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Application of Two Rowed Weyl Module in the Case of Partition (6, 6)/(1,U) when U= 0,1

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Abstract
The main aim of this paper is to study the application of Weyl module resolution in the case of two rows, which will be specified in the skew- partition (6, 6)/(1,1) and (6,6)/(1,0), by using the homological Weyl (i.e. the contracting homotopy and place polarization).

Keywords: Divided power algebra, resolution of Weyl module, place polarization, mapping Cone.

تطبيق مقاس وايل لصفين في حالة التجزئة (6,6)/(1, U) عندما U = 0, 1

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الخلاصة:
الغرض من هذا البحث هو دراسة تطبيق تحلل مقاس وايل في حالة الصفين والتي ستكون محددة في شبه التجزئة (6 ,6)/(1,1 و(6,6)/(1,0) وذلك باستخدام طرق همولوجية) أي التوافق الهوموتوبي ودالة المكان.

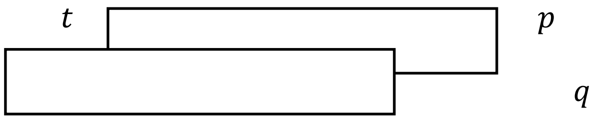
1. Introduction

Let R be a commutative ring with identity F and let it be a free R-module. Let $D_b F$ be the divided power of degree b.

Consider the figure below, which is associated with the resolution of the two-rowed Weyl module

$K_{\lambda/\mu} F$ = Im ($d'_{\lambda/\mu}$) where $d'_{\lambda/\mu}$ is the Weyl map that is described in [1] as follows:

$\lambda/\mu=$



(1)

We have:

$$\sum D_{p+k} \otimes D_{q-k} \xrightarrow{\boxdot} D_p \otimes D_q \xrightarrow{d'_{\lambda/\mu}} K_{\lambda/\mu} \to 0 \qquad (2)$$

And by using letter place, the maps will be explained now as follows:

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$$\begin{pmatrix} w \\ w' \end{pmatrix} \left| \begin{matrix} 1^{(p+k)} \\ 2^{(q-k)} \end{matrix} \right) \xrightarrow{\partial_{21}^{(k)}} \begin{pmatrix} w \\ w' \end{pmatrix} \left| \begin{matrix} 1^{(p)}2^{(k)} \\ 2^{(q-k)} \end{matrix} \right) \longrightarrow$$
$$\sum_w \begin{pmatrix} w_{(1)} \\ w'w_{(2)} \end{pmatrix} \left| \begin{matrix} (t+1)'(t+2)' \dots (p+t)' \\ 1'2'3' \dots q' \end{matrix} \right) \quad (3)$$

where

$$w \otimes w' \in D_{p+k} \otimes D_{q-k} \quad , \qquad \square = \sum_{k=t+1}^{q} \partial_{21}^{(k)}$$

and

$$d'_{\lambda/\mu} = \partial_{q'2} \dots \partial_{1'2} \partial_{(p+t)'1} \dots \partial_{(t+1)'1}$$

is the composition of place polarization, from positive places {1,2} to negative places {1', 2', … , (p+t)'}.

Also, as shown in [2], $\square$ delivers a component $x \otimes y$ of $D_{p+k} \otimes D_{q-k}$ to $\sum x_p \otimes x'_k \, y$, where $\sum x_p \otimes x'_k$ is the element of the diagonal of x in $D_p \otimes D_k$.

Let Z_{21} be the free generator of the divided power algebra D(Z_{21}) in one generator, then the divided power component $Z_{21}^{(k)}$ of degree k of the free generator Z_{21} acts on $D_{p+k} \otimes D_{q-k}$ by place polarization of degree k from place 1 to place 2.

Particularly, the graded algebra with identity $A = D(Z_{21})$ acts on the graded module $M = \sum D_{p+k} \otimes D_{q-k} = \sum M_{q-k}$, where the degree of the 2nd factor dictates the grading[3-5].

