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Reliability Estimation for the Exponential-Pareto Hybrid System

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Abstract

The reliability of hybrid systems is important in modern technology, specifically in engineering and industrial fields; it is an indicator of the machine's efficiency and ability to operate without interruption for an extended period of time. It also allows for the evaluation of machines and equipment for planning and future development. This study looked at reliability of hybrid (parallel series) systems with asymmetric components using exponential and Pareto distributions. Several simulation experiments were performed to estimate the reliability function of these systems using the Maximum Likelihood method (ML) and the Standard Bayes method (SB) with a quadratic loss (QL) function and two priors: non-informative (Jeffery) and informative (Conjugate). Different sample sizes and parameter values are used in these simulation experiments, and the Mean Squared Error (MSE) was used to compare those experiments. The simulation results showed that the standard Bayes method with Conjugate loss function is better than the results from the maximum likelihood method.

Keywords: Exponential distribution, Pareto distribution, hybrid (parallel-series) system, Reliability function, Maximum Likelihood Method, Standard Bayes, Quadratic Loss function, Simulation, Mean Squared Error.

تقدير معولية النظام الهجين للتوزيعين الأسي – باريتو

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الخلاصة

تم في هذا البحث تحليل موثوقية الانظمة الهجينة ذات السلاسل المتوازية للمكونات غير المتماثلة باستخدام التوزيع الأسي وتوزيع باريتو. ولتحقيق ذلك اجريت تجارب محاكاة لتقدير دالة المعولية للأنظمة الهجينة غير المتماثلة باستخدام طريقة الامكان الاعظم (ML) وطريقة بيز القياسية (SB) مع دالة الخسارة التربيعية (QLF) على اساس التوزيعين السابقين (Jeffery, Conjugate).

تمت المقارنة بين نتائج المحاكاة للمقدرات المختلفة باستخدام احجام مختلفة من العينات وقيم معلمات افتراضية، بالاعتماد على مقياس متوسط مربعات الخطأ (MSE) لاختيار افضل مقدر .

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1. Introduction

The interest in reliability studies in modern research on electronic devices and equipment has increased in the last century. Furthermore, rapid technological advancements and the use of complex systems in various areas of life have increased in this field. As failure to operate results in material losses and decreased production, there are many causes for the interruptions in various types of equipment and machinery. Few studies have been done in this area, and they have all focused on the reliability of simple systems composed of symmetric or asymmetric components with sequential-parallel reliability systems, despite the fact that there are many sources of production composed of complex, manifold, and non-complex work systems. Ali and Woo [1] performed studies in this field using maximum likelihood estimates of the threshold parameter with known parameters for the exponentiated Pareto distribution. Rahim [1] investigated the capabilities of series parallel hybrid reliability systems for carpet production plant machines using the Bayes method. Karam [2] investigated the posterior analysis of five exponentiated distributions (Weibull, Exponential, Inverted Weibull, Pareto, and Gumbel) with different prior distributions and loss functions. Al-Saady [3] modelled the reliability function of the series-parallel hybrid system in the case of asymmetric components. Shafay [4] considered the general form of the underlying distribution and the general conjugate prior, and developed a general procedure for deriving maximum likelihood and Bayesian estimators from an observed generalized Type-II hybrid censored sample. For modelling lifetime data, Maiti and Pramanik [5] proposed a new distribution called the Odd Generalized Exponential-Pareto distribution. Amin [6] discussed the upper record value distribution when the parent distribution is exponential Pareto. Akomolafe, Oladejo, Bello, and Ajiboye [7] looked at some results that describe the generalization of the exponential Pareto and negative Binomial distributions based on their distribution functions and asymptotic properties. Baharith et al. [8] studied the statistical properties of the odd exponential-Pareto IV distribution, which is a member of the odd family of distributions and characterized by decreasing, increasing, and upside down hazard functions. All of these attempts aided in the motivation for this work.

The aim of this study is to provide a reliability analysis of parallel-series systems with asymmetric components, representing the exponential and Pareto distributions. To accomplish this aim, simulation experiments carried out to estimate the reliability function of asymmetric hybrid systems using the maximum likelihood (ML) method and the standard Bayes method with loss function. Section 3 discusses reliability systems, while Section 4 discusses simulation results.

