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Some Results on Strongly Fully Stable Banach Γ –Algebra Modules Related To ΓA -deal

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Abstract
The main objective of this research is to study and to introduce a concept of strong fully stable Banach Γ-algebra modules related to an ideal.. Some properties and characterizations of full stability are studied.

Keywords : strongly fully stable Banach Γ-algebra modules related to an ΓA- ideal, fully stable Banach Γ-algebra modules related to an ΓA- ideal , Baer criterion related to an ΓA-ideal, strongly quasi α-injective related to an ΓA- ideal, fully stable Banach Γ-algebra modules

بعض نتائج مقاسات بناخ الاجبرا تامة الاستقرار ية بقوة من النمط ΓA- بالنسبة إلى مثالي من النمط ΓA

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الخلاصة
الهدف الرئيسي في هذا العمل هو دراسة مفهوم مقاسات بناخ الاجبرا تامة الاستقرار ية بقوة من النمط ΓA– بالنسبة إلى مثالي من النمط ΓAودراسة خواصه وبعض العلاقات المرتبطة بمفهوم مقاسات بناخ الاجبرا تامه الاستقرارية بقوة من النمط ΓA–

1. Introduction

The theory of Banach algebras is an abstract mathematical theory. Banach algebras started in the early twentieth century, when abstract concepts and structures were introduced, transforming both of the mathematical language and practice. A non-empty set $\mathcal{A}$ is an algebra with $(\mathcal{A},+,.)$ over a field $\mathcal{F}$ is a vector space and $\mathcal{A}$ is a ring with $+, \circ$ and $a \circ (\alpha b) = \alpha\ (a \circ b) = (\alpha a) \circ b$ for all α in $\mathcal{F}$, for each a, b in $\mathcal{A}$ [1]. In [2], a ring $\mathcal{R}$ is an algebra $< \mathcal{R}, +, \cdot, -, 0 >$ where $\mathcal{R}$ is a ring and binary operations $+$ and $\cdot$, unary is$-$ and nullary element is 0, when a commutative group is $< \mathcal{R}, +, -, 0 >$ and a semi-group is $< \mathcal{R}, . >$, $(x+y).z = (x.z) + (y+z)$also $x.(y+z) = (x.y) + (x.z)$. Assume that $\mathcal{A}$ is an algebra , recall that a Banach space $\mathcal{E}$ is a left B- algebra-module if $\|a\|\|x\| \leq \|a.x\|$(a in $\mathcal{A}$, x in $\mathcal{E}$) and $\mathcal{E}$ is a left $\mathcal{A}$ −module [1]. All modules over commutative Banach algebras are both left and right modules. A Banach algebra $\mathcal{A}$ is always a left and right $\mathcal{A}$ -

