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Studying the Chaotic Dynamics Using Rossler-Chua Systems Combined with A Semiconductor Laser

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Abstract

In this paper, two different chaotic dynamic systems are coupled using a semiconductor laser to produce a new chaotic system. These two chaotic systems are Rossler and Chua systems. X-dynamic of Rossler system was coupled optically using optical fiber as a carrier of signal with x, y, and z-dynamics of Chua system. The results were analyzed and the behavior of Chua system was found to be changing in time series which, in turn, changed the attractor. The Chua attractor was converted from double scroll to single scroll. The results obtained from connecting two different systems in chaotic behavior showed a remarkable increase in the bandwidth of Chua system. This increase in bandwidth opens up a wide field for many applications, the most important of which is in the field of secure communications.

Keywords: Chaos systems, Rossler circuit, Chua circuit, coupling system, nonlinear dynamics, and secure communication.

دراسة الديناميكية الفوضوية باستخدام نظامين تشوا- روزلار والمقترنة بليزر شبه الموصل

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الخلاصة

في هذا البحث، تمت عملية ربط نظامين ديناميين فوضويين مختلفين باستخدام ليزر شبه موصل لإنتاج نظام فوضوي جديد. هذان النظامان الفوضويان هما نظام تشوا وروزلار. الديناميك X لنظام روزلار يقترن بصرياً باستخدام الألياف الضوئية كحامل للإشارة مع ديناميكيات X و Y و Z لنظام تشوا ويتم تحليل النتائج. تم تغيير سلوك نظام تشوا في سلسلتها الزمنية والتي بدورها غيرت الجاذب. يتم تحويل جاذب تشوا من لفائف مزدوجة إلى لفائف فردية. ظهرت النتائج المستحصلة من ربط نظامين مختلفين في السلوك الفوضوي إلى حدوث زيادة ملحوظة في عرض النطاق الترددي لنظام تشوا وهذه الزيادة في عرض النطاق تفسح مجال واسع للتطبيقات كثيرة وأهما في مجال الاتصالات الآمنة.

1. Introduction

Chaos refers to "An irregular oscillation governed by a moderately basic principle" [1]. Here oscillation only refers to a variety of specific amounts. There are numerous sorts of chaotic systems; Lorenz system [2] was utilized in the climate science, the Rossler system in the field of chemical engineering, the Chua system for electrical circuits [3], in addition to the Duffing system in the field of mechanics [4, 5]. The chaotic systems have important properties; they are dependent on preliminary

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situations and pseudo-randomness, and system parameters. They also have no periodicity and feature huge variety of Fourier spectra [6]. The chaos might be located in a lot of research disciplines, including communication [7-11], electronics, engineering, economics, physics, biology, chemistry, mathematics, steganography [12], and encryption [13]. When engineers use the word chaos, they generally suggest that a predictable dynamics machine can provide unpredictable effects. The simplest way to examine chaos is in electronic circuits. This is because of its simplicity and lower cost and because electronic devices are nicely understood. Chua's and Rossler circuits are examples of chaotic circuits; however, Chua is less complicated than Rossler.

In this paper, two different chaotic dynamic systems (Chua-Rossler) are coupled using a semiconductor laser to produce a new chaotic system, where the influence of x-dynamic of the Rossler's circuit on the behavior of Chua's circuit was studied. There are many previous studies that have used chaos theory to solve and develop many security problems [14-15]. However, the most important of these studies is that being conducted presently, reflecting an attempt to hide important and confidential documents using chaotic technology, which is currently in the process of publication.

2. Chua's and Rossler's circuits

Chaotic dynamics may be defined in lots of unique differential equations that can be constructed into an electrical circuit. An instance of an electrical circuit, that is used in this research, is Chua circuit. The differential equations of Chua's circuit are presented below [16]:

$$C_1 \frac{dv_1}{dt} = \frac{1}{R}(v_2 - v_1) - f(v_1) \quad (1)$$

$$C_2 \frac{dv_2}{dt} = \frac{1}{R}(v_1 - v_2) + i_L \quad (2)$$

$$L \frac{di_L}{dt} = -v_2 \quad (3)$$

$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|v_1 + B_p| - |v_1 - B_p|) \quad (4)$$

In Figure-1, it is shown how the Chua's circuit includes three energy-storing components (2 capacitors C_1 and as C_2 , and one inductor L). C_1 and C_2 are the parameters of the conductors. R is the changeable resistance and L is the constant value of the coil. N_R is the value of the negative resistance. The values v_1 and v_2 represent the voltages on C_1 and C_2 , respectively, while i_L represents the current in the inductor L , which can be measured experimentally, as shown previously [11]. Nevertheless, it can be also theoretically calculated by the differential equation above. The values of G_a , G_b , and B_p are defined in Figure-2, whereas table (1) represents all Chua circuit components.

