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Topology on Rough Pentapartitioned Neutrosophic Set

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Abstract

The main focus of this article is to introduce the notion of rough pentapartitioned neutrosophic set and rough pentapartitioned neutrosophic topology by using rough pentapartitioned neutrosophic lower approximation, rough pentapartitioned neutrosophic upper approximation, and rough pentapartitioned neutrosophic boundary region. Then, we provide some basic properties, namely operations on rough pentapartitioned neutrosophic set and rough pentapartitioned neutrosophic topology. By defining rough pentapartitioned neutrosophic set and topology, we formulate some results in the form of theorems, propositions, etc. Further, we give some examples to justify the definitions introduced in this article.

Keywords: Neutrosophic Set; Pentapartitioned Neutrosophic Set; Rough Pentapartitioned Neutrosophic Set; RPNT-space; RPNO-set.

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1. Introduction

Smarandache [29] grounded the concept of neutrosophic set (in short NS) theory by extending the notion of fuzzy set (in short FS) [40] and intuitionistic fuzzy set (in short IFS) [1] to deal with the uncertainty events having indeterminacy. In the year 2010, Wang et al. [39] studied the notion of single valued neutrosophic set (in short SVNS). Thereafter, Salama and Alblowi [27] presented the idea of neutrosophic topological spaces via neutrosophic sets by extending the notion of intuitionistic fuzzy topological spaces. Thereafter, Imran et al. [19] grounded the neutrosophic generalized alpha generalized continuity via neutrosophic topological space. Santhi and Udhayarani [28] grounded the idea of Nω-closed set via neutrosophic topological space. Afterwards, Maheswari and Chandrasekar [20] introduced the concept of neutrosophic *gb*-closed set and continuous functions via neutrosophic topological space. In the year 2019, Pushpalatha and Nandhini [25] presented the idea of generalized closed set via the neutrosophic topological space. The idea of neutrosophic α^*m* continuity was introduced and studied by Dhavanseelan et al. [17]. In the year 2020, Das and Pramanik [9] studied the generalized neutrosophic *b*-open set via neutrosophic topological space. Later on, Hanif

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PAGE and Imran [18] studied the neutrosophic generalized homomorphism via neutrosophic topological space. Das and Pramanik [10] also presented the neutrosophic ϕ-open set and neutrosophic ϕ-continuous functions. Recently, Das and Tripathy [15] introduced the notion of neutrosophic simply *b*-open set in neutrosophic topological space. Later on, Ozturk and Ozkan [23] grounded the concept of bi-topological space under the neutrosophic set environment. Thereafter, Das and Tripathy [14] presented the notion of pairwise neutrosophic *b*-open set via neutrosophic bi-topological space. Tripathy and Das [33] grounded the concept of pairwise neutrosophic *b*-continuous mappings via neutrosophic bi-topological space. The idea of neutrosophic multiset topology was grounded by Das and Tripathy [13]. In the year 1982, Pawlak [24] introduced the concept of rough set for the processing of incomplete information system. Thereafter, Broumi et al. [3] presented the idea of rough neutrosophic set (in short R-NS) by extending the notion of fuzzy rough set. In the year 2018, Thivagar et al. [32] grounded the concept of nano topology via neutrosophic sets. Afterwards, Sweety and Arockiarani [31] studied the topological structures of fuzzy neutrosophic rough sets. Mukherjee and Das [22] introduced the neutrosophic bipolar vague soft set and proposed a multi attribute decision making strategy based on it. Smarandache et al. [30] studied the fuzzy soft topological space, intuitionistic fuzzy soft topological space and neutrosophic soft topological space. Later on, Riaz et al. [26] notion of neutrosophic soft rough topology and presented an application to decision making. In the year 2021, Das et al. [6] introduced the notion of quadripartitioned neutrosophic topological space. Recently, Mallick and Pramanik [21] introduced the notions of pentapartitioned neutrosophic set (in short P-NS) by splitting indeterminacy-membership into three independent components namely contradiction, ignorance and unknown membership. In the year 2021, Das and Tripathy [16] introduced the notion of pentapartitioned neutrosophic topological space. Recently, Das et al. [5] proposed a MADM-strategy based on tangent similarity measure under the pentapartitioned neutrosophic set environment. In the year 2021, Das et al. [7] introduced and studied the concept of pentapartitioned neutrosophic *Q*-ideals of *Q*-algebra.

In this paper, we introduce the notion of rough pentapartitioned neutrosophic set (in short R-P-NS) and applied the concept of topology to R-P-NS. Then, we establish some basic properties, operations, and examples of the proposed set and topology.

Research gap: No investigation on rough pentapartitioned neutrosophic set and rough pentapartitioned neutrosophic topology has been reported in the recent literature.

Motivation: To diminish the research gap, we procure the notion of rough pentapartitioned neutrosophic set and rough pentapartitioned neutrosophic topology.

The remaining part of this article is designed as follows:

In section 2, we recall some relevant definitions and results to the main results of this article. Section 3 introduces the notion of R-P-NS and some operations defined on them. In section 4, we apply the concept of topology to R-P-NSs and introduce rough pentapartitioned neutrosophic topology (in short RPNT) and its properties. In section 5, we conclude our work done in this article and state some future scope of research.

2. Preliminaries

In this section, we give some definitions and results on NSs, P-NSs and R-NSs, which are relevant to the main results of this paper.

Definition 2.1:[16]. A neutrosophic set V over a fixed set W is defined as follows:
$V = \{(r, T_V(r), I_V(r), F_V(r)): r\in W\}$, where $T, I, F : W\rightarrow[0, 1]$ are the truth, indeterminacy and falsity membership functions respectively.

Definition 2.2:[11]. Let W be a fixed set. Then, a pentapartitioned neutrosophic set (in short P-NS) Z over W is defined as follows:
$Z = \{(r,T_Z(r),C_Z(r),G_Z(r),U_Z(r),F_Z(r)): r\in W\}$, where $T_Z(r)$, $C_Z(r)$, $G_Z(r)$, $U_Z(r)$, $F_Z(r)$ ($\in[0, 1]$) are the truth, contradiction, ignorance, unknown, falsity membership values of each $n\in W$. So, $0 \leq T_Z(r)+C_Z(r)+G_Z(r)+U_Z(r)+F_Z(r) \leq 5$, for all $r\in W$.

Definition 2.3:[11]. Let W be a fixed set. Then, the absolute P-NS (1_{PN}) and the null P-NS (0_{PN}) over W are defined as follows:
(i) $1_{PN} = \{(r,1,1,0,0,0): r\in W\}$;
(ii) $0_{PN} = \{(r,0,0,1,1,1): r\in W\}$.
The absolute P-NS (1_{PN}) and the null P-NS (0_{PN}) have other seven types of representations. They are given below:

$1_{PN} = \{(r,1,1,0,0,1): r\in W\}$;	$0_{PN} = \{(r,0,0,1,1,0): r\in W\}$;
$1_{PN} = \{(r,1,1,0,1,0): r\in W\}$;	$0_{PN} = \{(r,0,0,1,0,1): r\in W\}$;
$1_{PN} = \{(r,1,1,1,0,0): r\in W\}$;	$0_{PN} = \{(r,0,0,0,1,1): r\in W\}$;
$1_{PN} = \{(r,1,1,0,1,1): r\in W\}$;	$0_{PN} = \{(r,0,0,1,0,0): r\in W\}$;
$1_{PN} = \{(r,1,1,1,0,1): r\in W\}$;	$0_{PN} = \{(r,0,0,0,1,0): r\in W\}$;
$1_{PN} = \{(r,1,1,1,1,0): r\in W\}$;	$0_{PN} = \{(r,0,0,0,0,1): r\in W\}$;
$1_{PN} = \{(r,1,1,1,1,1): r\in W\}$;	$0_{PN} = \{(r,0,0,0,0,0): r\in W\}$.