Therefore, M is a graded left A-module, where for $w = Z_{21}^{(k)} \in A$ and $v \in D_{\beta_1} \otimes D_{\beta_2}$, by definition, we have:

$$w(v) = Z_{21}^{(k)}(v) = \partial_{21}^{(k)}(v) . \quad (4)$$

And if we have (t^+), which is the graded strand of degree q

$$M_\bullet : 0 \to M_{q-t} \xrightarrow{\partial_s} \dots \to M_l \xrightarrow{\partial_s} \dots M_1 \xrightarrow{\partial_s} M_0 \quad (5)$$

of the normalized bar complex Bar(M, A; S, •), and S = {x}.

By definition, $M_\bullet$ is the complex:

$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots x Z_{21}^{(k_l)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_l}$$
$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots x Z_{21}^{(k_l - 1)} \varkappa D_{P+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_{l-1}}$$
$$\dots \xrightarrow{d_1} \sum_{k_i \geq 0} Z_{21}^{(t+k)} x D_{p+t+|k|} \otimes D_{q-t-k} \xrightarrow{d_0} D_p \otimes D_q \quad (6)$$

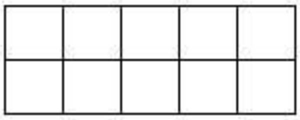
where $|k| = \sum k_i$ and d_l is the boundary operator $\partial_\varkappa$. Notice that (6) illustrates a left complex ($\partial_\varkappa^2 = 0$) over the Weyl module in terms of bar complex and letter-place algebra. Furthermore, when, in (6), the separator x disappears between $Z_{ab}^{(t)}$ and the components in the tensor product of the divided powers, this means that $\partial_{ab}^{(t)}$ is applied to the tensor product [1, 6].

The authors in earlier works [4, 5] exhibited the terms and the exactness of the Weyl module resolution in the cases of partition (8,7) and skew-shape (8,6)/(2,1) . In this work, we locate the terms and the exactness of the Weyl module resolution in the cases of skew-partition (6, 6)/(1,1) and (6,6)/(1,0).

2. Application of Weyl Module Resolution in the Case of Partition (6,6)/(1,1)

2.1 The Terms of Weyl Module Resolution in the Case of Partition (6, 6)/(1,1)

In this section, we find the term for the resolution of Weyl module in the case of the Partition (6,6)/(1,1).



$M_0 = D_5 \otimes D_5$

$M_1 = Z_{21}\varkappa D_6 \otimes D_4 \oplus Z_{21}^{(2)}\varkappa D_7 \otimes D_3 \oplus Z_{21}^{(3)}\varkappa D_8 \otimes D_2 \oplus Z_{21}^{(4)}\varkappa D_9 \otimes D_1 \oplus Z_{21}^{(5)}\varkappa D_{10} \otimes D_0$

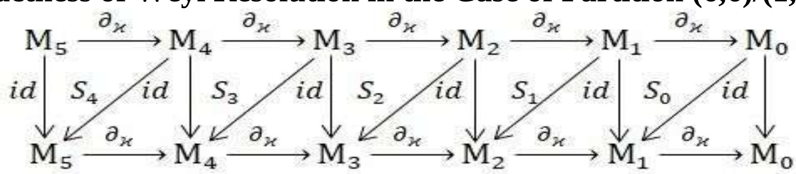
$M_2 = Z_{21}\varkappa Z_{21}\varkappa D_7 \otimes D_3 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_8 \otimes D_2 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_8 \otimes D_2 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_9 \otimes D_1 \oplus Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_9 \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_9 \otimes D_1 \oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{10} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}^{(4)}\varkappa D_{10} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa D_{10} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_0$

$M_3 = Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_8 \otimes D_2 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_9 \otimes D_1 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_9 \otimes D_1 \oplus Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_9 \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{10} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{10} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{10} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_0$