The dimensionless state equations (Wu, 2002)[17] are given by:

$$\dot{x}_1 = \alpha_1(y_1 - x_1 - g(x_1)) \quad (5)$$

$$\dot{y}_1 = x_1 - y_1 + z_1 \quad (6)$$

$$\dot{z}_1 = -\beta y_1 \quad (7)$$

$$g(x_1) = c_1 x_1 + \left(\frac{1}{2}\right)(d - c_1)(|x_1 + 1| - |x_1 - 1|) \quad (8)$$

where x_1 , y_1 , and z_1 represent C_1 , C_2 , and i_L , respectively, α_1 , β , c_1 , and d represent $1/R$, C_1 , $1/L$, G_b and G_a , respectively, and $g(x_1)$ is the nonlinear function (Sometimes called Chua's diode).

The second chaotic circuit that is used in our scheme is Rossler circuit [18]. The electronic circuit of a Rossler is shown in Figure-3. Table-2 presents all Rossler circuit components. The amplifier A4 is switched on when the voltage exceeds 3V.

Rossler circuit might be specified via the following three-dimensional system related to differential equations (Lynch, 2004) [19]:

$$\dot{x}_2 = -(y_2 + z_2) \quad (9)$$

$$\dot{y}_2 = x_2 - a_2 y_2 \quad (10)$$

$$\dot{z}_2 = b + x_2 y_2 - c_2 z_2 \quad (11)$$

The differential equations are defining continuous-time dynamical system which is showing chaotic dynamics related to attractor's fractal properties. It has seven terms, single quadratic non-linearity, and three parameters, in which a , b , $c \in \mathbb{R}$. There are also dimensionless parameters, x , y , and z which are three variables that are evolved with continues time. Firstly, the real parameter's values were examined via Otto. Furthermore, the E. Rossler parameters showed values of a and $b = 0.2$ and $c = 5.7$,

which implies that the systems is showing chaotic behavior. The initial two equations were linear terms which are creating oscillations in variable x and y . The last equation has only a single non-linear term (xz), thus the expected chaotic behavior was considered as a form of the system.

To employ the new chaotic scheme, two chaotic systems were connected, as shown in Figure-4. The Rossler circuit consists of three channels, with any channel can be used to couple with Chua circuit optically via optical fiber, using semiconductor laser as carrier source. It has a wavelength of 850 nm over a continuous and controlled power range of 1nW to 2mW. The description of other laser characteristics includes spectral half bandwidth of 2.5nm, multimode, maximum continuous output power of 2mW, beam diameter of 50mm, beam divergence of 0.2rad, threshold current of laser of 2.5mA. Then, the two circuits were coupled, i.e. the output variable x of Rossler circuit is the input variable x of Chua circuit, as shown in Figure-4.

Single mode optical fiber (SMF) can be considered as an optical fiber developed for carrying light just directly down the fiber. SMF might be carrying more information with a longer distance in comparison to the electrical signals carried in copper coaxial medium, with not much transmission losses, low interferences, as well as elevated bandwidth possibilities; the optical fiber is specified as the almost optimum transmission medium. PD's function is converting the optical signal which comes from Rossler circuit through the optical fiber to an electrical signal at Chua circuit. PD (InGaAs type) has many characteristics, including high-gain, low noise, as well as RF photoreception. The detector's typical bandwidth is 1GHz, current gain (40), bias of 12V battery, and peak response of 0.95A/W at 1550nm.

The Rossler and Chua circuits are disaccorded components. Then, the signal is driven out from the Rossler circuit and drives in the Chua circuit, thus each of the changes in any of the elements related to the first circuit are affecting the second one. The output voltage x , y , and z of Chua's circuit are recorded with a digital storage oscilloscope (Tektronix TDS 2024B model). The digital storage oscilloscope is connected with a personal computer (PC) to handle the resulting data.

