^9 & \sigma^5\\ \sigma^{14} & \sigma^{10} & \sigma^2\end{pmatrix}:2, \begin{pmatrix}\sigma^{10} & \sigma^{14} & \sigma^2\\ \sigma^9 & \sigma^{12} & \sigma^5\\ \sigma^7 & 1 & \sigma^4\end{pmatrix}:2, \begin{pmatrix}\sigma^9 & \sigma^{13} & \sigma\\ \sigma^6 & \sigma^{14} & \sigma^3\\ \sigma^8 & \sigma^{11} & \sigma^4\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{13} & \sigma^9 & \sigma\\ \sigma^{14} & \sigma^6 & \sigma^3\\ \sigma^{11} & \sigma^8 & \sigma^4\end{pmatrix}:2, \begin{pmatrix}\sigma^{14} & \sigma^{10} & \sigma^2\\ \sigma^{12} & \sigma^9 & \sigma^5\\ 1 & \sigma^7 & \sigma^4\end{pmatrix}:3, \begin{pmatrix}0 & \sigma^6 & 0\\ 0 & 0 & \sigma^6\\ \sigma^6 & 0 & 0\end{pmatrix}:3, \begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma^9 & \sigma^{12} & \sigma^5\\ \sigma^{10} & \sigma^{14} & \sigma^2\\ \sigma^7 & 1 & \sigma^4\end{pmatrix}:3, \begin{pmatrix}\sigma^{13} & \sigma & \sigma^9\\ \sigma^{11} & \sigma^4 & \sigma^8\\ \sigma^{14} & \sigma^3 & \sigma^6\end{pmatrix}:2$$

Therefore , the stabilizer groups of $\mathcal{F}_{32}$ which is denoted by $G_{\mathcal{F}_{32}}$ contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{32}}$ is isomorphic to $(\boldsymbol{Z_3} \times \boldsymbol{Z_3}) \rtimes \boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{32}} \cong (\boldsymbol{Z_3} \times \boldsymbol{Z_3}) \rtimes \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{33} = xyz + \sigma^8(x+y+z)^3$. The points of $PG(2,16)$ on $\mathcal{F}_{32}$ are $[\sigma^{11},1,1]$,$[\sigma^4,\sigma^4,1]$,$[\sigma^8,\sigma^3,1]$,$[\sigma^7,\sigma^{10},1]$,$[\sigma^3,\sigma^8,1]$, $[\sigma^{10},\sigma^7,1]$,$[1,1,0]$,$[0,1,1]$,$[1,\sigma^{11},1]$,$[\sigma^{12},\sigma^5,1]$,$[\sigma^5,\sigma^{12},1]$,$[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{33}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{33}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^{11} & 0 & 0\\ 0 & 0 & \sigma^{11}\\ 0 & \sigma^{11} & 0\end{pmatrix};2, \begin{pmatrix}0 & 0 & \sigma^2\\ 0 & \sigma^2 & 0\\ \sigma^2 & 0 & 0\end{pmatrix}:2, \begin{pmatrix}0 & 0 & \sigma^2\\ \sigma^2 & 0 & 0\\ 0 & \sigma^2 & 0\end{pmatrix}:3$$
$$\begin{pmatrix}0 & \sigma^{11} & 0\\ 0 & 0 & \sigma^{11}\\ \sigma^{11} & 0 & 0\end{pmatrix}:3, \begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{33}$ which is denoted by $G_{\mathcal{F}_{33}}$ contains

- 3 matrices of order 2.
- 2 matrix of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{33}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{33}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{33}^* = \{[\sigma^{11},1,1],[\sigma^8,\sigma^3,1],[1,\sigma^{11},1],[\sigma^{12},\sigma^5,1],[\sigma^5,\sigma^{12},1],[1,0,1]\}$ be a subset of $\mathcal{F}_{33}$ which is forming by partition $\mathcal{F}_{33}$ into two sets such that $\mathcal{F}_{33}^*$ dose not contains the inflexion points of $\mathcal{F}_{33}$, so we note that $\mathcal{F}_{33}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{33}^*$, by some calculation . we get that the matrix which is stabilizing of $\mathcal{F}_{33}^*$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1$$

Therefore, the stabilizer group of $\mathcal{F}_{33}^*$ which is denoted by $G_{\mathcal{F}_{33}^*}$ which contains

- The identity matrix.

Thus, $G_{\mathcal{F}_{33}^*}$ is isomorphic to $\boldsymbol{Z_1}$, that is $G_{\mathcal{F}_{33}^*} \cong \boldsymbol{Z_1}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{34} = xyz + \sigma^9(x+y+z)^3$. The points of $PG(2,16)$ on $\mathcal{F}_{34}$ are $[\sigma^2,\sigma^{10},1]$,$[\sigma^{12},\sigma^{12},1]$,$[\sigma^5,\sigma^7,1]$,$[1,\sigma^3,1]$,$[\sigma^{10},\sigma^2,1]$, $[\sigma^7,\sigma^8,1]$,$[\sigma^8,\sigma^{13},1]$,$[\sigma^8,\sigma^7,1]$,$[1,1,0]$,$[0,1,1]$,$[\sigma,\sigma^8,1]$,$[\sigma^8,\sigma,1]$,$[\sigma^{13},\sigma^8,1]$,$[\sigma^3,1,1]$, $[\sigma^7,\sigma^{14},1]$, $[\sigma^{14},\sigma^7,1]$,$[\sigma^7,\sigma^5,1]$,$[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{34}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{34}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&\sigma^{13}&0\\0&0&\sigma^{13}\\\sigma^{13}&0&0\end{pmatrix}:3,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{14}\\\sigma^{14}&0&0\\0&\sigma^{14}&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&0&\sigma^{14}\\0&\sigma^{14}&0\\\sigma^{14}&0&0\end{pmatrix}:2,\begin{pmatrix}\sigma^{13}&0&0\\0&0&\sigma^{13}\\0&\sigma^{13}&0\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{34}$ which is denoted by $G_{\mathcal{F}_{34}}$ contains

- 3 matrices of order 2.
- 2 matrix of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{34}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{34}} \cong \boldsymbol{S_3}$.

The points of $PG(2,16)$ on $\mathcal{F}_{35}$ are $[\sigma^3,\sigma^{13},1]$,$[\sigma^{12},\sigma^{10},1]$,$[\sigma^{12},\sigma,1]$,$[\sigma^7,\sigma^{12},1]$, $[\sigma^{10},\sigma^{12},1]$, $[\sigma^8,\sigma^5,1]$,$[\sigma^{13},\sigma^3,0]$,$[\sigma^4,\sigma^3,1]$,$[\sigma^{11},\sigma^{14},1]$,$[1,\sigma^{10},1]$,$[\sigma,\sigma^{12},1]$,$[1,1,0]$,$[0,1,1]$,$[\sigma^{14},\sigma^{11},1]$, $[\sigma^{10},\sigma^3,1]$, $[\sigma^{12},\sigma^7,1]$,$[\sigma^5,\sigma^2,1]$,$[\sigma^5,\sigma^8,1]$,$[\sigma^5,\sigma^5,1]$,$[\sigma^2,\sigma^{15},1]$,$[\sigma^{10},1,1]$, $[\sigma^3,\sigma^4,1]$, $[\sigma^3,\sigma^{10},1]$,$[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{35}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{35}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&0&\sigma^{12}\\0&\sigma^{12}&0\\\sigma^{12}&0&0\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{12}\\\sigma^{12}&0&0\\0&\sigma^{12}&0\end{pmatrix}:3,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma^5&0&0\\0&0&\sigma^5\\0&\sigma^5&0\end{pmatrix}:2,\begin{pmatrix}0&\sigma^5&0\\0&0&\sigma^5\\\sigma^5&0&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{35}$ which is denoted by $G_{\mathcal{F}_{35}}$ contains

- 3 matrices of order 2;
- 2 matrix of order 3;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{35}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{35}} \cong \boldsymbol{S_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{37} = xyz + \sigma^{12}(x+y+z)^3$. The points of $PG(2,16)$ on $\mathcal{F}_{37}$ are $[\sigma^8,\sigma^4,1]$,$[\sigma^6,\sigma^6,1]$,$[\sigma^4,\sigma^8,1]$,$[\sigma^{14},\sigma^4,1]$,$[\sigma^{10},\sigma^{11},1]$, $[1,\sigma^9,1]$,$[\sigma^4,\sigma^{14},1]$,$[\sigma^4,\sigma^{11},1]$,$[\sigma^{11},\sigma^4,0]$,$[1,1,0]$,$[0,1,1]$,$[\sigma^7,\sigma^4,1]$,$[\sigma^{11},\sigma^{10},1]$,$[\sigma^{11},\sigma^7,1]$, $[\sigma^9,1,1]$, $[\sigma,\sigma^5,1]$,$[\sigma^5,\sigma,1]$,$[1,0,1]$. To find the stabilizer group of $\mathcal{F}_{37}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{37}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}\sigma^8&0&0\\0&0&\sigma^8\\0&\sigma^8&0\end{pmatrix}:2,\begin{pmatrix}0&\sigma^8&0\\0&0&\sigma^8\\\sigma^8&0&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&0&\sigma^{13}\\0&\sigma^{13}&0\\\sigma^{13}&0&0\end{pmatrix}:2,\begin{pmatrix}0&0&\sigma^{13}\\\sigma^{13}&0&0\\0&\sigma^{13}&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{37}$ which is denoted by $G_{\mathcal{F}_{37}}$ contains

- 3 matrices of order 2.
- 2 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{37}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{37}} \cong \boldsymbol{S_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{40} = yz^2 + xyz + x^3 + xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{40}$ are $[0,1,0]$,$[0,0,1]$,$[\sigma^{14},\sigma^8,1]$,$[\sigma^{14},\sigma^5,1]$,$[\sigma^{13},\sigma^{10},1]$, $[\sigma^5,1,1]$, $[\sigma^7,\sigma^{10},1]$,$[\sigma^{10},\sigma^5,1]$,$[\sigma^{11},\sigma^5,0]$,$[\sigma^7,\sigma^4,0]$,$[1,1,0]$,$[\sigma^{11},\sigma^2,1]$,$[\sigma^5,\sigma^{10},1]$,$[\sigma^{13},\sigma,1]$,$[1,1,1]$, $[\sigma^{10},1,1]$. To find the stabilizer group of $\mathcal{F}_{40}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{40}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{14}&0&\sigma^{14}\\0&\sigma^{14}&0\\0&0&\sigma^{14}\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{40}$ which is denoted by $G_{\mathcal{F}_{40}}$ contains

- One matrix of order 2;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{40}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{40}} \cong \boldsymbol{Z_2}$.