2. The Exponential and Pareto Distributions

The exponential distribution is an effective continuous distribution that is widely used in failure and survival time studies. The probability density function (pdf) for an exponential random variable with a scale parameter is as follows: [9], [10]

$$f(t;\alpha) = \alpha\, e^{-\alpha t} \qquad ; \;\; t, \alpha > 0 \tag{1}$$

The reliability function can be defined as:

$$R(t;\alpha) = e^{-\alpha t} \tag{2}$$

The Pareto distribution parameter is a continuous and it’s used in a variety of applications, particularly in economics. The probability and reliability functions of the random variable (S) for one parameter of the Pareto distribution are as follows: [11], [12]

$$f(s;\varphi) = \varphi\, s^{-(\varphi+1)} \qquad ; \quad s \geq 0 \;, \qquad \varphi > 0 \tag{3}$$

$$R(s;\varphi) = \left(\frac{1}{s}\right)^{\varphi} \tag{4}$$

2.1 Maximum Likelihood (ML)Estimation

It is possible to explain parameter estimation (α) using the Maximum Likelihood (ML). For an independent random sample $(t_1, t_2, \dots, t_n)$ which is drawn from an exponential distributed random variable, the likelihood function of the observations can be expressed as:

$$L(t_i|\alpha) = \prod_{i=1}^{n} f(t_i|\alpha) = \prod_{i=1}^{n} \alpha\, e^{-\alpha t_i} = \alpha^n\, e^{-\alpha \sum_{i=1}^{n} t_i} \tag{5}$$

Then, ML estimator for the parameter α is:

$$\hat{\alpha}_{ML} = \frac{n}{\sum_{i=1}^{n} t_i} \tag{6}$$

Similarly, the MLE for a parameter φ from a Pareto independent random sample $(s_1, s_2, \dots, s_n)$ can be calculated as follows:

$$L(s_i|\varphi) = \prod_{i=1}^{n} f(s_i|\varphi) = \prod_{i=1}^{n} \varphi\, s_i^{-(\varphi+1)} = \varphi^n \prod_{i=1}^{n} s_i^{-(\varphi+1)} \tag{7}$$

$$\hat{\varphi}_{ML} = \frac{n}{\sum_{i=1}^{n} Ln\, s_i} \tag{8}$$

The reliability function for Exponential and Pareto distributions can then be estimated using the ML method by substituting equation (6) in equation (2), and equation (8) in equation (4) as follows: [13], [14]

$$\hat{R}(t)_{ML} = e^{-\left(\frac{n}{\sum_{i=1}^{n} t_i}\right) t} \tag{9}$$

$$\hat{R}(s)_{ML} = \left(\frac{1}{s}\right)^{\frac{n}{\sum_{i=1}^{n} Ln\, s_i}} \tag{10}$$

2.2 Standard Bayes (SB) Estimation

The Bayes method assumes that the estimated parameters are random variables. These parameters represented in a p.d.f. known as the prior distribution, and then a p.d.f. known as the posterior distribution can be obtained by combining the likelihood function and this prior using the Bayes inversion formula. By using a random sample $(v_1, v_2, \dots, v_n)$ of size n with p.d.f f(v; β), the inversion formula is: [13], [15], [16]

$$\pi(\beta|V) = \frac{L(V|\beta)\, P(\beta)}{\int_{\forall\beta} L(V|\beta)\, P(\beta) d\beta} \tag{11}$$

Where:

L(V|β) is the likelihood function for sample $v_1, v_2, \dots, v_n$.

P(β) is the Prior distribution, and π(β|V) is the posterior distribution.

Applying the formula (11) to Exponential and Pareto distributions respectively, we get:

$$\pi(\alpha|t) = \frac{L(t|\alpha)\, P(\alpha)}{\int_{\forall\alpha} L(t|\alpha)\, P(\alpha) d\alpha} \tag{12}$$

$$\pi(\varphi|s) = \frac{L(s|\varphi)\, P(\varphi)}{\int_{\forall\varphi} L(s|\varphi)\, P(\varphi) d\varphi} \tag{13}$$

2.2.1 Prior Distributions

There are several forms of prior distributions, in the absence of sufficient primary information on the estimated parameter, or when none is available, the prior distribution is referred to as a little informative prior. When information from previous experiments about this parameter is available, the prior distribution is known as the conjugate prior or informative prior. [13], [10], [15], [16]