The dimensionless states' equations of the coupled Chua circuit will be [9]:

$$\dot{x}_1 = \alpha_1(y_1 - x_1 - g(x_1)) - (y_2 + z_2) \quad (12)$$

$$\dot{y}_1 = x_1 - y_1 + z_1 \quad (13)$$

$$\dot{z}_1 = -\beta y_1 \quad (14)$$

$$g(x_1) = c_1 x_1 + \left(\frac{1}{2}\right)(d - c_1)(|x_1 + 1| - |x_1 - 1|) \quad (15)$$

3. Results and discussion

The time series of the Rossler circuit is shown in Figure-5, which represents the output voltage in x -dynamics, where the output voltage is in the range of +12 to -9V, while the time range is 1000 to 5000 μ s. Figure-6 demonstrates the three state variables x , y and z of Chua's system, where the output voltage values are +7 to -5.5V, +1.27 to -1.12V, and +8.85 to -8.78V, respectively.

The second main objective of this experiment is to produce a strange attractor of the Chua circuit. If the attractor exists, the process cannot be random. Since the Chua circuit has three degree of freedom, then its state space is three-dimensional (3D). One way to display a strange attractor is to take a pair of the state variable (e.g. x,y), as shown in Figure-7 that shows double-scroll strange attractor mode, where the variables x and y represent the voltages on the capacitors that can be tapped easily. Figures-8 and 9 present the strange attractors of the pairs (x,z) and (y,z) , respectively, that show also double-scroll strange.

When coupling the dynamic variable x of the Rossler circuit with all the dynamic variables of the Chua circuit (x , y , and z) visually, the time series of Chua circuit will change to a new behavior and become as shown in Figure-10, where it is completely different from the first sate. The range values of output voltage are changing as a result of this coupling. Moreover, the attractor of Chua system is showing "double scroll" pattern. This behavior converts to homoclinic chaotic by coupled x -dynamics of Chua circuit with x , y , or z dynamics of Rossler circuit, as shown in Figures-(11, 12, 13). Finally, by this scheme, novel chaotic dynamics can be obtained by laser-coupled two different chaotic systems.

Figures-(14, 15) show the importance of this work and the benefits of coupling the two different chaotic systems, through the increase in the bandwidth of the Chua system, characterized by a narrow bandwidth which prompts this system to be effective in different applications, the most important of

which is in secure communications. This fact confirms the results of Fourier transforms obtained before and after coupling. From the results, chaotic systems with attractor of higher dimensions are believed to have important and exciting applications. Figures-(14,15) refer to the spectra of the Fourier transforms of the uncoupling Chua system and the Chua system coupled with the Rossler system, respectively. The results show a wide spectrum in the case of coupling, with the disappearance of the distinctive frequencies evident in the Chua system. In these two figures, it is noticed that the dynamic bandwidth z-coupled became more broad than it is in the case of non-coupling, and this point is very important in the subject of secret communications. Thus, the characteristic frequency of the Chua system has completely disappeared within the wide range formed as a result of the coupling. Thus, the dynamic z-dynamic is more important than the other dynamics due to its discontinuous exponential dicey distribution [20].

4. Conclusions

Through the foregoing description, the process of coupling two different chaotic systems in a chaotic behavior can produce a new strong chaotic behavior with a wide bandwidth, especially in the dynamic z_2 , which leads us to use this new chaotic system in secure communications. The attractor of Chua system is showing a "double scroll" pattern. This behavior is converted to a homoclinic chaotic by the coupled x -dynamics of Chua circuit with x , y , or z dynamics of Rossler circuit

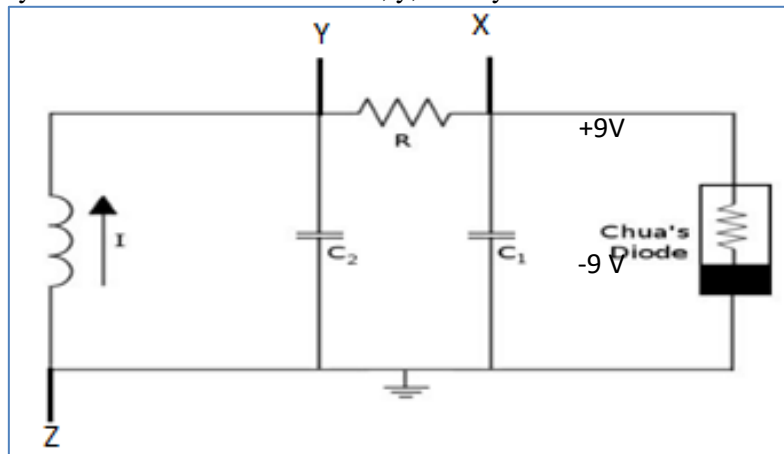


Figure 1- The schematic of Chua's circuit.

