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Weyl Module Resolution Res (6,6,4;0,0) in the Case of Characteristic Zero

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Abstract
In this work, we prove by employing mapping Cone that the sequence and the subsequence of the characteristic-zero are exact and subcomplex respectively in the case of partition (6,6,4) .

Keywords: resolution, Lascoux resolution, Weyl module, mapping Cone, exact sequence, subcomplex.

التحلل لمقاس وايل Res(6,6,4;0,0) في حالة المميز الصفري

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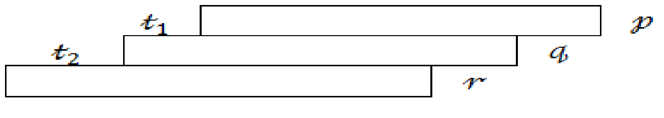
الخلاصة:
في هذا العمل، برهنا باستخدام تطبيق كون ان السلسلة والسلاسل الجزئية للمميز الصفري هي تامة ومعقدة جزئيا على التوالي في حالة التجزئة (6,6,4).

1. Introduction

In this work $\mathcal{R}$ is a abelian ring with 1 and $\mathcal{F}$ is a free R-module and $\mathcal{D}_i\mathcal{F}$ be the divided power algebra of degree i. The resolution of partition $(p+t_1+t_2, q+t_2, r)$ which represented by below diagram and in our case $t_1 = t_2 = 0$, [1]

$\lambda/\mu =$ 

Authors in [2 - 4] spoke about the partitions (4,4,4), (3,3,2), and (8,7,3), respectively. While in [5] the authors discussed by employing mapping Cone for the same partition.

2. The sequence of the characteristic-zero

The complex of Lascoux in the case of partition (6,6,4) is:

$$\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \qquad \mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}$$
$$\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \longrightarrow \quad \oplus \quad \longrightarrow \quad \oplus \quad \longrightarrow \mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_4\mathcal{F}$$
$$\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_5\mathcal{F}\otimes\mathcal{D}_3\mathcal{F} \qquad \mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_4\mathcal{F}$$

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By using the pursue diagram we find the characteristic-zero sequence and proof it is a resolution.

$$
\begin{array}{ccccc}
0 \longrightarrow & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} & \xrightarrow{z_1} & \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} & \\
 & w_1 \big\downarrow & \text{E} & \big\downarrow k_1 & \\
0 \longrightarrow & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & \xrightarrow{z_2} & \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & \\
 & w_2 \big\downarrow & \text{G} & \big\downarrow k_2 & \\
0 \longrightarrow & \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} & \xrightarrow{z_3} & \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} & \\
 & & & \big\downarrow & \\
 & & & \mathcal{K}_{(6,6,4)}(\mathcal{F}) & \\
 & & & \big\downarrow & \\
 & & & 0 &
\end{array}
$$

Diagram (1)

Now define the maps by

$z_1(\mathcal{s}) = \partial_{21}(\mathcal{s})$; $\mathcal{s} \in \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$,

$w_1(\mathcal{s}) = \partial_{32}(\mathcal{s})$; $\mathcal{s} \in \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$,

$z_2(\mathcal{s}) = \partial_{21}^{(2)}(\mathcal{s})$; $\mathcal{s} \in \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$, and

$k_2(\mathcal{s}) = \partial_{32}(\mathcal{s})$; $\mathcal{s} \in \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$

At this point we need to define k_1 which make subdiagram E in diagram (1) is commute. We define it as

$$k_1(\mathcal{s}) = \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(\mathcal{s})$$

Proposition (2.1): The subdiagram E in diagram (1) is commute.