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module with the module multiplication which is taken to be the multiplication in $\mathcal{A}$. A Banach space is a $\mathcal{A}$ -module with the module multiplication which is taken to be the scalar multiplication. So every closed left ideal of a Banach algebra $\mathcal{A}$ is in a natural way is a left $\mathcal{A}$ -module. If U is a submodule of an $\mathcal{A}$ -module V, then the quotient space V/U with the quotient norm is in a natural way an $\mathcal{A}$ -module. If $\mathcal{A}$ is a Banach algebra, and V is a Banach space, then V becomes an $\mathcal{A}$ -module if we define av = 0 for all a ∈A, v∈V. We call V a trivial $\mathcal{A}$ –module [3]. A function $\mathcal{T} : \mathcal{X} \to \mathcal{Y}$ (not necessarily $\mathcal{A}$ is commutative) is said to be (homomorphism) multiplier if $a.\mathcal{T}x = \mathcal{T}(a.x)\ \forall\ a \in \mathcal{A}, x \in \mathcal{X}$ [4]. Following [5], let $\mathcal{N}$ be sub-module of a module $\mathcal{M}$, if $f(\mathcal{N}) \subseteq \mathcal{N}$, $\mathcal{N}$ is said to be stable, for each $\mathcal{R}$ −homomorphism $f: \mathcal{N} \to \mathcal{M}$ an $\mathcal{R}$ −module $\mathcal{M}$ is called a fully stable module , if each sub-module of $\mathcal{M}$ is stable". In [6] , a B-A module $\mathcal{M}$ is said to be full stability B-algebra-module if each sub-module $\mathcal{N}$ of B-algebra-module $\mathcal{M}$ and for every multiplier θ from N to $\mathcal{M}$ such that $\mathcal{N} \supseteq \theta(\mathcal{N})$. From [7], a left B –algebra-module $\mathcal{X}$ is n −generated $n \in \mathcal{N}$ if there exists $x_1, \dots, x_n$ in $\mathcal{X}$ such that for all x in $\mathcal{X}$, can be written x as $x = a_1x_1, \dots\dots, a_kx_k$ for some $a_1, \dots, a_k$ in $\mathcal{A}$. A 1-generated is called cyclic module . Following [8,9,10] Let $\mathcal{X}$ be B-algebra module , $\mathcal{X}$ is called fully stable Banach $\mathcal{A}$ −module if for every submodule $\mathcal{N}$ of $\mathcal{X}$ and for each multiplier $\theta: \mathcal{N} \longrightarrow \mathcal{X}$ such that $\theta(\mathcal{N}) \subseteq \mathcal{N}$", suppose that $\mathcal{X}$ is B-algebra module, if for each sub-module $\mathcal{N}$ of $\mathcal{X}$ and for every multiplier θ from $\mathcal{N}$ to $\mathcal{X}$, such that $\mathcal{N} + \mathcal{K}\mathcal{X} \supseteq \theta(\mathcal{N})$, $\mathcal{X}$ is said to be fully stable Banach algebra-module relative to an ideal $\mathcal{K}$of $\mathcal{A}$. It is easy to see every fully stable Banach $\mathcal{A}$ −module is fully stable Banach $\mathcal{A}$module relative to an ideal. Assume that $\mathcal{X}$ is B-algebra module , if for every sub-module $\mathcal{N}$ of $\mathcal{X}$ and for every multiplier $\theta: \mathcal{N} \longrightarrow \mathcal{X}$ such that $\mathcal{N} \cap \mathcal{K}\mathcal{X} \supseteq \theta(\mathcal{X})$, $\mathcal{X}$ is said to be strongly fully stable B- algebra module relative to an ideal $\mathcal{K}$of $\mathcal{A}$, . It is an easy mater to see that every strongly fully stable B- algebra module relative to an ideal is fully stable Banach algebra module. Following [11], suppose that Γ- is a groupoid and $\mathcal{V}$ is a space of vectors over $\mathcal{F}$. Hence , $\mathcal{V}$ over $\mathcal{F}$ is said to be a Γ –algebra, if there is a mapping from $\mathcal{V} \times \Gamma \times \mathcal{V}$ to $\mathcal{V}$ (we denoted the image by $x\ \alpha\ y$ for x , y in $\mathcal{V}$ and α in Γ) such that :(1)) $x\ (\alpha + \beta)\ y = x\ \alpha\ y + x\ \beta\ y$, (2) $(c\ x)\alpha\ y = c(x\ \alpha\ y) = x\ \alpha(c\ y)$, (3)$(x + y)\ \alpha z = x\ \alpha z + y\ \alpha z$, $x\ \alpha\ (y + z) = x\ \alpha\ y + x\ \alpha z$, (4) $0\alpha\ y = y\ \alpha 0 = 0$, α, β ∈ Γ and for all $x, y, z \in \mathcal{V}$, c ∈ $\mathcal{V}$. A Γ - algebra is said to be associative if (5) $(x\ \alpha\ y)\beta z = x\ \alpha(y\ \beta z)$, and unital if for every α, β ∈ Γ, there is an element 1_α in V such that $1_\alpha\ \alpha\ v = v = v\alpha 1_\alpha$ for every nonzero elements of $\mathcal{V}$. The concept of strongly fully stable B- ΓA-modules related to an ideal have been introduced and is proving anther characterization of strong fully stable B- ΓA-modules related to ΓA-ideal A Banach ΓA-module $\mathcal{X}$ is strong fully stable B- ΓA-modules related to ΓA-ideal if and only if for each $\mathcal{N}_{\alpha x}, \mathcal{K}_{\beta y}$ subsets of $\mathcal{X}$, $y \notin \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\ \mathcal{X}$ implies $\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x}) \nsubseteq \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y})$.

2- Strongly Fully Stability Banach Γ- Algebra Modules Related to an Γ-ideal.

In this section the concept of strongly fully stable Banach Γ- Algebra Modules Related to an ideal is introduced and other characterizations of this concept have been studied.