Let $\mathcal{F}^*_{40} = \{[\sigma^{14},\sigma^8,1],[\sigma^{14},\sigma^5,1],[\sigma^{11},\sigma^5,1],[1,1,0],[\sigma^{11},\sigma^2,1],[\sigma^5,\sigma^{10},1],[1,1,1], [\sigma^{10},1,1]\}$ be a subset of $\mathcal{F}_{40}$ which is forming by partition $\mathcal{F}_{40}$ into two sets such that $\mathcal{F}^*_{40}$ dose not contains the inflexion points of $\mathcal{F}_{40}$, so we note that $\mathcal{F}^*_{40}$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}^*_{40}$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}^*_{40}$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{12}&0&\sigma^{12}\\0&\sigma^{12}&0\\0&0&\sigma^{12}\end{pmatrix}:2$$

Therefore, the stabilizer group of $\mathcal{F}^*_{40}$ which is denoted by $G_{\mathcal{F}^*_{40}}$ which contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}^*_{40}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}^*_{40}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{41} = yz^2 + xyz + x^3 + \sigma xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{41}$ are $[0,1,0]$,$[0,0,1]$,$[\sigma^9,\sigma^9,1]$,$[\sigma^9,\sigma^8,1]$,$[\sigma^5,\sigma^{13},1]$,$[\sigma^5,\sigma^{11},1]$, $[\sigma^3,\sigma^5,1]$,$[\sigma^4,\sigma,1]$,$[\sigma^4,\sigma^6,0]$,$[\sigma^{10},\sigma,1]$,$[\sigma^2,\sigma,1]$,$[\sigma^{10},\sigma^3,1]$,$[\sigma^2,\sigma^2,1]$,$[\sigma^3,1,1]$,$[\sigma^8,1,0]$, $[1,\sigma^7,1]$. To find the stabilizer group of $\mathcal{F}_{41}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{41}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^6&0&\sigma^6\\0&\sigma^6&0\\0&0&\sigma^6\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{41}$ which is denoted by $G_{\mathcal{F}_{41}}$ contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{41}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{41}} \cong \boldsymbol{Z_2}$.

Let $\mathcal{F}^*_{41} = \{[\sigma^9,\sigma^9,1],[\sigma^5,\sigma^{13},1],[\sigma^5,\sigma^{11},1],[\sigma^{10},\sigma,1],[\sigma^{10},\sigma^3,1],[\sigma^2,\sigma^2,1],[\sigma^8,1,0], [1,\sigma^7,1]\}$ be a subset of $\mathcal{F}_{41}$ which is forming by partition $\mathcal{F}_{41}$ into two sets such that $\mathcal{F}^*_{41}$ dose not contains the inflexion points of $\mathcal{F}_{41}$, so we note that $\mathcal{F}^*_{41}$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}^*_{41}$, by some calculation .we get that the matrix which is stabilizing of $\mathcal{F}^*_{41}$ is

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^{13} & 0 & \sigma^{13}\\0 & \sigma^{13} & 0\\0 & 0 & \sigma^{13}\end{pmatrix}:2$$

Therefore, the stabilizer group of $\mathcal{F}_{41}^*$ which is denoted by $G_{\mathcal{F}_{41}^*}$ which contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{41}^*}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{41}^*} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{42} = yz^2 + xyz + x^3 + \sigma^2 xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{42}$ are [0,1,0],[0,0,1],$[1,\sigma^{14},1]$,$[\sigma^6,\sigma^{10},1]$,$[\sigma^{10},\sigma^{11},1]$, $[\sigma^4,\sigma^4,1]$,$[\sigma,1,0]$,$[\sigma^8,\sigma^2,1]$,$[\sigma^8,\sigma^{12},1]$,$[\sigma^3,\sigma^3,1]$,$[\sigma^{10},\sigma^7,1]$,$[\sigma^6,1,1]$,$[\sigma^3,\sigma,1]$,$[\sigma^4,\sigma^2,1]$, $[\sigma^5,\sigma^6,1]$, $[\sigma^5,\sigma^2,1]$. To find the stabilizer group of $\mathcal{F}_{42}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{42}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^{9} & 0 & \sigma^{9}\\0 & \sigma^{9} & 0\\0 & 0 & \sigma^{9}\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{42}$ which is denoted by $G_{\mathcal{F}_{42}}$ contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{42}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{42}} \cong \boldsymbol{Z_2}$.

Let $\mathcal{F}_{42}^* = \{[1,\sigma^{14},1],[\sigma^{10},\sigma^{11},1],[\sigma^4,\sigma^4,1],[\sigma,1,0],[\sigma^3,\sigma^3,1][\sigma^{10},\sigma^7,1],[\sigma^5,\sigma^6,1], [\sigma^5,\sigma^2,1]\}$ be a subset of $\mathcal{F}_{42}$ which is forming by partition $\mathcal{F}_{42}$ into two sets such that $\mathcal{F}_{42}^*$ dose not contains the inflexion points of $\mathcal{F}_{42}$, so we note that $\mathcal{F}_{42}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{42}^*$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}_{42}^*$ is

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^{9} & 0 & \sigma^{9}\\0 & \sigma^{9} & 0\\0 & 0 & \sigma^{9}\end{pmatrix}:2$$

Therefore, the stabilizer group of $\mathcal{F}_{42}^*$ which is denoted by $G_{\mathcal{F}_{42}^*}$ which contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{42}^*}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{42}^*} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{43} = yz^2 + xyz + x^3 + \sigma^3 xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{43}$ are [0,1,0],[0,0,1],$[\sigma^3,\sigma^{11},1]$,$[\sigma^9,\sigma^{12},1]$,$[\sigma^4,\sigma^8,1]$, $[\sigma^8,1,1]$,$[\sigma^{14},\sigma^{12},1]$,$[\sigma^{11},\sigma^9,1]$,$[\sigma^9,\sigma^3,1]$,$[\sigma^8,\sigma^{13},1]$,$[\sigma^9,1,0]$,$[\sigma^6,\sigma^{11},1]$,$[\sigma^{11},\sigma^{10},1]$,$[1,\sigma^6,1]$, $[\sigma^4,\sigma^{12},1]$, $[\sigma^6,\sigma^{13},1]$,$[\sigma^{14},\sigma^{13},1]$,$[\sigma^2,\sigma^5,1]$,$[\sigma^2,\sigma^{11},1]$,$[\sigma^3,\sigma^7,1]$. To find the stabilizer group of $\mathcal{F}_{43}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{43}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^{12} & 0 & \sigma^{12}\\0 & \sigma^{12} & 0\\0 & 0 & \sigma^{12}\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{43}$ which is denoted by $G_{\mathcal{F}_{43}}$ contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{43}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{43}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{44} = yz^2 + xyz + x^3 + \sigma^4 xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{44}$ are [0,1,0],[0,0,1],$[\sigma^8,\sigma^4,1]$,$[\sigma^6,\sigma^6,1]$,$[\sigma^2,1,0]$, $[\sigma^5,\sigma^7,1]$,$[\sigma,\sigma^9,1]$,$[\sigma^{10},\sigma^{12},1]$,$[\sigma^6,\sigma^2,1]$,$[1,\sigma^{13},1]$,$[\sigma^{12},1,1]$,$[\sigma^8,\sigma^8,1]$,$[\sigma^{10},\sigma^4,1]$,$[\sigma^5,\sigma^{14},1]$, $[\sigma,\sigma^4,1]$, $[\sigma^{12},\sigma^5,1]$. To find the stabilizer group of $\mathcal{F}_{44}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{44}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^{3} & 0 & \sigma^{3}\\0 & \sigma^{3} & 0\\0 & 0 & \sigma^{3}\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{44}$ which is denoted by $G_{\mathcal{F}_{44}}$ contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{44}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{44}} \cong \boldsymbol{Z_2}$.

Let $\mathcal{F}^*_{44} = \{[\sigma^5,\sigma^7,1],[\sigma,\sigma^9,1],[\sigma^{10},\sigma^{12},1],[\sigma^{12},1,1],[\sigma^{10},\sigma^4,1],[\sigma^5,\sigma^{14},1],[\sigma,\sigma^4,1],[\sigma^{12},\sigma^5,1]\}$ be a subset of $\mathcal{F}_{44}$ which is forming by partition $\mathcal{F}_{44}$ into two sets such that $\mathcal{F}^*_{44}$ dose not contains the inflexion points of $\mathcal{F}_{44}$, so we note that $\mathcal{F}^*_{44}$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}^*_{44}$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}^*_{44}$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}:2$$

Therefore, the stabilizer group of $\mathcal{F}^*_{44}$ which is denoted by $G_{\mathcal{F}^*_{44}}$ which contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}^*_{44}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}^*_{44}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{45} = yz^2 + xyz + x^3 + \sigma^5 xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{45}$ are $[0,1,0],[0,0,1],[\sigma^{13},\sigma^{13},1],[\sigma,\sigma^7,1],[\sigma^{12},\sigma,1],[\sigma^{12},\sigma^3,1],[\sigma^7,\sigma^2,1],[\sigma^{10},\sigma^6,1],[\sigma^4,\sigma^{13},1],[\sigma^3,\sigma^{12},1],[\sigma^{14},\sigma,1],[\sigma^{11},\sigma^4,1],[\sigma^{11},\sigma^{13},1],[\sigma^{10},\sigma^9,1],[\sigma^5,\sigma^4,1],[1,\sigma^5,1],[\sigma^7,\sigma^7,1],[\sigma,\sigma^5,1],[\sigma^{13},\sigma^8,1],[\sigma^{14},\sigma^7,1],[\sigma^{10},1,0],[\sigma^5,\sigma,1],[\sigma^4,\sigma^5,1],[\sigma^3,\sigma^4,1]$. To find the stabilizer group of $\mathcal{F}_{45}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{45}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{10}&0&\sigma^{10}\\0&\sigma^{10}&0\\0&0&\sigma^{10}\end{pmatrix}:2,\begin{pmatrix}\sigma^5&\sigma^3&\sigma^2\\\sigma^{14}&\sigma^{14}&\sigma\\\sigma^4&\sigma^{14}&\sigma^{12}\end{pmatrix}:3,\begin{pmatrix}\sigma^8&1&\sigma^7\\\sigma^{14}&\sigma^{14}&\sigma\\\sigma^4&\sigma^{14}&\sigma^{12}\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma^{13}&\sigma^{11}&\sigma^9\\\sigma^7&\sigma^7&1\\\sigma^{12}&\sigma^7&\sigma^{14}\end{pmatrix}:2,\begin{pmatrix}\sigma^5&\sigma^{12}&\sigma^8\\\sigma^{11}&\sigma^{11}&\sigma^4\\\sigma&\sigma^{11}&\sigma^3\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{45}$ which is denoted by $G_{\mathcal{F}_{45}}$ contains

