Statistical Study of Nuclear Energy Levels for 6 Particles in The Nuclear *N82* Model Space

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Abstract

The statistical fluctuations of nuclear excitation energies in ^{138}Ba nucleus are investigated. This nucleus is described as a core of ^{132}Sn with 6 active protons move in the space of $N82$. The OXBASH shell-model program together with the interaction of $N82K$ are used to obtain the excitation energies in ^{138}Ba. To reflect the transition from regular (ordered) to chaos in ^{138}Ba, we consider different strengths parameter (β) to the off diagonal elements of $N82K$. The level density for the states $J^{\pi}T = 2^+3$ is found to have a Gaussian shape, which is in agreement with the predicted studies for a many-body system with two-body residual interaction. Both distributions for the level spacing $P(s)$ and $\Delta_3 -$ statistics show an ordered performance at strength parameter $\beta = 0$, a chaotic performance at $\beta \geq 0.3$ and an in-between situation at $0 < \beta < 0.3$.

Keywords: Chaos in many body quantum systems; Transition from ordered to chaos; Level density of states; Spectral fluctuations; Calculations of the shell- model with N82 space PACS number(s): 24.60.Lz, 21.60.Cs, 21.10.Ky

دراسة أحصائية لمستويات الطاقة النووية لِستة جسيمات في إنموذج الفضاء النووي-N82

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الخلاصة

تم دراسة التموجات الأحصائية لمستويات الطاقة النووية لِستة جسيمات تتحرك ضمن إنموذج الفضاء-N82 (والتي تُكوَن النواة ^{138}Ba مع قلب مغلق ^{132}Sn) . أجرينا حسابات إنموذج القشرة النووي لإيجاد مستويات الطاقة النووية للنواة ^{138}Ba عن طريق أستخدام التفاعل المؤثر N82K مع شدات (β) مختلفة لعناصر المصفوفة الغير قطرية لهذا التفاعل.

في هذه الدراسة تم البحث عن السلوك الفوضوي للنواة ^{138}Ba وكيفية أنتقالها الى السلوك المنتظم. وجدنا بأن كثافة مستويات الطاقة للحالات النووية $J^{\pi}T = 2^+3$ لها شكل كاوسي (Gaussian shape) والتي تتفق تماماً مع الدراسات النظرية الأخرى لنظام متعدد الجسيمات ذو التفاعل المتبقي للجسيمتين. أظهرت هذه

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الدراسة بأن الأحصاء $P(s)$ والأحصاء Δ_3 لهما سلوك منتظم عندما $\beta = 0$ ، ولهما سلوك فوضوي عندما

$\beta \geq 0.3$ ، ولهما حالة وسطية عندما $0.3 > \beta > 0$.

1. Introduction

Chaotic properties of many body quantum systems were searched tremendously in the previous thirty years [1]. A relationship between chaos in the classical-system and the spectral fluctuations of the similar quantum-system is recommended by Bohigas et al. [2], where a systematic confirmation of this recommendation [2] is presented in [3]. At the present, it is naturally recognized that quantum equivalents of classically chaotic-systems reveal statistical fluctuations that reach agreement with the random matrix theory (RMT) [4, 5] whereas quantum equivalents of classically ordered-systems reveal statistical fluctuations that reach agreement with a Poisson distribution. For systems that are invariant under time reversal, the proper formula of the RMT is the Gaussian orthogonal ensemble (GOE). RMT had been primarily utilized to illustrate the behavior of fluctuations in the neutron resonances of compound-nuclei [6]. RMT has come to be a typical implement for investigating the general manners of fluctuations in chaotic systems [7-10].