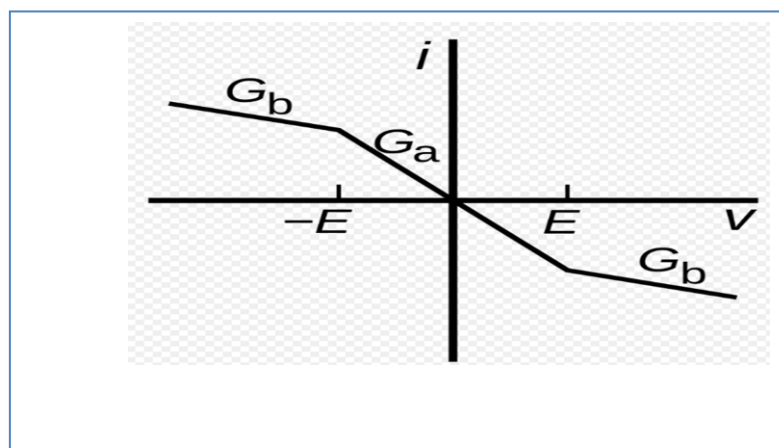


Figure 2- The current–voltage characteristic of the Chua diode.

Table 1- Values of components used in Chua circuit implementation

Parameters	C_1	C_2	L	R	Op-amp.	Power supply
Value	10nf	100nf	15mH	2.5k Ω	TLO82	± 9 V

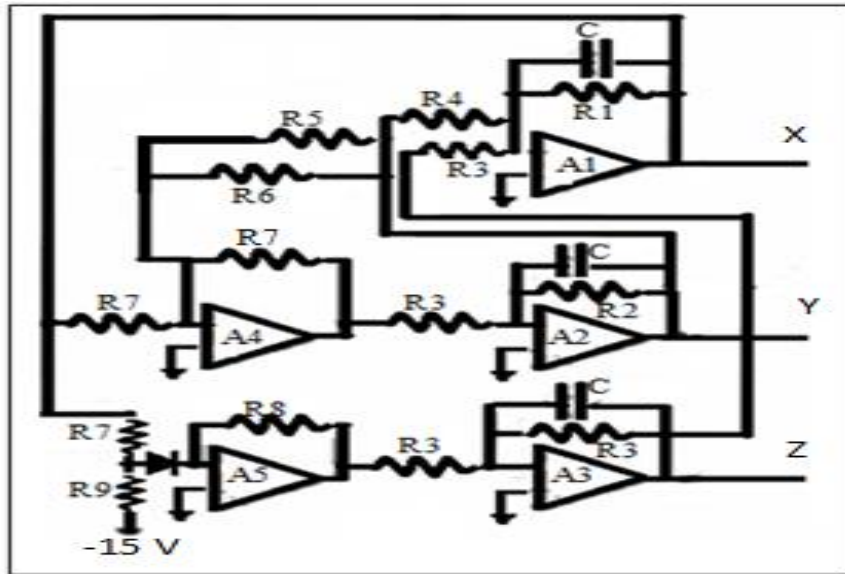


Figure 3- The schematic of Rossler's circuit [11].

