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# On the Embedding of an Arc Into a Cubic Curves in a Finite Projective Plane of Order Five

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**Abstract:**
The main aims of this research is to find the stabilizer groups of a cubic curves over a finite field of order 5, studying the properties of their groups and then constructing the arcs of degree 2 which are embedding in a cubic curves of even size which are considering as the arcs of degree 3. Also drawing all these arcs.

**Keywords:** Stabilizer groups , arcs , cubic curves .

# حول غمر القوس الى المنحنيات المكعبة في المستوي الأسقاطي المنتهي من الرتبة الخامسة

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**الخلاصة**
الاهداف الرئيسية لهذا البحث هو ايجاد الزمر المثبتة للمنحنيات المكعبة حول الحقل المنتهي من الرت 5 ، ودراسة الخواص لهذه الزمر، وكذلك تشكيل الاقواس من الدرجة الثانية والتي تغمر في المنحنيات المكعبة ذات الحجم الزوجي والتي نفسها تعتبر كأقواس من الدرجة الثانية . كذلك رسم كل هذه الأقواس .

## 1 . Introduction
The subject of this research depends on themes of
- Projective geometry over a finite field ;
- Group theory ;
- Field theory ;
- Linear algebra .

The strategy to construct the stabilizer groups and also to embedded the arcs is given as following: Constructing the linear transformations group $PGL(3,q)$ of $PG(2,q)$, where $q = 5$. Which its elements are considering the non-singular matrices $A_n = [a_{ij}]$, $a_{i,j}$ in $F_q$ , $i,j = 1,2,3$ for some $n$ in $\mathbb{N}$ and satisfying $K(tA_n) = K$ for all $t$ in $F_q \backslash \{0\}$ and $K$ be any arc. The set of all matrices $A_n$ which construct the group, and according to the number of $A_n$ and its order, we are doing comparison with the groups in [1], so we can find which one of them similar with it. In another hand, we have found the arcs which are embedding in a cubic curves which are splitted into two sets, one of them contains the inflection points and the other does not, the set which does not contain the inflection points is considering the arc of degree two.

The summary history of this theme is shown as follows
- The ideas and definitions of this research are taken from James Hirshfeld [2] ;

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- In 2010, Najm Al-Seraji [3] has been studied the cubic curves over a finite field of order 17;
- In 2011, Emad Al-Zangana [4] has been shown the cubic curves over a finite field of order 19;
- In 2013, Emad Al-Zangana [5] has been described the cubic curves over a finite field of orders 2 , 3 , 5 , 7;
- In 2013, Emad Al-Zangana [6] has been classified the cubic curves over a finite field of order 11 , 13. For more details and information see also [7-9].

Now, we introduce the definitions which are using in this research as follows:

**Definition $(\mathbf{1.1})[\mathbf{2}]$:** Denote by $S$ and $S^*$ two subspaces of $P(n,K)$, A projectivity $\beta: S \rightarrow S^*$ is a bijection given by a matrix $T$, necessarily non-singular, where $P(X) = P(X)\beta$ if $tX^* = XT$, with $t \in K$. Write $= M(T)$ ; then $\beta = M(\lambda T)$ for any $\lambda$ in $K$. The group of projectivities of $PG(n,K)$ is denoted by $PGL(n+1,K)$.

**Definition $(\mathbf{1.2})[\mathbf{2}]$:**The stabilizer of $x$ in $\Lambda$ (any non-empty set) under the action of $G$ is the group $G_x = \{g \in G | xg = x\}$.

**Definition $(\mathbf{1.3})[\mathbf{2}]$:** An (n; r) arc $K$ or arc of degree $r$ in $PG(k,q)$ with $n \geq r+1$ is a set of points with property that every hyperplane meets $K$ in at most $r$ points of $K$ and there is some hyperplane meeting $K$ in exactly $r$ points.

**2. The classification of cubic curves over a finite field of order 5**

The polynomial of degree three $g_3(x) = x^3 - 4x - 1$ is primitive in $F_5 = \{0,1,2,3,4\}$, since $g_3(0) = -1$, $g_3(1) = 1$, $g_3(2) = -1$, $g_3(3) = -1$, $g_3(4) = 2$, also $g_3(\alpha^{48}) = 0$, $g_3(\alpha^{84}) = 0$, $g_3(\alpha^{116}) = 0$, this means $\alpha^{48}, \alpha^{84}, \alpha^{116}$ are roots of $g_3$ in $F_{5^3}$ .

The companion matrix of $g_3(x) = x^3 - 4x - 1$ in $F_5[x]$ generated the points and lines of $PG(2,5)$ as follows:

$$P(k) = [1,0,0]C(g)^k = [1,0,0]\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 4 & 0 \end{pmatrix}^k, \ k=0,1,\dots,30 .$$

With selecting the points in $PG(2,5)$ which are the third coordinate equal to zero, this means belong to $L_0 = v(z)$, that is $v(z) = tz = z$ for all $t$ in $F_5 \backslash \{0\}$ and with $P(k) = k$, we obtain $L_0$={0,1,3,8,12, 18}, that is

$$L_{k=}L_0 C(g)^k = L_0 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 4 & 0 \end{pmatrix}^k, \ k=0,1,\dots,30 .$$

The number of distinct cubic curves in $PG(2,5)$ is 16 see [4], one of them is given as follows:

$$\theta_1 = xyz - (x+y+z)^3. \qquad \dots(1)$$

The points of $PG(2,5)$ on $\theta_1$ in equation (1) are $[4,1,0], [0,4,1], [4,0,1], [3,2,1], [2,3,1], [2,4,1], [4,2,1], [3,4,1], [4,3,1]$. To find the stabilizer group of $\theta_1$ in equation (1), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_1$ with their orders are shown as follows:

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}:2, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}:3, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}:3, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}:2,$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}:2, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1, \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}:2, \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}:6,$$
$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix}:6, \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}:2, \begin{pmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}:2, \begin{pmatrix} 1 & 3 & 3 \\ 3 & 3 & 1 \\ 3 & 1 & 3 \end{pmatrix}:2.$$

Therefore, the stabilizer group of $\theta_1$ in equation (1) which is denoted by $G_{\theta_1}$ which contains

- 7 matrices of order 2;
- 2 matrices of order 3;
- 2 matrices of order 6;
- The identity matrix.

Form [1], $G_{\theta_1}$ is isomorphic to $\boldsymbol{D_6}$, that is $G_{\theta_1} \cong \boldsymbol{D_6}$. $\theta_1$ in equation (1) is drawn in Figure-1:

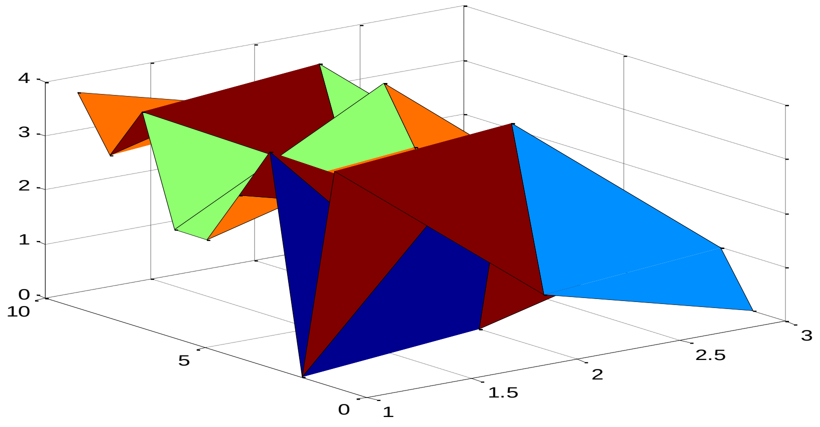


**Figure 1-**Drawing of $\boldsymbol{\theta_1}$

Another one of cubic curve which is given in [4] is:

$\theta_2 = xyz + 2(x+y+z)^3$ . …(2)

The points of $PG(2,5)$ on $\theta_2$ in equation (2) are [4,1,0], [0,4,1], [4,0,1], [1,2,1], [3,3,1], [2,1,1], [1,1,1]. To find the stabilizer group of $\theta_2$ in equation (2), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_2$ with their orders are shown as follows:

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}:2,\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}:3,\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}:3,$$
$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1.$$

Therefore, the stabilizer group of $\theta_2$ in equation (2) which is denoted by $G_{\theta_2}$ which contains

- 3 matrices of order 2;
- 2 matrices of order 3;
- The identity matrix.