In general, one can use the mean-field approach to investigate the chaotic performances for dynamics of the single-particle in atomic nuclei. On the other hand, the mixing of various mean-field configurations by the residual interaction leads to affect the behavior of fluctuations of many-particle excitation energies and wave functions. In addition, one can utilize different nuclear models to analyze these fluctuations. Alhassid et al. [11, 12] used the framework of the interacting boson-model (IBM) to study the fluctuation properties of the low-lying energy spectrum, where the nuclear space is drawn onto a considerably lesser space of bosonic-degrees of freedom. As a result of the comparatively slight number of degrees of freedom in the IBM, it had been also probable to connect the statistics to the essential mean-field collective dynamic. At the region of high-lying energy spectrum, extra degrees of freedom come to be significant [13], as well as the influences of interactions on the statistical properties have to be searched in bigger spaces. The nuclear shell model is considered as a good context for these studies, where many effective interactions for various model spaces are existed as well as the basis states are characterized via good quantum numbers of total angular momentum (J), isospin (T) and parity (π) [14].

The distribution of the wave function components [15-19] was analyzed by the perspective of the shell model. Brown and Bertsch [17] indicated that the distribution of the basis vector amplitudes is in agreement with that of Gaussian (i.e., consistent with the prediction of the GOE) at the region of higher excitation energies and departed from Gaussian performance in further regions unless the evaluations assumes degenerate single particle energies. Zelevinsky et al. [19] as well recommended that evaluations with degenerate single particle energies have chaotic performance at lower excitation energies than other realistic evaluations.

It is known that the nuclear observables of electromagnetic transition intensities are sensible to the nuclear wave functions. Therefore, the analysis of their statistical fluctuations should supply the analysis of spectral fluctuations and assist as an additional mark of chaos in quantum systems. Hamoudi et al. [20] performed calculations in the fp-shell to analyze the fluctuation properties of excitation energies and electromagnetic intensities in some of ^{60}A nuclei using an effective interaction of F5P [21]. The estimated outcomes were consistent with RMT and with the earlier outcomes of [15-19]. The influence of one body hamiltonian on the statistical fluctuations of excitation energies and electromagnetic intensities in ^{136}Xe was investigated by Hamoudi [22], where an explicit quantum mark for violation the chaoticity was remarked with enlarging the values of single particle energies. Later, full fp shell calculations were carried out by Hamoudi et al. [23]. They [23] used the FPD6 [24] as an effective interaction and looked for the transition from regular to chaos in the calculated excitation energies and electromagnetic intensities of ^{44}V nucleus. The outcomes revealed that the transition from regular to chaos is possible through implementing different strengths to the off diagonal interaction of FPD6. Recently, Hamoudi et al [25] have accomplished calculations in the space of sd shell to analyze the chaotic properties of energy spectra in ^{32}A (^{32}S, ^{32}P and ^{32}Si) nuclei. They [25] have adopted an empirical effective interaction of W [26] in the isospin formalism. The results have been well described by the GOE of random matrices with no dependency on J and T. Subsequently, Hamoudi et al [27] have repeated their study as in [25] but this time chosen an

empirical effective interaction of WPN [26] in the proton-neutron formalism. The spectral fluctuations in ^{32}A nuclei have been found to have an intermediate behavior between Wigner and Poisson limits. Besides, they move gradually toward the GOE limit when going over ^{32}S, ^{32}P and ^{32}Si nuclei, respectively. Moreover, they are independent of the spin J.

There has been no comprehensive investigation for chaotic dynamics of the nuclear energy spectrum in ^{138}Ba nucleus. We thus, in this research, examine the effect of interaction strengths (β) on the chaotic properties of excitation energies in ^{138}B. Here, the nucleus ^{138}B is described as an inert core of ^{132}Sn with 6 protons in the $N82$ space.