Table 2- Values of components used in Rossler circuit implementation

Parameters	C	$R1$	$R2$	$R3$	$R4$	$R5$	$R6$
Value	0.001μ	$2M\Omega$	$5M\Omega$	$100k\Omega$	$200k\Omega$	$47k\Omega$	$75k\Omega$
Parameters	$R7$	$R8$	$R9$	diode	Amplifier (A1-A5)	voltage	
Value	$10k\Omega$	$150k\Omega$	$68k\Omega$		LM 741	-15	

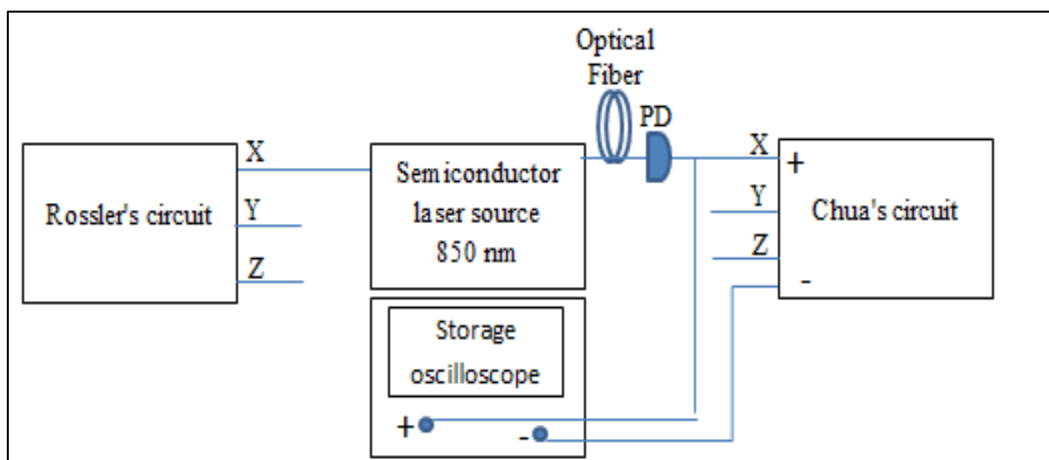


Figure 4- New chaotic dynamic scheme.

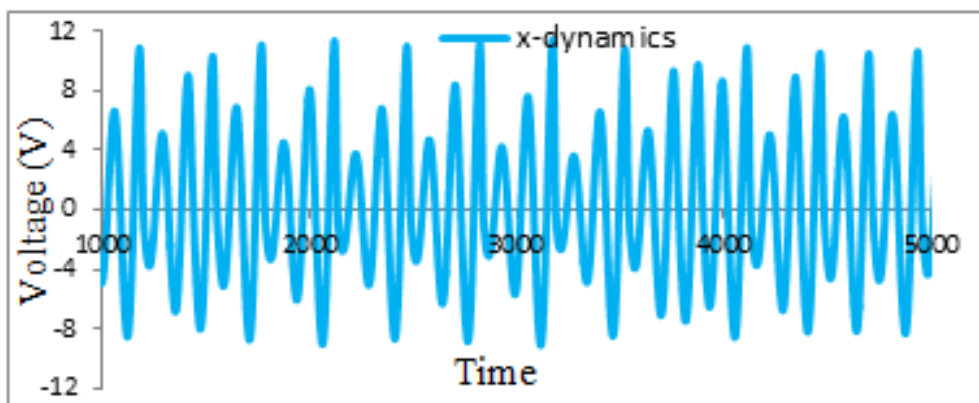


Figure 5- Time series of x-dynamics of Rossler system.

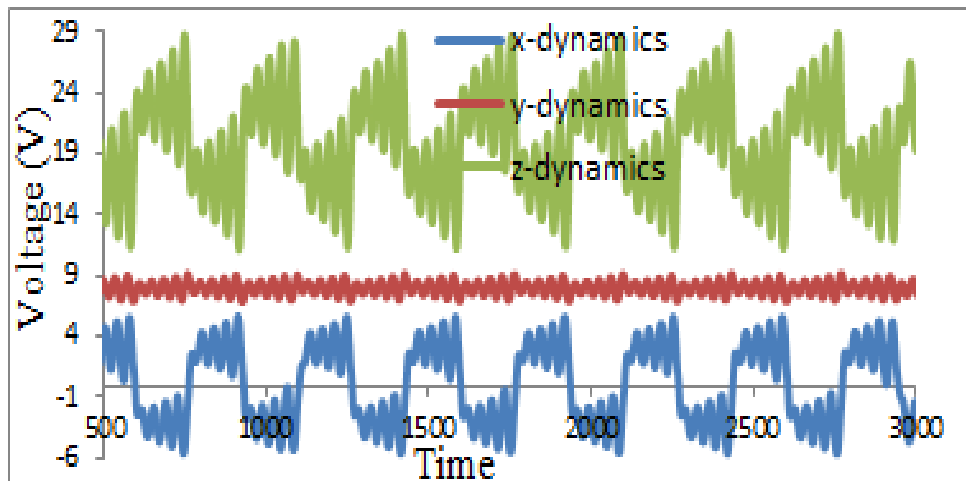


Figure 6- Time series of x, y, and z-dynamics of Chua system.