Proof:

$$(z_2 \circ w_1)(\mathcal{s}) = \left(\partial_{21}^{(2)}\,\partial_{32}\right)(\mathcal{s})$$

From Capelli identities

$$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\partial_{31}$$

Thus

$$(z_2 \circ w_1)(\mathcal{s}) = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\partial_{31}$$
$$= \left(\partial_{32}\left(\frac{1}{2}\partial_{21}\partial_{21}\right) - \partial_{31}\,\partial_{21}\right)(\mathcal{s})$$
$$= \left(\frac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{31}\,\partial_{21}\right)(\mathcal{s})$$
$$= \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\partial_{21}(\mathcal{s})$$
$$= (k_2 \circ z_1)(\mathcal{s}) \quad \blacksquare$$

Now from the subdiagram E

$$
\begin{array}{ccccc}
0 \longrightarrow & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} & \xrightarrow{z_1} & \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\
 & w_1 \big\downarrow & \text{E} & \big\downarrow k_1 \\
0 \longrightarrow & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & \xrightarrow{z_2} & \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
\end{array}
$$

We have the subsequence

$$0 \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\psi_3} \begin{array}{c} \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{array} \xrightarrow{\lambda_1} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Where

$\psi_3(s) = \big(-\partial_{21}(s)\,,\partial_{32}(s)\big)$ and $\lambda_1(s_1, s_2) = \partial_{21}^{(2)}(s_2) + \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s_1)$

Proposition (2.2): The subsequence

$$0 \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\psi_3} \begin{matrix} \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix} \xrightarrow{\lambda_1} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Is a subcomplex.

Proof:

$(\lambda_1 \circ \psi_3)(s) = \lambda_1\big(-\partial_{21}(s)\,,\partial_{32}(s)\big)$

$= \partial_{21}^{(2)}\big(\partial_{32}(s)\big) + \left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\big(-\partial_{21}(s)\big)\right)$

$= \left(\partial_{21}^{(2)}\partial_{32}\right)(s) - \left(\partial_{32}\,\partial_{21}^{(2)}\right)(s) + (\partial_{31}\partial_{21})(s)$

By using Capelli identities we get

$(\lambda_1 \circ \psi_3)(s) = \left(\partial_{32}\,\partial_{21}^{(2)}\right)(s) - (\partial_{21}\partial_{31})(s) - \left(\partial_{32}\,\partial_{21}^{(2)}\right)(s) + \left(\partial_{31}^{(2)}\right)(s)$

$= 0$ ■

Now consider the sequence

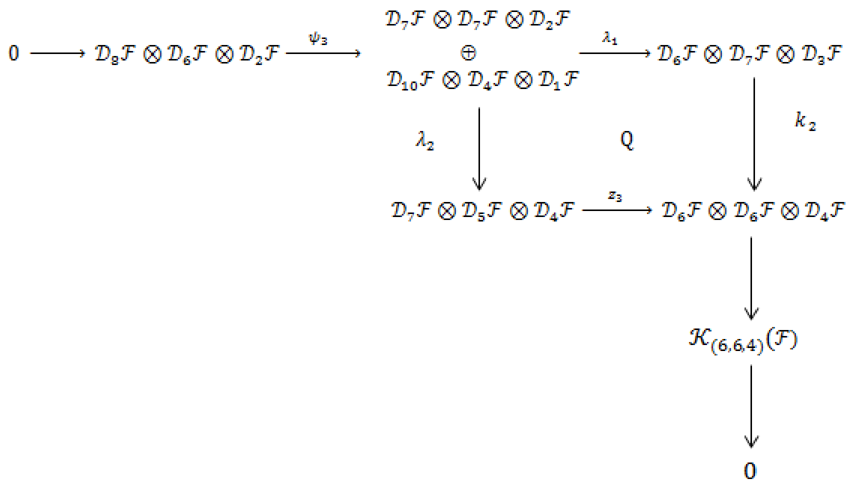

Diagram (2)

Now define the maps by

$z_3(s) = \partial_{21}(s)$; $s \in \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_4\mathcal{F}$,

$\lambda_2(s_1, s_2) = \partial_{32}^{(2)}(s_1) + \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(s_2)$; $s_1 \in \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$, $s_2 \in \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$,

Proposition (2.3): The subdiagram Q in diagram (2) is commute.