2. Theory

In the viewpoint of the shell model, the effective hamiltonian of many body quantum system is given by [14]

$$H = H_0 + H', \tag{1}$$

where H_0 is the one body term given by

$$H_0 = \sum_{\lambda} e_{\lambda} a_{\lambda}^+ a_{\lambda} \tag{2}$$

and H' is the two body residual interaction specified by

$$H' = \frac{1}{4}\sum V_{\lambda\mu;\nu\rho} a_{\lambda}^+ a_{\mu}^+ a_{\nu} a_{\rho}. \tag{3}$$

Here the symbols λ, μ, ν and ρ exemplify the single particle orbitals which have for example $\lambda \equiv (ljm\tau)$. The quantum numbers l, j, m and τ is the orbital, total angular momentum, projection ($j_z = m$) and isospin projection, respectively.

The wave functions for many body quantum systems are constructed in terms of the $m-$scheme picture. For certain J and T, with single-particle maximum spin and isospin projection [14],

$$\left| M = J, \; T = T_3; m \right\rangle. \tag{4}$$

In eq. (4), m denotes the $m-$scheme picture.

The many body hamiltonian

$$H_{kk'}^{JT} = \sum_k \left\langle JT; k \left| H \right| JT; k' \right\rangle \tag{5}$$

is ultimately diagonalized to get the energies E_{α} and wave functions

$$\left| JT; \alpha \right\rangle = \sum_k C_k^{\alpha} \left| JT; k \right\rangle, \tag{6}$$

where the energies E_{α} are the essential physical quantities of this study.

The statistical fluctuations of nuclear excitation energies are investigated by two statistics: these are the nearest neighbor's level spacing $P(s)$ and Δ_3 (Dyson-Mehta) - statistics [4, 29]. The staircase function $N(E)$ of calculated energy spectrum is initially constructed, where $N(E)$ is specified as the energy levels number N with excitation energies $\leq E$. A smooth fitting to $N(E)$ is made through appropriate polynomial fit. The calculated spectrum is then unfolded via the mapping [12]

$$\tilde{E}_i = \tilde{N}(E_i). \tag{7}$$

It is relevant to indicate that the unfolded energy levels $\tilde{E}_i$ possess a constant average spacing while the real spacings reveal strong fluctuations.

The $P(s)$ distribution (which characterizes the fluctuations of the short range relationships among energy levels) is specified by the possibility of two adjacent levels to be a distance s away from each other. The spacings s_i are calculated from the unfolded energy levels via the relation: $s_i = \tilde{E}_{i+1} - \tilde{E}_i$. For classically regular system, one expects to find the Poisson distribution

$$P(s) = \exp(-s). \tag{8}$$

Whereas for classically chaotic system, one expects to have the Wigner distribution

$$P(s) = (\pi/2)s\exp(-\pi s^2/4),\tag{9}$$

which is in agreement with the GOE limit.

In this study, the calculated distribution of $P(s)$ is fitted to the Brody distribution

$$P(s,\omega) = \alpha(\omega+1)s^\omega \exp(-\alpha s^{\omega+1}),\tag{10}$$

where

$$\alpha = \left(\Gamma\left[\frac{\omega+2}{\omega+1}\right]\right)^{\omega+1}\tag{11}$$

The distribution of Brody interposes between the Poisson (with $\omega = 0$) and Wigner (with $\omega = 1$) distributions. In this work, the limitation ω is employed as a simple numerical measure for the degree of chaoticity.

The $\Delta_3 -$ statistic (which exemplifies the fluctuations of the long range relationships among energy levels) is utilized to find the rigidity of the nuclear energy levels and given by [4]

$$\Delta_3(\alpha, L) = \min_{A,B}\frac{1}{L}\int_\alpha^{\alpha+L}\left[N(\tilde{E}) - (A\tilde{E} + B)\right]^2 d\tilde{E}.\tag{12}$$

This statistic determines the abnormality of the staircase function (of the unfolded spectrum) from a straight line. A rigid spectrum corresponds to smaller values of Δ_3 whereas a soft spectrum has a larger Δ_3. To get a smoother function $\overline{\Delta}_3(L)$, we average $\Delta_3(L)$ over several n_α intervals $(\alpha, \alpha + L)$

$$\overline{\Delta}_3(L) = \frac{1}{n_\alpha}\sum_\alpha \Delta_3(\alpha, L).\tag{13}$$