2.2.1.1 Jeffery Prior

If there is insufficient or no information about the parameter, the prior distribution can be calculated using Jefferys' formula, which is dependent on the parameter's zone. Given that the parameter field is $(0, \infty)$, the prior distribution based on fisher information $I(\beta)$, which is stated as follows: [17], [15], [16]

$$P(\beta) \propto \sqrt{I(\beta)} = h \,.\sqrt{I(\beta)} \qquad , h \; is \; constsnt \tag{14}$$

$$where, \qquad I(\beta) = -nE\left[\frac{\partial^2 \ln f(v;\beta)}{\partial \beta^2}\right]$$

The Jeffery prior is obtained by substituting $I(\beta)$ to the equation (14):

$$P(\beta) = h\,\frac{\sqrt{n}}{\beta} \propto \sqrt{n}\,\beta^{-1} \tag{15}$$

Applying the prior distribution based on the fisher information $I(\alpha)$ to the exponential distribution gives the following:

$$P(\alpha) \propto \sqrt{I(\alpha)} = h \,.\sqrt{I(\alpha)} \qquad , h \; is \; constsnt \tag{16}$$

$$I(\alpha) = -n\,E\left[\frac{\partial^2 \ln f(t;\alpha)}{\partial \alpha^2}\right] = \frac{n}{\alpha^2} \tag{17}$$

Substituting equation (17) into equation (16), obtaining:

$$P(\alpha) = h\,\frac{\sqrt{n}}{\alpha} \propto \sqrt{n}\,\alpha^{-1} \tag{18}$$

And for the Pareto distribution, the following results are obtained:

$$P(\varphi) \propto \sqrt{I(\varphi)} = h \,.\sqrt{I(\varphi)} \qquad , h \; is \; constsnt \;, \tag{19}$$

$$I(\varphi) = -n\,E\left[\frac{\partial^2 \ln f(s;\varphi)}{\partial \varphi^2}\right] = \frac{n}{\varphi^2} \tag{20}$$

Equation (21) is obtained by substituting equation (20) into equation (19).

$$P(\varphi) = h\,\frac{\sqrt{n}}{\varphi} \propto \sqrt{n}\,\varphi^{-1} \tag{21}$$

2.2.1.2 Conjugate Prior

Conjugate prior is a known, specific, and appropriate probability function to the parameter, it depends on the likelihood function of the observations. In this study, we depend on previous studies to select the conjugate priors for the exponential and Pareto distributions: [12], [15], [16]

$$P(\alpha) \propto \mu\, e^{-\mu\alpha} \qquad ; \; \mu, \alpha > 0 \tag{22}$$

$$P(\varphi) \propto \sigma^2\, \varphi\, e^{-\sigma\varphi} \qquad ; \; \sigma, \varphi > 0 \tag{23}$$

2.2.2 Posterior Distribution

the Exponential distribution $\pi_1(\alpha|t)$ is derived by substituting (5) and (18) in (12) and using the posterior distribution of the random parameter for the Jefferys' prior defined in (11) as follows: [18], [15], [16]

$$\pi_1(\alpha|t) = \frac{\sqrt{n}\,\alpha^{-1}\,\alpha^n\, e^{-\alpha\sum_{i=1}^n t_i}}{\int_0^\infty \sqrt{n}\;\alpha^{-1}\,\alpha^n\, e^{-\alpha\sum_{i=1}^n t_i}\, d\alpha} = \frac{\alpha^{n-1}\, e^{-\alpha\sum_{i=1}^n t_i}}{\int_0^\infty \alpha^{n-1}\, e^{-\alpha\sum_{i=1}^n t_i}\, d\alpha}$$

$$\pi_1(\alpha|t) = \frac{(\sum_{i=1}^n t_i)^n}{\Gamma(n)}\,\alpha^{n-1}\, e^{-\alpha\sum_{i=1}^n t_i} \tag{24}$$