Remark 2.1: Clearly, $0_{PN} \subseteq X \subseteq 1_{PN}$, for every P-NS X over W.

Definition 2.4:[11]. Let $M=\{(r,T_M(r),C_M(r),G_M(r),U_M(r),F_M(r)): r\in W\}$ and $N=\{(r,T_N(r),C_N(r), G_N(r),U_N(r),F_N(r)): r\in W\}$ be two P-NSs over W. Then, $M \subseteq N$ iff $T_M(r) \leq T_N(r)$, $C_M(r) \leq C_N(r)$, $G_M(r) \geq G_N(r)$, $U_M(r) \geq U_N(r)$, $F_M(r) \geq F_N(r)$, for all $r\in W$.

Example 2.1: Consider two P-NSs $X=\{(r,0.3,0.4,0.5,0.7,0.3), (m,0.3,0.6,0.4,0.8,0.4)\}$ and $Y=\{(r,0.4,0.7,0.1,0.5,0.2), (m,0.8,0.9,0.2,0.1,0.2)\}$ over a fixed set $W=\{r, m\}$. Then, $X\subseteq Y$.

Definition 2.5:[11]. Let $M=\{(r,T_M(r),C_M(r),G_M(r),U_M(r),F_M(r)): r\in W\}$ and $N=\{(r,T_N(r),C_N(r), G_N(r),U_N(r),F_N(r)): r\in W\}$ be two P-NSs over W. Then, the intersection of X and Y is $X\cap Y=\{(r, \min\{T_M(r), T_N(r)\}, \min\{C_M(r), C_N(r)\}, \max\{G_M(r), G_N(r)\}, \max\{U_M(r), U_N(r)\}, \max\{F_M(r), F_N(r)\}): r\in W\}$.

Example 2.2: Consider two P-NSs $X=\{(r,0.4,0.7,0.4,0.2,0.9), (m,0.5,0.6,0.7,0.8,0.5)\}$ and $Y=\{(r,0.9,0.2,0.8,0.7,0.8), (m,0.5,0.8,0.7,0.2,0.9)\}$ over $W=\{r, m\}$. Then, the intersection of X and Y is $X\cap Y=\{(r,0.4,0.2,0.8,0.7,0.9), (m,0.5,0.6,0.7,0.8,0.9)\}$.

Definition 2.6:[11]. Let $M=\{(r,T_M(r),C_M(r),G_M(r),U_M(r),F_M(r)): r\in W\}$ and $N=\{(r,T_N(r),C_N(r), G_N(r),U_N(r),F_N(r)): r\in W\}$ be two P-NSs over W. Then, the union of X and Y is $X\cup Y=\{(r, \max\{T_M(r), T_N(r)\},\max\{C_M(r), C_N(r)\},\min\{G_M(r), G_N(r)\},\min\{U_M(r), U_N(r)\},\min\{F_M(r), F_N(r)\}): r\in W\}$.

Example 2.3: Consider two P-NSs $X=\{(r,0.5,0.4,0.7,0.7,0.5), (m,0.8,0.5,0.9,1.0,0.5)\}$ and $Y=\{(r,0.6,0.7,0.1,0.5,0.2), (m,1.0,0.9,0.4,0.0,0.1)\}$ over $W=\{r, m\}$. Then, their union is $X\cup Y=\{(r,0.6,0.7,0.1,0.5,0.2), (m,1.0,0.9,0.4,0.0,0.1)\}$.

Definition 2.7:[11]. Let $M=\{(r,T_M(r),C_M(r),G_M(r),U_M(r),F_M(r)): r\in W\}$ be a P-NS over a fixed set W. Then, $M^c=\{(r,F_M(r),U_M(r),1-G_M(r),C_M(r),T_M(r)): r\in W\}$.

Example 2.4: Let $M=\{(r,0.4,0.5,0.9,0.7,0.8), (m,0.7,0.1,0.5,0.7,0.1)\}$ be a P-NS over $W=\{r, m\}$. Then, $M^c=\{(r,0.8,0.5,0.1,0.7,0.4), (m,0.1,0.7,0.5,0.1,0.7)\}$.

Definition 2.8:[2]. Let ρ be an equivalence relation on W. Let Q={(r,$T_Q(r)$,$I_Q(r)$,$F_Q(r)$): $r\in W$} be a NS over W. Then, the lower approximation ($\underline{N}(Q)$) and the upper approximation ($\overline{N}(Q)$) of Q in the approximation space (W, ρ) are defined as follows:

$\underline{N}(Q)$={(r,$T_{\underline{N}(Q)}(r)$,$I_{\underline{N}(Q)}(r)$,$F_{\underline{N}(Q)}(r)$): $p\in[r]_\rho$, $r\in W$};

$\overline{N}(Q)$={(r,$T_{\overline{N}(Q)}(r)$,$I_{\overline{N}(Q)}(r)$,$F_{\overline{N}(Q)}(r)$): $p\in[r]_\rho$, $r\in W$},

where $T_{\underline{N}(Q)}=\wedge_{p\in[r]_\rho} T_Q(p)$, $I_{\underline{N}(Q)}=\vee_{p\in[r]_\rho} I_Q(p)$, $F_{\underline{N}(Q)}=\vee_{p\in[r]_\rho} F_Q(p)$, $T_{\overline{N}(Q)}=\vee_{p\in[r]_\rho} T_Q(p)$, $I_{\overline{N}(Q)}=\wedge_{p\in[r]_\rho} I_Q(p)$, $F_{\overline{N}(Q)}=\wedge_{p\in[r]_\rho} F_Q(p)$.

So, $0 \le T_{\underline{N}(Q)}(r) + I_{\underline{N}(Q)}(r) + F_{\underline{N}(Q)}(r) \le 3$ and $0 \le T_{\overline{N}(Q)}(r) + I_{\overline{N}(Q)}(r) + F_{\overline{N}(Q)}(r) \le 3$.

Clearly, the lower approximation [$\underline{N}(Q)$] and the upper approximation [$\overline{N}(Q)$] are the NSs over W. The pair ($\underline{N}(Q)$, $\overline{N}(Q)$) is said to be a rough neutrosophic set (in short R-NS) in the approximation space (W, ρ).