- 3 matrices of order 2.
- 2 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{45}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{45}} \cong \boldsymbol{S_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{46} = yz^2 + xyz + x^3 + \sigma^6 xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{46}$ are $[0,1,0],[0,0,1],[1,\sigma^{12},1],[\sigma,\sigma^{11},1],[\sigma^{12},\sigma^{11},1],[\sigma^4,\sigma^7,1],[\sigma^6,\sigma^{14},1],[\sigma,1,1],[\sigma^{13},\sigma^9,1],[\sigma^{13},\sigma^{11},1],[\sigma^7,\sigma^3,1],[\sigma^3,\sigma^6,1],[\sigma^8,\sigma,1],[\sigma^3,\sigma^9,1],[\sigma^6,\sigma^7,1],[\sigma^3,1,0],[\sigma^4,\sigma^{10},1],[\sigma^8,\sigma^9,1],[\sigma^{12},\sigma^7,1],[\sigma^7,\sigma^5,1]$. To find the stabilizer group of $\mathcal{F}_{46}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{46}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^3&0&\sigma^3\\0&\sigma^3&0\\0&0&\sigma^3\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{46}$ which is denoted by $G_{\mathcal{F}_{46}}$ contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{46}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{46}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{47} = yz^2 + xyz + x^3 + \sigma^7 xy^2$.The points of $PG(2,16)$ on $\mathcal{F}_{47}$ are $[0,1,0],[0,0,1],[\sigma^8,\sigma^6,1],[\sigma^8,\sigma^3,1],[\sigma^{11},1,0],[\sigma^7,\sigma^9,1],[\sigma^{13},1,1],[\sigma^7,\sigma^{13},1],[\sigma^6,\sigma,1],[\sigma^6,\sigma^4,1],[1,\sigma^4,1],[\sigma^{13},\sigma^4,1]$. To find the stabilizer group of $\mathcal{F}_{47}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{47}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{10}&0&\sigma^{10}\\0&\sigma^{10}&0\\0&0&\sigma^{10}\end{pmatrix}:2,\begin{pmatrix}\sigma^{6}&1&\sigma\\\sigma^{9}&\sigma^{4}&\sigma\\\sigma^{3}&\sigma&\sigma^{12}\end{pmatrix}:3,\begin{pmatrix}\sigma^{14}&\sigma&\sigma^{10}\\\sigma^{6}&\sigma&\sigma^{13}\\1&\sigma^{13}&\sigma^{9}\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma^{11}&\sigma^{5}&\sigma\\\sigma^{14}&\sigma^{9}&\sigma^{8}\\\sigma^{8}&\sigma^{6}&1\end{pmatrix}:2,\begin{pmatrix}\sigma^{14}&\sigma&\sigma^{11}\\\sigma^{6}&\sigma&1\\1&\sigma^{13}&\sigma^{7}\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{47}$ which is denoted by $G_{\mathcal{F}_{47}}$ contains

- 3 matrices of order 2.
- 2 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{47}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{47}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{47}^{*} = \{[\sigma^8,\sigma^6,1],[\sigma^8,\sigma^3,1],[\sigma^7,\sigma^9,1],[\sigma^{13},1,1],[\sigma^7,\sigma^{13},1],[\sigma^6,\sigma,1]\}$ be a subset of $\mathcal{F}_{47}$ which is forming by partition $\mathcal{F}_{47}$ into two sets such that $\mathcal{F}_{47}^{*}$ dose not contains the inflexion points of $\mathcal{F}_{47}$, so we note that $\mathcal{F}_{47}^{*}$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{47}^{*}$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}_{47}^{*}$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1$$

Therefore, the stabilizer group of $\mathcal{F}_{47}^{*}$ which is denoted by $G_{\mathcal{F}_{47}^{*}}$ which contains

- The identity matrix .

Thus, $G_{\mathcal{F}_{47}^{*}}$ is isomorphic to $\boldsymbol{Z_1}$, that is $G_{\mathcal{F}_{47}^{*}} \cong \boldsymbol{Z_1}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{48} = yz^2 + xyz + x^3 + \sigma^8 xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{48}$ are $[0,1,0]$,$[0,0,1]$,$[\sigma^4,1,0]$,$[\sigma^9,\sigma^{10},1]$,$[\sigma^{12},\sigma^{12},1]$, $[\sigma^{10},\sigma^{14},1]$,$[\sigma^5,\sigma^9,1]$,$[\sigma^2,\sigma^8,1]$,$[\sigma,\sigma^8,1]$,$[\sigma^9,1,1]$,$[\sigma,\sigma,1]$,$[1,\sigma^{11},1]$,$[\sigma^5,\sigma^8,1]$,$[\sigma^{10},\sigma^{13},1]$, $[\sigma^2,\sigma^3 1]$,$[\sigma^{12},\sigma^4,1]$. To find the stabilizer group of $\mathcal{F}_{48}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{48}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{14}&0&\sigma^{14}\\0&\sigma^{14}&0\\0&0&\sigma^{14}\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{48}$ which is denoted by $G_{\mathcal{F}_{48}}$ contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{48}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{48}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{49} = yz^2 + xyz + x^3 + \sigma^9 xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{49}$ are $[0,1,0]$,$[0,0,1]$,$[\sigma^9,\sigma^{13},1]$,$[\sigma^3,\sigma^{13},1]$,$[\sigma^4,1,1]$, $[\sigma^3,\sigma^{14},1]$, $[1,\sigma^3,1]$,$[\sigma^4,\sigma^{14},1]$,$[\sigma^7,\sigma^6,1]$,$[\sigma^9,\sigma^{11},1]$,$[\sigma^{13},\sigma^{12},1]$,$[\sigma,\sigma^{10},1]$,$[\sigma^{12},\sigma^9,1]$,$[\sigma^{13},\sigma^5,1]$, $[\sigma^{12},1,0]$,$[\sigma,\sigma^{13},1]$,$[\sigma^{12},\sigma^6,1]$,$[\sigma^7,\sigma^{14},1]$,$[\sigma^2,\sigma^6,1]$,$[\sigma^2,\sigma^4,1]$. To find the stabilizer group of $\mathcal{F}_{49}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{49}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{11}&0&\sigma^{11}\\0&\sigma^{11}&0\\0&0&\sigma^{11}\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{49}$ which is denoted by $G_{\mathcal{F}_{49}}$ contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{49}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{49}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{50} = yz^2 + xyz + x^3 + \sigma^{10}xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{50}$ are [0,1,0],[0,0,1],$[\sigma^5,1,0]$,$[\sigma^2,\sigma^{10},1]$,$[\sigma^{14},\sigma^{14},1]$, $[\sigma^6,\sigma^8,1]$,$[\sigma^{14},\sigma^4,1]$,$[\sigma^{10},\sigma^8,1]$,$[\sigma^{10},\sigma^2,1]$,$[\sigma^5,\sigma^3,1]$,$[\sigma^7,\sigma^8,1]$,$[\sigma^{13},\sigma^{14},1]$,$[1,\sigma^{10},1]$, $[\sigma^{11},\sigma^{11},1]$,$[\sigma^{11},\sigma,1]$,$[\sigma^7,\sigma^{11},1]$,$[\sigma^{13},\sigma^2,1]$,$[\sigma^8,\sigma^{11},1]$,$[\sigma^9,\sigma^6,1]$,$[\sigma^2,\sigma^{14},1]$,$[\sigma^8,\sigma^{10},1]$, $[\sigma^6,\sigma^9,1]$,$[\sigma^5,\sigma^{12},1]$,$[\sigma^9,\sigma^2,1]$. To find the stabilizer group of $\mathcal{F}_{50}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{50}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1, \begin{pmatrix}\sigma^{14}&0&\sigma^{14}\\0&\sigma^{14}&0\\0&0&\sigma^{14}\end{pmatrix}:2, \begin{pmatrix}\sigma^{6}&\sigma^{2}&\sigma^{13}\\\sigma^{9}&\sigma^{9}&\sigma^{10}\\\sigma^{4}&\sigma^{9}&\sigma^{8}\end{pmatrix}:2, \begin{pmatrix}\sigma^{4}&\sigma^{3}&\sigma^{10}\\\sigma&\sigma&\sigma^{2}\\\sigma^{11}&\sigma&1\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{12}&\sigma^{8}&\sigma^{6}\\1&1&\sigma^{4}\\\sigma^{10}&1&\sigma^{11}\end{pmatrix}:3, \begin{pmatrix}\sigma^{14}&\sigma^{13}&\sigma^{12}\\\sigma^{11}&\sigma^{11}&1\\\sigma^{6}&\sigma^{11}&\sigma^{7}\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{50}$ which is denoted by $G_{\mathcal{F}_{50}}$ contains

- 3 matrices of order 2.
- 2 matrix of order 3.
- The identity matrix.

Form [6] , $G_{\mathcal{F}_{50}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{50}} \cong \boldsymbol{S_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{51} = yz^2 + xyz + x^3 + \sigma^{11}xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{51}$ are [0,1,0],[0,0,1],$[\sigma^4,\sigma^9,0]$,$[\sigma^4,\sigma^3,1]$,$[\sigma^{11},\sigma^{14},1]$, $[\sigma^3,\sigma^8,1]$,$[\sigma^{14},1,1]$,$[1,\sigma^2,1]$,$[\sigma^{14},\sigma^2,1]$,$[\sigma^3,\sigma^2,1]$,$[\sigma^{13},1,0]$,$[\sigma^{11},\sigma^{12},1]$. To find the stabilizer group of $\mathcal{F}_{51}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{51}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1, \begin{pmatrix}\sigma^{4}&0&\sigma^{4}\\0&\sigma^{4}&0\\0&0&\sigma^{4}\end{pmatrix}:2, \begin{pmatrix}\sigma^{11}&\sigma^{12}&\sigma^{9}\\\sigma^{7}&\sigma^{12}&\sigma^{3}\\\sigma^{4}&\sigma^{3}&\sigma\end{pmatrix}:2, \begin{pmatrix}\sigma^{5}&\sigma^{2}&\sigma^{10}\\\sigma^{14}&\sigma^{4}&\sigma^{10}\\\sigma^{11}&\sigma^{10}&\sigma^{8}\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{8}&\sigma^{5}&\sigma^{3}\\\sigma^{2}&\sigma^{7}&\sigma^{14}\\\sigma^{14}&\sigma^{13}&\sigma^{10}\end{pmatrix}:2, \begin{pmatrix}\sigma^{14}&1&\sigma^{5}\\\sigma^{10}&1&\sigma^{7}\\\sigma^{7}&\sigma^{6}&\sigma^{3}\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{51}$ which is denoted by $G_{\mathcal{F}_{51}}$ contains

- 3 matrices of order 2.
- 2 matrix of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{51}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{51}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{51}^* = \{[0,1,0],[\sigma^4,\sigma^9,1],[\sigma^{11},\sigma^{14},1],[\sigma^{14},1,1],[1,\sigma^2,1],[\sigma^{13},1,0]\}$ be a subset of $\mathcal{F}_{51}$ which is forming by partition $\mathcal{F}_{51}$ into two sets such that $\mathcal{F}_{51}^*$ dose not contains the inflexion points of $\mathcal{F}_{51}$, so we note that $\mathcal{F}_{51}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{51}^*$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}_{51}^*$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1, \begin{pmatrix}\sigma^{14}&1&\sigma^{12}\\\sigma^{10}&1&\sigma^{6}\\\sigma^{7}&\sigma^{6}&\sigma^{4}\end{pmatrix}:2$$

Therefore, the stabilizer group of $\mathcal{F}_{51}^*$ which is denoted by $G_{\mathcal{F}_{51}^*}$ which contains