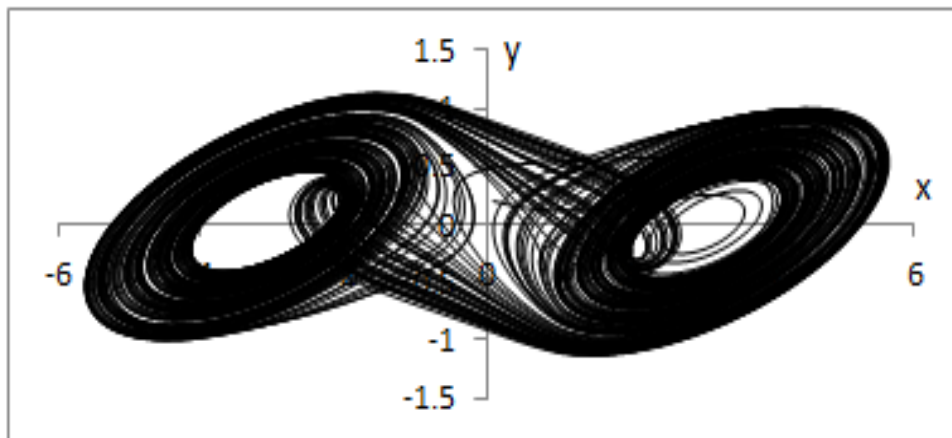


Figure 7- Strange attractor in(x-y) dynamics of Chua system.

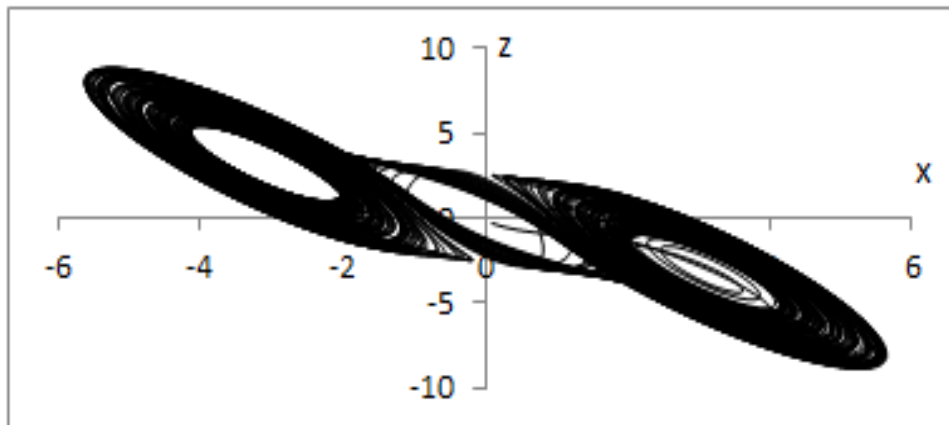


Figure 8- Strange attractor in(x-z) dynamics of Chua system.

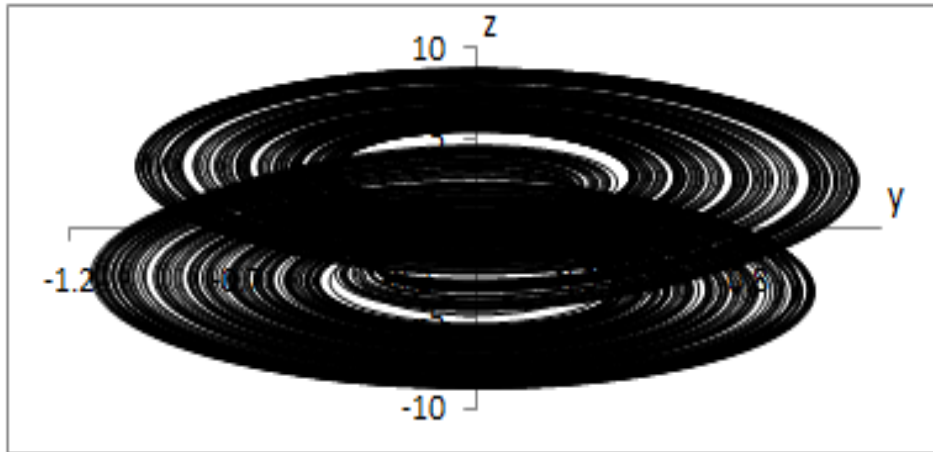


Figure 9- Strange attractor in (y-z) dynamics of Chua system.