2.1 Definition: Suppose that $\mathcal{X}$ is B-algebra- module , $\mathcal{X}$ is said to be strongly fully stable B-$\Gamma\mathcal{A}$ −module related to Γ −ideal $\mathcal{J}$ of $\Gamma\ \mathcal{A}$, if for each sub-module N of B-algebra- module $\mathcal{X}$ and for all $\Gamma\mathcal{A}$ -multiplier θ from $\mathcal{N}$ to $\mathcal{X}$ such that $\mathcal{N} \cap \Gamma\mathcal{J}\Gamma\mathcal{X} \supseteq \theta(\mathcal{N})$. It is easy to see that every strongly fully stable Banach $\mathcal{A}$-modules related to an ideal is fully stable Banach $\Gamma\mathcal{A}$-modules. Therefore $\mathcal{X}$ is strongly fully stable Banach $\Gamma\mathcal{A}$-modules related to $\Gamma\mathcal{A}$ − ideal, if and if for each 1-generated sub-module L in $\mathcal{X}$ and for every $\Gamma\mathcal{A}$ − multiplier $\theta: L \longrightarrow \mathcal{X}$ such that $\theta(L) \subseteq L \cap K\mathcal{X}$.

Let $\mathcal{X}$ be a Banach $\mathcal{A}$-modules and $\mathcal{J}$ be a nonzero Γ −ideal of algebra $\mathcal{A}$. If $\mathcal{X}$ is fully stable B- $\Gamma\mathcal{A}$-modules and $\mathcal{X} = \Gamma\mathcal{J}\Gamma\ \mathcal{X}$ then $\mathcal{X}$ is strong fully stable B- ΓA-modules related to an ideal K, since for each 1-generated sub-module N of $\mathcal{X}$ and ΓA-homomorphism f from $\mathcal{N}$ to $\mathcal{X}$, $\mathcal{N} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$=. $\mathcal{N} \cap X \supseteq f(\mathcal{N})$.

Suppose that $\mathcal{X}$ is a B-$\Gamma\mathcal{A}-$ module, let $\mathcal{N}$ and $\mathcal{K}$ be two subsets of $\mathcal{X}$, then

1) $\mathcal{N}_{\alpha x} = \{n_{\alpha x} \mid n \in \mathcal{N}, \alpha \in \Gamma, x \in \mathcal{X}\}$,and

2) $ann_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x}) = \{a.\alpha \in \Gamma\mathcal{A} \mid a.\alpha.n_{\alpha x} = 0, \forall n_{\alpha x} \in \mathcal{N}_{\alpha x}\}$.

2.2 Proposition: A Banach ΓA-module $\mathcal{X}$ is strong fully stable B- ΓA-modules related to ΓA-ideal if and only if for each $\mathcal{N}_{\alpha x}, \mathcal{K}_{\beta y}$ subsets of $\mathcal{X}$, y $\notin \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$ implies $\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x}) \nsubseteq \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y})$.

Proof :- Assume that $\mathcal{X}$ is is strong fully stable B- ΓA-modules related to ΓA-ideal K, there is $\mathcal{N}_{\alpha x}, \mathcal{K}_{\beta y}$ subsets of $\mathcal{X}$, such that y $\notin \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$ and $\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x}) \subseteq \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y})$. Define $\theta: < \mathcal{N}_{\alpha x} > \longrightarrow X$ by $\theta(\delta a.n_{\alpha x}) = \delta a.\kappa_{\beta y}$, for all $\delta a \in \Gamma\mathcal{A}$, if $\delta a.n_{\alpha x} = 0$ then $\delta a \in \mathrm{ann}_{\Gamma A}(\mathcal{N}_{\alpha x}) \subseteq \mathrm{ann}_{\Gamma A}(\mathcal{K}_{\beta y})$. This implies that $\delta a.\kappa_{\beta y} = 0$, hence θ is well define. θ is a $\Gamma-$multiplier, since $\mathcal{X}$ is strong fully stable B-ΓA-modules related to ΓA-idea , there is an element $\gamma t \in \Gamma\mathcal{A}$ s.t $\theta(m_{\alpha x}) = \gamma t m_{\alpha x}$ for each $m_{\alpha x} \in \mathcal{N}_{\alpha x}$. In particular, $\kappa_{\beta y} = \theta(n_{\alpha x}) = \gamma t n_{\alpha x} \in \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$. This gives a contradiction. Hence $\mathcal{X}$ is strong fully stable B- ΓA-modules related to ΓA-idea. Conversely, suppose that there exists a $\Gamma-$multiplier $\theta: < \mathcal{N}_{\alpha x} > \longrightarrow X$ and a subset $N_{\alpha x}$ in $\mathcal{X}$ such that $\theta(\mathcal{N}_{\alpha x}) \nsubseteq \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$ then there exists an element $m_{\alpha x} \in N_{\alpha x}$ such that $(m_{\alpha x}) \notin \mathcal{N}_{\alpha x} \cap \Gamma K\Gamma\mathcal{X}$. Let $\eta s \in \mathrm{ann}_{\Gamma A}(\mathcal{N}_{\alpha x})$. Therefore $\eta s n_{\alpha x} = 0$, $\eta s\theta(m_{\alpha x}) = \theta(\eta s m_{x}) = \theta(\eta s\gamma t n_{x}) = \theta(\gamma t\eta s n_{x})\theta(\gamma\eta t s n_{x}) = \theta(0) = 0$. Hence $\mathrm{ann}_{\Gamma A}(\mathcal{N}_{\alpha x}) \subseteq \mathrm{ann}_{\Gamma A}(\theta(m_{\alpha x}))$which is contradiction .