- One matrix of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{51}^*}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{51}^*} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{52} = yz^2 + xyz + x^3 + \sigma^{12}xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{52}$ are [0,1,0],[0,0,1],$[\sigma^{11},\sigma^3,1]$,$[\sigma^{12},\sigma^{13},1]$,$[\sigma^{14},\sigma^{10},1]$, $[\sigma^6,1,0]$,$[\sigma^8,\sigma^5,1]$,$[1,\sigma^9,1]$,$[\sigma^8,\sigma^{14},1]$,$[\sigma^{12},\sigma^{14},1]$,$[\sigma^2,1,1]$,$[\sigma^2,\sigma^7,1]$,$[\sigma^{14},\sigma^6,1]$,$[\sigma,\sigma^3,1]$, $[\sigma^9,\sigma^{14},1]$,$[\sigma^{11},\sigma^7,1]$,$[\sigma^6,\sigma^{12},1]$,$[\sigma^6,\sigma^3,1]$,$[\sigma,\sigma^2,1]$,$[\sigma^9,\sigma^7,1]$. To find the stabilizer group of $\mathcal{F}_{52}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{52}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^6&0&\sigma^6\\0&\sigma^6&0\\0&0&\sigma^6\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{52}$ which is denoted by $G_{\mathcal{F}_{52}}$ contains

- One matrices of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{52}}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{52}} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{53} = yz^2 + xyz + x^3 + \sigma^{13}xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{53}$ are [0,1,0],[0,0,1],$[\sigma^{13},\sigma^7,0]$,$[\sigma^9,\sigma,1]$,$[\sigma^7,\sigma,1]$, $[\sigma^2,\sigma^{12},1]$,$[\sigma^9,\sigma^4,1]$,$[1,\sigma,1]$,$[\sigma^{13},\sigma^6,1]$,$[\sigma^2,\sigma^9,1]$,$[\sigma^7,1,1]$,$[\sigma^{14},1,0]$. To find the stabilizer group of $\mathcal{F}_{53}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{53}$ and their orders are shown as follows :

$$\begin{pmatrix}\sigma^{13}&0&\sigma^{13}\\0&\sigma^{13}&0\\0&0&\sigma^{13}\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^7&1&\sigma^6\\\sigma^5&1&\sigma^3\\\sigma^{11}&\sigma^3&\sigma^2\end{pmatrix}:2,\begin{pmatrix}\sigma^{12}&\sigma^3&\sigma^7\\\sigma^9&\sigma^4&\sigma^7\\1&\sigma^7&\sigma^6\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^6&\sigma^{12}&\sigma^{11}\\\sigma^3&\sigma^{13}&\sigma^9\\\sigma^9&\sigma&\sigma^7\end{pmatrix}:2,\begin{pmatrix}\sigma&\sigma^9&\sigma^4\\\sigma^{14}&\sigma^9&\sigma^5\\\sigma^5&\sigma^{12}&\sigma^3\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{53}$ which is denoted by $G_{\mathcal{F}_{53}}$ contains

- 3 matrices of order 2.
- 2 matrix of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{53}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{53}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{53}^* = \{[\sigma^{13},\sigma^7,1],[\sigma^9,\sigma,1],[\sigma^7,\sigma,1],[\sigma^2,\sigma^{12},1],[\sigma^2,\sigma^9,1],[\sigma^7,1,1]\}$ be a subset of $\mathcal{F}_{53}$ which is forming by partition $\mathcal{F}_{53}$ into two sets such that $\mathcal{F}_{53}^*$ dose not contains the inflexion points of $\mathcal{F}_{53}$, so we note that $\mathcal{F}_{53}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{53}^*$, by some calculation. we get that the matrix which is stabilizing of $\mathcal{F}_{53}^*$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1$$

Therefore, the stabilizer group of $\mathcal{F}_{53}^*$ which is denoted by $G_{\mathcal{F}_{53}^*}$ which contains

- The identity matrix.

Thus, $G_{\mathcal{F}_{53}^*}$ is isomorphic to $\boldsymbol{Z_1}$, that is $G_{\mathcal{F}_{53}^*} \cong \boldsymbol{Z_1}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{54} = yz^2 + xyz + x^3 + \sigma^{14}xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{54}$ are [0,1,0],[0,0,1],$[\sigma^7,1,0]$,$[1,\sigma^8,1]$,$[\sigma^{12},\sigma^8,1]$, $[\sigma^{14},\sigma^3,1]$,$[\sigma^{11},\sigma^8,1]$,$[\sigma^{11},1,1]$,$[\sigma,\sigma^{12},1]$,$[\sigma^{12},\sigma^2,1]$,$[\sigma^{14},\sigma^{11},1]$,$[\sigma,\sigma^6,1]$. To find the stabilizer group of $\mathcal{F}_{54}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{54}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma&0&\sigma\\0&\sigma&0\\0&0&\sigma\end{pmatrix}:2,\begin{pmatrix}\sigma^4&\sigma^8&\sigma^{13}\\\sigma^3&\sigma^8&\sigma^6\\\sigma^6&\sigma^2&\sigma^5\end{pmatrix}:3,\begin{pmatrix}\sigma^9&\sigma^{12}&\sigma^4\\1&\sigma^5&\sigma^3\\\sigma^3&\sigma^{14}&\sigma^2\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma&\sigma^5&\sigma^8\\1&\sigma^5&\sigma^{14}\\\sigma^3&\sigma^{14}&\sigma^6\end{pmatrix}:2,\begin{pmatrix}\sigma^6&\sigma^9&\sigma^{11}\\\sigma^{12}&\sigma^2&\sigma^{11}\\1&\sigma^{11}&\sigma^3\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{54}$ which is denoted by $G_{\mathcal{F}_{54}}$ contains

- 3 matrices of order 2.
- 2 matrix of order 3.
- The identity matrix .

Form [6], $G_{\mathcal{F}_{54}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\mathcal{F}_{54}} \cong \boldsymbol{S_3}$.

Let $\mathcal{F}_{54}^* = \{[\sigma^{14},\sigma^3,1],[\sigma^{11},\sigma^8,1],[\sigma^{11},1,1],[\sigma^{12},\sigma^2,1],[\sigma^{14},\sigma^{11},1],[\sigma,\sigma^6,1]\}$ be a subset of $\mathcal{F}_{54}$ which is forming by partition $\mathcal{F}_{54}$ into two sets such that $\mathcal{F}_{54}^*$ dose not contains the inflexion points of $\mathcal{F}_{54}$, so we note that $\mathcal{F}_{54}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\mathcal{F}_{54}^*$, by some calculation .we get that the matrix which is stabilizing of $\mathcal{F}_{54}^*$ is

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^7&0&\sigma^7\\0&\sigma^7&0\\0&0&\sigma^7\end{pmatrix}:2$$

Therefore, the stabilizer group of $\mathcal{F}_{54}^*$ which is denoted by $G_{\mathcal{F}_{54}^*}$ which contains

- One matrices of order 2.
- The identity matrix.

Thus, $G_{\mathcal{F}_{54}^*}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\mathcal{F}_{54}^*} \cong \boldsymbol{Z_2}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{55} = z^2y + zy^2 + x^3$. The points of $PG(2,16)$ on $\mathcal{F}_{55}$ are $[0,1,0],[0,0,1],[\sigma^{10},\sigma^{10},1],[\sigma^{10},\sigma^5,1],[1,\sigma^{10},1],[0,1,1]$, $[\sigma^5,\sigma^{10},1],[1,\sigma^5,1],[\sigma^5,\sigma^5,1]$. After calculations with computer help, we are note that the number of matrices which are stabilizing of $\mathcal{F}_{55}$and their orders is 216, and we can not write them, because they are too much.

Therefore, the stabilizer group of $\mathcal{F}_{55}$ which is denoted by $G_{\mathcal{F}_{55}}$ contains

- 9 matrices of order 2.
- 80 matrix of order 3.
- 54 matrix of order 4.
- 72 matrix of order 6.
- The identity matrix.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{56} = z^2y + zy^2 + x^3 + xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{56}$ are $[0,1,0],[0,0,1],[\sigma^3,\sigma^{13},1],[\sigma^5,\sigma^{13},1],[\sigma^{12},\sigma^3,1],[\sigma^7,\sigma,1]$, $[\sigma^{10},\sigma^{14},1],[\sigma^{11},\sigma^8,1],[\sigma^5,\sigma^7,1],[\sigma^{10},\sigma^{11},1],[\sigma^3,\sigma^{12},1],[\sigma^{13},\sigma^{14},1],[1,1,0],[0,1,1]$, $[\sigma^6,\sigma^{11},1],[\sigma^7,\sigma^{11},1],[\sigma^{11},\sigma^{13},1],[\sigma^9,\sigma^{14},1],[\sigma^9,\sigma^6,1],[\sigma^{12},\sigma^7,1],[\sigma^{14},\sigma^7,1],[\sigma^{14},\sigma^2,1]$, $[\sigma^6,\sigma^9,1],[\sigma^{13},\sigma^4,1],[1,1,1]$. To find the stabilizer group of $\mathcal{F}_{56}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{56}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad\begin{pmatrix}\sigma^9&0&\sigma^4\\\sigma^{14}&\sigma^9&\sigma^{11}\\0&0&\sigma^9\end{pmatrix}:4,\quad\begin{pmatrix}\sigma^4&0&1\\\sigma&\sigma^9&\sigma^{11}\\0&0&\sigma^9\end{pmatrix}:6$$

$$\begin{pmatrix}1&0&\sigma^3\\\sigma^{11}&\sigma^5&\sigma^4\\0&0&\sigma^5\end{pmatrix}:3,\quad\begin{pmatrix}\sigma^4&0&\sigma^2\\\sigma^{10}&\sigma^{14}&1\\0&0&\sigma^{14}\end{pmatrix}:6,\quad\begin{pmatrix}\sigma^{12}&0&\sigma^6\\\sigma^{10}&\sigma^7&\sigma^{14}\\0&0&\sigma^7\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^9 & 0 & \sigma^8\\ \sigma^{12} & \sigma^{14} & \sigma^7\\ 0 & 0 & \sigma^{14}\end{pmatrix}: 6, \quad \begin{pmatrix}\sigma & 0 & \sigma^8\\ \sigma^{10} & \sigma^{11} & 1\\ 0 & 0 & \sigma^{11}\end{pmatrix}: 6, \quad \begin{pmatrix}\sigma^6 & 0 & \sigma^9\\ \sigma^2 & \sigma^{11} & \sigma^{14}\\ 0 & 0 & \sigma^{11}\end{pmatrix}: 6$$

$$\begin{pmatrix}\sigma & 0 & \sigma^8\\ \sigma^{10} & \sigma^{11} & \sigma^{12}\\ 0 & 0 & \sigma^{11}\end{pmatrix}: 3, \quad \begin{pmatrix}\sigma^{10} & 0 & \sigma^{10}\\ \sigma^{10} & \sigma^{10} & 0\\ 0 & 0 & \sigma^{10}\end{pmatrix}: 4, \quad \begin{pmatrix}\sigma^{12} & 0 & 0\\ 0 & \sigma^{12} & \sigma^{12}\\ 0 & 0 & \sigma^{12}\end{pmatrix}: 2$$

$$\begin{pmatrix}\sigma^{12} & 0 & \sigma^2\\ \sigma^7 & \sigma^{12} & \sigma\\ 0 & 0 & \sigma^{12}\end{pmatrix}: 4, \quad \begin{pmatrix}\sigma^{13} & 0 & \sigma^{11}\\ \sigma^4 & \sigma^8 & \sigma^{12}\\ 0 & 0 & \sigma^8\end{pmatrix}: 3, \quad \begin{pmatrix}\sigma^4 & 0 & \sigma^3\\ \sigma^7 & \sigma^9 & \sigma^{11}\\ 0 & 0 & \sigma^9\end{pmatrix}: 3$$