$M_4 = Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_9 \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{10} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_0$

$M_5 = Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_0$

2.2 The Exactness of Weyl Resolution in the Case of Partition (6,6)/(1,1)



The contracting homotopies $\{S_i\}$, where i=1,2,…,4, are:

$S_0 : D_5 \otimes D_5 \longrightarrow \sum_{k>0} Z_{21}^{(k+1)}\varkappa\, D_{5+k} \otimes D_{5-k}$, such that:

$$S_0\left(\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(5)}2^{(k)} \\ 2^{(5-k)} \end{vmatrix}\right) = \begin{cases} Z_{21}^{(k)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(5+k)} \\ 2^{(5-k)} \end{vmatrix} & ; if\ k > 0 \\ 0 & ; if\ k = 0 \end{cases}$$

$S_1 : \sum_{k>0} Z_{21}^{(k)}\varkappa\, D_{5+k} \otimes D_{5-k} \longrightarrow Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa\, D_{5+k} \otimes D_{5-k}$, such that:

$$S_1\left(Z_{21}^{(k)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(5+k)}2^{(m)} \\ 2^{(5-k-m)} \end{vmatrix}\right) = \begin{cases} Z_{21}^{(k)}\varkappa Z_{21}^{(m)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(5+k+m)} \\ 2^{(5-k-m)} \end{vmatrix} & ; if\ m = 1,2,3 \\ 0 & ; if\ m = 0 \end{cases}$$

$S_2 : \sum_{k_i>0} Z_{21}^{(k_1)}\varkappa\, Z_{21}^{(k_2)}\varkappa\, D_{5+|k|} \otimes D_{5-|k|} \longrightarrow Z_{21}^{(k_1)}\varkappa\, Z_{21}^{(k_2)}\varkappa\, Z_{21}^{(k_3)}\varkappa\, D_{5+|k|} \otimes D_{5-|k|}$, such that:

$$S_2\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(5+|m|)}2^{(m)} \\ 2^{(5-|k|-m)} \end{vmatrix}\right) =$$

$$\begin{cases} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(m)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(5+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} & ; if\ m = 1,2, \\ 0 & ; if\ m = 0 \end{cases} \quad ; \text{where } |k| = k_1 + k_2$$

$S_3 : \sum_{k_i>0} Z_{21}^{(k_1)}\varkappa \quad Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa \quad D_{5+|k|} \otimes D_{5-|k|} \longrightarrow Z_{21}^{(k_1)}\varkappa \quad Z_{21}^{(k_2)}\varkappa \quad Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\, D_{5+|k|} \otimes D_{5-|k|}$, such that:

$$S_3\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$$
$$=\begin{cases}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right); & if\ m=1\\ 0 & ; if\ m=0\end{cases};$$

where $|k|=k_1+k_2+k_3$

$$S_4:\sum_{k_i>0}Z_{21}^{(k_1)}\varkappa\ Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\ Z_{21}^{(k_4)}\varkappa\,\mathrm{D}_{5+|k|}\otimes\mathrm{D}_{5-|k|}\longrightarrow Z_{21}^{(k_1)}\varkappa\ Z_{21}^{(k_2)}\varkappa\ Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\ Z_{21}^{(k_5)}\varkappa\,\mathrm{D}_{5+|k|}\otimes\mathrm{D}_{5-|k|}$$

$$S_4\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$$
$$=\begin{cases}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right) & ; if\ m=1\\ 0 & ; if\ m=0\end{cases}$$

where $|k|=k_1+k_2+k_3+k_4$

$$S_0\partial_\varkappa\left(Z_{21}^{(k)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k)}2^{(m)}\\2^{(5-k-m)}\end{matrix}\right)\right)=S_0\partial_{21}^{(k)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k)}2^{(m)}\\2^{(5-k-m)}\end{matrix}\right)$$
$$=\binom{k+m}{m}Z_{21}^{(k+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k+m)}\\2^{(5-k-m)}\end{matrix}\right)$$