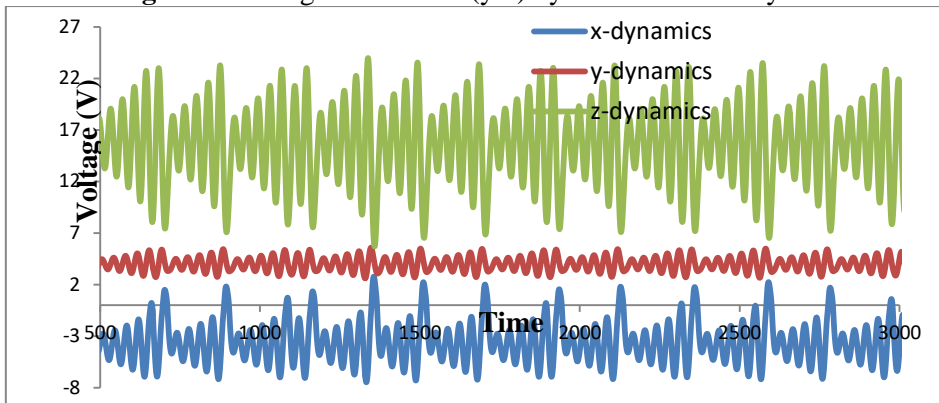


Figure 10- Time series in x, y, and z-dynamics of Chua system with new scheme.

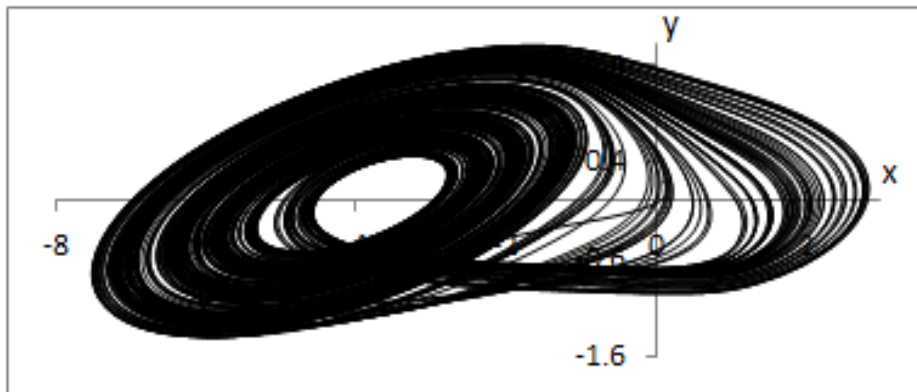


Figure 11- Strange attractor in (x-y) dynamics of Chua system with new scheme.

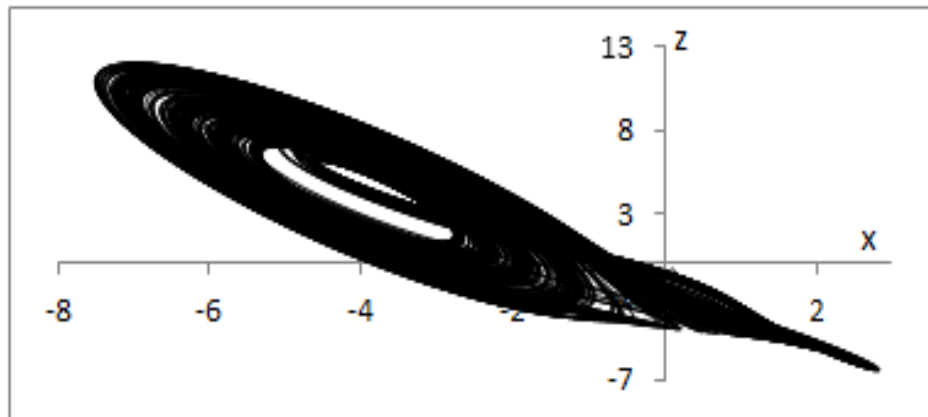


Figure 12- Strange attractor in(x-z) dynamics of Chua system with new scheme.