The successive intervals are taken to overlap by $L/2$.
For regular system, we guess to find the Poisson limit
$$\Delta_3(L) = L/15.\tag{14}$$
But for chaotic system, we foresee to obtain the GOE limit

$$\Delta_3 \approx \begin{bmatrix} L/15 & (for\ smal\ L) \\ \pi^{-2}\ln L & (for\ l\arg e\ L) \end{bmatrix}\tag{15}$$

3. Results and discussion

The statistical fluctuations of the nuclear excitation energies in ^{138}Ba nucleus are investigated by means of the interacting shell model. This nucleus is supposed to have a core of ^{132}Sn and the remaining six protons move in the $N82$ space defined by $2d_{5/2}, 1g_{7/2}, 1h_{11/2}, 3s_{1/2}$ and $2d_{3/2}$ orbitals. The isospin-conserving $N82K$ interaction [28] is taken as an effective interaction together with realistic single particle energies. Shell model calculations for ^{138}Ba nucleus are carried out by the code OXBASH [30] using various strengths β to the off diagonal elements of $N82K$ interaction. The statistical properties of nuclear energy levels are studied for states which possess similar spin (J), parity (π) and isospin (T). In this study, we generally use the class of states 2^+3 (which possesses a dimension of 2134) as a case test and involve all obtainable energy levels in the analysis.

3.1. Level density

In Figure-1, the calculated level density $\rho(E)$ (histograms) for 2^+3 class of states in ^{138}Ba is displayed with different interaction strength (β). The Gaussian fit [31] (the dashed line) is also displayed for comparison. Fig. 1(a) demonstrates the result (histograms) of the diagonalization with no off-diagonal residual interaction (i.e., with $\beta = 0$) and Fig. 1(i) characterizes the result of the diagonalization with the full hamiltonian (i.e., with $\beta = 1$). It is obvious that the histograms in