Form [1], $G_{\theta_2}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\theta_2} \cong \boldsymbol{S_3}$. $\theta_2$ in equation (2) is drawn in Figure-2:

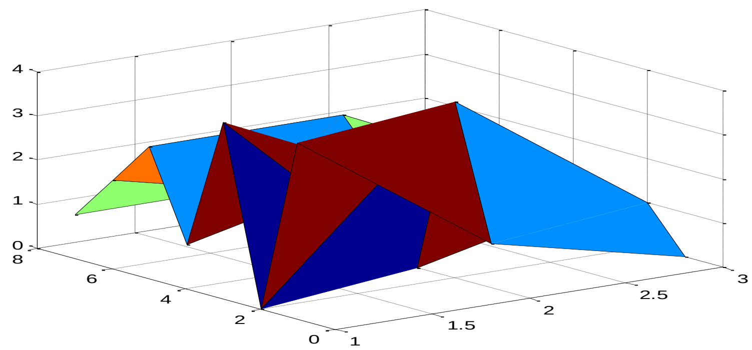


**Figure 2-**Drawing of $\boldsymbol{\theta_2}$

Another one of cubic curve which is given in [4] is :

$\theta_3 = xy(x+y) + z^3$. (3)

The points of $PG(2,5)$ on $\theta_3$ in equation (3) are [1,0,0], [0,1,0], [4,1,0], [3,3,1], [3,4,1], [4,3,1]. To find the stabilizer group of $\theta_3$ in equation (3), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_3$ with their orders are shown as follows:

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&1&0\\2&2&4\\3&1&3\end{pmatrix}:3,\begin{pmatrix}0&1&0\\3&4&1\\2&4&2\end{pmatrix}:4,\begin{pmatrix}0&1&0\\4&1&0\\0&0&4\end{pmatrix}:3,$$
$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\1&4&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}1&0&0\\2&2&4\\1&3&3\end{pmatrix}:2,\begin{pmatrix}1&0&0\\4&3&1\\4&2&2\end{pmatrix}:2,$$
$$\begin{pmatrix}1&1&2\\0&3&0\\4&3&4\end{pmatrix}:2,\begin{pmatrix}1&1&2\\1&3&2\\1&2&1\end{pmatrix}:3,\begin{pmatrix}1&1&2\\3&0&0\\3&4&4\end{pmatrix}:3,\begin{pmatrix}1&1&2\\3&1&2\\2&1&1\end{pmatrix}:4,$$
$$\begin{pmatrix}1&2&4\\2&1&4\\4&4&2\end{pmatrix}:2,\begin{pmatrix}1&2&4\\2&2&4\\4&2&2\end{pmatrix}:3,\begin{pmatrix}1&2&4\\4&0&0\\1&3&3\end{pmatrix}:4,\begin{pmatrix}1&2&4\\4&1&0\\1&1&3\end{pmatrix}:3,$$
$$\begin{pmatrix}1&3&2\\0&2&0\\4&3&4\end{pmatrix}:2,\begin{pmatrix}1&3&2\\1&1&2\\1&2&1\end{pmatrix}:4,\begin{pmatrix}1&3&2\\3&1&2\\2&2&1\end{pmatrix}:2,\begin{pmatrix}1&3&2\\3&2&0\\3&3&4\end{pmatrix}:4,$$
$$\begin{pmatrix}1&4&0\\0&4&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&4&0\\1&0&0\\0&0&4\end{pmatrix}:3,\begin{pmatrix}1&4&0\\2&1&4\\1&1&3\end{pmatrix}:3,\begin{pmatrix}1&4&0\\4&3&1\\4&4&2\end{pmatrix}:4.$$

Therefore, the stabilizer group of $\theta_3$ in equation(3) which is denoted by $G_{\theta_3}$ which contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix.

Form [1], $G_{\theta_3}$ is isomorphic to $\boldsymbol{S_4}$, that is $G_{\theta_3} \cong \boldsymbol{S_4}$. $\theta_3$ in equation (3) is drawn in Figure-3:

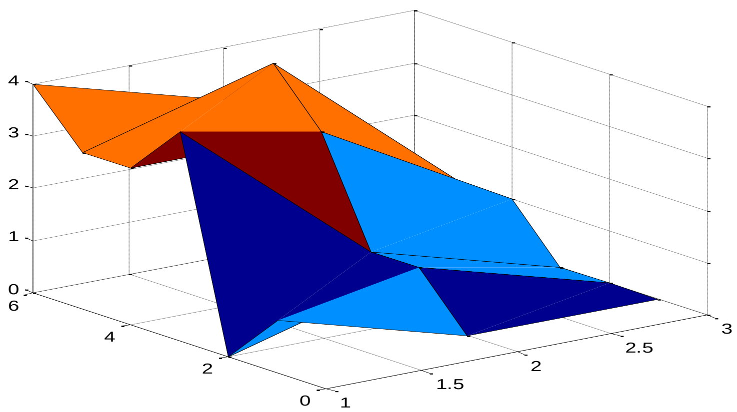


**Figure 3**-Drawing of $\boldsymbol{\theta_3}$

Let $\theta_3^* = \{[3,3,1],[3,4,1],[4,3,1]\}$ be a subset of $\theta_3$ in equation (3) which is forming by partition the $\theta_3$ into two sets such that $\theta_3^*$ does not contains the inflection points of $\theta_3$, so we note that $\theta_3^*$ represents an arc of degree two. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\theta_3^*$ is 96, and we can not write them, because they are too much. Moreover, the stabilizer group of $\theta_3^*$ which is denoted by $G_{\theta_3^*}$ which contains

- 15 matrix of order 2;
- 32 matrix of order 3;
- 24 matrix of order 4;
- 24 matrix of order 8;
- The identity matrix.

Drawing of $\theta_3^*$ is given in Figure-4 as following:

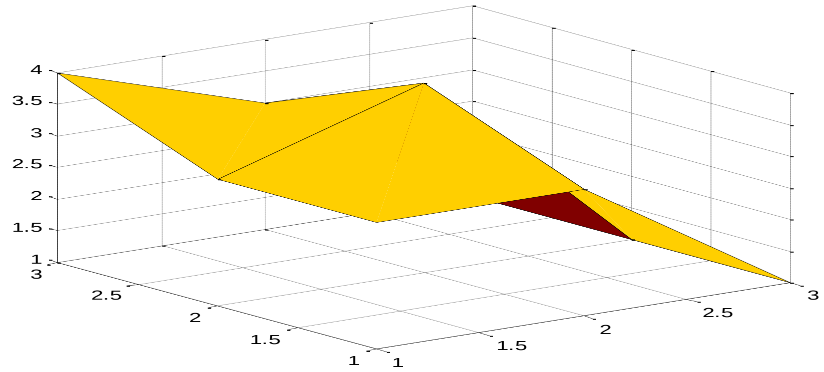


**Figure 4-**Drawing of $\boldsymbol{\theta_3^*}$

Another one of cubic curve which is given in [4] is:
$\theta_4 = xyz + (x + y + z)^3$ . (4)

The points of $PG(2,5)$ on $\theta_4$ in equation (4) are [4,1,0], [0,4,1], [4,1,1], [1,4,1], [4,0,1], [4,4,1]. To find the stabilizer group of $\theta_4$ in equation (4), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_4$ with their orders are shown as follows:

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}:2,\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}:3,\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}:3,\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,$$
$$\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&1&3\\3&4&1\\4&3&1\end{pmatrix}:4,\begin{pmatrix}1&1&3\\4&3&1\\3&4&1\end{pmatrix}:3,$$
$$\begin{pmatrix}1&2&2\\2&1&3\\2&3&1\end{pmatrix}:2,\begin{pmatrix}1&2&2\\2&3&1\\2&1&3\end{pmatrix}:2,\begin{pmatrix}1&2&3\\2&1&2\\3&2&1\end{pmatrix}:2,\begin{pmatrix}1&2&3\\3&2&1\\1&2&1\end{pmatrix}:3,$$
$$\begin{pmatrix}1&2&4\\2&1&4\\4&4&2\end{pmatrix}:2,\begin{pmatrix}1&2&4\\4&4&2\\2&1&4\end{pmatrix}:4,\begin{pmatrix}1&3&1\\3&1&4\\4&3&1\end{pmatrix}:3,\begin{pmatrix}1&3&1\\4&1&3\\3&1&4\end{pmatrix}:4,$$
$$\begin{pmatrix}1&3&2\\2&2&1\\3&1&2\end{pmatrix}:3,\begin{pmatrix}1&3&2\\3&1&2\\2&2&1\end{pmatrix}:2,\begin{pmatrix}1&3&4\\1&4&3\\3&1&1\end{pmatrix}:4,\begin{pmatrix}1&3&4\\3&1&1\\1&4&3\end{pmatrix}:3,$$
$$\begin{pmatrix}1&4&2\\2&4&1\\4&2&4\end{pmatrix}:4,\begin{pmatrix}1&4&2\\4&2&4\\2&4&1\end{pmatrix}:2,\begin{pmatrix}1&4&3\\1&3&4\\3&1&1\end{pmatrix}:3,\begin{pmatrix}1&4&3\\3&1&1\\1&3&4\end{pmatrix}:4\ .$$

Therefore, the stabilizer group of $\theta_4$ in equation (4) which is denoted by $G_{\theta_4}$ which contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix.

Form [1], $G_{\theta_4}$ is isomorphic to $\boldsymbol{S_4}$, that is $G_{\theta_4} \cong \boldsymbol{S_4}$. $\theta_4$ in equation(4) is drawn in Figure-5:

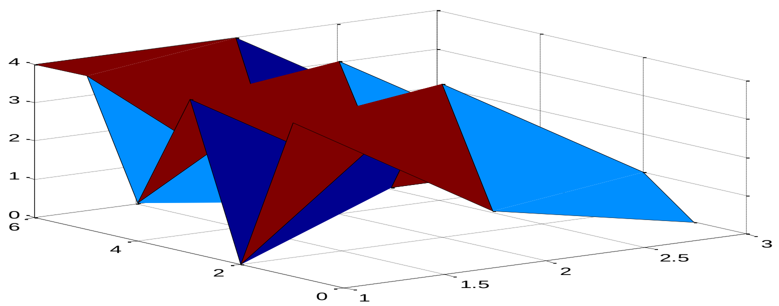


**Figure 5-**Drawing of $\boldsymbol{\theta_4}$

Let $\theta_4^* = \{[1,4,1],[4,0,1],[4,4,1]\}$ be a subset of $\theta_4$ in equation (4) which is forming by partition $\theta_4$ into two sets such that $\theta_4^*$ does not contains the inflection points of $\theta_4$, so we note that $\theta_4^*$ represents an arc of degree two. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\theta_4^*$ is 96, and we can not write them, because they are too much. Moreover, the stabilizer group of $\theta_4^*$ which is denoted by $G_{\theta_4^*}$ which contains

- 15 matrix of order 2;
- 32 matrix of order 3;
- 24 matrix of order 4;
- 24 matrix of order 8;
- The identity matrix.

Drawing of $\theta_4^*$ is given in Figure-6 as following:

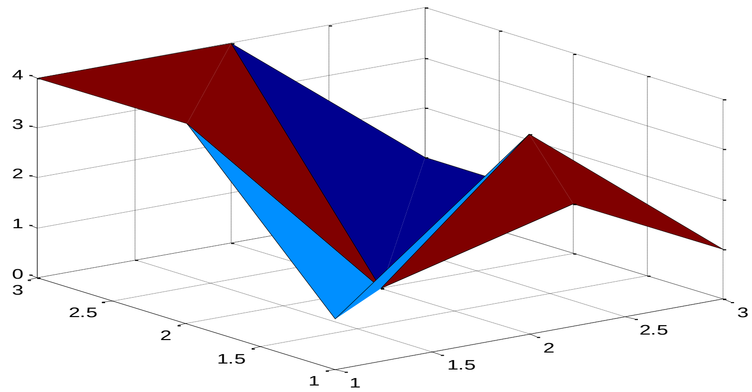


**Figure 6 -**Drawing of $\boldsymbol{\theta_4^*}$

Another one of cubic curve which is given in [4] is:

$$\theta_5 = yz^2 + x^3 - 2xy^2 - 2y^3 . \quad (5)$$

The points of $PG(2,5)$ on $\theta_5$ in equation (5) are $[0,0,1],[4,1,1],[1,4,1],[3,1,1],\ [2,4,1]$.To find the stabilizer group of $\theta_5$ in equation (5), we are doing calculations by help the computer.Thus the transformation matrices which stabilizing of $\theta_5$ with their orders are shown as follows:

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&1&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}1&0&0\\3&4&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\3&4&0\\0&0&4\end{pmatrix}:2,$$
$$\begin{pmatrix}1&1&0\\0&4&0\\0&0&2\end{pmatrix}:4,\begin{pmatrix}1&1&0\\0&4&0\\0&0&3\end{pmatrix}:4,\begin{pmatrix}1&1&0\\3&4&0\\0&0&2\end{pmatrix}:2,\begin{pmatrix}1&1&0\\3&4&0\\0&0&3\end{pmatrix}:2\ .$$

Therefore, the stabilizer group of $\theta_5$ in equation (5) which is denoted by $G_{\theta_5}$which contains

- 5 matrices of order 2;
- 2 matrices of order 4;
- The identity matrix.

Form [1], $G_{\theta_5}$is isomorphic to $\boldsymbol{D_4}$, that is $G_{\theta_5} \cong \boldsymbol{D_4}$. $\theta_5$ in equation (5) is drawn in Figure-7:

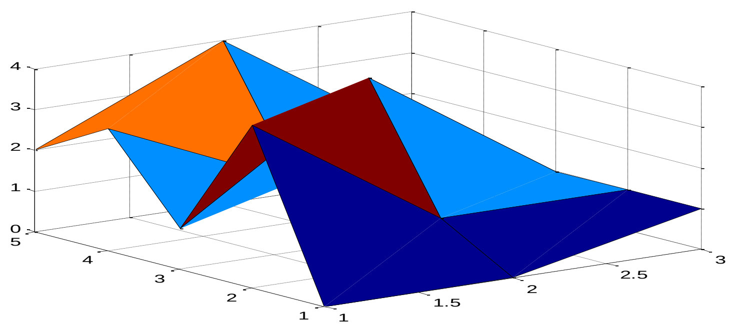


**Figure 7-**Drawing of $\theta_5$

Another one of cubic curve which is given in [4] is:

$$\theta_6 = yz^2 + x^3 - xy^2 - y^3 \ . \qquad (6)$$

The points of $PG(2,5)$ on $\theta_6$ in equation (6) are $[0,0,1], [0,4,1], [4,1,1], [0,1,1], [1,4,1], [2,1,0], [4,4,1], [1,1,1]$. To find the stabilizer group of $\theta_6$ in equation (6), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_6$ with their orders are shown as follows:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}:2 \ .$$

Therefore, the stabilizer group of $\theta_6$ in equation (6) which is denoted by $G_{\theta_6}$ which contains

- 1 matrices of order 2;
- The identity matrix.