Proof:

$(k_2 \circ \lambda_1)(s_1, s_2) = k_2\left(\partial_{21}^{(2)}(s_2) + \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s_1)\right)$

$= \partial_{32}\left(\partial_{21}^{(2)}(s_2) + \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s_1)\right)$

$= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(s_2) + \left(\frac{1}{2}\partial_{32}\partial_{32}\partial_{21} - \partial_{32}\partial_{31}\right)(s_1)$

$= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(s_2) + \left(\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31}\right)(s_1)$

From Capelli identities we have

$\partial_{32}^{(2)}\partial_{21} = \partial_{21}\,\partial_{32}^{(2)} + \partial_{32}\partial_{31}$ and $\partial_{32}\,\partial_{21}^{(2)} = \partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31}$

Thus

$(k_2 \circ \lambda_1)(s_1, s_2) = \left(\partial_{21}^{(2)}\partial_{32}\right)(s_2) + (\partial_{21}\partial_{31})(s_2) + \left(\partial_{21}\,\partial_{32}^{(2)}\right)(s_1) + (\partial_{32}\partial_{31})(s_1)$

$$= \left(\frac{1}{2}\partial_{21}\partial_{21}\partial_{32}\right)(s_2) + (\partial_{21}\partial_{31})(s_2) + \left(\partial_{21}\,\partial_{32}^{(2)}\right)(s_1)$$

$= \partial_{21}\left(\left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(s_2) + \partial_{32}^{(2)}(s_1)\right) = (z_3 \circ \lambda_2)(s_1, s_2)$

We have the following sequence

$$0 \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\psi_3} \begin{matrix} \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix} \xrightarrow{\psi_2} \begin{matrix} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \end{matrix} \xrightarrow{\psi_1} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \xrightarrow{d'_{(6,6,4)}(\mathcal{F})} \mathcal{K}_{(6,6,4)}(\mathcal{F}) \longrightarrow 0$$

Where

$\psi_3(s_1, s_2) = \left(-\partial_{21}^{(2)}(s_2) - \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s_1), \partial_{32}^{(2)}(s_1) + \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(s_2)\right)$

$\psi_1(s_1, s_2) = \partial_{32}(s_1) + \partial_{21}(s_2)$

Proposition (2.4): $\psi_2 \circ \psi_3 = 0$.

Proof:

$(\psi_2 \circ \psi_3)(s_1) = \psi_2(-\partial_{21}(s), \partial_{32}(s)),\ s \in \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

$= \left(\left(-\partial_{21}^{(2)}\partial_{32}\right)(s_1) + \left(\frac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{31}\partial_{21}\right)(s_1), (\partial_{32}\partial_{21}(s_1) + \left(\frac{1}{2}\partial_{21}\partial_{32}\partial_{32} + \partial_{31}\partial_{32}\right)(s_1)\right)$

$= \left(\left(-\partial_{21}^{(2)}\partial_{32}\right)(s_1) + \left(\partial_{32}\,\partial_{21}^{(2)} - \partial_{31}\partial_{21}\right)(s_1), -\left(\partial_{32}^{(2)}\partial_{21}(s_1) + \left(\partial_{21}\,\partial_{32}^{(2)} + \partial_{31}\partial_{32}\right)(s_1)\right)$

By using Capelli identities we get

$(\psi_2 \circ \psi_3)(s_1) = \left(\left(-\partial_{21}^{(2)}\partial_{32}\right)(s_1) + \left(\partial_{21}^{(2)}\partial_{32}\right)(s_1) + (\partial_{21}\partial_{31})(s_1) - (\partial_{21}\partial_{31})(s_1),\right.$

$\left.-\left(\partial_{32}^{(2)}\partial_{21}(s_1) + \left(\partial_{32}^{(2)}\partial_{21}\right)(s_1) - (\partial_{32}\partial_{31})(s_1) + (\partial_{32}\partial_{31})(s_1)\right)\right) = (0,0)$

Proposition (2.5): $\psi_1 \circ \psi_2 = 0$.