$$\begin{pmatrix}1 & 0 & \sigma^5\\ \sigma^{10} & 1 & \sigma\\ 0 & 0 & 1\end{pmatrix}: 4, \quad \begin{pmatrix}\sigma^4 & 0 & \sigma^{13}\\ \sigma^2 & \sigma^{14} & \sigma^8\\ 0 & 0 & \sigma^{14}\end{pmatrix}: 6, \quad \begin{pmatrix}\sigma^8 & 0 & \sigma^3\\ \sigma^{13} & \sigma^8 & \sigma\\ 0 & 0 & \sigma^8\end{pmatrix}: 4$$

$$\begin{pmatrix}\sigma^{12} & 0 & \sigma^8\\ \sigma^9 & \sigma^2 & \sigma^{10}\\ 0 & 0 & \sigma^2\end{pmatrix}: 3, \quad \begin{pmatrix}\sigma^6 & 0 & \sigma^{12}\\ \sigma^{13} & \sigma & \sigma^{14}\\ 0 & 0 & \sigma\end{pmatrix}: 3, \quad \begin{pmatrix}\sigma^7 & 0 & \sigma^{13}\\ \sigma^{14} & \sigma^2 & \sigma^8\\ 0 & 0 & \sigma^2\end{pmatrix}: 6$$

$$\begin{pmatrix}\sigma^7 & 0 & \sigma^4\\ \sigma^6 & \sigma^{12} & \sigma^8\\ 0 & 0 & \sigma^{12}\end{pmatrix}: 3, \quad \begin{pmatrix}\sigma^7 & 0 & \sigma^7\\ \sigma^7 & \sigma^7 & \sigma^7\\ 0 & 0 & \sigma^7\end{pmatrix}: 4, \quad \begin{pmatrix}\sigma^{11} & 0 & \sigma^8\\ \sigma^{10} & \sigma & \sigma^{13}\\ 0 & 0 & \sigma\end{pmatrix}: 6$$

Therefore, the stabilizer groups of $\mathcal{F}_{56}$ which is denoted by $G_{\mathcal{F}_{56}}$ contains

- One matrix of order 2.
- 8 matrices of order 3.
- 6 matrices of order 4.
- 8 matrices of order 6.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{56}}$ is isomorphic to $\boldsymbol{SL_2(F_3)}$, that is $G_{\mathcal{F}_{56}} \cong \boldsymbol{SL_2(F_3)}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{57} = z^2y + zy^2 + x^3 + \sigma xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{57}$ are [0,1,0],[0,0,1],$[\sigma^9, \sigma^{13}, 1]$,$[\sigma^9, \sigma^9, 1]$,$[\sigma^8, 1, 1]$, $[\sigma^{14}, \sigma^{12}, 1]$, $[1, \sigma^9, 1]$,$[\sigma^8, \sigma^2, 1]$,$[\sigma^7, \sigma^{10}, 1]$,[0,1,1],$[\sigma^7, \sigma^9, 1]$,$[\sigma^3, \sigma, 1]$,$[1, \sigma^2, 1]$,$[\sigma^2, \sigma^2, 1]$,$[\sigma^8 1, 0]$,$[\sigma^2, \sigma^5, 1]$, $[\sigma^3, \sigma^7, 1]$. To find the stabilizer group of $\mathcal{F}_{57}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{57}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}: 1, \begin{pmatrix}\sigma^9 & 0 & \sigma^9\\ \sigma^9 & \sigma^9 & 0\\ 0 & 0 & \sigma^9\end{pmatrix}: 4, \begin{pmatrix}\sigma^4 & 0 & 0\\ 0 & \sigma^4 & \sigma^4\\ 0 & 0 & \sigma^4\end{pmatrix}: 2, \begin{pmatrix}\sigma^6 & 0 & \sigma^6\\ \sigma^6 & \sigma^6 & \sigma^4\\ 0 & 0 & \sigma^6\end{pmatrix}: 4$$

Therefore, the stabilizer groups of $\mathcal{F}_{57}$ which is denoted by $G_{\mathcal{F}_{57}}$ contains

- One matrix of order 2.
- 2 matrix of order 4.
- The identity matrix.

Thus, $G_{\mathcal{F}_{57}}$ is isomorphic to $\boldsymbol{Z_4}$, that is $G_{\mathcal{F}_{57}} \cong \boldsymbol{Z_4}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{71} = z^2y + zy^2 + \sigma x^3$. The points of $PG(2,16)$ on $\mathcal{F}_{71}$ are [0,1,0],[0,0,1],$[\sigma^6, \sigma^6, 1]$,$[\sigma^5, \sigma^7, 1]$,$[1, \sigma^9, 1]$,$[\sigma^8, \sigma^2, 1]$, $[\sigma^3, \sigma^8, 1]$,$[\sigma^{10}, \sigma^7, 1]$,[0,1,1],$[\sigma^{11}, \sigma^{13}, 1]$,$[\sigma^8, \sigma^8, 1]$,$[\sigma^{10}, \sigma^9, 1]$,$[\sigma^5, \sigma^9, 1]$,$[\sigma^{13}, \sigma^2, 1]$,$[\sigma^6, \sigma^{13}, 1]$, $[\sigma^{11}, \sigma^6, 1]$,$[\sigma, \sigma^6, 1]$,$[\sigma, \sigma^{13}, 1]$,$[\sigma^{13}, \sigma^8, 1]$,$[\sigma^3, \sigma^2, 1]$,$[1, \sigma^7, 1]$. To find the stabilizer group of $\mathcal{F}_{71}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{71}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^5&0&0\\0&\sigma^{10}&0\\0&0&\sigma^{10}\end{pmatrix}:3,\begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^5&0\\0&0&\sigma^5\end{pmatrix}:3,\begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^5&0\\0&\sigma^5&\sigma^5\end{pmatrix}:6$$

$$\begin{pmatrix}\sigma^5&0&0\\0&\sigma^{10}&0\\0&\sigma^{10}&\sigma^{10}\end{pmatrix}:6,\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix}:2,\begin{pmatrix}\sigma^{13}&0&0\\0&0&\sigma^{13}\\0&\sigma^{13}&0\end{pmatrix}:2,\begin{pmatrix}\sigma^8&0&0\\0&0&\sigma^3\\0&\sigma^3&0\end{pmatrix}:6$$

$$\begin{pmatrix}\sigma^3&0&0\\0&0&\sigma^8\\0&\sigma^8&0\end{pmatrix}:6,\begin{pmatrix}\sigma^{11}&0&0\\0&0&\sigma\\0&\sigma&\sigma\end{pmatrix}:3,\begin{pmatrix}\sigma^6&0&0\\0&0&\sigma^6\\0&\sigma^6&\sigma^6\end{pmatrix}:3,\begin{pmatrix}\sigma&0&0\\0&0&\sigma^{11}\\0&\sigma^{11}&\sigma^{11}\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^8&0&0\\0&\sigma^3&\sigma^3\\0&\sigma^3&0\end{pmatrix}:3,\begin{pmatrix}\sigma^{13}&0&0\\0&\sigma^{13}&\sigma^{13}\\0&\sigma^{13}&0\end{pmatrix}:3,\begin{pmatrix}\sigma^3&0&0\\0&\sigma^8&\sigma^8\\0&\sigma^8&0\end{pmatrix}:3,\begin{pmatrix}\sigma^6&0&0\\0&\sigma^6&\sigma^6\\0&0&\sigma^6\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma&0&0\\0&\sigma^{11}&\sigma^{11}\\0&0&\sigma^{11}\end{pmatrix}:6,\begin{pmatrix}\sigma^{11}&0&0\\0&\sigma&\sigma\\0&0&\sigma\end{pmatrix}:6$$

Therefore, the stabilizer groups of $\mathcal{F}_{71}$ which is denoted by $G_{\mathcal{F}_{71}}$ contains

- 3 matrices of order 2.
- 8 matrices of order 3.
- 6 matrices of order 6.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{71}}$ is isomorphic to $\boldsymbol{S_3}\times\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{71}}\cong\boldsymbol{S_3}\times\boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{73}=z^2y+zy^2+\sigma x^3+\sigma xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{73}$ are $[0,1,0],[0,0,1],[1,\sigma^{12},1],[\sigma^{11},\sigma^9,1],[\sigma^{11},\sigma^{14},1]$, $[\sigma^8,\sigma^7,1],[1,1,0],[0,1,1],[\sigma^4,\sigma,1],[\sigma^4,\sigma^2,1],[\sigma^8,\sigma^{11},1],[\sigma^{14},\sigma^{13},1],[1,1,1]$. To find the stabilizer group of $\mathcal{F}_{73}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{73}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\sigma^{10}&0&\sigma^4\\\sigma^7&\sigma^5&\sigma^{11}\\0&0&\sigma^5\end{pmatrix}:3,\begin{pmatrix}\sigma^7&0&\sigma^6\\\sigma^9&\sigma^{12}&\sigma^{13}\\0&0&\sigma^{12}\end{pmatrix}:3,\begin{pmatrix}\sigma^3&0&0\\0&\sigma^3&\sigma^3\\0&0&\sigma^3\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma^7&0&\sigma^4\\\sigma^4&\sigma^2&0\\0&0&\sigma^2\end{pmatrix}:6,\begin{pmatrix}\sigma^{12}&0&\sigma^{11}\\\sigma^{14}&\sigma^2&\sigma^6\\0&0&\sigma^2\end{pmatrix}:6$$

Therefore, the stabilizer groups of $\mathcal{F}_{73}$ which is denoted by $G_{\mathcal{F}_{73}}$ contains

- One matrix of order 2.
- 2 matrix of order 3.
- 2 matrix of order 6.
- The identity matrix.