Form[1], $G_{\theta_6}$ is isomorphic to $\boldsymbol{Z_2}$, that is $G_{\theta_6} \cong \boldsymbol{Z_2}$. $\theta_6$ in equation (6) is drawn in Figure-8:

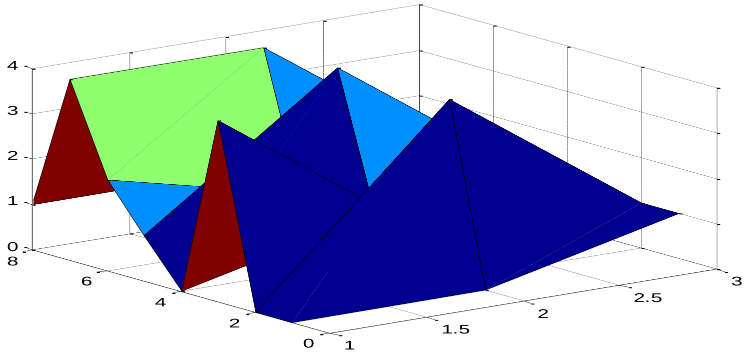


**Figure 8-**Drawing of $\theta_6$

Let $\theta_6^* = \{[1,4,1], [2,1,0], [4,4,1], [1,1,1]\}$ be a subset of $\theta_6$ in equation (6) which is forming by partition $\theta_6$ into two sets such that $\theta_6^*$ does not contains the inflection points of $\theta_6$, so we note that $\theta_6^*$ represents an arc of degree two. Also, to find the stabilizer group of $\theta_6^*$, by some calculation, we obtain

$$\begin{pmatrix} 0 & 0 & 1 \\ 4 & 1 & 2 \\ 3 & 0 & 2 \end{pmatrix}:4, \begin{pmatrix} 0 & 0 & 1 \\ 4 & 2 & 3 \\ 3 & 1 & 3 \end{pmatrix}:3, \begin{pmatrix} 0 & 1 & 3 \\ 4 & 2 & 3 \\ 3 & 0 & 1 \end{pmatrix}:4, \begin{pmatrix} 0 & 1 & 3 \\ 4 & 4 & 3 \\ 3 & 2 & 1 \end{pmatrix}:3,$$
$$\begin{pmatrix} 0 & 1 & 4 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{pmatrix}:2, \begin{pmatrix} 0 & 1 & 4 \\ 1 & 4 & 3 \\ 2 & 4 & 3 \end{pmatrix}:2, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}:3,$$
$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 2 & 1 & 2 \end{pmatrix}:3, \begin{pmatrix} 1 & 0 & 2 \\ 4 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix}:4, \begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix}:2, \begin{pmatrix} 1 & 0 & 4 \\ 4 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}:4,$$
$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{pmatrix}:3, \begin{pmatrix} 1 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 4 & 1 \end{pmatrix}:2, \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 4 & 3 \end{pmatrix}:4, \begin{pmatrix} 1 & 2 & 1 \\ 4 & 4 & 3 \\ 1 & 0 & 0 \end{pmatrix}:3,$$
$$\begin{pmatrix} 1 & 2 & 2 \\ 0 & 3 & 3 \\ 0 & 4 & 1 \end{pmatrix}:3, \begin{pmatrix} 1 & 2 & 2 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{pmatrix}:2, \begin{pmatrix} 1 & 2 & 4 \\ 0 & 3 & 0 \\ 2 & 3 & 4 \end{pmatrix}:2, \begin{pmatrix} 1 & 2 & 4 \\ 4 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix}:3,$$
$$\begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 3 \end{pmatrix}:3, \begin{pmatrix} 1 & 3 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 0 \end{pmatrix}:2, \begin{pmatrix} 1 & 4 & 2 \\ 0 & 0 & 4 \\ 2 & 4 & 2 \end{pmatrix}:4, \begin{pmatrix} 1 & 4 & 2 \\ 4 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix}:2 \ .$$

Therefore, the stabilizer group of $\theta_6^*$ which is denoted by $G_{\theta_6^*}$ which contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix.

Form[1], $G_{\theta_6^*}$ is isomorphic to $\boldsymbol{S_4}$, that is $G_{\theta_6^*} \cong \boldsymbol{S_4}$. Drawing of $\theta_6^*$ is given in Figure-9:

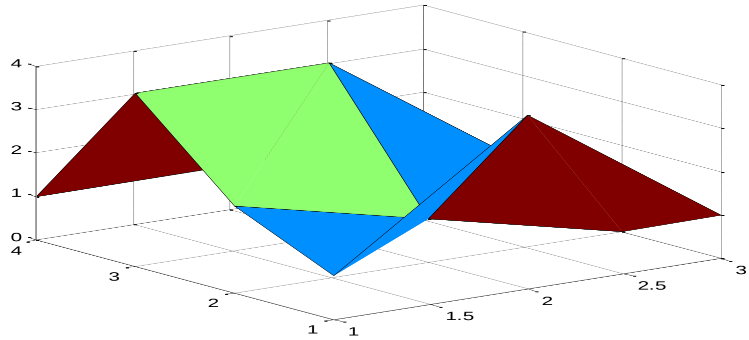


**Figure 9-**Drawing of $\theta_6^*$

Another one of cubic curve which is given in [4] is:

$$\theta_7 = yz^2 + x^3 + xy^2 - 2y^3 \;. \tag{7}$$

The points of $PG(2,5)$ on $\theta_7$ in equation (7) are $[0,0,1], [1,1,0], [3,2,1], [2,3,1]$. To find the stabilizer group of $\theta_7$ in equation(7), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_7$ with their orders are shown as follows:

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&1&0\\1&0&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}0&1&2\\1&0&3\\2&3&1\end{pmatrix}:4,\begin{pmatrix}0&1&2\\1&0&3\\3&2&4\end{pmatrix}:12,$$
$$\begin{pmatrix}0&1&3\\1&0&2\\2&3&4\end{pmatrix}:12,\begin{pmatrix}0&1&3\\1&0&2\\3&2&1\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&1&0\\0&0&4\end{pmatrix}:2,$$
$$\begin{pmatrix}1&0&2\\0&1&3\\2&3&4\end{pmatrix}:4,\begin{pmatrix}1&0&2\\0&1&3\\3&2&1\end{pmatrix}:12,\begin{pmatrix}1&0&3\\0&1&2\\2&3&1\end{pmatrix}:12,\begin{pmatrix}1&0&3\\0&1&2\\3&2&4\end{pmatrix}:4,$$
$$\begin{pmatrix}1&2&0\\2&1&0\\0&0&1\end{pmatrix}:4,\begin{pmatrix}1&2&0\\2&1&0\\0&0&4\end{pmatrix}:4,\begin{pmatrix}1&2&2\\2&1&3\\2&3&1\end{pmatrix}:2,\begin{pmatrix}1&2&2\\2&1&3\\3&2&4\end{pmatrix}:6,$$
$$\begin{pmatrix}1&2&3\\2&1&2\\2&3&4\end{pmatrix}:6,\begin{pmatrix}1&2&3\\2&1&2\\3&2&1\end{pmatrix}:2,\begin{pmatrix}1&3&0\\3&1&0\\0&0&2\end{pmatrix}:4,\begin{pmatrix}1&3&0\\3&1&0\\0&0&3\end{pmatrix}:4,$$
$$\begin{pmatrix}1&3&1\\3&1&4\\1&4&2\end{pmatrix}:2,\begin{pmatrix}1&3&1\\3&1&4\\4&1&2\end{pmatrix}:3,\begin{pmatrix}1&3&4\\3&1&1\\1&4&3\end{pmatrix}:3,\begin{pmatrix}1&3&4\\3&1&1\\4&1&2\end{pmatrix}:2\;.$$

Therefore, the stabilizer group of $\theta_7$ in equation (7) which is denoted by $G_{\theta_7}$ which contains

- 7 matrices of order 2;
- 2 matrices of order 3;
- 8 matrices of order 4;
- 2 matrices of order 6;
- 4 matrices of order 12;
- The identity matrix.