Also, if equations (7) and (21) are substituted into equation (13), the posterior distribution for
the Pareto distribution is obtained $\pi_1(\varphi|s)$ as follows:

$$\pi_1(\varphi|s) = \frac{\sqrt{n}\,\varphi^{-1}\varphi^n(\sum_{i=1}^n \ln(s_i))^{-(\varphi+1)}}{\int_0^\infty \sqrt{n}\,\varphi^{-1}\varphi^n(\sum_{i=1}^n \ln(s_i))^{-(\varphi+1)}\, d\varphi}\;,$$

$$\pi_1(\varphi|s) = \frac{\left(\sum_{i=1}^n \ln(s_i)\right)^n}{\Gamma n}\;\varphi^{n-1} e^{-\varphi\sum_{i=1}^n \ln(s_i)} \tag{25}$$

Equation (11) defines the posterior distribution of the random parameter that depends on the conjugate prior, so the exponential distribution $\pi_2(\alpha|t)$ can be found by substituting equations (5) and (22) into equation (12), as follows:

$$\pi_2(\alpha|t) = \frac{\mu\, e^{-\mu\alpha}\,\alpha^n\, e^{-\alpha\sum_{i=1}^n t_i}}{\int_0^\infty \mu\, e^{-\mu\alpha}\,\alpha^n\, e^{-\alpha\sum_{i=1}^n t_i}\, d\alpha} = \frac{\alpha^n\, e^{-\alpha(\mu+\sum_{i=1}^n t_i)}}{\int_0^\infty \alpha^n\, e^{-\alpha(\mu+\sum_{i=1}^n t_i)} d\alpha}$$

$$\pi_2(\alpha|t) = \frac{(\mu+\sum_{i=1}^n t_i)^{n+1}}{\Gamma(n+1)}\;\alpha^n\, e^{-\alpha(\mu+\sum_{i=1}^n t_i)} \tag{26}$$

Similarly, if equations (7) and (23) are substituted into equation (13), the Pareto distribution's posterior distribution $\pi_2(\varphi|s)$ is obtained as follows:

$$\pi_2(\varphi|s) = \frac{\sigma^2\varphi\, e^{-\sigma\varphi}\,\varphi^n\, e^{-\varphi\sum_{i=1}^n \ln(s_i)}}{\int_0^\infty \sigma^2\varphi\, e^{-\sigma\varphi}\,\varphi^n\, e^{-\varphi\sum_{i=1}^n \ln(s_i)} d\varphi},$$

$$\pi_2(\varphi|s) = \frac{\varphi^{n+1}\, e^{-\sigma\varphi}\, e^{-\varphi\sum_{i=1}^n \ln(s_i)}}{\int_0^\infty \varphi^{n+1}\, e^{-\sigma\varphi}\, e^{-\varphi\sum_{i=1}^n \ln(s_i)}\, d\varphi} = \frac{\varphi^{n+1}\, e^{-\varphi[\sigma+\sum_{i=1}^n \ln(s_i)]}}{\int_0^\infty \varphi^{n+1}\, e^{-\varphi[\sigma+\sum_{i=1}^n \ln(s_i)]} d\varphi},$$

$$\pi_2(\varphi|s) = \frac{[\sigma+\sum_{i=1}^n \ln(s_i)]^{n+2}}{\Gamma(n+2)}\;\varphi^{n+1}\, e^{-\varphi[\sigma+\sum_{i=1}^n \ln(s_i)]} \tag{27}$$

2.2.3 Bayes Estimation under Quadratic Loss Functions

The reliability Bayesian estimator for the exponential distribution under Jeffery prior and quadratic loss function can be obtained as: [12], [2], [17]

$$L(\hat{\delta}, \delta) = (\hat{\delta} - \delta)^2 \tag{28}$$

The Bayes estimator for the parameter (δ) under the quadratic loss function say ($\hat{\delta}_B$) is actually the posterior expectation for the parameter (δ):

$$\hat{\delta}_B = E(\delta|v) = \int_{\forall\delta} \delta\, \pi(\delta|v) d\delta \tag{29}$$

and the reliability function is a function of the parameter $(\delta|v)$, then the reliability function of the Bayes estimator under the quadratic loss function can be given as:

$$\hat{R}(v,\delta)_B = E(R|v) = \int_{\forall\delta} R(v;\delta)\, \pi(\delta|v) d\delta \tag{30}$$

$$\hat{R}_{1B} = E(R|t) = \int_{\forall\alpha} R(t\,;\alpha)\, \pi_1(\alpha|t)\, d\alpha = \int_0^\infty e^{-\alpha t}\, \frac{(\sum_{i=1}^n t_i)^n}{\Gamma(n)} \alpha^{n-1}\, e^{-\alpha \sum_{i=1}^n t_i}\, d\alpha$$