2.3 Corollary : Suppose that $\mathcal{X}$ is a related to strong fully stable B- ΓA-modules related to ΓA-ideal K .Then $\mathcal{N}_{\alpha x}, \mathcal{K}_{\beta y}$ subsets of $\mathcal{X}, \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y}) = \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})$ implies $\mathcal{K}_{\beta y} \cap \Gamma K\Gamma\mathcal{X} = \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$.

Proof:- Suppose that there are two elements $x, y \in \mathcal{X} \ni \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y}) = \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})$ and $\mathcal{K}_{\beta y} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X} \neq \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$. Therefore without loss of generality there exists $z_{\alpha x} \in \mathcal{N}_{\alpha x}$and $z_{\alpha x} \notin \mathcal{K}_{\beta y}$ .By using proposition (4.2) we get $\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y}) \nsubseteq \mathrm{ann}_{\Gamma A}(z_{\alpha x})$, but $\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x}) \subseteq \mathrm{ann}_{\Gamma A}(z_{\alpha x})$, hence $\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y}) \nsubseteq \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})$ which is contradiction.

2. 4 Definition : A sub-modue N of B- $\Gamma\mathcal{A}$-module is called pure $\Gamma-$submodule if $\Gamma K\Gamma\, \mathcal{N} = \Gamma\, \mathcal{N} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X}$ for each $\Gamma-$ideal $\mathcal{J}$ of $\mathcal{A}$.

When the sub-module of strong fully stable B- ΓA-modules related to ΓA-ideal have been partial answer in the next result.

2.5 Proposition : Suppose that $\mathcal{X}$ is a strong fully stable B- ΓA-modules related to a nonzero $\Gamma-$ideal $\mathcal{J}$ of $\mathcal{A}$. Then every pure $\Gamma-$submodule is strong fully stable B- ΓA-modules related to ΓA-ideal.

Proof : Assume that $\mathcal{N}$ is pure $\Gamma-$sub-module in $\mathcal{X}$. For all $\Gamma-$sub-module L in $\mathcal{N}$ and f : L $\rightarrow$ N a $\Gamma-$multiplier, set i o f = g : L $\rightarrow$ X , i is the inclusion mapping from $\mathcal{N}$ to $\mathcal{X}$, then from assumption f(L) = g(L) $\subseteq \Gamma\mathcal{J}\Gamma\, \mathcal{X}$, and f(L) $\subseteq$ N. Hence f (L) $\subseteq$ L $\cap\, \Gamma\mathcal{J}\Gamma\mathcal{X} \cap \mathcal{N}$. Because of $\mathcal{N}$ is pure $\Gamma-$sub-module in $\mathcal{X}$,we have $\Gamma\, \mathcal{N} \cap \Gamma\mathcal{J}\Gamma\, \mathcal{X} = \Gamma\mathcal{J}\Gamma\, \mathcal{N}$, for each $\Gamma-$ideal $\mathcal{J}$ of $\Gamma\mathcal{A}$, therefore f(L) $\subseteq$ L $\cap\, \Gamma\mathcal{J}\Gamma\, \mathcal{N}$. Thus $\mathcal{N}$ is strong fully stable B- ΓA-modules related to ΓA-ideal $\mathcal{J}$.