3. Rough Pentapartitioned Neutrosophic Set

The notion of rough pentapartitioned neutrosophic set (in short R-P-NS) and its properties are defined as follows:

Definition 3.1: Suppose that ρ be an equivalence relation on a fixed set W. Assume that Q = {<r,$T_Q(r)$,$C_Q(r)$,$G_Q(r)$,$U_Q(r)$,$F_Q(r)$> : $r\in W$} be a P-NS over W. Then, the lower approximation set [$\underline{N}(Q)$] and the upper approximation set [$\overline{N}(Q)$] of Q in the approximation space (W, ρ) are defined as follows:

$\underline{N}(Q)$ = {<r,$T_{\underline{N}(Q)}(r)$,$C_{\underline{N}(Q)}(r)$,$G_{\underline{N}(Q)}(r)$,$U_{\underline{N}(Q)}(r)$,$F_{\underline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$};

$\overline{N}(Q)$ = {<r,$T_{\overline{N}(Q)}(r)$,$C_{\overline{N}(Q)}(r)$,$G_{\overline{N}(Q)}(r)$,$U_{\overline{N}(Q)}(r)$,$F_{\overline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$},

where $T_{\underline{N}(Q)}=\wedge_{p\in[r]_\rho} T_Q(r)$, $C_{\underline{N}(Q)}=\wedge_{p\in[r]_\rho} C_Q(r)$, $G_{\underline{N}(Q)}=\wedge_{p\in[r]_\rho} G_Q(r)$, $U_{\underline{N}(Q)}=\wedge_{p\in[r]_\rho} U_Q(r)$, $F_{\underline{N}(Q)}=\wedge_{p\in[r]_\rho} F_Q(r)$, $T_{\overline{N}(Q)}=\vee_{p\in[r]_\rho} T_Q(r)$, $C_{\overline{N}(Q)}=\vee_{p\in[r]_\rho} C_Q(r)$, $G_{\overline{N}(Q)}=\vee_{p\in[r]_\rho} G_Q(r)$, $U_{\overline{N}(Q)}=\vee_{p\in[r]_\rho} U_Q(r)$, $F_{\overline{N}(Q)}=\vee_{p\in[r]_\rho} F_Q(r)$.

So, $0 \le T_{\underline{N}(Q)}(r)+C_{\underline{N}(Q)}(r)+G_{\underline{N}(Q)}(r)+U_{\underline{N}(Q)}(r)+F_{\underline{N}(Q)}(r) \le 5$,

and $0 \le T_{\overline{N}(Q)}(r)+C_{\overline{N}(Q)}(r)+G_{\overline{N}(Q)}(r)+U_{\overline{N}(Q)}(r)+F_{\overline{N}(Q)}(r) \le 5$.

Here, the operators "$\vee$" and "$\wedge$" means "max" or "join" and "min" or "meet" operators respectively. Clearly, $\underline{N}(Q)$ and $\overline{N}(Q)$ are two P-NSs over W. The pair ($\underline{N}(Q)$, $\overline{N}(Q)$) is called the rough pentapartitioned neutrosophic set (in short R-P-NS) in (W, ρ).

Example 3.1: Let W = {r_1, r_2, r_3, r_4, r_5} be a fixed set. Let ρ be an equivalence relation, where its partition of W is given by W/ρ = {(r_1, r_3), (r_2, r_5), (r_4)}. Suppose that Q = {<r_1,0.5,0.4,0.2,0.3,0.6>, <r_2,0.8,0.2,0.6,0.6,0.4>, <r_3,0.2,0.3,0.4,0.7,0.6>, <r_4,0.9,0.8,0.7,0.1, 0.8>, <r_5,0.6,0.3,0.7,0.2,0.5>} be a P-NS over W. Then, the lower approximation set of the P-NS Q is $\underline{N}(Q)$={< r_1,0.2,0.3,0.2,0.3,0.6 >, < r_2,0.6,0.2,0.6,0.2,0.4 >, < r_3,0.2,0.3,0.2,0.3, 0.6 >, < r_4,0.9,0.8,0.7,0.1,0.8 >, < r_5,0.6,0.2,0.6,0.2,0.4 >}, and

The upper approximation set of the P-NS Q is $\overline{N}$ (Q)={<r_1,0.5,0.4,0.4,0.7,0.6>, <r_2,0.8,0.3,0.7,0.6,0.5>, <r_3,0.5,0.4,0.4,0.7,0.6>, <r_4,0.9,0.8,0.7,0.1,0.8>, <r_5,0.8,0.3,0.7,0.6, 0.5>}. Therefore, ($\underline{N}$ (Q), $\overline{N}$ (Q)) = ({<r_1,0.2,0.3,0.2,0.3,0.6>, <r_2,0.6,0.2,0.6,0.2,0.4>, <r_3,0.2,0.3,0.2,0.3,0.6>, <r_4,0.9,0.8,0.7,0.1,0.8>, <r_5,0.6,0.2,0.6,0.2,0.4>}, {<r_1,0.5,0.4,0.4, 0.7,0.6>, <r_2,0.8,0.3,0.7,0.6,0.5>, <r_3,0.5,0.4,0.4,0.7,0.6>, < r_4,0.9,0.8,0.7,0.1, 0.8>, <r_5,0.8, 0.3,0.7,0.6,0.5>}) is a R-P-NS in (W, ρ).

Definition 3.2: Assume that $N(Q)$ = ($\underline{N}$ (Q), $\overline{N}$ (Q)) = ({<r, $T_{\underline{N}(Q)}(r)$, $C_{\underline{N}(Q)}(r)$, $G_{\underline{N}(Q)}(r)$, $U_{\underline{N}(Q)}(r)$,$F_{\underline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$}, {<r,$T_{\overline{N}(Q)}(r)$,$C_{\overline{N}(Q)}(r)$,$G_{\overline{N}(Q)}(r)$,$U_{\overline{N}(Q)}(r)$,$F_{\overline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$}) be a R-P-NS in the approximation space (W, ρ). Then, [<$T_{\underline{N}(Q)}(r)$,$C_{\underline{N}(Q)}(r)$,

$G_{\underline{N}(Q)}(r), U_{\underline{N}(Q)}(r), F_{\underline{N}(Q)}(r)$>, <$T_{\overline{N}(Q)}(r), C_{\overline{N}(Q)}(r), G_{\overline{N}(Q)}(r), U_{\overline{N}(Q)}(r), F_{\overline{N}(Q)}(r)$>] is called a single valued rough pentapartitioned neutrosophic number (in short SVRPNN) for all $r \in W$.

Example 3.2: Let $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ be a R-P-NS in the approximation space (W, ρ) as it is shown in Example 3.1. Then, [<0.5,0.4, 0.4,0.7,0.6>, <0.5,0.4,0.4,0.7,0.6>] is a SVRPNN in the approximation space (W, ρ).

Definition 3.3: Let $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ be a R-P-NS in the approximation space (W, ρ). Then, the complement of $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ is defined as follows:

$N(Q)^c = (\underline{N}(Q)^c, \overline{N}(Q)^c)$, where $\underline{N}(Q)^c = \{<r, F_{\underline{N}(Q)}(r), U_{\underline{N}(Q)}(r), 1-G_{\underline{N}(Q)}(r), C_{\underline{N}(Q)}(r), T_{\underline{N}(Q)}(r)> : p \in [r]_\rho, r \in W\}$ and $\overline{N}(Q)^c = \{<r, F_{\overline{N}(Q)}(r), U_{\overline{N}(Q)}(r), 1-G_{\overline{N}(Q)}(r), C_{\overline{N}(Q)}(r), T_{\overline{N}(Q)}(r)> : p \in [r]_\rho, r \in W\}$.