$$\hat{R}_{1B} = \frac{(\sum_{i=1}^n t_i)^n}{\Gamma(n)}\, \frac{\Gamma(n)}{(t+\sum_{i=1}^n t_i)^n} \int_0^\infty \frac{(t+\sum_{i=1}^n t_i)^n}{\Gamma(n)}\, \alpha^{n-1}\, e^{-\alpha(t+\sum_{i=1}^n t_i)} d\alpha$$

$$\hat{R}_{1B} = \left(\frac{\sum_{i=1}^n t_i}{t+\sum_{i=1}^n t_i}\right)^n \tag{31}$$

Furthermore, using the exponential distribution's reliability Bayesian estimator with conjugate prior and quadratic loss function and applying equations (2), (26), get the following result:

$$\hat{R}_{2B} = E(R|t) = \int_{\forall\alpha} R(t;\alpha)\, \pi_2(\alpha|t)\, d\alpha\,,$$

$$\hat{R}_{2B} = \int_0^\infty e^{-\,\alpha t}\, \frac{(\mu+\sum_{i=1}^n t_i)^{n+1}}{\Gamma(n+1)}\, \alpha^n\, e^{-\alpha\,(\mu+\sum_{i=1}^n t_i)}\, d\alpha$$

$$\hat{R}_{2B} = \frac{(\mu+\sum_{i=1}^n t_i)^{n+1}}{\Gamma(n+1)} \int_0^\infty \alpha^n\, e^{-\alpha(\mu+t+\sum_{i=1}^n t_i)}\, d\alpha$$

But

$$\int_0^\infty \alpha^n\;\, e^{-\alpha(\mu+t+\sum_{i=1}^n t_i)}\, d\alpha = \frac{\Gamma(n+1)}{(\mu+t+\sum_{i=1}^n t_i)^{n+1}}$$

So,

$$\hat{R}_{2B} = \frac{(\mu+\sum_{i=1}^n t_i)^{n+1}}{\Gamma(n+1)}\, \frac{\Gamma(n+1)}{(\mu+t+\sum_{i=1}^n t_i)^{n+1}}$$

$$\hat{R}_{2B} = \left(\frac{\mu+\sum_{i=1}^n t_i}{\mu+t+\sum_{i=1}^n t_i}\right)^{n+1} \tag{32}$$

The reliability Bayesian estimator for the Pareto distribution under Jeffery prior and quadratic loss function is obtained by using equations (4), (25) as follows:

$$\hat{R}_{1B} = E(R|s) = \int_{\forall\varphi} R(s;\varphi)\, \pi_1(\varphi|s\,)\, d\varphi \;= \int_0^\infty s^{-\varphi}\, \frac{(\sum_{i=1}^n ln\,(s_i))^n}{\Gamma n}\;\, \varphi^{n-1} e^{-\,\varphi \sum_{i=1}^n ln\,(s_i)}\, d\varphi$$

$$\hat{R}_{1B} = \frac{(\sum_{i=1}^n ln\,(s_i))^n}{\Gamma n} \int_0^\infty e^{-\varphi ln\,(s)}\, \varphi^{n-1} e^{-\,\varphi \sum_{i=1}^n ln\,(s_i)}\, d\varphi$$

$$\hat{R}_{1B} = \frac{(\sum_{i=1}^n ln\,(s_i))^n}{\Gamma n} \int_0^\infty \varphi^{n-1}\, e^{-\,\varphi(ln\,(s)+\sum_{i=1}^n ln\,(s_i))}\, d\varphi \quad ,$$

$$\hat{R}_{1B} = \left(\frac{\sum_{i=1}^{n} ln\,(s_i)}{ln\,(s) + \sum_{i=1}^{n} ln\,(s_i)}\right)^{n} \tag{33}$$