and

$$\partial_\varkappa S_1\left(Z_{21}^{(k)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k)}2^{(m)}\\2^{(5-k-m)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(k)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k+m)}\\2^{(5-k-m)}\end{matrix}\right)\right)$$
$$=-\binom{k+m}{m}Z_{21}^{(k+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k+m)}\\2^{(5-k-m)}\end{matrix}\right)+Z_{21}^{(k)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k)}2^{(m)}\\2^{(5-k-m)}\end{matrix}\right)$$

It is clear that $S_0\partial_\varkappa+\partial_\varkappa S_1=id_{M_1}$.

$$S_1\partial_\varkappa\left(-\binom{|k|}{k_2}Z_{21}^{|k|}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1)}\varkappa\,\partial_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$$
$$=-\binom{|k|+k_2}{k_2}Z_{21}^{|k|}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_2+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right),$$

and

$$\partial_\varkappa S_2\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k+m)}\\2^{(5-k-m)}\end{matrix}\right)\right)$$
$$=\binom{|k|}{k_2}Z_{21}^{|k|}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_2+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1)}\varkappa\,Z_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right),$$

where $|k|=k_1+k_2$.

It is clear that $S_1\partial_\varkappa+\partial_\varkappa S_2=id_{M_2}$.

$$S_2\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$$
$$=S_2\left(\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)-\right.$$
$$\left.\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$$

It is clear that $S_0\partial_\varkappa + \partial_\varkappa S_1 = id_{M_1}$.

$S_1\partial_\varkappa\left(-\binom{|k|}{k_2}Z_{21}^{|k|}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1)}\varkappa\,\partial_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=-\binom{|k|+k_2}{k_2}Z_{21}^{|k|}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_2+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

and

$\partial_\varkappa S_2\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+k+m)}\\2^{(5-k-m)}\end{matrix}\right)\right)$

$=\binom{|k|}{k_2}Z_{21}^{|k|}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_2+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1)}\varkappa\,Z_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

where $|k|=k_1+k_2$.

It is clear that $S_1\partial_\varkappa + \partial_\varkappa S_2 = id_{M_2}$.

$S_2\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$= S_2\left(\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)-\right.$

$\left.\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\,Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_3+k_4}{k_4}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_4+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

and

$\partial_\varkappa S_3\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)=$

$\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_3+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\partial_{21}^{(m)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)$

$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(5+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$$\binom{k_3+m}{m} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)} x \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) +$$
$$Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|)}2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right),$$

where $|k| = k_1+k_2+k_3$.

It is clear that $S_2\partial_\varkappa + \partial_\varkappa S_3 = id_{M_3}$.

and

$$\partial_\varkappa S_4\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)} x \left(\begin{array}{c|c} w & 1^{(5+|k|)}2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)$$
$$= \partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)$$
$$= \binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) -$$
$$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) +$$
$$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) -$$
$$\binom{k_4+m}{m} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) +$$
$$Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa \partial_{21}^{(m)} \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$$
$$= \binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) -$$
$$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) +$$
$$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) -$$
$$\binom{k_4+m}{m} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+m)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|+m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right) +$$
$$Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa \left(\begin{array}{c|c} w & 1^{(5+|k|)}2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right),$$

where $|k| = k_1+k_2+k_3+k_4$.

It is clear that $S_3\partial_\varkappa + \partial_\varkappa S_4 = id_{M_4}$.

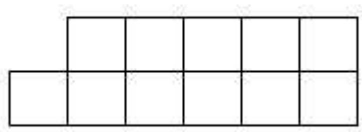
From the above homotopies, we obtain that $\{S_0, S_1, S_2, S_3, S_4\}$ is a contracting homotopy [7], which means that the complex

$0 \to M_4 \to M_3 \to M_2 \to M_1 \to M_0$ is exact.