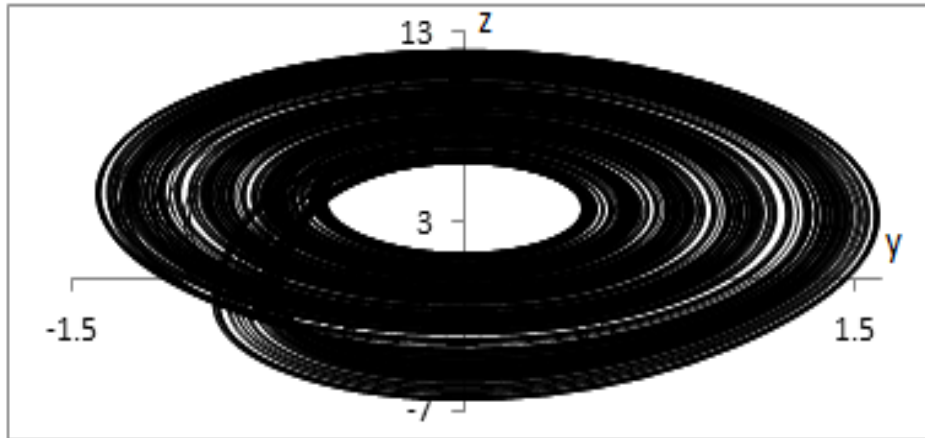


Figure 13- Strange attractor in(y-z) dynamics of Chua system with new scheme.

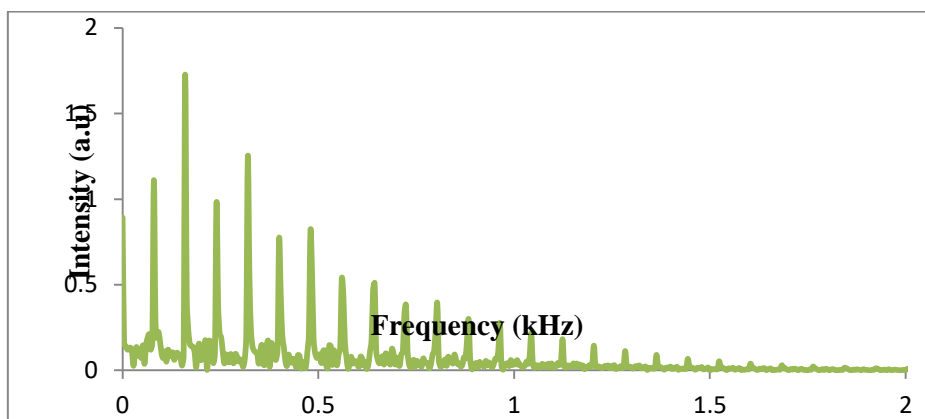


Figure 14- FFT spectrum of z-dynamics of Chua system.

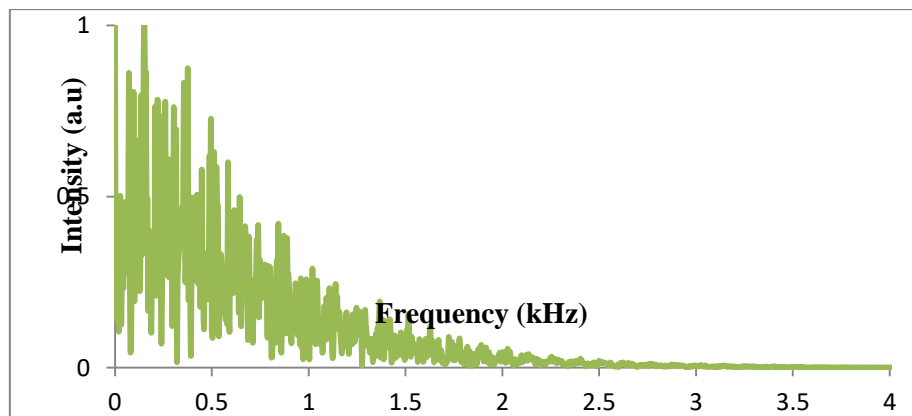


Figure 15- FFT spectrum of z-dynamics of Chua system with new scheme.

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