Thus, $G_{\mathcal{F}_{73}}$ is isomorphic to $\boldsymbol{Z_6}$, that is $G_{\mathcal{F}_{73}}\cong\boldsymbol{Z_6}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{87}=z^2y+zy^2+\sigma^2x^3$. The points of $PG(2,16)$ on $\mathcal{F}_{87}$ are $[0,1,0],[0,0,1],[1,\sigma^{14},1],[\sigma^{12},\sigma^{12},1],[\sigma^{10},\sigma^{14},1]$, $[1,\sigma^3,1]$, $[\sigma^{12},\sigma^{11},1],[\sigma^7,\sigma^{12},1],[\sigma^2,\sigma^{12},1],[\sigma^5,\sigma^3,1],[\sigma^{11},\sigma^4,1],[0,1,1],[\sigma^{11},1,1],[\sigma^7,\sigma^{11},1],[\sigma^6,\sigma,1]$, $[\sigma^6,\sigma^4,1],[\sigma^{10},\sigma^3,1],[\sigma^5,\sigma^{14},1],[\sigma,\sigma,1],[\sigma,\sigma^4,1],[\sigma^2,\sigma^{11},1]$. To find the stabilizer group of $\mathcal{F}_{87}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{87}$ and their orders are shown as follows :

$$\begin{pmatrix}\sigma^{10} & 0 & 0\\ 0 & \sigma^5 & 0\\ 0 & 0 & \sigma^5\end{pmatrix}:3, \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^5 & 0 & 0\\ 0 & \sigma^{10} & 0\\ 0 & 0 & \sigma^{10}\end{pmatrix}:3, \begin{pmatrix}\sigma^{10} & 0 & 0\\ 0 & \sigma^5 & 0\\ 0 & \sigma^5 & \sigma^5\end{pmatrix}:6$$

$$\begin{pmatrix}\sigma^5 & 0 & 0\\ 0 & \sigma^{10} & 0\\ 0 & \sigma^{10} & \sigma^{10}\end{pmatrix}:6, \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 1 & 1\end{pmatrix}:2, \begin{pmatrix}\sigma^{13} & 0 & 0\\ 0 & 0 & \sigma^{13}\\ 0 & \sigma^{13} & 0\end{pmatrix}:2, \begin{pmatrix}\sigma^3 & 0 & 0\\ 0 & 0 & \sigma^8\\ 0 & \sigma^8 & 0\end{pmatrix}:6$$

$$\begin{pmatrix}\sigma^8 & 0 & 0\\ 0 & 0 & \sigma^3\\ 0 & \sigma^3 & 0\end{pmatrix}:6, \begin{pmatrix}\sigma^8 & 0 & 0\\ 0 & 0 & \sigma^3\\ 0 & \sigma^3 & \sigma^3\end{pmatrix}:3, \begin{pmatrix}\sigma^{13} & 0 & 0\\ 0 & 0 & \sigma^{13}\\ 0 & \sigma^{13} & \sigma^{13}\end{pmatrix}:3, \begin{pmatrix}\sigma^3 & 0 & 0\\ 0 & 0 & \sigma^8\\ 0 & \sigma^8 & \sigma^8\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{13} & 0 & 0\\ 0 & \sigma^{13} & \sigma^{13}\\ 0 & \sigma^{13} & 0\end{pmatrix}:3, \begin{pmatrix}\sigma^3 & 0 & 0\\ 0 & \sigma^8 & \sigma^8\\ 0 & \sigma^8 & 0\end{pmatrix}:3, \begin{pmatrix}\sigma^8 & 0 & 0\\ 0 & \sigma^3 & \sigma^3\\ 0 & \sigma^3 & 0\end{pmatrix}:3, \begin{pmatrix}\sigma^8 & 0 & 0\\ 0 & \sigma^3 & \sigma^3\\ 0 & 0 & \sigma^3\end{pmatrix}:6$$

$$\begin{pmatrix}\sigma^{13} & 0 & 0\\ 0 & \sigma^{13} & \sigma^{13}\\ 0 & 0 & \sigma^{13}\end{pmatrix}:2, \begin{pmatrix}\sigma^3 & 0 & 0\\ 0 & \sigma^8 & \sigma^8\\ 0 & 0 & \sigma^8\end{pmatrix}:6$$

Therefore, the stabilizer groups of $\mathcal{F}_{87}$ which is denoted by $G_{\mathcal{F}_{87}}$ contains

- 3 matrices of order 2.
- 8 matrices of order 3.
- 6 matrices of order 6.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{87}}$ is isomorphic to $\boldsymbol{S_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{87}} \cong \boldsymbol{S_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{89} = z^2y + zy^2 + \sigma^2x^3 + \sigma xy^2$. The points of $PG(2,16)$ on $\mathcal{F}_{89}$ are $[0,1,0]$,$[0,0,1]$,$[\sigma^7,1,0]$,$[\sigma^9,\sigma^8,1]$,$[\sigma^{14},\sigma^{14},1]$, $[\sigma^9,\sigma,1]$,$[\sigma^5,\sigma^7,1]$, $[0,1,1]$,$[\sigma^7,1,1]$,$[\sigma^7,\sigma^6,1]$,$[\sigma^{11},\sigma^{11},1]$,$[\sigma^{11},\sigma^{13},1]$,$[\sigma^5,\sigma^{12},1]$. To find the stabilizer group of $\mathcal{F}_{89}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{89}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1, \begin{pmatrix}\sigma^4 & 0 & \sigma^9\\ \sigma^2 & \sigma^9 & \sigma^8\\ 0 & 0 & \sigma^9\end{pmatrix}:6, \begin{pmatrix}\sigma^2 & 0 & \sigma^2\\ \sigma^{10} & \sigma^{12} & \sigma^5\\ 0 & 0 & \sigma^{12}\end{pmatrix}:3, \begin{pmatrix}\sigma^8 & 0 & 0\\ 0 & \sigma^8 & \sigma^8\\ 0 & 0 & \sigma^8\end{pmatrix}:2$$

$$\begin{pmatrix}\sigma^8 & 0 & \sigma^8\\ \sigma & \sigma^3 & \sigma^5\\ 0 & 0 & \sigma^3\end{pmatrix}:6, \begin{pmatrix}\sigma^8 & 0 & \sigma^{13}\\ \sigma^6 & \sigma^{13} & \sigma\\ 0 & 0 & \sigma^{13}\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{89}$ which is denoted by $G_{\mathcal{F}_{89}}$ contains

- One matrix of order 2.
- 2 matrix of order 3.
- 2 matrix of order 6.
- The identity matrix.

Thus, $G_{\mathcal{F}_{89}}$ is isomorphic to $\boldsymbol{Z_6}$, that is $G_{\mathcal{F}_{89}} \cong \boldsymbol{Z_6}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{103} = x^3 + \sigma y^3 + \sigma^2 z^3$. The points of $PG(2,16)$ on $\mathcal{F}_{103}$ are $[\sigma^9,\sigma^{12},1]$,$[\sigma^{14},\sigma^{12},1]$,$[\sigma^4,\sigma^7,1]$,$[\sigma^4,\sigma^{12},1]$,$[\sigma^4,\sigma^2,1]$, $[\sigma^{14},\sigma^7,1]$,$[\sigma^{14},\sigma^2,1]$,$[\sigma^9,\sigma^7,1]$,$[\sigma^9,\sigma^2,1]$. After calculations with computer help, we are note that the number of matrices which are stabilizing of $\mathcal{F}_{103}$ and their order is 54, and we can not write them , because they are too much.

Therefore, the stabilizer groups of $\mathcal{F}_{103}$ which is denoted by $G_{\mathcal{F}_{103}}$ contains

- 9 matrices of order 2.
- 26 matrix of order 3.
- 18 matrix of order 6.

- The identity matrix.

Form [6], $G_{\mathcal{F}_{103}}$ is isomorphic to $\boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{103}} \cong \boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{104} = x^3 + \sigma y^3 + \sigma^2 z^3 + xyz$.The points of $PG(2,16)$ on $\mathcal{F}_{104}$ are $[\sigma^6,\sigma^6,1]$,$[\sigma,\sigma^{11},1]$,$[\sigma^{11},1,1]$,$[\sigma^7,\sigma^{12},1]$,$[\sigma,\sigma^{14},1]$, $[1,\sigma^{10},1]$,$[\sigma^{11},\sigma^4,1]$,$[\sigma^{11},\sigma,1]$,$[\sigma^6,\sigma^5,1]$,$[\sigma^{14},1,1]$,$[\sigma,\sigma^{10},1]$,$[\sigma^4,\sigma^{10},1]$,$[\sigma^9,\sigma^5,1]$,$[\sigma^2,\sigma^2,1]$, $[\sigma^{12},\sigma^7,1]$,$[\sigma^6,\sigma^9,1]$,$[\sigma^5,\sigma^5,1]$,$[\sigma^{10},1,1]$. To find the stabilizer group of $\mathcal{F}_{104}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{104}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^5&0\\0&0&1\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^9\\\sigma^5&0&0\\0&1&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&1&0\\0&0&\sigma^5\\\sigma^{11}&0&0\end{pmatrix}:3,\quad \begin{pmatrix}\sigma^5&0&0\\0&\sigma^{10}&0\\0&0&1\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^9\\1&0&0\\0&\sigma^5&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&0&\sigma^{10}\\\sigma^{10}&0&0\\0&\sigma^{10}&0\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^{10}&0\\0&0&\sigma^5\\\sigma&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^5&0\\0&0&\sigma^5\\\sigma^6&0&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{104}$ which is denoted by $G_{\mathcal{F}_{104}}$ contains

- 8 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{104}}$ is isomorphic to $\boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{104}} \cong \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{106} = x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma^2 xyz$. The points of $PG(2,16)$ on $\mathcal{F}_{106}$ are $[\sigma^{12},\sigma^{13},1]$,$[\sigma^9,\sigma^9,1]$,$[\sigma,\sigma^7,1]$,$[\sigma^{12},\sigma^{10},1]$, $[\sigma^{14},\sigma^4,1]$,$[\sigma^3,\sigma^{14},1]$,$[\sigma^4,\sigma^{14},1]$,$[\sigma^6,\sigma^2,1]$,$[\sigma^7,1,1]$,$[\sigma^2,\sigma^8,1]$,$[\sigma^7,\sigma^3,1]$,$[\sigma^{12},\sigma^9,1]$,$[\sigma^8,\sigma^9,1]$, $[\sigma^7,\sigma^{14},1]$,$[\sigma^{13},\sigma^4,1]$,$[\sigma^2,\sigma^5,1]$,$[\sigma^{11},\sigma^{12},1]$,$[\sigma^2,\sigma^4,1]$. To find the stabilizer group of $\mathcal{F}_{106}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{106}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}0&\sigma^6&0\\0&0&\sigma^6\\\sigma^7&0&0\end{pmatrix}:3,\quad \begin{pmatrix}\sigma^5&0&0\\0&\sigma^{10}&0\\0&0&1\end{pmatrix}:3$$

$$\begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^5&0\\0&0&1\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^9\\\sigma^5&0&0\\0&1&0\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma&0\\0&0&\sigma^6\\\sigma^{12}&0&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&0&\sigma^9\\1&0&0\\0&\sigma^5&0\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^{11}&0\\0&0&\sigma^6\\\sigma^2&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^9\\\sigma^{10}&0&0\\0&\sigma^{10}&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{106}$ which is denoted by $G_{\mathcal{F}_{106}}$ contains

- 8 matrices of order 3;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{106}}$ is isomorphic to $\boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{106}} \cong \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{107} = x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma^3 xyz$. The points of $PG(2,16)$ on $\mathcal{F}_{107}$ are $[\sigma^{13},\sigma^7,1]$,$[\sigma^{13},\sigma^{13},1]$,$[\sigma^{11},\sigma^8,1]$,$[\sigma^8,\sigma^3,1]$, $[\sigma^8,\sigma^{12},1]$,$[\sigma^{14},\sigma,1]$,$[\sigma^4,\sigma^{11},1]$,$[\sigma^3,\sigma^8,1]$,$[1,\sigma^{13},1]$,$[\sigma,\sigma^3,1]$,$[\sigma^{13},\sigma^5,1]$,$[\sigma^6,\sigma^{13},1]$,$[\sigma^{10},\sigma^3,1]$,$[\sigma^9,\sigma^{16},1]$,$[\sigma^3,1,1]$,$[\sigma^8,\sigma^{10},1]$,$[\sigma^5,\sigma^8,1]$,$[\sigma^3,\sigma^2,1]$. To find the stabilizer group of $\mathcal{F}_{107}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{107}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^5&0\\0&0&1\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^8&0\\0&0&\sigma^{13}\\\sigma^4&0&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&\sigma^3&0\\0&0&\sigma^{13}\\\sigma^9&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^{14}\\\sigma^5&0&0\\0&\sigma^{10}&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^{14}\\1&0&0\\0&1&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&\sigma^{13}&0\\0&0&\sigma^{13}\\\sigma^{14}&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^{14}\\\sigma^{10}&0&0\\0&\sigma^5&0\end{pmatrix}:3,\quad \begin{pmatrix}\sigma^5&0&0\\0&\sigma^{10}&0\\0&0&1\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{107}$ which is denoted by $G_{\mathcal{F}_{107}}$ contains