Form[1], $G_{\theta_7}$ is isomorphic to $\boldsymbol{S_3} \times \boldsymbol{Z_4}$, that is $G_{\theta_7} \cong \boldsymbol{S_3} \times \boldsymbol{Z_4}$. $\theta_7$ in equation (7) is drawn in Figure-10:

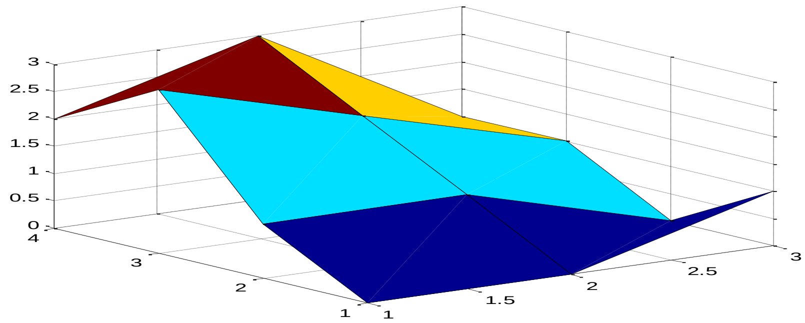


**Figure 10-**Drawing of $\theta_7$

Let $\theta_7^* = \{[3,2,1],[2,3,1]\}$ be a subset of $\theta_7$ in equation (7) which is forming by partition $\theta_7$ into two sets such that $\theta_7^*$ does not contains the inflection points of $\theta_7$, so we note that $\theta_7^*$ represents an arc of degree two. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\theta_7^*$ is 800, and we can not write them, because they are too much. Moreover, the stabilizer group of $\theta_7^*$ which is denoted by $G_{\theta_7^*}$ which contains

- 55 matrix of order 2;
- 320 matrix of order 4;
- 24 matrix of order 5;
- 200 matrix of order 8;
- 120 matrix of order 10;
- 80 martrix of order 20;
- The identity matrix.

Drawing of $\theta_7^*$ is given in Figure-11 as following:

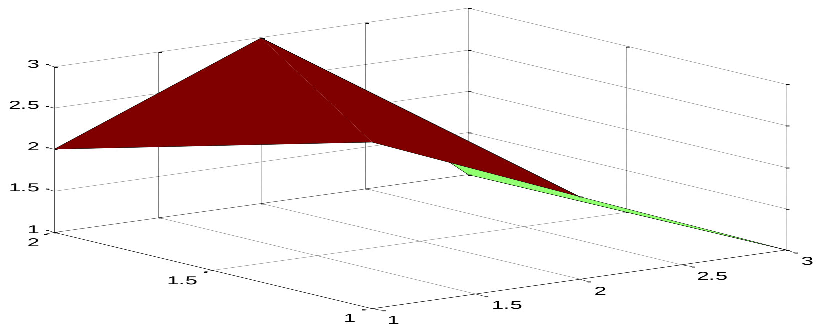


**Figure 11-**Drawing of $\theta_7^*$

Another one of cubic curve which is given in [4] is:

$$\theta_8 = yz^2 + x^3 + 2xy^2 - y^3 \,. \qquad (8)$$

The points of $PG(2,5)$ on $\theta_8$ in equation (8) are $[0,0,1],[0,4,1],[1,3,1],[0,1,1],[3,2,1],[2,3,1],[4,2,1]$. To find the stabilizer group of $\theta_8$ in equation (8), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_8$ with their orders are shown as follows:

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1, \begin{pmatrix}1&0&0\\0&1&0\\0&0&4\end{pmatrix}:2, \begin{pmatrix}1&1&0\\3&4&0\\0&0&2\end{pmatrix}:2, \begin{pmatrix}1&1&0\\3&4&0\\0&0&3\end{pmatrix}:2.$$

Therefore, the stabilizer group of $\theta_8$ in equation (8) which is denoted by $G_{\theta_8}$ which contains

- 3 matrices of order 2;
- The identity matrix.

Form[1], $G_{\theta_8}$is isomorphic to $\boldsymbol{Z_2} \times \boldsymbol{Z_2}$, that is $G_{\theta_8} \cong \boldsymbol{Z_2} \times \boldsymbol{Z_2}$. $\theta_8$ in equation (8) is drawn in Figure−12:

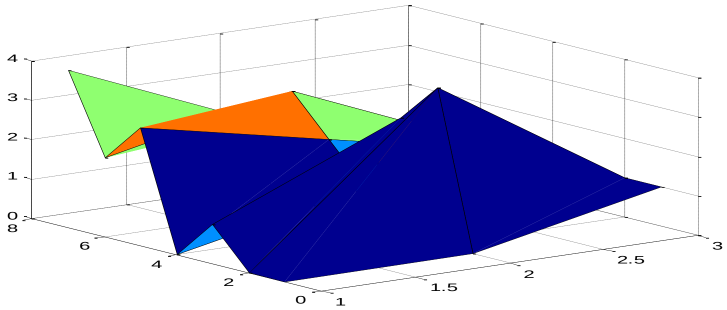


**Figure 12-**Drawing of $\theta_8$

Another one of cubic curve which is given in [4] is:

$$\theta_9 = yz^2 + x^3 + xy^2 \,. \qquad (9)$$

The points of $PG(2,5)$ on $\theta_9$ in equation (9) are[0,1,0], [0,0,1], [2,1,0], [3,1,0]. To find the stabilizer group of $\theta_9$ in equation(9), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_9$ with their orders are shown as follows:

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&1&0\\0&0&2\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&1&0\\0&0&3\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&1&0\\0&0&4\end{pmatrix}:2,$$
$$\begin{pmatrix}1&0&0\\0&4&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&4&0\\0&0&2\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&4&0\\0&0&3\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&4&0\\0&0&4\end{pmatrix}:2,$$
$$\begin{pmatrix}1&1&0\\2&1&0\\0&0&1\end{pmatrix}:12,\begin{pmatrix}1&1&0\\2&1&0\\0&0&2\end{pmatrix}:6,\begin{pmatrix}1&1&0\\2&1&0\\0&0&3\end{pmatrix}:3,\begin{pmatrix}1&1&0\\2&1&0\\0&0&4\end{pmatrix}:12,$$
$$\begin{pmatrix}1&1&0\\3&4&0\\0&0&1\end{pmatrix}:4,\begin{pmatrix}1&1&0\\3&4&0\\0&0&2\end{pmatrix}:2,\begin{pmatrix}1&1&0\\3&4&0\\0&0&3\end{pmatrix}:2,\begin{pmatrix}1&1&0\\3&4&0\\0&0&4\end{pmatrix}:4,$$
$$\begin{pmatrix}1&4&0\\2&4&0\\0&0&1\end{pmatrix}:4,\begin{pmatrix}1&4&0\\2&4&0\\0&0&2\end{pmatrix}:2,\begin{pmatrix}1&4&0\\2&4&0\\0&0&3\end{pmatrix}:2,\begin{pmatrix}1&4&0\\2&4&0\\0&0&4\end{pmatrix}:4,$$
$$\begin{pmatrix}1&4&0\\3&1&0\\0&0&1\end{pmatrix}:12,\begin{pmatrix}1&4&0\\3&1&0\\0&0&2\end{pmatrix}:6,\begin{pmatrix}1&4&0\\3&1&0\\0&0&3\end{pmatrix}:3,\begin{pmatrix}1&4&0\\3&1&0\\0&0&4\end{pmatrix}:12\,.$$

Therefore, the stabilizer group of $\theta_9$ in equation (9) which is denoted by $G_{\theta_9}$which contains

- 7 matrices of order 2;
- 2 matrices of order 3;
- 8 matrices of order 4;
- 2 matrices of order 6;
- 4 matrices of order 12;
- The identity matrix.