Example 3.3: Let $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ be a R-P-NS in the approximation space (W, ρ) as it is shown in Example 3.1. Then, the complement of $N(Q)$ is $N(Q)^c = (\underline{N}(Q)^c, \overline{N}(Q)^c)$,
where, $\underline{N}(Q)^c$ = {<r_1,0.6,0.3,0.8,0.3,0.2>, <r_2,0.4,0.2,0.4,0.2,0.6>, <r_3,0.6,0.3,0.8,0.3,0.2>, <r_4, 0.8,0.1,0.3,0.8,0.9>, <r_5,0.4,0.2,0.4,0.2,0.6>} and $\overline{N}(Q)$ = {<r_1,0.6,0.7,0.6,0.4,0.5>, <r_2,0.5, 0.6,0.3,0.3,0.8>, <r_3,0.6,0.7,0.6,0.4,0.5>, <r_4,0.8,0.1,0.3,0.8,0.9>, <r_4,0.5,0.6,0.3,0.3,0.8>}.

Definition 3.4: Let $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ and $N(V) = (\underline{N}(V), \overline{N}(V))$ be two R-P-NSs in the approximation space (W, ρ). Then, $N(Q) \subseteq N(V)$ if and only if $\underline{N}(Q) \subseteq \underline{N}(V)$ and $\overline{N}(Q) \subseteq \overline{N}(V)$, i.e., $T_{\underline{N}(Q)}(r) \le T_{\underline{N}(Q)}(v)$, $C_{\underline{N}(Q)}(r) \le C_{\underline{N}(Q)}(v)$, $G_{\underline{N}(Q)}(r) \ge G_{\underline{N}(Q)}(v)$, $U_{\underline{N}(Q)}(r) \ge U_{\underline{N}(Q)}(v)$, $F_{\underline{N}(Q)}(r) \ge F_{\underline{N}(Q)}(v)$, $T_{\overline{N}(V)}(r) \le T_{\overline{N}(V)}(v)$, $C_{\overline{N}(V)}(r) \le C_{\overline{N}(V)}(v)$, $G_{\overline{N}(V)}(r) \ge G_{\overline{N}(V)}(v)$, $U_{\overline{N}(V)}(r) \ge U_{\overline{N}(V)}(v)$, $F_{\overline{N}(V)}(r) \ge F_{\overline{N}(V)}(v)$, for all $r \in W$.

Example 3.4: Let $N(Q)$ = ({<r_1,0.3,0.5,0.2,0.3,0.6>, <r_2,0.3,0.3,0.6,0.2,0.4>, <r_3,0.2,0.3,0.2, 0.3,0.6>, <r_4,0.9,0.8,0.7,0.1,0.8>, <r_5,0.6,0.2,0.6,0.2,0.4>}, {<r_1,0.5,0.4,0.4,0.7,0.6>, <r_2,0.8, 0.3,0.7,0.6,0.5>, <r_3,0.5,0.4,0.4,0.7,0.6>, <r_4,0.9,0.8,0.7,0.1,0.8>, <r_5,0.8,0.3,0.7,0.6,0.5>}) and $N(V)$ = ({<r_1,0.3,0.5,0.1,0.2,0.5>, <r_2,0.7,0.2,0.4,0.0,0.3>, <r_3,0.5,0.3,0.0,0.0,0.1>, <r_4, 0.9,0.9,0.5,0.0,0.3>, <r_5,0.8,0.3,0.2,0.0,0.2>}, {<r_1,0.5,0.5,0.3,0.3,0.6>, <r_2,0.8,0.3,0.5,0.1, 0.3>, <r_3,0.5,0.4,0.0,0.1,0.2>, <r_4,1.0,0.9,0.6,0.0,0.4>, <r_5,0.8,0.3,0.4,0.1,0.3>}) be two R-P-NSs in (W, ρ). Clearly, $N(Q) \subseteq N(V)$.

Definition 3.5: Let $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ and $N(V) = (\underline{N}(V), \overline{N}(V))$ be two R-P-NSs in the approximation space (W, ρ). Then, $N(Q) = N(V)$ if and only if $\underline{N}(Q) = \underline{N}(V)$ and $\overline{N}(Q) = \overline{N}(V)$, i.e., $T_{\underline{N}(Q)}(r) = T_{\underline{N}(Q)}(v)$, $C_{\underline{N}(Q)}(r) = C_{\underline{N}(Q)}(v)$, $G_{\underline{N}(Q)}(r) = G_{\underline{N}(Q)}(v)$, $U_{\underline{N}(Q)}(r) = U_{\underline{N}(Q)}(v)$, $F_{\underline{N}(Q)}(r) = F_{\underline{N}(Q)}(v)$, $T_{\overline{N}(V)}(r) = T_{\overline{N}(V)}(v)$, $C_{\overline{N}(V)}(r) = C_{\overline{N}(V)}(v)$, $G_{\overline{N}(V)}(r) = G_{\overline{N}(V)}(v)$, $U_{\overline{N}(V)}(r) = U_{\overline{N}(V)}(v)$, $F_{\overline{N}(V)}(r) = F_{\overline{N}(V)}(v)$, for all $r \in W$.

Example 3.5: Let $N(Q)$ = ({<r_1,0.2,0.3,0.2,0.3,0.6>, <r_2,0.6,0.2,0.6,0.2,0.4>, <r_3,0.2,0.3,0.2, 0.3,0.6>, <r_4,0.9,0.8,0.7,0.1,0.8>, <r_5,0.6,0.2,0.6,0.2,0.4>}, {<r_1,0.5,0.4,0.4,0.7,0.6>, <r_2,0.8, 0.3,0.7,0.6,0.5>, <r_3,0.5,0.4,0.4,0.7,0.6>, <r_4,0.9,0.8,0.7,0.1,0.8>, <r_5,0.8,0.3,0.7,0.6,0.5 >}) and $N(V)$ = ({<r_1,0.2,0.3,0.2,0.3,0.6>, <r_2,0.6,0.2,0.6,0.2,0.4>, <r_3,0.2,0.3,0.2,0.3,0.6>, <r_4, 0.9,0.8,0.7,0.1,0.8>, <r_5,0.6,0.2,0.6,0.2,0.4>}, {<r_1,0.5,0.4,0.4,0.7,0.6>, <r_2,0.8,0.3,0.7,0.6, 0.5>, <r_3,0.5,0.4,0.4,0.7,0.6>, <r_4,0.9,0.8,0.7,0.1,0.8>, <r_5,0.8,0.3,0.7,0.6,0.5>}) be two SVRPNS in the approximation space (W, ρ). Clearly, $N(Q) = N(V)$.