2. 6 Definition : A Banach $\Gamma\mathcal{A}-$module $\mathcal{X}$ is called Baer criterion relative to an ideal $\mathcal{J}$ in A satisfied, if every sub-module of $\mathcal{X}$ Baer criterion relative to an ideal $\mathcal{J}$ in A satisfied, this mean that, for each 1-generated sub-module $\mathcal{N}$ in $\mathcal{X}$ and θ from $\mathcal{N}$ to $\mathcal{X}$ $\mathcal{A}-$ multiplier, there is τa in$\in \Gamma\mathcal{A}$ s.t $\theta(n) = \tau a n \in \Gamma\mathcal{J}\Gamma\mathcal{X}$ for all n $\in \mathcal{N}$.

The next proposition and corollary give new characterization of strong fully stable B- ΓA-modules related to ΓA-ideal.

2.7 Proposition :If $\mathcal{X}$ is a B- algebra –module, then the Baer criterion relative to an ideal $\mathcal{J}$ in A is satisfied for 1-generated sub-module in $\mathcal{X}$ if and only if $\mathrm{ann}_{\Gamma\chi}\left(\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})\right) = \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\mathcal{X}$ for each $\alpha x \in \Gamma\mathcal{X}$.

Proof :- Suppose that the Baer criterion relative to an ideal $\mathcal{J}$ in A holds for 1-generated sub-module of $\mathcal{X}$. Let $y \in \mathrm{ann}_{\Gamma\chi}\left(\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})\right)$ and define $\theta: < \mathcal{N}_{\alpha x} > \longrightarrow X$ by $\theta(\gamma a. n_{\alpha x}) = \gamma a. \kappa_{\beta y}$, for all $\gamma a \in \Gamma\mathcal{A}$. Let $\gamma a_1. n_{\alpha x} = \gamma a_2. n_{\alpha x}$, thus $\gamma(a_1 - a_2)n_{\alpha x} = 0$ where $\gamma(a_1 - a_2) \in \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})$, so that $\gamma(a_1 - a_2) \in \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{K}_{\beta y})$. Therefore $\gamma(a_1 - a_2)\kappa_{\beta y} = 0$,and $a_1\kappa_{\beta y} = a_2\kappa_{\beta y}$, then θ is well define. It is easy to see *that* θ is an $\Gamma\mathcal{A} -$ multiplier. There exists an element $\delta t \in \Gamma\mathcal{A}$ from the assumption that $\theta(m_{\alpha x}) = \delta t m_{\alpha x} \in K\mathcal{X}$ for each $m_{\alpha x} \in \mathcal{N}_{\alpha x}$, we have in particular, $\kappa_{\beta y} = \theta(n_{\alpha x}) = \delta t n_{\alpha x} \in \Gamma\mathcal{J}\Gamma\mathcal{X}$, therefore $\mathrm{ann}_{\Gamma\chi}\left(\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})\right) \subseteq \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\mathcal{X}$, and $\mathrm{ann}_{\Gamma\chi}\left(\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})\right) = \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma$. Conversely, assume that $\mathrm{ann}_{\Gamma\chi}\left(\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})\right) = \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\mathcal{X}$, for each $\mathcal{N}_{\alpha x} \subseteq X$, then for each $\mathcal{A} -$ multiplier $\theta: \mathcal{N}_{\alpha x} \to \mathcal{X}$, and $\mu s \in \mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})$ we have $\mu s\theta(n_{\alpha x}) = \theta(\mu s n_{\alpha x}) = 0$. Thus $\theta(n_{\alpha x}) \in \mathrm{ann}_{\Gamma\chi}\left(\mathrm{ann}_{\Gamma\mathcal{A}}(\mathcal{N}_{\alpha x})\right) = \mathcal{N}_{\alpha x} \cap \Gamma\mathcal{J}\Gamma\mathcal{X}$, and $\theta(n_{\alpha x}) = \delta t\, n_{\alpha x} \in \Gamma\mathcal{J}\Gamma\mathcal{X}$ for some $\delta t \in \Gamma\mathcal{A}$, hence Baer criterion relative to an ideal $\mathcal{J}$ in A holds.