Form[1], $G_{\theta_9}$is isomorphic to $\boldsymbol{S_3} \times \boldsymbol{Z_4}$, that is $G_{\theta_9} \cong \boldsymbol{S_3} \times \boldsymbol{Z_4}$. $\theta_9$ in equation (9) is drawn in Figure-13:

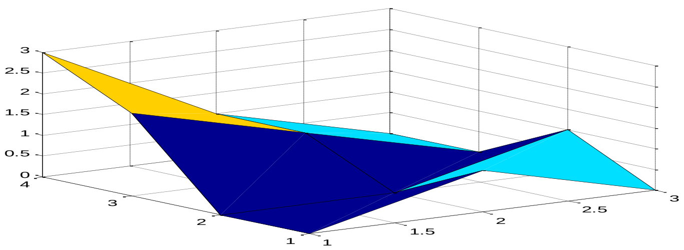


**Figure 13 -**Drawing of $\theta_9$

Let $\theta_9^* = \{[2,1,0],[3,1,0]\}$ be a subset of $\theta_9$ in equation (9) which is forming by partition $\theta_9$ into two sets such that $\theta_9^*$ does not contains the inflection points of $\theta_9$, so we note that $\theta_9^*$ represents an arc of degree two. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\theta_9^*$ is 800, and we can not write them, because they are too much. Moreover, the stabilizer group of $\theta_9^*$ which is denoted by $G_{\theta_9^*}$ which contains

- 55 matrix of order 2;
- 320 matrix of order 4;
- 24 matrix of order 5;
- 200 matrix of order 8;
- 120 matrix of order 10;
- 80 martrix of order 20;
- The identity matrix.

Drawing of $\theta_9^*$ is given in Figure-14 as following:

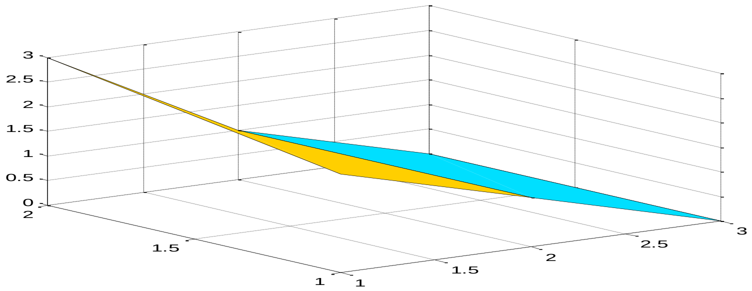

**Figure 14-**Drawing of $\theta_9^*$

Another one of cubic curve which is given in [4] is:

$$\theta_{10} = yz^2 + x^3 + 2xy^2 \,. \qquad (10)$$

The points of $PG(2,5)$ on $\theta_{10}$ in equation (10) are $[0,1,0],[0,0,1]$. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\theta_{10}$ in equation (10) is 800, and we can not write them, because they are too much.

Moreover, the stabilizer group of $\theta_{10}$ in equation (26) which is denoted by $G_{\theta_{10}}$ which contains

- 55 matrix of order 2;
- 320 matrix of order 4;
- 24 matrix of order 5;
- 200 matrix of order 8;
- 120 matrix of order 10;
- 80 martrix of order 20;
- The identity matrix.

The set of points on $\theta_{10}$ represents the arc of degree two and size 2.

Drawing of $\theta_{10}$ in equation (10) is given in Figure-15 as following:

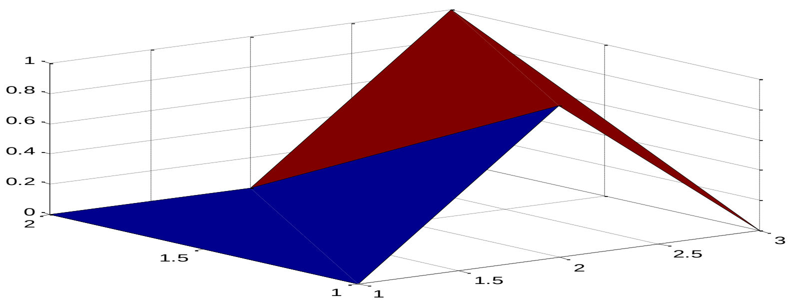

**Figure 15 -**Drawing of $\theta_{10}$

Another one of cubic curve which is given in [4] is:

$$\theta_{11} = yz^2 + x^3 - 2xy^2 \,. \qquad (11)$$

The points of $PG(2,5)$ on $\theta_{11}$ in equation (11) are [0,1,0], [0,0,1], [3,2,1], [1,2,1], [2,3,1], [4,4,1], [3,4,1], [4,3,1], [2,1,1], [1,1,1]. To find the stabilizer group of $\theta_{11}$ in equation (11), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_{11}$ with their orders are shown as follows:

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&1&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}0&1&0\\0&4&0\\0&0&2\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&4&0\\0&0&3\end{pmatrix}:4.$$

Therefore, the stabilizer group of $\theta_{11}$ in equation(11) which is denoted by $G_{\theta_{11}}$ which contains

- 1 matrices of order 2;
- 2 matrices of order 4;
- The identity matrix.

Form[1], $G_{\theta_{11}}$ is isomorphic to $\boldsymbol{Z_4}$, that is $G_{\theta_{11}} \cong \boldsymbol{Z_4}$. $\theta_{11}$ in equation (11) is drawn in Figure-16

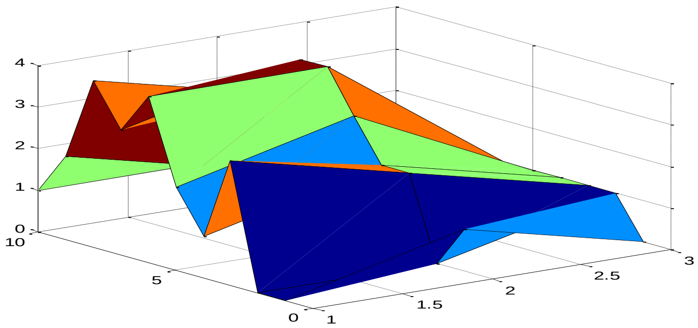


**Figure 16-**Drawing of $\theta_{11}$

Let $\theta_{11}^* = \{[2,3,1],[4,4,1],[4,3,1],[2,1,1],[1,1,1]\}$ be a subset of $\theta_{11}$ in equation (11) which is forming by partition the $\theta_{11}$ into two sets such that $\theta_{11}^*$ does not contains the inflection points of $\theta_{11}$, so we note that $\theta_{11}^*$ represents an arc of degree two. Also, to find the stabilizer group of $\theta_{11}^*$, by some calculation, we obtain

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}0&1&0\\1&1&2\\0&1&1\end{pmatrix}:5,\begin{pmatrix}0&1&3\\0&2&3\\2&4&2\end{pmatrix}:4,\begin{pmatrix}0&1&3\\1&0&2\\3&2&1\end{pmatrix}:4,$$
$$\begin{pmatrix}0&1&4\\0&3&4\\4&3&4\end{pmatrix}:4,\begin{pmatrix}0&1&4\\2&3&0\\1&3&0\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\1&1&3\\1&0&4\end{pmatrix}:2,$$
$$\begin{pmatrix}1&0&1\\3&0&1\\3&4&1\end{pmatrix}:4,\begin{pmatrix}1&0&1\\3&2&0\\3&1&0\end{pmatrix}:4,\begin{pmatrix}1&0&2\\0&1&3\\2&3&4\end{pmatrix}:4,\begin{pmatrix}1&0&2\\2&0&2\\4&2&3\end{pmatrix}:4,$$
$$\begin{pmatrix}1&1&2\\0&1&0\\0&4&4\end{pmatrix}:2,\begin{pmatrix}1&1&2\\1&4&4\\1&2&3\end{pmatrix}:5,\begin{pmatrix}1&1&3\\1&0&0\\4&0&1\end{pmatrix}:5,\begin{pmatrix}1&1&3\\4&1&1\\2&1&2\end{pmatrix}:2,$$
$$\begin{pmatrix}1&4&0\\0&3&2\\2&1&0\end{pmatrix}:4,\begin{pmatrix}1&4&0\\2&0&2\\4&3&0\end{pmatrix}:4,\begin{pmatrix}1&4&4\\1&1&2\\4&3&2\end{pmatrix}:2,\begin{pmatrix}1&4&4\\4&4&2\\2&1&2\end{pmatrix}:5\ .$$

Therefore, the stabilizer group of $\theta_{11}^*$ which is denoted by $G_{\theta_{11}^*}$ which contains

- 5 matrices of order 2;
- 10 matrix of order 4;
- 4 matrices of order 5;
- The identity matrix.