Figures-1(a) and 1(i) are indistinguishable, with the exception of a shift in energy as a whole. This figure reveals that the level density suddenly evolves along with the excitation energy, comes to its maximum at the central of the spectrum and then decreases again for the highest energy. This behavior of the high energy, and the rough symmetry relative to the mid of the spectrum, are fake features of models with finite Hilbert space which is in disagreement to actual many-body systems. This figure as well exhibits that the calculated histograms possesses a Gaussian shape, which is in accordance with the prediction of Brody et al. [7] for a many-body system with two-body residual interaction.
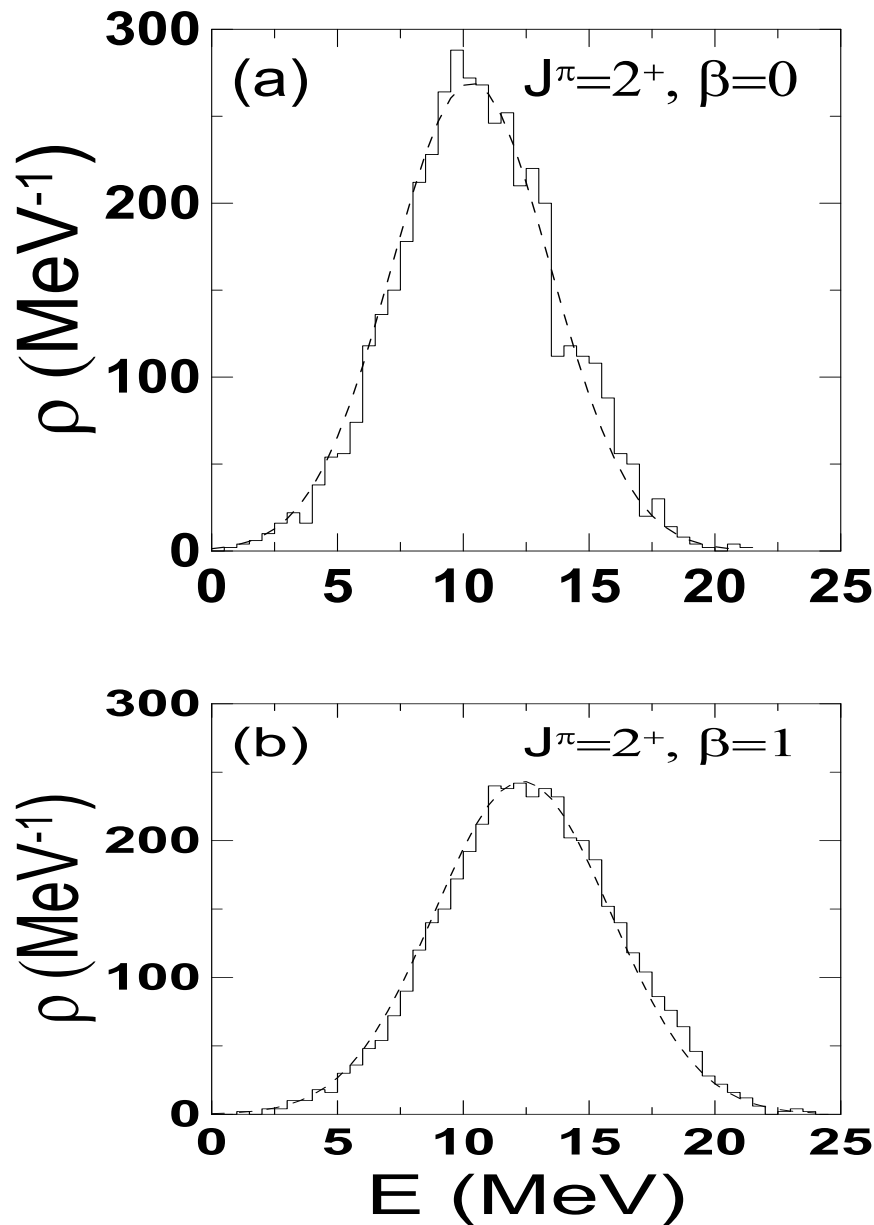
Figure 1- The level density in ^{138}Ba nucleus for the $J^\pi T = 2^+3$ class of levels: (a) corresponds to the result obtained with the absence of the off-diagonal interaction ($\beta = 0$); (b) demonstrates the result obtained with the full Hamiltonian ($\beta = 1$).

3.2. Level spacing distribution

In Figure-2, the calculated $P(s)$ distribution (histogram) for the unfolded 2^+3 class of energy levels in ^{138}Ba is presented with various interaction strengths β. The GOE limit (which defines chaotic systems) is displayed in the above figure via the solid curve. The Poisson limit (which refers to

ordered or regular systems) is exhibited by the dashed curve. Figure-2(a) reveals the outcome of the unperturbed hamiltonian calculations for the $P(s)$ distribution calculated with the interaction strength $\beta = 0$. The calculated histogram in Figure- 2(a) reveals ordered performance (which is in agreement with the Poisson limit) because of the deficiency of mixing as well as repulsion among energy levels. In fact, this deficiency is due to the nonattendance of the off diagonal interaction matrix elements (only diagonal elements of the N82K interaction are considered in the calculations). Figs. 2(b)-2(h) present the outcome of the full hamiltonian calculations for $P(s)$ distributions calculated with various strengths $(0 < \beta < 1)$. These figures exhibit the histograms move clearly away from the Poisson limit. They also show the level repulsion at small spacings, which is a typical feature of chaotic systems, growths more and more with increasing the strength β, as a consequence the computed histograms transfers regularly in the direction of the GOE limit. To compute the degree of chaos in the $P(s)$ via a limitation, we display in Figures-2(b)-2(h) the fitted results of Brody distribution (the red dash dotted curves) with their fitted limitations ω. It is realized from Fig. 2 that the transition from regular (order) to chaos arises at comparatively small strength of about $\beta = 0.3$. However, the outcome of Fig. 2 confirms the works achieved by Zelevinsky et al. [14] and Hamoudi et al. [23] for analyzing the $P(s)$ distribution in the $sd-$ and full $fp-$ shell model calculations, respectively.