Reliability Bayesian estimator of the Pareto distribution under conjugate prior and quadratic loss function is obtained by using equations (4), (27) as follows:

$$\hat{R}_{2B} = E(R|s) = \int_{\forall\varphi} R(s;\varphi)\, \pi_2(\varphi|s\,)\, d\varphi$$

$$\hat{R}_{2B} = \int_{0}^{\infty} e^{-\varphi\, ln\,(s)}\, \frac{(\sigma + \sum_{i=1}^{n} ln\,(s_i))^{n+2}}{\Gamma(n+2)}\, \varphi^{n+1}\, e^{-\varphi\,(\sigma + \sum_{i=1}^{n} ln\,(s_i))}\, d\varphi$$

$$\hat{R}_{2B} = \frac{(\sigma + \sum_{i=1}^{n} ln\,(s_i))^{n+2}}{\Gamma(n+2)} \int_{0}^{\infty} \varphi^{n+1}\, e^{-\varphi\,(\sigma + ln\,(s) + \sum_{i=1}^{n} ln\,(s_i))}\, d\varphi$$

$$\hat{R}_{2B} = \left(\frac{\sigma + \sum_{i=1}^{n} ln\,(s_i)}{\sigma + ln\,(s) + \sum_{i=1}^{n} ln\,(s_i)}\right)^{n+2} \tag{34}$$

3 Systems Reliability

According to the definition of system reliability, it is the probability that a system will continue to function properly after a specified period of time (v) or that it won't fail during the period $[0, v]$. A number of components make up the system, and the nature of the connections between those components determines the importance of the system's reliability. It is important to know the behaviour pattern of the components and their impact on the system behaviour.

3.1 Types of Systems

Systems can be classified into four types:

a) Series System: The components in this system are connected respectively, and the failure of any component causes the entire system to fail. If the system has (m) components (devices), then the reliability of the series system is expressed mathematically as follows:

$$R_s(v) = R_1(v).\, R_2(v) \dots R_m(v) = \prod_{j=1}^{m} R_j(v) \tag{35}$$

b) Parallel System: A system in which the components are interconnected so that a failure does not cause the system to fail. The mathematical formula for a parallel system's reliability is:

$$R_p(v) = 1 - \prod_{j=1}^{m}\left[1 - R_j(v)\right] \tag{36}$$

c) Hybrid system: A system contains many partial systems and each partial system contains many compounds within each partial system, and a system made up of several partial systems connected in series or parallel, and within each partial system. The number of compounds that are linked in series or parallel. The series-parallel hybrid system, on the other hand, will be investigated in this study.

d) Hybrid (Parallel-Series) System: This hybrid system is made up of (l) partial systems connected in series, and within each partial system, there are (m) partial systems that are connected in parallel. The system continues to function even when only one component is active, whereas the system ceases to function if one of the partial systems fails and figure 1 shows an example of this system in action.

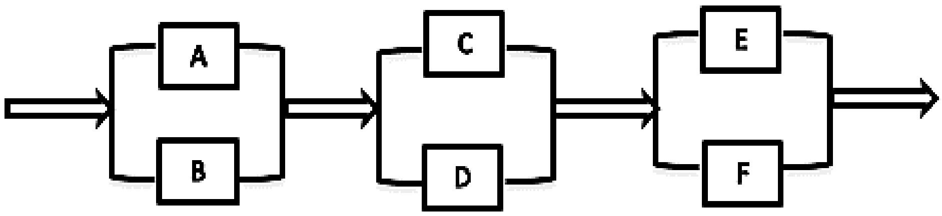

Figure 1: Hybrid (Parallel - Series) System

A system of (k) series bound partial systems' reliability function is defined as follows:

$$R_j(v) = 1 - \prod_{j=1}^{m} \{[1 - R_{kj}(v)]\} \ ; \ j = 1,2,\dots,m \ , \qquad k = 1,2,\dots,l \tag{37}$$

$\mathrm{R_{kj}}(v)$ represents the reliability function of the compound (k) in parallel to the partial system (j).

Further information in Al-Saady (2016), Martz and Waller (1982), Raheem (2014), and Rausand and Hoylan (2004). When the system consists of two series interconnected partial systems and each partial system consists of two components; the first follows an exponential distribution with parameter (α), and the second follows a Pareto distribution with parameter (φ), then the reliability function of each component in the partial system is defined by equations (2) and (4) respectively, and the reliability function of the partial system according to equation (37) is obtained as:

$R_j(v) = \{1 - [(1 - e^{-\alpha t})\ (1 - s^{-\varphi})]\}$.