Definition 3.6: Let $N(Q)=(\underline{N}(Q), \overline{N}(Q))$ and $N(V)=(\underline{N}(V), \overline{N}(V))$ be two R-P-NSs in the approximation space (W, ρ). Then, the intersection and union of the R-P-NSs $N(Q)$ and $N(V)$ are defined as follows:

$N(Q \cap V) = (\underline{N}(Q \cap V), \overline{N}(Q \cap V))$ and $N(Q \cup V) = (\underline{N}(Q \cup V), \overline{N}(Q \cup V))$,
where,

$\underline{N}(Q\cap V)$ = {<r, $T_{\underline{N}(Q)}(r)\wedge T_{\underline{N}(V)}(r)$, $C_{\underline{N}(Q)}(r)\wedge C_{\underline{N}(V)}(r)$, $G_{\underline{N}(Q)}(r)\vee G_{\underline{N}(V)}(r)$, $U_{\underline{N}(Q)}(r)\vee U_{\underline{N}(V)}(r)$, $F_{\underline{N}(Q)}(r)\vee F_{\underline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$};
$\overline{N}(Q\cap V)$ = {<r, $T_{\overline{N}(Q)}(r)\wedge T_{\overline{N}(V)}(r)$, $C_{\overline{N}(Q)}(r)\wedge C_{\overline{N}(V)}(r)$, $G_{\overline{N}(Q)}(r)\vee G_{\overline{N}(V)}(r)$, $U_{\overline{N}(Q)}(r)\vee U_{\overline{N}(V)}(r)$, $F_{\overline{N}(Q)}(r)\vee F_{\overline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$};
$\underline{N}(Q\cup V)$ = {<r, $T_{\underline{N}(Q)}(r)\vee T_{\underline{N}(V)}(r)$, $C_{\underline{N}(Q)}(r)\vee C_{\underline{N}(V)}(r)$, $G_{\underline{N}(Q)}(r)\wedge G_{\underline{N}(V)}(r)$, $U_{\underline{N}(Q)}(r)\wedge U_{\underline{N}(V)}(r)$, $F_{\underline{N}(Q)}(r)\wedge F_{\underline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$};
and
$\overline{N}(Q\cup V)$ = {<r, $T_{\overline{N}(Q)}(r)\vee T_{\overline{N}(V)}(r)$, $C_{\overline{N}(Q)}(r)\vee C_{\overline{N}(V)}(r)$, $G_{\overline{N}(Q)}(r)\wedge G_{\overline{N}(V)}(r)$, $U_{\overline{N}(Q)}(r)\wedge U_{\overline{N}(V)}(r)$, $F_{\overline{N}(Q)}(r)\wedge F_{\overline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$}.

Example 3.6: Let $N(Q)=(\underline{N}(Q), \overline{N}(Q))$ and $N(V)=(\underline{N}(V), \overline{N}(V))$ be two R-P-NSs in (W, ρ) as they are given in Example 3.4. Then,

$N(Q\cap V)$ = ($\underline{N}(Q\cap V)$, $\overline{N}(Q\cap V)$) = ({<r_1,0.2,0.2,0.2,0.3,0.6>, <r_2,0.5,0.2,0.6,0.2,0.4>, <r_3,0.2,0.3,0.2,0.3,0.6>, <r_4,0.9,0.2,0.7,0.1,0.8>, <r_5,0.5,0.2,0.6,0.2,0.4>}, {<r_1,0.5,0.2,0.4, 0.7,0.6>, <r_2,0.8,0.3,0.7,0.6,0.5>, <r_3,0.5,0.4,0.4,0.7,0.6>, <r_4,0.9,0.8, 0.7,0.1,0.8>, <r_5,0.8, 0.3,0.7,0.6,0.5>}),

and $N(Q\cup V)$ = ($\underline{N}(Q\cup V)$, $\overline{N}(Q\cup V)$) = ({<r_1,0.3,0.3,0.1,0.2,0.5>, <r_2,0.6,0.2,0.4,0.0,0.3>, <r_3,0.5,0.3,0.0,0.0,0.1>, <r_4,0.9,0.8,0.5,0.0,0.3>, <r_5,0.6,0.2,0.2,0.0,0.2>}, {<r_1,0.5,0.4,0.3, 0.3,0.6>, <r_2,0.8,0.3,0.5,0.1,0.3>, <r_3,0.5,0.4,0.0,0.1,0.2>, <r_4,1.0,0.9,0.6,0.0,0.4>, <r_5,0.8,0.3, 0.4,0.1,0.3>}).

Definition 3.7: Let $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ be a R-P-NSs in the approximation space (W, ρ). Then, the boundary region of the R-P-NSs $N(Q)$ is denoted by $N_B(Q)$ and defined as follows:
$N_B(Q) = \overline{N}(Q) - \underline{N}(Q)$, where $\overline{N}(Q) - \underline{N}(Q) = \overline{N}(Q) \cap \underline{N}(Q)^c$.

Theorem 3.1: Let $N(Q)=(\underline{N}(Q), \overline{N}(Q))$ and $N(V)=(\underline{N}(V), \overline{N}(V))$ be two R-P-NSs in the approximation space (W, ρ). Then, the following holds:
(i) $N(Q\cap V)^c = N(Q)^c\cup N(V)^c$;
(ii) $N(Q\cup V)^c= N(Q)^c\cap N(V)^c$.

Proof. Let $N(Q) = (\underline{N}(Q), \overline{N}(Q))$ = ({<r,$T_{\underline{N}(Q)}(r)$,$C_{\underline{N}(Q)}(r)$,$G_{\underline{N}(Q)}(r)$,$U_{\underline{N}(Q)}(r)$,$F_{\underline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$}, {<r,$T_{\overline{N}(Q)}(r)$,$C_{\overline{N}(Q)}(r)$,$G_{\overline{N}(Q)}(r)$,$U_{\overline{N}(Q)}(r)$,$F_{\overline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$}), and N(V) = ($\underline{N}(V)$, $\overline{N}(V)$) = ({<r, $T_{\underline{N}(V)}(r)$,$C_{\underline{N}(V)}(r)$,$G_{\underline{N}(V)}(r)$,$U_{\underline{N}(V)}(r)$,$F_{\underline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$}, {<r,$T_{\overline{N}(V)}(r)$, $C_{\overline{N}(V)}(r)$, $G_{\overline{N}(V)}(r)$, $U_{\overline{N}(V)}(r)$,$F_{\overline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$}) be two R-P-NS in the approximation space (W, ρ).

Then, $N(Q\cap V) = (\underline{N}(Q\cap V), \overline{N}(Q\cap V))$
= ({<r,$T_{\underline{N}(Q)}(r)\wedge T_{\underline{N}(V)}(r)$,$C_{\underline{N}(Q)}(r)\wedge C_{\underline{N}(V)}(r)$, $G_{\underline{N}(Q)}(r)\vee G_{\underline{N}(V)}(r)$, $U_{\underline{N}(Q)}(r)\vee U_{\underline{N}(V)}(r)$, $F_{\underline{N}(Q)}(r)\vee F_{\underline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$}, {<r, $T_{\overline{N}(Q)}(r)\wedge T_{\overline{N}(V)}(r)$, $C_{\overline{N}(Q)}(r)\wedge C_{\overline{N}(V)}(r)$, $G_{\overline{N}(Q)}(r)\vee G_{\overline{N}(V)}(r)$, $U_{\overline{N}(Q)}(r)\vee U_{\overline{N}(V)}(r)$, $F_{\overline{N}(Q)}(r)\vee F_{\overline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$}).

This implies,
$N(Q\cap V)^c$=({<r, $F_{\underline{N}(Q)}(r)\vee F_{\underline{N}(V)}(r)$, $U_{\underline{N}(Q)}(r)\vee U_{\underline{N}(V)}(r)$, 1-($G_{\underline{N}(Q)}(r)\vee G_{\underline{N}(V)}(r)$), $C_{\underline{N}(Q)}(r)\wedge C_{\underline{N}(V)}(r)$, $T_{\underline{N}(Q)}(r)\wedge T_{\underline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$}, {<r, $F_{\overline{N}(Q)}(r)\vee F_{\overline{N}(V)}(r)$, $U_{\overline{N}(Q)}(r)\vee U_{\overline{N}(V)}(r)$, 1-($G_{\overline{N}(Q)}(r)\vee G_{\overline{N}(V)}(r)$), $C_{\overline{N}(Q)}(r)\wedge C_{\overline{N}(V)}(r)$, $T_{\overline{N}(Q)}(r)\wedge T_{\overline{N}(V)}(r)$> : $p\in[r]_\rho$, $r\in W$}).