Form[1], $G_{\theta_{11}^*}$ is isomorphic to $\boldsymbol{Z_5} \rtimes \boldsymbol{Z_4}$, that is $G_{\theta_{11}^*} \cong \boldsymbol{Z_5} \rtimes \boldsymbol{Z_4}$. Drawing of $\theta_{11}^*$ is given in Figure-17 as following:

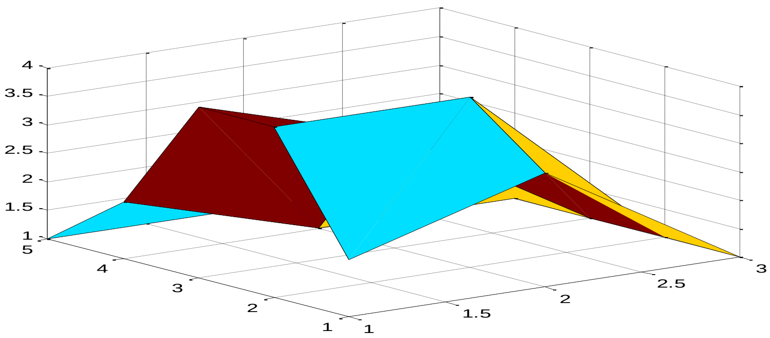


**Figure 17**-Drawing of $\theta_{11}^{*}$

Another one of cubic curve which is given in [4] is:

$$\theta_{12} = yz^2 + x^3 - xy^2 \; . \qquad (12)$$

The points of $PG(2,5)$ on $\theta_{12}$ in equation (12) are $[0,1,0], [0,0,1], [4,1,0], [1,3,1], [1,1,0], [3,1,1], [2,4,1], [4,2,1]$. To find the stabilizer group of $\theta_{12}$ in equation (12), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_{12}$ with their orders are shown as follows:

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&1&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}0&1&0\\0&4&0\\0&0&2\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&4&0\\0&0&3\end{pmatrix}:4.$$

Therefore, the stabilizer group of $\theta_{12}$in equation (12) which is denoted by $G_{\theta_{12}}$which contains

- 1 matrices of order 2;
- 2 matrices of order 4;
- The identity matrix.

Form[1], $G_{\theta_{12}}$is isomorphic to $\boldsymbol{Z_4}$, that is $G_{\theta_{12}} \cong \boldsymbol{Z_4}$. $\theta_{12}$in equation (12) is drawn in Figure-18:

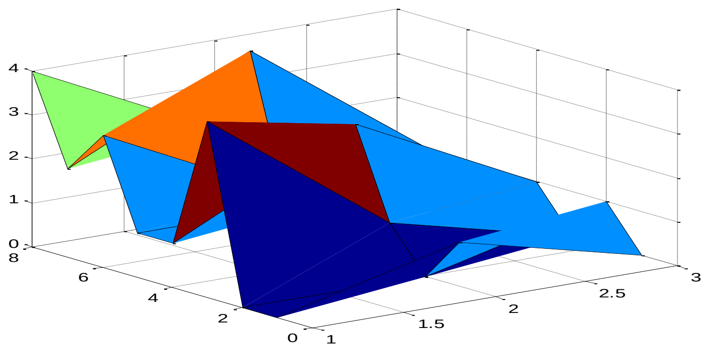


**Figure 18-**Drawing of $\theta_{12}$

Let $\theta_{12}^{*} = \{[1,3,1], [3,1,1], [2,4,1], 4,2,1]\}$ be a subset of $\theta_{12}$ in equation (12) which is forming by partition $\theta_{12}$ into two sets such that $\theta_{12}^{*}$ does not contains the inflection points of $\theta_{12}$, so we note that $\theta_{12}^{*}$ represents an arc of degree two. Also, to find the stabilizer group of $\theta_{12}^{*}$, by some calculation, we obtain

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&1&0\\1&0&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}0&1&0\\4&0&0\\0&0&2\end{pmatrix}:2,\begin{pmatrix}0&1&0\\4&0&0\\0&0&3\end{pmatrix}:2$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&1&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&4&0\\0&0&2\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&4&0\\0&0&3\end{pmatrix}:4,$$

$$\begin{pmatrix}1&1&1\\1&1&4\\2&3&0\end{pmatrix}:2,\begin{pmatrix}1&1&1\\1&1&4\\3&2&0\end{pmatrix}:4,\begin{pmatrix}1&1&1\\4&4&1\\1&4&0\end{pmatrix}:3,\begin{pmatrix}1&1&1\\4&4&1\\4&1&0\end{pmatrix}:3,$$

$$\begin{pmatrix}1&1&4\\1&1&1\\2&3&0\end{pmatrix}:4,\begin{pmatrix}1&1&4\\1&1&1\\3&2&0\end{pmatrix}:2,\begin{pmatrix}1&1&4\\4&4&4\\1&4&0\end{pmatrix}:3,\begin{pmatrix}1&1&4\\4&4&4\\4&1&0\end{pmatrix}:3,$$
$$\begin{pmatrix}1&4&2\\1&4&3\\2&2&0\end{pmatrix}:3,\begin{pmatrix}1&4&2\\1&4&3\\3&3&0\end{pmatrix}:3,\begin{pmatrix}1&4&2\\4&1&2\\1&1&0\end{pmatrix}:2,\begin{pmatrix}1&4&2\\4&1&2\\4&4&0\end{pmatrix}:4,$$
$$\begin{pmatrix}1&4&3\\1&4&2\\2&2&0\end{pmatrix}3,\begin{pmatrix}1&4&3\\1&4&2\\2&2&0\end{pmatrix}:3,\begin{pmatrix}1&4&3\\4&1&3\\1&1&0\end{pmatrix}:4,\begin{pmatrix}1&4&3\\4&1&3\\4&4&0\end{pmatrix}:2\ .$$

Therefore, the stabilizer group of $\theta_{12}^*$ which is denoted by $G_{\theta_{12}^*}$ which contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix.

Form[1], $G_{\theta_{12}^*}$ is isomorphic to $\boldsymbol{S_4}$, that is $G_{\theta_{12}^*} \cong \boldsymbol{S_4}$. Drawing of $\theta_{12}^*$ is given in Figure-19:

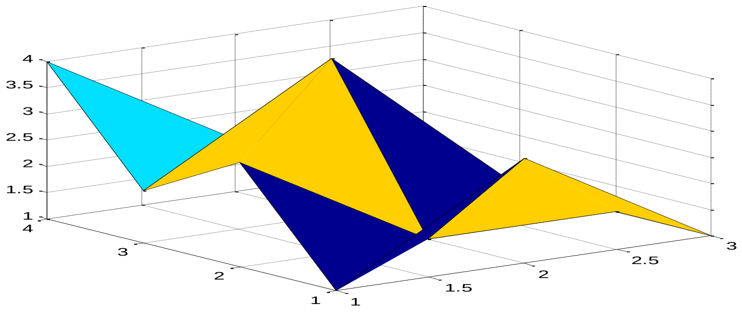

**Figure 19-**Drawing of $\theta_{12}^*$

Another one of cubic curve which is given in [4] is:

$$\theta_{13} = z^3 - 3(x^2 - xy + y^2)z - (x^3 - 3xy^2 + y^3)\ . \qquad (13)$$

The points of $PG(2,5)$ on $\theta_{13}$ in equation (13) are $[1,4,1], [1,2,1], [3,4,1]$. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\theta_{13}$ in equation (13) is 96, and we can not write them, because they are too much.