3. Application of Weyl Module Resolution in the Case of the skew- Partition (6, 6)/(1, 0)

3.1 The Terms of Weyl Module Resolution in the Case of the skew- Partition (6, 6)/(1, 0)

The resolution of Weyl Module associated to this case has the following terms.

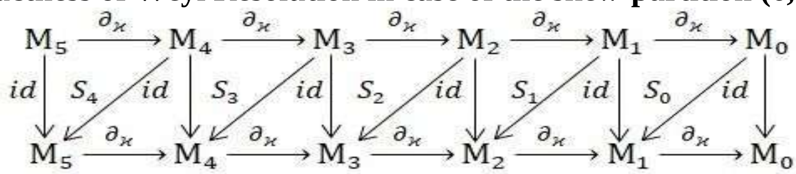


$M_0 = D_5 \otimes D_6$

$M_1 = Z_{21}^{(2)}\varkappa D_7 \otimes D_4 \ \oplus \ Z_{21}^{(3)}\varkappa D_8 \otimes D_3 \ \oplus \ Z_{21}^{(4)}\varkappa D_9 \otimes D_2$
$\quad \oplus Z_{21}^{(5)}\varkappa D_{10} \otimes D_1 \oplus \ Z_{21}^{(6)}\varkappa D_{11} \otimes D_0$

$M_2 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_8 \otimes D_3 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_9 \otimes D_2 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_9 \otimes D_2$
$\oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{10} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa D_{10} \otimes D_1$
$\oplus Z_{21}^{(5)}\varkappa Z_{21}\varkappa D_{11} \otimes D_0 \oplus Z_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(4)}\varkappa D_{11} \otimes D_0$
$\oplus Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa D_{11} \otimes D_0$
$M_3 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_9 \otimes D_2 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_1$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{10} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_1$
$\oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{11} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{11} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{11} \otimes D_0$
$M_4 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{11} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_0$
$M_5 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_0$

3.2 The Exactness of Weyl Resolution in case of the skew-partition (6,6)/(1,0)



The contracting homotopies $\{S_i\}$, where i =1,2,3,4, are

$S_0: D_5 \otimes D_6 \longrightarrow \sum_{k>0} Z_{21}^{(k+1)}\varkappa\, D_{5+k} \otimes D_{6-k}$

$$S_0\left(\begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(5)}2^{(k)} \\ 2^{(6-k)} \end{matrix}\right.\right) = \begin{cases} Z_{21}^{(k)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(5+k)} \\ 2^{(6-k)} \end{matrix}\right. & ; if\ 1 < k \le 5 \\ 0 & ; if\ k \le 1 \end{cases}$$

$S_1: \sum_{k>0} Z_{21}^{(k+1)}\varkappa\, D_{6+k} \otimes D_{5-k} \longrightarrow Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa\, D_{6+k} \otimes D_{5-k}$ such that:

$$S_1\left(Z_{21}^{(k+1)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(6+k)}2^{(m)} \\ 2^{(5-k-m)} \end{matrix}\right.\right) = \begin{cases} Z_{21}^{(k+1)}\varkappa Z_{21}^{(m)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(6+k+m)} \\ 2^{(5-k-m)} \end{matrix}\right. & ; if\ m = 1,2,3,4 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1 + k_2$

$S_2: \sum_{k_i>0} Z_{21}^{(k_1+1)}\varkappa\, Z_{21}^{(k_2)}\varkappa\, D_{6+|k|} \otimes D_{5-|k|} \longrightarrow Z_{21}^{(k_1+1)}\varkappa\, Z_{21}^{(k_2)}\varkappa\, Z_{21}^{(k_3)}\varkappa\, D_{6+|k|} \otimes D_{5-|k|}$