The reliability function of the parallel-series hybrid system based on equation (36) would be:

$$R(v) = \prod_{k=1}^{l} \left[1 - \prod_{j=1}^{m} \left(1 - R_{kj}(v)\right)\right],$$

$$R(v) = \prod_{k=1}^{2} \{1 - [(1 - R_{k1}(v))(1 - R_{k2}(v))]\},$$

$$R(v) = \{1 - [(1 - R_1(v))(1 - R_2(v))]\}^2 \tag{38}$$

3.2 Reliability estimators for the parallel-series hybrid system

The ML method is used to estimate the reliability function of the parallel-series hybrid system by substituting equations (9) and (10) in (38) as follows:

$$\hat{R}(v)_{ML} = \left\{1 - \left[\left(1 - e^{-(\frac{n}{\sum_{i=1}^{n} t_i})\, t}\right)\left(1 - \left(\frac{1}{s}\right)^{\frac{n}{\sum_{i=1}^{n} Ln\, s_i}}\right)\right]\right\}^2 \tag{39}$$

And the reliability function of the parallel-series hybrid system using the Bayes method with Jeffery prior and quadratic loss function is obtained by substituting equations (31) and (33) into equation (38) as follows:

$$\hat{R}(v)_{B1} = \left\{1 - \left\{\left[1 - \left(\frac{\sum_{i=1}^{n} t_i}{t + \sum_{i=1}^{n} t_i}\right)^n\right]\left[1 - \left(\frac{\sum_{i=1}^{n} ln\,(s_i)}{ln\,(s_i) + \sum_{i=1}^{n} ln\,(s_i)}\right)^n\right]\right\}\right\}^2 \tag{40}$$

In addition, by substituting equations (32) and (34) into equation (38), the reliability function of the parallel-series hybrid system using the Bayes method with conjugate prior and quadratic loss function is obtained:

$$\hat{R}(v)_{B2} = \left\{1 - \left\{\left[1 - \left(\frac{\mu + \sum_{i=1}^{n} t_i}{\mu + t + \sum_{i=1}^{n} t_i}\right)^{n+1}\right]\left[1 - \left(\frac{\sigma + \sum_{i=1}^{n} \ln (s_i)}{\sigma + \ln (s) + \sum_{i=1}^{n} \ln (s_i)}\right)^{n+2}\right]\right\}\right\}^2 \quad (41)$$

4. Simulations and Results

Monte Carlo Simulation Experiments are used to estimate the reliability function of the series-parallel hybrid system and are based on the R 3.5.1 program as follows: [19], [20]

I. Select the default values for the parameters of the Exponential and the Pareto distributions, as well as the parameters of the natural accompanying function as shown in Table 1.

Table 1: PARAMETER VALUES OF SIMULATION EXPERIMENTS

Experiment	$\alpha = \mu$	φ	σ
1	0.1	0.2	0.2
2	0.25	0.5	0.5
3	0.5	0.75	1

DIFFERENT SAMPLE SIZES ARE CHOSEN FOR THE FOUR COMPOUNDS $(n = 25,50,100)$. ESTIMATE RELIABILITY FUNCTION OF THE SYSTEM (PARALLEL SERIES) BASED ON THE THEORETICAL ASPECT AND AS AN INDICATOR IN THE FORMULAS (39), (40), AND (41).

II. Perform the parallel-series hybrid system operation $(v = 2, 5, 8, 10)$ times and repeat the experiments $(B = 5000)$ times.

$$\hat{R}(v) = \frac{1}{B}\sum_{b=1}^{B} \hat{R}_b(v)$$

III. The Mean Squared Error (MSE) estimation for these estimators is as follows:

$$MSE\left[\hat{R}(v)\right] = \frac{1}{B}\sum_{b=1}^{B}\left[\hat{R}_b(v) - R_b\,(v)\right]^2 \quad (42)$$

Table 2 displays the simulation results for estimating and comparing the parallel-series hybrid system reliability function estimators by MLE method and Bayes method with the quadratic loss function and two priors.