3.3. Spectral rigidity

In Figure- 3, the influence of the strength β on the $\Delta_3(L)$ statistics (spectral rigidity) is examined. In this figure, the calculated distribution of the $\Delta_3(L)$ statistics (denoted by open circles) for the unfolded 2^+3 class of energy levels in ^{138}Ba is exhibited with several interaction strengths β. The Poisson limit (signified by the dashed curve) and the GOE limit (signified by the solid curve) are likewise exhibited for comparison. It is noticeable that the calculated distribution of $\Delta_3(L)$ in Fig. 3(a) (obtained with zero strength $\beta = 0$) reveals a good agreement with the Poisson distribution whereas those in Figs. 3(b)-3(h) (obtained with $0 < \beta < 1$) show important abnormality from the Poisson distribution. Increasing the interaction strength β across Figs. 3(a)-3(i) leads to transfer the calculated distribution of $\Delta_3(L)$ in the direction of the GOE distribution. Fig. 3 also demonstrates that the calculated distribution of $\Delta_3(L)$ goes forward to the GOE limit at small strength of about $\beta = 0.3$. This also approves the result that we have gained in Fig. 2 from the examination of the level spacing $P(s)$. Besides, the outcome of Fig. 3 also approves the works done by Hamoudi et al. [23] for the distribution of $\Delta_3(L)$ statistics in ^{44}V nucleus.