Moreover, the stabilizer group of $\theta_{13}$ in equation (13) which is denoted by $G_{\theta_{13}}$which contains

- 15 matrix of order 2;
- 32 matrix of order 3;
- 24 matrix of order 4;
- 24 matrix of order 8;
- The identity matrix.

Drawing of $\theta_{13}$ in equation (13) is given in Figure-20 as following:

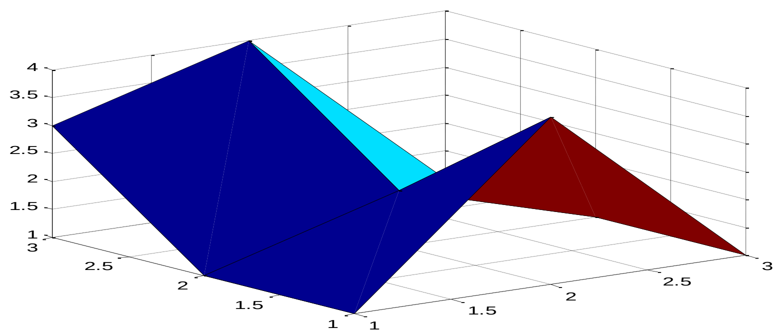

**Figure 20-**Drawing of $\theta_{13}$

Another one of cubic curve which is given in [4] is:

$$\theta_{14} = z^3 - (x^2 - xy + y^2)z - (x^3 - 3xy^2 + y^3)\ . \qquad (14)$$

The points of $PG(2,5)$ on $\theta_{14}$ in equation (14) are [4,1,1], [3,2,1], [3,1,1], [3,0,1], [0,3,1], [2,2,1]. To find the stabilizer group of $\theta_{14}$ in equation (14), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_{14}$with their orders are shown as follows:

$$\begin{pmatrix} 0 & 1 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}:3,\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1,\begin{pmatrix} 1 & 1 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}:3 .$$

Therefore, the stabilizer group of $\theta_{14}$ in equation (14) which is denoted by $G_{\theta_{14}}$contains

- 2 matrices of order 3;
- The identity matrix.

Form[6], $G_{\theta_{14}}$is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\theta_{14}} \cong \boldsymbol{Z_3}$. $\theta_{14}$ in equation(14) is drawn in Figure-21:

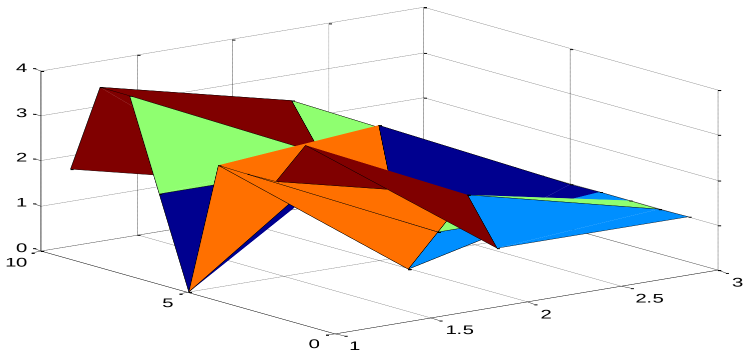


**Figure 21-**Drawing of $\theta_{14}$

Another one of cubic curve which is given in [4] is:

$$\theta_{15} = z^3 - (x^3 - 3xy^2 + y^3) . \qquad (15)$$

The points of $PG(2,5)$ on $\theta_{15}$ in equation (15) are [1,3,1], [0,1,1], [2,3,1], [2,4,1], [4,4,1], [1,0,1]. To find the stabilizer group of $\theta_{15}$ in equation (15), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_{15}$ with their orders are shown as follows:

$$\begin{pmatrix} 0 & 1 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}:3,\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}:1,\begin{pmatrix} 1 & 1 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}:3 .$$

Therefore, the stabilizer group of $\theta_{15}$in equation (15) which is denoted by $G_{\theta_{15}}$which contains

- 2 matrices of order 3;
- The identity matrix.

Form[1], $G_{\theta_{15}}$is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\theta_{15}} \cong \boldsymbol{Z_3}$. $\theta_{15}$ in equation (15) is drawn in Figure-22:

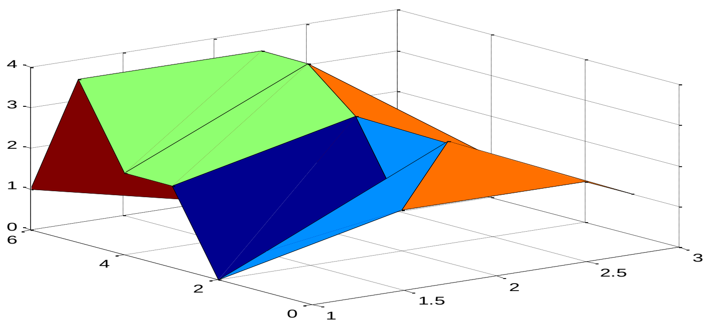


**Figure 22-**Drawing of $\theta_{15}$

Another one of cubic curve which is given in [4] is:

$$\theta_{16} = z^3 + 3(x^2 - xy + y^2)z - (x^3 - 3xy^2 + y^3) . \qquad (16)$$

The points of $PG(2,5)$ on $\theta_{16}$ in equation (16) are [0,4,1], [2,0,1], [4,0,1], [0,2,1], [3,3,1], [1,1,1]. To find the stabilizer group of $\theta_{16}$ in equation (16), we are doing calculations by help the computer. Thus the transformation matrices which stabilizing of $\theta_{16}$ with their orders are shown as follows:

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}0&1&0\\4&4&0\\0&0&1\end{pmatrix}:3,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,$$
$$\begin{pmatrix}1&0&0\\4&4&0\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&1&0\\0&4&0\\0&0&4\end{pmatrix}:2,\begin{pmatrix}1&1&0\\4&0&0\\0&0&4\end{pmatrix}:3.$$

Therefore, the stabilizer group of $\theta_{16}$ in equation (16) which is denoted by $G_{\theta_{16}}$ which contains

- 3 matrices of order 2;
- 2 matrices of order 3;
- The identity matrix.

Form [1], $G_{\theta_{16}}$ is isomorphic to $\boldsymbol{S_3}$, that is $G_{\theta_{16}} \cong \boldsymbol{S_3}$. $\theta_{16}$ in equation (16) is drawn in Figure-23:

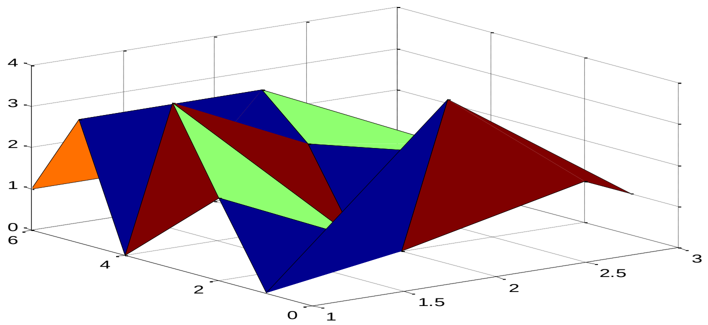


**Figure 23-**Drawing of $\theta_{16}$

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