2.8 Corollary: X is strong fully stable B- ΓA-modules related to ΓA-ideal K if and only if $\mathrm{ann}_X(\mathrm{ann}_A(N_x)) \subseteq N_x \cap \Gamma\mathcal{J}\Gamma\mathcal{X}$ for each $x \in \mathcal{X}$.

In[8], the authors assume that $\mathcal{A}$ be a unital B-Γ −algebra. Algebra-module $\mathcal{X}$ is said to be quasi α-injective if, φ from $\mathcal{N}$ to $\mathcal{X}$ is algebra-module homomorphism (multiplier) such that $\| \varphi \| \leq 1$, there is algebra-module homomorphism (multiplier) θ from $\mathcal{X}$ to $\mathcal{X}$, ∃θ o i = φ and $\| \theta \| \leq \alpha$, i is an iso-metry ,algebra-module isomorphism is an iso-metry algebra-multiplier , from sub-module $\mathcal{N}$ in $\mathcal{X}$ to $\mathcal{X}$, and $\mathcal{X}$ is said to be quasi injective if $\mathcal{X}$ is quasi α - injective for some α

The concept of strongly quasi α-injective related to an $\Gamma\mathcal{A} -$ ideal $\mathcal{J}$ of $\mathcal{A}$ is introduced.

2.9 Definition: Assume that $\mathcal{A}$ be a unital B-Γ −algebra. $\Gamma\mathcal{A}$ −module $\mathcal{X}$ is said to be strongly quasi α-injective related to an $\Gamma\mathcal{A} -$ ideal $\mathcal{J}$ of $\mathcal{A}$ if, φ from $\mathcal{N}$ to $\mathcal{X}$ is $\Gamma\mathcal{A}$ −multiplier such that $\| \varphi \| \leq 1$, there exists $\Gamma\mathcal{A}$ −multiplier θ from $\mathcal{X}$to $\mathcal{X}$, such that (θ o i)(n) = φ(n) $\in \Gamma\mathcal{J}\Gamma\mathcal{X}$ and $\| \theta \| \leq \alpha$ i is an iso-metry from sub-module $\mathcal{N}$ to $\mathcal{X}$. $\mathcal{X}$ is said to be strongly quasi injective related to $\Gamma\mathcal{A} -$ ideal if it is strongly quasi α – injective related to $\Gamma\mathcal{A} -$ ideal for some α.

In the following proposition we give the relationship between strongly quasi α-injective B-$\Gamma\mathcal{A}$ −module related to $\Gamma\mathcal{A} -$ ideal and strongly fully stable B- $\Gamma\mathcal{A}$ −module related to an $\Gamma\mathcal{A} -$ ideal K of A.

2.10 Proposition: Assume that $\mathcal{X}$ be B- $\Gamma\mathcal{A}$ −module and $\mathcal{J}$ be a non-zero ideal of $\Gamma\mathcal{A}$. If $\mathcal{X}$ is strong fully stable B- ΓA-modules related to ΓA-ideal then $\mathcal{X}$ is strongly quasi injective B-$\Gamma\mathcal{A}$ −module related to $\Gamma\mathcal{A}$ −ideal.

Proof: Suppose that $\mathcal{N}$is sub-module in $\mathcal{X}$ and f : $\mathcal{N} \to \mathcal{X}$ be any algebra-module homomrphism. Because *of* $\mathcal{X}$ is a fully stable B- $\Gamma\mathcal{A}$-module related to $\Gamma\mathcal{A}$ −ideal $\mathcal{J}$, therefore $f(\mathcal{N}) \subseteq \mathcal{N} \cap \Gamma\mathcal{J}\Gamma\mathcal{X}$, hence there exists $\lambda t \in \Gamma\mathcal{A}$ such that f(n) = λtn . Define g : $\mathcal{X} \to \mathcal{X}$ by λtx = g (x), it is easy to see that g is a well defined $\Gamma\mathcal{A}$ −multipler, f(x) = g(x) = λtx $\in \Gamma\mathcal{J}\Gamma\mathcal{X}$, and for all y in $\mathcal{N}$, (f o i) (y) – g(y) = f(y) – g(y) $\in \Gamma\mathcal{J}\Gamma\mathcal{X}$, i is iso-metry, and for some α, $\| g \| \leq \alpha$ Therefore $\mathcal{X}$ is strongly quasi injective B- $\Gamma\mathcal{A}$ −module related to $\Gamma\mathcal{A} -$ ideal.

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