4. Conclusions

The fluctuation properties ($P(s)$ and Δ_3 statistics) of the excitation energies in ^{138}Ba have been searched through the context of the shell model, utilzing the isopsin-conserving $N82K$ as an effective interaction for 6 protons in the $N82$ space with a core of ^{132}Sn. We have searched for a transition from ordered to chaos in ^{138}Ba through performing shell-model calculations with several strengths β. The level density has been found to have a Gaussian shape, which is in agreement with the predicted studies for a many-body system with two-body residual interaction. Both distributions of the $P(s)$ and Δ_3 statistics have been found to own an ordered dynamic at strength $\beta = 0$, a disordered dynamic at $\beta \geq 0.3$ and an intermediate limit at $0 < \beta < 0.3$. Moreover, they have been well characterized by the GOE prediction once the strength becomes $\beta \geq 0.4$.
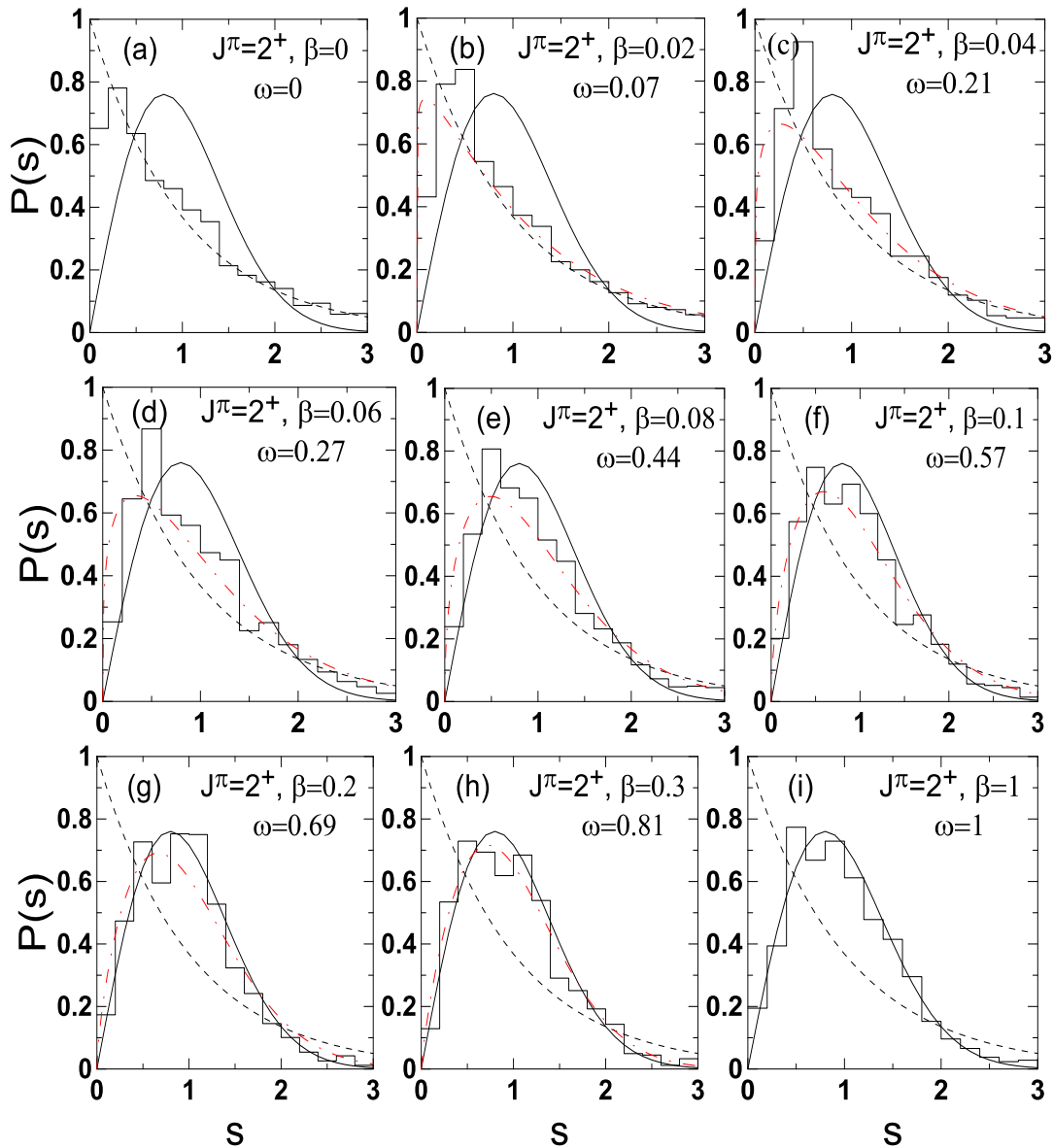
Figure 2-The nearest neighbor level spacing $P(s)$ distributions in ^{138}Ba nucleus for the unfolded $J^{\pi}T = 2^{+}3$ states (histograms) calculated with interaction strength $\beta = 0$ (a), $\beta = 0.02$ (b), $\beta = 0.04$ (c), $\beta = 0.06$ (d), $\beta = 0.08$ (e), $\beta = 0.1$ (f), $\beta = 0.2$ (g), $\beta = 0.3$ (h) and $\beta = 1$ (i).