$S_3: \sum_{k_i>0} Z_{21}^{(k_1+1)}\varkappa\ Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\ D_{6+|k|} \otimes D_{5-|k|} \longrightarrow Z_{21}^{(k_1+1)}\varkappa\ Z_{21}^{(k_2)}\varkappa\ Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\ D_{6+|k|} \otimes D_{5-|k|}$

$$S_3\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(6+|k|)}2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right.\right)$$

$$= \begin{cases} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right. & ; if\ m = 1 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1+k_2+k_3$

Now, we have

$$S_0\partial_\varkappa\left(Z_{21}^{(k+1)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(6+k)}2^{(m)} \\ 2^{(5-k-m)} \end{matrix}\right.\right) = S_0\partial_{21}^{(k+1)} \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(6)}2^{(k+m)} \\ 2^{(5-k-m)} \end{matrix}\right. =$$

$$= \binom{k+1+m}{m} Z_{21}^{(k+1+m)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\left|\begin{matrix} 1^{(6+k+m)} \\ 2^{(5-k-m)} \end{matrix}\right.,$$

It is clear that $S_0\partial_\varkappa + \partial_\varkappa S_1 = id_{M_1}$.

$S_1\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=S_1\left(-\binom{|k|+1}{k_2}Z_{21}^{|k|+1}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+\quad Z_{21}^{(k_1+1)}\varkappa\,\partial_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=-\binom{|k|+1}{k_2}Z_{21}^{|k|+1}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$=-$ (|k|k+ 1) Z|21k|+1ϰZ(21m)ϰ (ww′|21((65−+||kk||+−mm))) +

$\binom{k_2+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

and

$\partial_\varkappa S_2\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+k+m)}\\2^{(5-k-m)}\end{matrix}\right)\right)$

$=\binom{|k|+1}{k_2}Z_{21}^{|k|+1}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_2+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1+1)}\varkappa\,Z_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

where $|k|=k_1+k_2$.

It is clear that $S_1\partial_\varkappa + \partial_\varkappa S_2 = id_{M_2}$.

$S_2\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$= S_2\left(\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)-\right.$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\left.Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_3+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

and

$S_3\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$= S_3\left(-\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+\right.$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa)\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_3+k_4}{k_4}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\partial_{21}^{(k_4)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)$

$=-\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(6+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa \quad Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) -$

$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) +$

$\binom{k_4+m}{m} Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)$

$\partial_\varkappa S_3 \left(Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|)} 2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) \right) =$

$\partial_\varkappa \left(Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) \right)$

$= -\binom{k_1+1+k_2}{k_2} Z_{21}^{(k_1+1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) +$

$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) -$

$\binom{k_3+m}{m} Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) +$

$Z_{21}^{(k_1+1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)$

where $|k| = k_1 + k_2 + k_3$.

It is clear that $S_2\partial_\varkappa + \partial_\varkappa S_3 = id_{M_3}$.

$S_4: \sum_{k_i>0} Z_{21}^{(k_1)} \varkappa \quad Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa \quad Z_{21}^{(k_4)} \varkappa\, \mathcal{D}_{6+|k|} \otimes \mathcal{D}_{5-|k|} \longrightarrow Z_{21}^{(k_1)} \varkappa \quad Z_{21}^{(k_2)} \varkappa \quad Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa$
$Z_{21}^{(k_5)} \varkappa\, \mathcal{D}_{6+|k|} \otimes \mathcal{D}_{5-|k|}$

$S_4 \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|)} 2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) \right)$

$= \begin{cases} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) & ; if\ m = 1 \\ 0 & ; if\ m = 0 \end{cases}$

where $|k| = k_1 + k_2 + k_3 + k_4$

$\partial_\varkappa S_4 \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|)} 2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) \right)$

$= \partial_\varkappa \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) \right)$

$= \binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) -$

$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) +$

$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) -$

$\binom{k_4+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) +$

$Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)$

From the above homotopies, we have that $\{S_0, S_1, S_2, S_3, S_4\}$ is a contracting homotopy [7], which means that our complex is exact.

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