Also, we have
$N(Q)^c$=({<r, $F_{\underline{N}(Q)}(r)$, $U_{\underline{N}(Q)}(r)$, 1-$G_{\underline{N}(Q)}(r)$, $C_{\underline{N}(Q)}(r)$, $T_{\underline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$}, {<r, $F_{\overline{N}(Q)}(r)$, $U_{\overline{N}(Q)}(r)$, 1-$G_{\overline{N}(Q)}(r)$, $C_{\overline{N}(Q)}(r)$, $T_{\overline{N}(Q)}(r)$> : $p\in[r]_\rho$, $r\in W$})

and $N(V)^c=(\{<r, F_{\underline{N}(V)}(r), U_{\underline{N}(V)}(r), 1\text{-}G_{\underline{N}(V)}(r), C_{\underline{N}(V)}(r), T_{\underline{N}(V)}(r)> \,:\, p\in[r]_\rho, r\in W\}, \{<r, F_{\overline{N}(V)}(r), U_{\overline{N}(V)}(r), 1\text{-}G_{\overline{N}(V)}(r), C_{\overline{N}(V)}(r), T_{\overline{N}(V)}(r)> : p\in[r]_\rho, r\in W\})$.

Now, $N(Q)^c \cup N(V)^c$

$= N(Q^c \cup V^c)$

$= (\underline{N}(Q^c \cup V^c), \overline{N}(Q^c \cup V^c))$

$= (\{<r, F_{\underline{N}(Q)}(r)\vee F_{\underline{N}(V)}(r), U_{\underline{N}(Q)}(r)\vee U_{\underline{N}(V)}(r), (1\text{-}G_{\underline{N}(Q)}(r))\wedge(1\text{-}G_{\underline{N}(V)}(r)), C_{\underline{N}(Q)}(r)\wedge C_{\underline{N}(V)}(r), T_{\underline{N}(Q)}(r)\wedge T_{\underline{N}(V)}(r)> \,:\, p\in[r]_\rho, r\in W\}, \{<r, F_{\overline{N}(Q)}(r)\vee F_{\overline{N}(V)}(r), U_{\overline{N}(Q)}(r)\vee U_{\overline{N}(V)}(r), (1\text{-}G_{\overline{N}(Q)}(r))\wedge(1\text{-}G_{\overline{N}(V)}(r)), C_{\overline{N}(Q)}(r)\wedge C_{\overline{N}(V)}(r), T_{\overline{N}(Q)}(r)\wedge T_{\overline{N}(V)}(r)> : p\in[r]_\rho, r\in W\})$

$= (\{<r, F_{\underline{N}(Q)}(r)\vee F_{\underline{N}(V)}(r), U_{\underline{N}(Q)}(r)\vee U_{\underline{N}(V)}(r), 1\text{-}(G_{\underline{N}(Q)}(r)\vee G_{\underline{N}(V)}(r)), C_{\underline{N}(Q)}(r)\wedge C_{\underline{N}(V)}(r), T_{\underline{N}(Q)}(r)\wedge T_{\underline{N}(V)}(r)> \,:\, p\in[r]_\rho, r\in W\}, \{<r, F_{\overline{N}(Q)}(r)\vee F_{\overline{N}(V)}(r), U_{\overline{N}(Q)}(r)\vee U_{\overline{N}(V)}(r), 1\text{-}(G_{\overline{N}(Q)}(r)\vee G_{\overline{N}(V)}(r)), C_{\overline{N}(Q)}(r)\wedge C_{\overline{N}(V)}(r), T_{\overline{N}(Q)}(r)\wedge T_{\overline{N}(V)}(r)> : p\in[r]_\rho, r\in W\})$

$= N(Q\cap V)^c$.

Hence, $N(Q\cap V)^c = N(Q)^c \cup N(V)^c$.

(*ii*) Similarly, it can be established that $N(Q\cup V)^c = N(Q)^c \cap N(V)^c$.

4. Rough Pentapartitioned Neutrosophic Topology

In this section, we introduce the idea of rough pentapartitioned neutrosophic topology and study some of its properties.

Definition 4.1: Let $N(Q)=(\underline{N}(Q), \overline{N}(Q))$ be a R-P-NS in the approximation space (W, ρ). Then, $\tau_{PRNS}(\rho) = \{1_{PN}, 0_{PN}, \underline{N}(Q), \overline{N}(Q), N_B(Q)\}$ is called a rough pentapartitioned neutrosophic topology (RPNT) which guarantees the following postulates:

(*i*) 1_{PN} and 0_{PN} belongs to $\tau_{\mathrm{RPNT}}(\rho)$;

(*ii*) Arbitrary union of members of $\tau_{\mathrm{RPNT}}(\rho)$ belongs to $\tau_{\mathrm{RPNT}}(\rho)$;

(*iii*) Finite intersection of members of $\tau_{\mathrm{RPNT}}(\rho)$ belongs to $\tau_{\mathrm{RPNT}}(\rho)$.

Then, $(W, \tau_{\mathrm{RPNT}}(\rho))$ is called a rough pentapartitioned neutrosophic topological space (in short RPNT-space), if $\tau_{\mathrm{RPNT}}(\rho)$ is a rough pentapartitioned neutrosophic topology (in short RPNT).

Definition 4.2: Let $(W, \tau_{\mathrm{RPNT}}(\rho))$ be a RPNT-space. Then, the members of $\tau_{\mathrm{RPNT}}(\rho)$ are called rough pentapartitioned neutrosophic open set (in short RPNO-set). A R-P-NS is said to be a rough pentapartitioned neutrosophic closed set (in short RPNC-set) if its complement belongs to $\tau_{\mathrm{RPNT}}(\rho)$.

Proposition 4.3: Let $(W, \tau_{\mathrm{RPNT}}(\rho))$ be a RPNT-space. Then,

(*i*) Both 1_{PN} and 0_{PN} are NSR-closed sets;

(*ii*) Arbitrary intersection of RPN-closed sets is also a RPN-closed set;

(*iii*) Finite union of RPN-closed sets is also a RPN-closed set.

Definition 4.3: Let $(W, \tau_{\mathrm{RPNT}}(\rho))$ be a RNPT-space such that $\tau_{\mathrm{RPNT}}(\rho)=\{1_{PN}, 0_{PN}\}$. Then, $\tau_{\mathrm{RPNT}}(\rho)$ is called a RPN-indiscrete topology on W w.r.t ρ and corresponding space is said to be a RPN-indiscrete topological space.

Definition 4.4: Let $(W, \tau_{\mathrm{RPNT}}(\rho))$ be a RPNT-space and A be a rough pentapartitioned neutrosophic set over W. Then, the collection $\tau_{\mathrm{RPNT}}(A) = \{ B_i \cap A : B_i \in \tau_{\mathrm{RPNT}}(\rho), i \in N \}$ is also a rough pentapartitioned neutrosophic topology on W. Then, $(A, \tau_{\mathrm{RPNT}}(\rho))$ is called a rough pentapartitioned neutrosophic topological subspace (in short RPNT-subspace) of $(W, \tau_{\mathrm{RPNT}}(\rho))$.