The solid and dashed lines are the GOE and Poisson distributions, respectively. The red dash-dotted line is the best-fitted Brody distribution with the quoted ω.
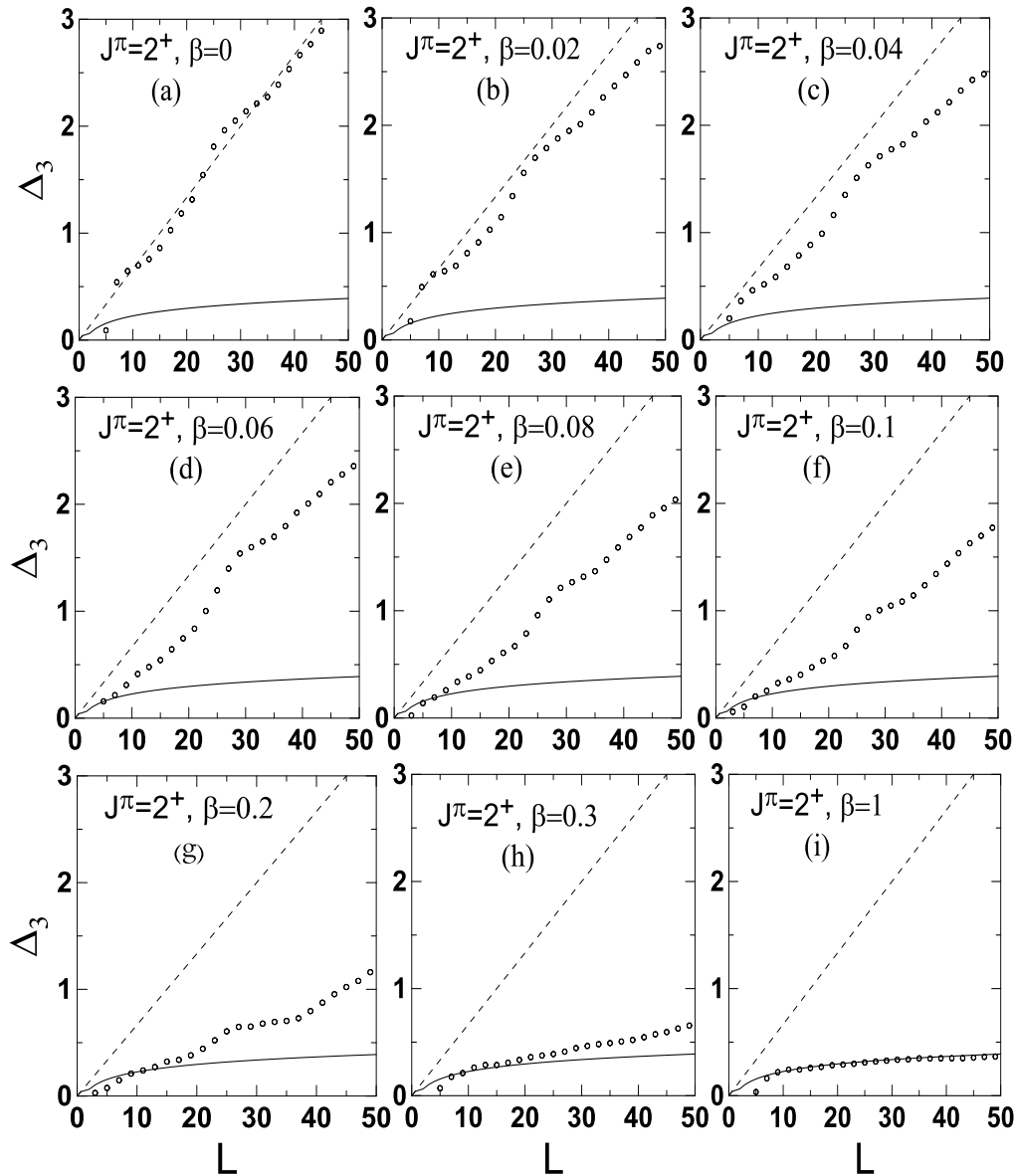
Figure 3-The average Δ_3 statistic in ^{138}Ba nucleus for the unfolded $J^{\pi}T = 2^+3$ states (open circles) calculated with interaction strength $\beta = 0$ (a), $\beta = 0.02$ (b), $\beta = 0.04$ (c), $\beta = 0.06$ (d), $\beta = 0.08$ (e), $\beta = 0.1$ (f), $\beta = 0.2$ (g), $\beta = 0.3$ (h) and $\beta = 1$ (i). The solid and dashed lines are the GOE and Poisson distributions, respectively.

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