- 8 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{107}}$ is isomorphic to $\boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{107}} \cong \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{108} = x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma^3 xyz$. The points of $PG(2,16)$ on $\mathcal{F}_{108}$ are $[1,\sigma^{14},1]$,$[\sigma^9,\sigma^8,1]$,$[\sigma^{14},\sigma^3,1]$,$[\sigma^{10},\sigma^{12},1]$, $[\sigma^{10},\sigma^6,1]$,$[\sigma^4,\sigma^{13},1]$,$[\sigma^5,\sigma^{11},1]$,$[1,\sigma,1]$,$[\sigma^7,\sigma^{11},1]$,$[\sigma^{13},\sigma^{11},1]$,$[\sigma^5,\sigma^9,1]$,$[\sigma^3,\sigma^6,1]$,$[\sigma^8,\sigma,1]$, $[\sigma^{12},\sigma,1]$,$[\sigma^{10},\sigma^4,1]$,$[\sigma^{12},\sigma^6,1]$,$[\sigma^5,\sigma^2,1]$,$[1,\sigma^7,1]$. To find the stabilizer group of $\mathcal{F}_{108}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{108}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}0&0&\sigma^9\\\sigma^5&0&0\\0&1&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^9\\1&0&0\\0&\sigma^5&0\end{pmatrix}:3$$

$$\begin{pmatrix}0&0&\sigma^9\\\sigma^{10}&0&0\\0&\sigma^{10}&0\end{pmatrix}:3,\quad \begin{pmatrix}0&1&0\\0&0&\sigma^5\\\sigma^{11}&0&0\end{pmatrix}:3,\quad \begin{pmatrix}\sigma^5&0&0\\0&\sigma^{10}&0\\0&0&1\end{pmatrix}:3$$

$$\begin{pmatrix}0&\sigma^5&0\\0&0&\sigma^5\\\sigma^6&0&0\end{pmatrix}:3,\quad \begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^5&0\\0&0&1\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^{10}&0\\0&0&\sigma^5\\\sigma&0&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{108}$ which is denoted by $G_{\mathcal{F}_{108}}$ contains

- 8 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{108}}$ is isomorphic to $\boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{108}} \cong \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{231} = xy^2 + x^2z + \sigma yz^2$.The points of $PG(2,16)$ on $\mathcal{F}_{231}$ are $[1,0,0]$,$[0,1,0]$,$[0,0,1]$,$[\sigma^9,\sigma^9,1]$,$[\sigma,\sigma^7,1]$,$[\sigma^{12},\sigma^{10},1]$, $[\sigma^{14},\sigma^{10},1]$,$[\sigma^{14},\sigma^4,1]$,$[\sigma,\sigma^9,1]$,$[\sigma^2,\sigma^{12},1]$,$[\sigma^4,\sigma^{14},1]$,$[\sigma^6,\sigma^2,1]$,$[\sigma^{11},\sigma^{14},1]$,$[\sigma^7,1,1]$, $[\sigma^{12},\sigma^2,1]$,$[\sigma^6,\sigma^4,1]$,$[\sigma^9,1,1]$,$[\sigma^7,\sigma^7,1]$,$[\sigma^2,\sigma^5,1]$, $[\sigma^4,\sigma^5,1]$,$[\sigma^{11},\sigma^{12},1]$. To find the stabilizer group of $\mathcal{F}_{231}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{231}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}\sigma^5&0&0\\0&\sigma^{10}&0\\0&0&1\end{pmatrix}:3,\quad \begin{pmatrix}\sigma^{10}&0&0\\0&\sigma^5&0\\0&0&1\end{pmatrix}:3$$

$$\begin{pmatrix}0&\sigma^{14}&0\\0&0&\sigma^{14}\\1&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^{14}&0\\0&0&\sigma^4\\\sigma^4&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^{14}&0\\0&0&\sigma^9\\\sigma^5&0&0\end{pmatrix}:3$$

$$\begin{pmatrix} 0 & 0 & \sigma^{13} \\ \sigma^4 & 0 & 0 \\ 0 & \sigma^9 & 0 \end{pmatrix}:3, \quad \begin{pmatrix} 0 & 0 & \sigma^{3} \\ \sigma^4 & 0 & 0 \\ 0 & \sigma^4 & 0 \end{pmatrix}:3, \quad \begin{pmatrix} 0 & 0 & \sigma^{8} \\ \sigma^4 & 0 & 0 \\ 0 & \sigma^{14} & 0 \end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{231}$ which is denoted by $G_{\mathcal{F}_{231}}$ contains

- 8 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{231}}$ is isomorphic to $\boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{231}} \cong \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{232} = xy^2 + x^2z + \sigma yz^2 + (x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{232}$ are $[\sigma^9,\sigma^{10},1]$,$[\sigma^{10},\sigma^{11},1]$, $[\sigma^4,\sigma^4,1]$,$[\sigma^{13},0,1]$,$[\sigma^6,\sigma^6,1]$,$[\sigma^2,\sigma^7,1]$,$[1,\sigma^{10},1]$,$[\sigma,\sigma^{12},1]$,$[\sigma^{11},\sigma,1]$,$[\sigma^{12},1,1]$,$[\sigma^6,\sigma^5,1]$, $[\sigma^2,\sigma^8,1]$,$[\sigma^3,1,0]$,$[0,\sigma^3,1]$,$[\sigma^5,\sigma^{14},1]$,$[\sigma^8,\sigma^9,1]$,$[\sigma^7,\sigma^{14},1]$,$[\sigma^2,\sigma^6,1]$. To find the stabilizer group of $\mathcal{F}_{232}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{232}$ and their orders are shown as follows :

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1, \quad \begin{pmatrix} 0 & \sigma^4 & 0 \\ 0 & 0 & \sigma^4 \\ \sigma^5 & 0 & 0 \end{pmatrix}:3, \quad \begin{pmatrix} 0 & 0 & \sigma^{5} \\ \sigma^6 & 0 & 0 \\ 0 & \sigma^6 & 0 \end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{232}$ which is denoted by $G_{\mathcal{F}_{232}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{232}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{232}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{233} = xy^2 + x^2z + \sigma yz^2 + \sigma(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{233}$ are $[\sigma^5,1,0]$,$[0,\sigma^5,1]$,$[1,\sigma^{12},1]$, $[\sigma^{14},\sigma^3,1]$,$[\sigma^4,0,1]$,$[\sigma^{10},\sigma^8,1]$,$[\sigma^4,\sigma^3,1]$,$[\sigma^3,\sigma^{12},1]$$[\sigma^8,\sigma^2,1]$,$[\sigma^{14},\sigma^6,1]$,$[\sigma^{13},\sigma^{11},1]$,$[\sigma^{13},\sigma,1]$,$[\sigma^4,\sigma^6,1]$,$[\sigma^2,0,1]$,$[\sigma^{14},1,0]$,$[0,\sigma^{14},1]$,$[\sigma^{12},1,0]$,$[0,\sigma^{12},1]$,$[\sigma^{14},\sigma^7,1]$, $[\sigma^{11},0,1]$,$[\sigma^5,\sigma^2,1]$, $[\sigma^{10},\sigma^{13},1]$,$[\sigma^9,\sigma^7,1]$,$[\sigma^9,\sigma^2,1]$. To find the stabilizer group of $\mathcal{F}_{233}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{233}$ and their orders are shown as follows :

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1, \quad \begin{pmatrix} 0 & \sigma^{14} & 0 \\ 0 & 0 & \sigma^{14} \\ 1 & 0 & 0 \end{pmatrix}:3, \quad \begin{pmatrix} 0 & 0 & \sigma^{12} \\ \sigma^{13} & 0 & 0 \\ 0 & \sigma^{13} & 0 \end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{233}$ which is denoted by $G_{\mathcal{F}_{233}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{233}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{233}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{234} = xy^2 + x^2z + \sigma yz^2 + \sigma^2(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{234}$ are $[\sigma^4,\sigma^9,1]$,$[\sigma^{12},\sigma^3,1]$, $[\sigma^6,\sigma^8,1]$,$[\sigma,\sigma^{14},1]$,$[\sigma^7,\sigma^{10},1]$,$[\sigma^8,\sigma^{13},1]$,$[\sigma^7,\sigma^4,1]$,$[\sigma^{11},1,0]$,$[0,\sigma^{11},1]$,$[\sigma^{13},\sigma^9,1]$, $[\sigma^{11},\sigma^{10},1]$,$[\sigma^{14},1,1]$,$[1,\sigma^5,1]$,$[\sigma^6,\sigma,1]$,$[\sigma^6,\sigma^{12},1]$,$[\sigma^5,0,1]$,$[\sigma^2,\sigma^2,1]$,$[\sigma^3,\sigma^{10},1]$.To find the stabilizer group of $\mathcal{F}_{234}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{234}$ and their orders are shown as follows :