Definition 4.5: Let $(W, \tau'_{\mathrm{RPNT}}(\rho))$ and $(W, \tau_{\mathrm{RPNT}}(\rho))$ be two RPNT-spaces. Then, $(W, \tau'_{\mathrm{RPNT}}(\rho))$ is finer than $(W, \tau_{\mathrm{RPNT}}(\rho))$ if and only if $\tau'_{\mathrm{RPNT}}(\rho) \supseteq \tau_{\mathrm{RPNT}}(\rho)$.

Definition 4.6: Let $(W, \tau_{\mathbf{RPNT}}(\rho))$ be a RPNT-space w.r.t ρ and K be an arbitrary rough pentapartitioned neutrosophic subset of W. Then, the RPN-interior (in short Int_{RPN}) of K is union of all rough pentapartitioned neutrosophic open (in short RPN-O) subsets of K.
Clearly, $Int_{\mathrm{RPN}}(K)$ is the largest RPN-O set contained in K.

Theorem 4.1: Let $(W, \tau_{\mathbf{RPNT}}(\rho))$ be a RPNT-space w.r.t ρ. Let M and N be two RPN-sets over W. Then,

(*i*) $Int_{\mathrm{RPN}}(0_N) = 0_N$ and $Int_{\mathrm{RPN}}(1_N) = 1_N$.
(*ii*) $Int_{\mathrm{RPN}}(M) \subseteq M$.
(*iii*) M is RPN-O set if and only if $Int_{\mathrm{RPN}}(M) = M$.
(*iv*) $Int_{\mathrm{RPN}}(Int_{\mathrm{RPN}}(M)) = Int_{\mathrm{RPN}}(M)$.
(*v*) $M \subseteq N$ implies $Int_{\mathrm{RPN}}(M) \subseteq Int_{\mathrm{RPN}}(N)$.
(*vi*) $Int_{\mathrm{RPN}}(M) \cup Int_{\mathrm{RPN}}(N) \subseteq Int_{\mathrm{RPN}}(M \cup N)$.
(*vii*) $Int_{\mathrm{RPN}}(M) \cap Int_{\mathrm{RPN}}(N) = Int_{\mathrm{RPN}}(M \cap N)$.

Proof. (*i*) By Definition 4.6., $Int_{\mathrm{RPN}}(\mathrm{A}) \subseteq \mathrm{A}$. If we put $\mathrm{A}=0_{PN}$ in $Int_{\mathrm{RPN}}(\mathrm{A}) \subseteq \mathrm{A}$, we have $Int_{\mathrm{RPN}}(0_{PN}) \subseteq 0_{PN}$. Further, it is known that $0_{PN} \subseteq Int_{\mathrm{RPN}}(0_{PN})$. Therefore, $Int_{\mathrm{RPN}}(0_{PN}) = 0_{PN}$. Similarly, it can be shown that $Int_{\mathrm{RPN}}(1_{PN})=1_{PN}$.
(*ii*) By Definition 4.6., $Int_{\mathrm{RPN}}(M)$ is the largest RPN-O set which is contained in M. Hence, $Int_{\mathrm{RPN}}(M) \subseteq M$.
(*iii*) For any RPN-set M. we have, $Int_{\mathrm{RPN}}(M) \subseteq M$. Since, M is a RPN-O set, so it is the largest RPN-O set contained in M. Therefore, $Int_{\mathrm{RPN}}(M)=M$.
(*iv*) For any RPN-set M, we have $Int_{\mathrm{RPN}}(M) \subseteq M$. Now, $Int_{\mathrm{RPN}}(M)$ is the largest RPN-O set contained in M, and $Int_{\mathrm{RPN}}(Int_{\mathrm{RPN}}(M)) \subseteq Int_{\mathrm{RPN}}(M)$. Hence, by using the third part of this theorem $Int_{\mathrm{RPN}}(Int_{\mathrm{RPN}}(M))=Int_{\mathrm{RPN}}(M)$.
(*v*) Let M and N be two RPN-sets over W such that $M \subseteq N$. Therefore, $Int_{\mathrm{RPN}}(M) \subseteq M$ and $Int_{\mathrm{RPN}}(N) \subseteq N$. Now, $Int_{\mathrm{RPN}}(M) \subseteq M \subseteq N$. This implies, $Int_{\mathrm{RPN}}(M) \subseteq N$. Therefore, $Int_{\mathrm{RPN}}(M)$ is a RPN-O set contained in N. Again, $Int_{\mathrm{RPN}}(N)$ be the largest RPN-O set contained in N. Hence, $Int_{\mathrm{RPN}}(M) \subseteq Int_{\mathrm{RPN}}(N)$.
(*vi*) For any two RPN-sets M and N, we have $M \subseteq M \cup N$ and $N \subseteq M \cup N$.
By using the above results we have, $Int_{\mathrm{RPN}}(M) \subseteq Int_{\mathrm{RPN}}(M \cup N)$ and $Int_{\mathrm{RPN}}(N) \subseteq Int_{\mathrm{RPN}}(M \cup N)$.
This implies, $Int_{\mathrm{RPN}}(M) \cup Int_{\mathrm{RPN}}(N) \subseteq Int_{\mathrm{RPN}}(M \cup N)$ (1)
It is known that $Int_{\mathrm{RPN}}(M) \subseteq M$ and $Int_{\mathrm{RPN}}(N) \subseteq N$. This implies, $Int_{\mathrm{RPN}}(M) \cup Int_{\mathrm{RPN}}(N) \subseteq M \cup N$. Since, the union of two RPN-O sets is again a RPN-O set in $(W, \tau_{\mathbf{RPNT}}(\rho))$, so $Int_{\mathrm{RPN}}(M) \cup Int_{\mathrm{RPN}}(N)$ is a RPN-O set. Therefore, $Int_{\mathrm{RPN}}(M) \cup Int_{\mathrm{RPN}}(N)$ is a RPN-O set contained in $M \cup N$. But we know that $Int_{\mathrm{RPN}}(M \cup N)$ is the largest RPN-O set contained in $M \cup N$. Therefore, $Int_{\mathrm{RPN}}(M \cup N) \subseteq Int_{\mathrm{RPN}}(M) \cup Int_{\mathrm{RPN}}(N)$ (2)
From eq. (1) and eq. (2), we have $Int_{\mathrm{RPN}}(M \cup N)=Int_{\mathrm{RPN}}(M) \cup Int_{\mathrm{RPN}}(N)$.
(*vii*) For any two RPN sets M and N we have, $M \cap N \subseteq M$ and $M \cap N \subseteq N$.
By using the above results we have, $Int_{\mathrm{RPN}}(M \cap N) \subseteq Int_{\mathrm{RPN}}(M)$ and $Int_{\mathrm{RPN}}(M \cap N) \subseteq Int_{\mathrm{RPN}}(N)$.
This implies, $Int_{\mathrm{RPN}}(M \cap N) \subseteq Int_{\mathrm{RPN}}(M) \cap Int_{\mathrm{RPN}}(N)$ (3)
It is known that $Int_{\mathrm{RPN}}(M) \subseteq M$ and $Int_{\mathrm{RPN}}(N) \subseteq N$. This implies, $Int_{\mathrm{RPN}}(M) \cap Int_{\mathrm{RPN}}(N) \subseteq M \cap N$. Since, the intersection of two RPN-O sets is also a RPN-O set, so $Int_{\mathrm{RPN}}(M) \cap Int_{\mathrm{RPN}}(N)$ is a RPN-O set. It is known that $Int_{\mathrm{RPN}}(M \cap N)$ is the largest RPN-O set which is contained in $M \cap N$. Therefore, $Int_{\mathrm{RPN}}(M) \cap Int_{\mathrm{RPN}}(N) \subseteq Int_{\mathrm{RPN}}(M \cap N)$ (4)
From (3) and (4), we have $Int_{\mathrm{RPN}}(M) \cap Int_{\mathrm{RPN}}(N) \subseteq Int_{\mathrm{RPN}}(M \cap N)$.