Proof:

$(\psi_1 \circ \psi_2)(s_1, s_2) = \psi_1\left(-\partial_{21}^{(2)}(s_2) - \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s_1), \partial_{32}(s_1) + \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(s_2)\right),$

$= \left(-\partial_{32}\,\partial_{21}^{(2)}\right)(s_2) - \left(\frac{1}{2}\partial_{32}\partial_{32}\partial_{21}\right)(s_1) - (\partial_{32}\partial_{31})(s_1) + (\partial_{21}\partial_{32})(s_1) + \left(\frac{1}{2}\partial_{21}\partial_{21}\partial_{32}\right)(s_2) +$
$(\partial_{21}\partial_{31})(s_2)$

$= \left(-\partial_{32}\,\partial_{21}^{(2)}\right)(s_2) - \left(\partial_{32}^{(2)}\partial_{21}\right)(s_1) - (\partial_{32}\partial_{31})(s_1) + \left(\partial_{21}\,\partial_{32}^{(2)}\right)(s_1) + \left(\partial_{21}^{(2)}\partial_{32}\right)(s_1) +$
$\left(\partial_{21}^{(2)}\partial_{32}\right)(s_2) + (\partial_{21}\partial_{31})(s_2)$

Again from Capelli identities we get

$(\psi_1 \circ \psi_2)(s_1, s_2) = \left(-\partial_{21}^{(2)}\partial_{32}\right)(s_2) - (\partial_{21}\partial_{31})(s_2) - \left(\partial_{21}\,\partial_{32}^{(2)}\right)(s_1) - (\partial_{32}\partial_{31})(s_1) +$
$(\partial_{32}\partial_{31})(s_1) + (\partial_{21}\partial_{32})(s_1) + \left(\partial_{21}^{(2)}\partial_{32}\right)(s_2) + (\partial_{21}\partial_{31})(s_2)$

$= 0$ ■

Theorem (2.6):.The sequence

$$0 \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\psi_3} \begin{matrix} \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix} \xrightarrow{\psi_2} \begin{matrix} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \end{matrix} \xrightarrow{\psi_1} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \xrightarrow{d'_{(6,6,4)}(\mathcal{F})} \mathcal{K}_{(6,6,4)}(\mathcal{F}) \longrightarrow 0$$

Is exact.

Proof:

Since the diagrams, E and Q in a diagrams (1) and (2) are commute and the maps of place polarization are injective [6], [7] then the maps

$z_1: \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \longrightarrow \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$; and

$z_2: \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \longrightarrow \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$ are injective [8], [9] so we have a commuting diagram with exact rows. But from Proposition (2.2) $\lambda_1 \circ \psi_3 = 0$ so the mapping Cone conditions are satisfied then the complex

$$0 \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\psi_3} \begin{array}{c} \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{array} \xrightarrow{\lambda_1} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Is exact.

Now consider the diagram (2), from Proposition (2.3) we have the diagram Q is commute and the map $z_3: \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \longrightarrow \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F}$ is injective [10], [11], then we have the diagram (2) commute with exact rows. But $\psi_2 \circ \psi_3 = 0$ (Proposition (2.4)) and $\psi_1 \circ \psi_2 = 0$ (Proposition (2.5)), then again the mapping Cone conditions are satisfied which implies the complex

$$0 \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\psi_3} \begin{array}{c} \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{array} \xrightarrow{\psi_2} \begin{array}{c} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \end{array} \xrightarrow{\psi_1} \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \xrightarrow{d'_{(6,6,4)}(\mathcal{F})} \mathcal{K}_{(6,6,4)}(\mathcal{F}) \longrightarrow 0$$

Is exact.

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