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1, \quad \begin{pmatrix} 0 & \sigma^1 & 0 \\ 0 & 0 & \sigma^1 \\ \sigma & 0 & 0 \end{pmatrix}:3, \quad \begin{pmatrix} 0 & 0 & \sigma^{7} \\ \sigma^8 & 0 & 0 \\ 0 & \sigma^8 & 0 \end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{234}$ which is denoted by $G_{\mathcal{F}_{234}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{234}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{234}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{235} = xy^2 + x^2z + \sigma yz^2 + \sigma^3(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{235}$ are $[\sigma^{13},\sigma^7,1]$,$[\sigma^6,\sigma^{10},1]$, $[\sigma^8,1,1]$,$[\sigma^3,\sigma^8,1]$,$[\sigma^6,\sigma^{11},1]$,$[\sigma^5,\sigma^{10},1]$,$[\sigma^8,\sigma^8,1]$,$[\sigma,\sigma^8,1]$,$[\sigma^6,\sigma^{13},1]$,$[\sigma^{10},\sigma^3,1]$,$[\sigma^9,\sigma^6,1]$, $[\sigma^8,\sigma^{10},1]$. To find the stabilizer group of $\mathcal{F}_{235}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{235}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1, \quad \begin{pmatrix}0 & 0 & \sigma^2\\ \sigma^3 & 0 & 0\\ 0 & \sigma^3 & 0\end{pmatrix}:3, \quad \begin{pmatrix}0 & \sigma^8 & 0\\ 0 & 0 & \sigma^8\\ \sigma^9 & 0 & 0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{235}$ which is denoted by $G_{\mathcal{F}_{235}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{235}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{235}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{236} = xy^2 + x^2z + \sigma yz^2 + \sigma^4(x^3 + \sigma y^3 + \sigma^2 z^3 + \sigma xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{236}$ are $[\sigma^{10},\sigma^{14},1]$,$[\sigma^{10},\sigma^6,1]$, $[\sigma^{13},\sigma^{14},1]$,$[\sigma^{10},\sigma^4,1]$,$[\sigma^3,\sigma,1]$,$[1,\sigma^2,1]$,$[\sigma^{12},\sigma^6,1]$,$[\sigma^2,\sigma^{14},1]$,$[\sigma^{14},\sigma^{13},1]$,$[\sigma^5,\sigma^6,1]$, $[\sigma^2,\sigma^3,1]$,$[\sigma^2,\sigma^{11},1]$.To find the stabilizer group of $\mathcal{F}_{236}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{236}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1, \quad \begin{pmatrix}0 & 0 & \sigma^9\\ \sigma^{10} & 0 & 0\\ 0 & \sigma^{10} & 0\end{pmatrix}:3, \quad \begin{pmatrix}0 & \sigma^8 & 0\\ 0 & 0 & \sigma^8\\ \sigma^9 & 0 & 0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{236}$ which is denoted by $G_{\mathcal{F}_{236}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{236}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{236}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{247} = xy^2 + x^2z + \sigma^2 yz^2$. The points of $PG(2,16)$ on $\mathcal{F}_{247}$ are $[1,0,0]$,$[0,1,0]$,$[0,0,1]$,$[\sigma^4,\sigma^9,1]$,$[\sigma^{14},\sigma^{14},1]$,$[\sigma^{12},\sigma^8,1]$, $[\sigma^8,\sigma^{13},1]$,$[\sigma^9,\sigma^4,1]$,$[\sigma^3,\sigma^3,1]$,$[\sigma^7,\sigma^9,1]$,$[\sigma^7,\sigma^{13},1]$,$[\sigma^{14},1,1]$,$[\sigma^{13},\sigma^5,1]$,$[\sigma^4,\sigma^{10},1]$, $[\sigma^2,\sigma^{14},1]$,$[\sigma^{13},\sigma^8,1]$,$[\sigma^9,\sigma^5,1]$,$[\sigma^3,1,1]$,$[\sigma^8,\sigma^{10},1]$,$[\sigma^2,\sigma^3,1]$,$[\sigma^{12},\sigma^4,1]$. To find the stabilizer group of $\mathcal{F}_{247}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{247}$ and their orders are shown as follows :

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}:1, \quad \begin{pmatrix}\sigma^{10} & 0 & 0\\ 0 & \sigma^5 & 0\\ 0 & 0 & 1\end{pmatrix}:3, \quad \begin{pmatrix}\sigma^5 & 0 & 0\\ 0 & \sigma^{10} & 0\\ 0 & 0 & 1\end{pmatrix}:3$$

$$\begin{pmatrix}0 & \sigma & 0\\ 0 & 0 & \sigma^6\\ \sigma^{13} & 0 & 0\end{pmatrix}:3, \quad \begin{pmatrix}0 & \sigma^{11} & 0\\ 0 & 0 & \sigma^6\\ \sigma^3 & 0 & 0\end{pmatrix}:3, \quad \begin{pmatrix}0 & \sigma^6 & 0\\ 0 & 0 & \sigma^6\\ \sigma^8 & 0 & 0\end{pmatrix}:3$$

$$\begin{pmatrix}0 & 0 & \sigma^{11}\\ \sigma^3 & 0 & 0\\ 0 & \sigma^8 & 0\end{pmatrix}:3, \quad \begin{pmatrix}0 & 0 & \sigma^{11}\\ \sigma^{13} & 0 & 0\\ 0 & \sigma^{13} & 0\end{pmatrix}:3, \quad \begin{pmatrix}0 & 0 & \sigma^{11}\\ \sigma^{13} & 0 & 0\\ 0 & \sigma^{13} & 0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{247}$ which is denoted by $G_{\mathcal{F}_{247}}$ contains

- 8 matrices of order 3.
- The identity matrix.

Form [6], $G_{\mathcal{F}_{247}}$ is isomorphic to $\boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{247}} \cong \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{248} = xy^2 + x^2z + \sigma^2yz^2 + (x^3 + \sigma^2y^3 + \sigma^4z^3 + \sigma^4xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{248}$ are $[\sigma^{12},\sigma^{13},1]$,$[\sigma^{12},\sigma^{10},1]$, $[\sigma^7,\sigma^2,1]$,$[\sigma^5,\sigma^7,1]$,$[\sigma^6,1,0]$,$[0,\sigma^6,1]$,$[\sigma^4,\sigma^{14},1]$,$[\sigma^3,\sigma^5,1]$,$[\sigma^2,\sigma^9,1]$,$[\sigma,\sigma^3,1]$,$[\sigma^4,\sigma,1]$, $[\sigma^8,\sigma^8,1]$,$[1,\sigma^5,1]$,$[\sigma^4,\sigma^{12},1]$,$[\sigma^9,1,1]$,$[\sigma^{14},\sigma^{13},1]$,$[\sigma^{11},0,1]$,$[\sigma^{10},\sigma^{13},1]$. To find the stabilizer group of $\mathcal{F}_{248}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{248}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}0&\sigma^3&0\\0&0&\sigma^3\\\sigma^5&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^2\\\sigma^4&0&0\\0&\sigma^4&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{248}$ which is denoted by $G_{\mathcal{F}_{248}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{248}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{248}} \cong .\boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{249} = xy^2 + x^2z + \sigma^2yz^2 + \sigma(x^3 + \sigma^2y^3 + \sigma^4z^3 + \sigma^4xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{249}$ are $[\sigma^6,\sigma^{10},1]$,$[\sigma,\sigma^9,1]$, $[\sigma^2,1,1]$,$[\sigma^8,\sigma^7,1]$,$[\sigma^6,1,1]$,$[\sigma^{11},\sigma^{11},1]$,$[\sigma^6,\sigma^{11},1]$,$[\sigma^7,\sigma^{11},1]$,$[\sigma^{10},\sigma,1]$,$[\sigma^2,\sigma^2,1]$,$[\sigma^2,\sigma^6,1]$, $[1,1,1]$. To find the stabilizer group of $\mathcal{F}_{249}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{249}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}0&\sigma^{14}&0\\\sigma&0&0\\0&\sigma&0\end{pmatrix}:3,\quad \begin{pmatrix}0&\sigma^2&0\\0&0&\sigma^2\\\sigma^4&0&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{249}$ which is denoted by $G_{\mathcal{F}_{249}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{249}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{249}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{250} = xy^2 + x^2z + \sigma^2yz^2 + \sigma^2(x^3 + \sigma^2y^3 + \sigma^4z^3 + \sigma^4xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{250}$ are $[\sigma^8,0,1]$,$[\sigma^8,\sigma^6,1]$, $[\sigma^3,\sigma^{14},1]$,$[\sigma^4,0,1]$,$[1,\sigma^9,1]$,$[\sigma^8,\sigma^{12},1]$,$[\sigma^5,\sigma^{11},1]$,$[\sigma^9,1,0]$,$[0,\sigma^9,1]$,$[\sigma^{13},\sigma^6,1]$,$[\sigma^{13},\sigma^{14},1]$, $[\sigma^{11},\sigma^2,1]$,$[\sigma^{13},\sigma^{12},1]$,$[\sigma^{11},\sigma^7,1]$,$[\sigma^{10},\sigma^4,1]$,$[\sigma,\sigma^4,1]$,$[\sigma^{10},1,0]$,$[0,\sigma^{10},1]$,$[\sigma^5,\sigma,1]$,$[\sigma^6,\sigma^9,1]$, $[\sigma^7,0,1]$,$[\sigma^{13},1,0]$,$[0,\sigma^{13},1]$,$[\sigma^3,\sigma^4,1]$. To find the stabilizer group of $\mathcal{F}_{250}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{250}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}0&\sigma^3&0\\0&0&\sigma^3\\\sigma^5&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^3\\\sigma^5&0&0\\0&\sigma^5&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{250}$ which is denoted by $G_{\mathcal{F}_{250}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{250}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{250}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table-2 is $\mathcal{F}_{251} = xy^2 + x^2z + \sigma^2yz^2 + \sigma^3(x^3 + \sigma^2y^3 + \sigma^4z^3 + \sigma^4xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{251}$ are $[\sigma^9,\sigma^8,1]$,$[\sigma^9,\sigma,1]$, $[\sigma^{10},\sigma^8,1]$,$[\sigma^{10},\sigma^7,1]$,$[\sigma^3,\sigma^6,1]$,$[\sigma,\sigma^8,1]$,$[\sigma^{10},\sigma^3,1]$,$[\sigma^5,\sigma^{14},1]$,$[\sigma^{14},\sigma^7,1]$,$[\sigma^9,\sigma^2,1]$, $[\sigma^{11},\sigma^{12},1]$,$[1,\sigma^7,1]$. To find the stabilizer group of $\mathcal{F}_{251}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{251}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\quad \begin{pmatrix}0&\sigma^9&0\\0&0&\sigma^9\\\sigma^{11}&0&0\end{pmatrix}:3,\quad \begin{pmatrix}0&0&\sigma^{12}\\\sigma^{14}&0&0\\0&\sigma^{14}&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{251}$ which is denoted by $G_{\mathcal{F}_{251}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{251}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{251}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 2 is $\mathcal{F}_{252} = xy^2 + x^2z + \sigma^2yz^2 + \sigma^4(x^3 + \sigma^2y^3 + \sigma^4z^3 + \sigma^4xyz)$. The points of $PG(2,16)$ on $\mathcal{F}_{252}$ are $[\sigma^7,1,0]$,$[0,\sigma^7,1]$, $[\sigma^{11},\sigma^3,1]$,$[\sigma^{10},0,1]$,$[\sigma^{12},\sigma,1]$,$[\sigma^{14},\sigma^8,1]$,$[\sigma^{14},\sigma^5,1]$,$[\sigma^2,\sigma^{13},1]$,$[\sigma,\sigma^{11},1]$,$[\sigma^4,\sigma^4,1]$,$[\sigma^8,\sigma^3,1]$, $[1,\sigma^{10},1]$,$[\sigma^{13},1,1]$,$[\sigma^{12},\sigma^2,1]$,$[\sigma^6,\sigma^5,1]$,$[\sigma^{12},\sigma^9,1]$,$[\sigma^9,\sigma^6,1]$,$[\sigma^7,\sigma^5,1]$. To find the stabilizer group of $\mathcal{F}_{252}$, we are doing calculations with computer help, thus the transformation matrices which stabilizing of $\mathcal{F}_{252}$ and their orders are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1, \qquad \begin{pmatrix}0&\sigma&0\\0&0&\sigma\\\sigma^3&0&0\end{pmatrix}:3, \qquad \begin{pmatrix}0&0&\sigma^{14}\\\sigma^{14}&0&0\\0&\sigma&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{252}$ which is denoted by $G_{\mathcal{F}_{252}}$ contains

- 2 matrices of order 3.
- The identity matrix.

Thus, $G_{\mathcal{F}_{252}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{252}} \cong \boldsymbol{Z_3}$.

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