Definition 4.7: Let $(\mathrm{W}, \tau_{\mathbf{RPNT}}(\rho))$ be a RPNT-space. Suppose that K be a RPN-subset of W. Then, *RPN*-closure (Cl_{RPN}) of K is the intersection of all RPN-C supersets of K.

Theorem 4.2: Let $(W, \tau_{\mathbf{RPNT}}(\rho))$ be a RPNT-space over W. Suppose that M and N be two RPN-subsets of W. Then, the following holds:
(*i*) $Cl_{\mathrm{RPN}}(0_{PN})=0_{PN}$ and $Cl_{\mathrm{RPN}}(1_{PN})=1_{PN}$;

(*ii*) $M \subseteq Cl_{\text{RPN}}(M)$;
(*iii*) M is RPN-C set if and only if $M = Cl_{\text{RPN}}(M)$;
(*iv*) $Cl_{\text{RPN}}(Cl_{\text{RPN}}(M)) = Cl_{\text{RPN}}(M)$;
(*v*) $M \subseteq N$ implies $Cl_{\text{RPN}}(M) \subseteq Cl_{\text{RPN}}(N)$;
(*vi*) $Cl_{\text{RPN}}(M \cup N) = Cl_{\text{RPN}}(M) \cup Cl_{\text{RPN}}(N)$;
(*vii*) $Cl_{\text{RPN}}(M \cap N) \subseteq Cl_{\text{RPN}}(M) \cap Cl_{\text{RPN}}(N)$.

Proof. (*i*) By definition of RPNT, 0_{PN} and 1_{PN} are the smallest and largest RPN-O set as well as RPN-C set. Therefore, $Cl_{\text{RPN}}(0_{PN}) = 0_{PN}$ and $Cl_{\text{RPN}}(1_{PN}) = 1_{PN}$.

(*ii*) It is known that $Cl_{\text{RPN}}(M)$ is the smallest RPN-C set containing M, for any RPN-set M. Therefore, $M \subseteq Cl_{\text{RPN}}(M)$.

(*iii*) Since, the smallest RPN-C set which contains M is $Cl_{\text{RPN}}(M)$. Again, M is closed. So, the only possible case is $M = Cl_{\text{RPN}}(M)$.

Conversely, let $M = Cl_{\text{RPN}}(M)$. Since, $Cl_{\text{RPN}}(M)$ is the RPN-C set, so M is a RPN-C set.

(*iv*) For any RPN-set M, $Cl_{\text{RPN}}(M)$ is the smallest RPN-C set which contains M. Again, $Cl_{\text{RPN}}(M) = M$, for any RPN-C set M. Hence, $Cl_{\text{RPN}}(Cl_{\text{RPN}}(M)) = Cl_{\text{RPN}}(M)$.

(*v*) Assume that M and N be two RPN-subsets of a RPNT-space $(W, \tau_{\text{RPNT}}(\rho))$ such that $M \subseteq N$.

Now $Cl_{\text{RPN}}(M) = \cap\{Z: Z \text{ is a RPN-C set in } (W, \tau_{\text{RPNT}}(\rho)) \text{ and } M \subseteq Z\}$
$\subseteq \cap\{Z: Z \text{ is a RPN-C set in } (W, \tau_{\text{RPNT}}(\rho)) \text{ and } N \subseteq Z\}$ [since $M \subseteq N$]
$= Cl_{\text{RPN}}(N)$

This implies, $Cl_{\text{RPN}}(M) \subseteq Cl_{\text{RPN}}(N)$.

Hence, $M \subseteq N \Rightarrow Cl_{\text{RPN}}(M) \subseteq Cl_{\text{RPN}}(N)$.

(*vi*) Let M and N be two RPN-subsets of a RPNT-space $(W, \tau_{\text{RPNT}}(\rho))$. Clearly, $M \subseteq M \cup N$ and $N \subseteq M \cup N$. It is known that $Cl_{\text{RPN}}(M) \subseteq Cl_{\text{RPN}}(M \cup N)$ and $Cl_{\text{RPN}}(N) \subseteq Cl_{\text{RPN}}(M \cup N)$. This implies, $Cl_{\text{RPN}}(M) \cup Cl_{\text{RPN}}(N) \subseteq Cl_{\text{RPN}}(M \cup N)$ (5)

It is also known that, $M \subseteq Cl_{\text{RPN}}(M)$ and $N \subseteq Cl_{\text{RPN}}(N)$. This implies, $M \cup N \subseteq Cl_{\text{RPN}}(M) \cup Cl_{\text{RPN}}(N)$. Since, the union of two RPN-C sets is again a RPN-C set in $(W, \tau_{\text{RPNT}}(\rho))$, so $Cl_{\text{RPN}}(M) \cup Cl_{\text{RPN}}(N)$ is a RPN-C set. Therefore, $Cl_{\text{RPN}}(M) \cup Cl_{\text{RPN}}(N)$ is a RPN-C set which contains $M \cup N$. But we know that $Cl_{\text{RPN}}(M \cup N)$ is the smallest RPN-C set which contains $M \cup N$. Therefore, $Cl_{\text{RPN}}(M \cup N) \subseteq Cl_{\text{RPN}}(M) \cup Cl_{\text{RPN}}(N)$. (6)

From (5) and (6), we have $Cl_{\text{RPN}}(M \cup N) \subseteq Cl_{\text{RPN}}(M) \cup Cl_{\text{RPN}}(N)$.

(*vii*) Let M and N be two RPN-subsets of a RPNT-space $(W, \tau_{\text{RPNT}}(\rho))$. It is known that $M \cap N \subseteq M$ and $M \cap N \subseteq N$. By a known result, we have $Cl_{\text{RPN}}(M \cap N) \subseteq Cl_{\text{RPN}}(M)$ and $Cl_{\text{RPN}}(M \cap N) \subseteq Cl_{\text{RPN}}(N)$. This implies, $Cl_{\text{RPN}}(M \cap N) \subseteq Cl_{\text{RPN}}(M) \cap Cl_{\text{RPN}}(N)$.

5. Conclusions

In this article, we have established the concept of R-P-NS and studied several operations on them. Further, we have applied the concept of topology on R-P-NSs and introduced the notions of RPNT-space, and studied its basic properties, operations. In the future, we hope that based on the concept of RPNT, researchers can solve many complicated problems involving truth, contradiction, ignorance, unknown and falsity membership functions and many multi attribute decision making strategy can be formed.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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