Table 2: Reliability Estimators of (Parallel-Series) Hybrid System and MSE

α, φ	n	v	R(v)	$\hat{R}\,(v)_{MLE}$	MSE	$\hat{R}\,(v)_{B1}$	MSE	$\hat{R}\,(v)_{B2}$	MSE
0.1, 0.2	25	2	0.95362	0.93848	0.02294	0.93999	0.02063	0.94612	0.01130

		5	0.79515	0.78152	0.02063	0.78288	0.01855	0.80165	0.01016
		8	0.66038	0.64825	0.01834	0.64946	0.01649	0.65481	0.00904
		10	0.58786	0.57719	0.01612	0.57825	0.01450	0.58325	0.00795
	50	2	0.95362	0.94029	0.02062	0.96628	0.01856	0.94725	0.01016
		5	0.79515	0.80701	0.01833	0.80641	0.01650	0.80048	0.00903
		8	0.66038	0.64995	0.01612	0.65047	0.01451	0.65604	0.00794
		10	0.58786	0.57879	0.01401	0.59648	0.01261	0.58449	0.00691
	100	2	0.95362	0.94243	0.01648	0.94265	0.01631	0.94916	0.00862
		5	0.79515	0.80499	0.01302	0.80479	0.01288	0.79867	0.00724
		8	0.66038	0.66893	0.01015	0.66876	0.01005	0.65777	0.00608
		10	0.58786	0.58051	0.00781	0.58066	0.00773	0.58614	0.00504
$0.25, 0.5$	25	2	0.78279	0.79418	0.01721	0.79304	0.01548	0.78843	0.00849
		5	0.36674	0.35649	0.01548	0.35751	0.01392	0.37163	0.00764
		8	0.19452	0.18540	0.01377	0.18631	0.01238	0.19033	0.00679
		10	0.13865	0.13062	0.01211	0.13142	0.01089	0.13518	0.00597
	50	2	0.78279	0.79281	0.01548	0.77327	0.01393	0.78758	0.00763
		5	0.36674	0.37566	0.01376	0.37521	0.01239	0.37075	0.00679
		8	0.19452	0.20237	0.01210	0.20197	0.01089	0.19125	0.00597
		10	0.13865	0.13183	0.01052	0.14513	0.00947	0.14118	0.00519
	100	2	0.78279	0.79121	0.01237	0.79104	0.01225	0.78614	0.00648
		5	0.36674	0.37414	0.00977	0.37399	0.00967	0.36939	0.00544
		8	0.19452	0.20095	0.00762	0.20083	0.00754	0.19256	0.00457
		10	0.13865	0.13312	0.00587	0.13323	0.00581	0.13994	0.00379
$0.5, 0.75$	25	2	0.55315	0.54586	0.01099	0.54659	0.00988	0.54954	0.00543
		5	0.12717	0.13373	0.00988	0.13307	0.00889	0.12404	0.00488
		8	0.05049	0.05633	0.00879	0.05575	0.00791	0.05317	0.00434
		10	0.03362	0.02848	0.00773	0.02900	0.00696	0.03140	0.00382
	50	2	0.55315	0.54673	0.00988	0.55924	0.00889	0.55008	0.00488
		5	0.12717	0.12146	0.00879	0.12175	0.00791	0.12460	0.00434
		8	0.05049	0.04547	0.00773	0.04572	0.00696	0.05258	0.00382
		10	0.03362	0.02925	0.00672	0.03777	0.00605	0.03524	0.00332
	100	2	0.55315	0.54776	0.00790	0.54787	0.00782	0.55100	0.00415
		5	0.12717	0.12243	0.00624	0.12253	0.00618	0.12548	0.00348
		8	0.05049	0.04637	0.00487	0.04645	0.00482	0.05174	0.00292
		10	0.03362	0.03008	0.00375	0.03015	0.00371	0.03279	0.00243

5. Conclusions

The Exponential and Pareto hybrid systems have been considered in this study. The performance of the Maximum Likelihood estimator was compared with the Bayes estimators using the quadratic loss function and two priors in simulation studies. It was discovered that as the size of the sample increases, the estimated values approach the default values for all methods. The Standard Bayes method estimators with the conjugate prior are closest to the reliability function default values, followed by the Standard Bayes method with the Jeffery prior, then the ML estimator which were in the order. The mean squared error (MSE) values of the Bayes method estimators with conjugate prior were the lowest for all sample sizes, followed by the

Standard Bayes method with the Jeffery prior, and then the ML method, whereas the MSE converge when the sample size is increased, also the MSE for the reliability estimators reduces as the parameter and time values increases.

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