ISSN: 0067-2904

Application of Quasi Subordination Associated with Generalized Sakaguchi Type Functions

Nihad Hameed Shehab*, Abdul Rahman S. Juma
Department of Mathematics, College of Education for Pure Sciences, University of Anbar, Anbar, Iraq

Received: 2/12/2020 Accepted: 27/2/2021

Abstract

In this article, a new class $\mathcal{G}_q^{\beta}(H,2u,v)$ of analytic functions which is defined by terms of a quasi-subordination is introduced. The coefficient estimates, including the classical Fekete-Szegö inequality of functions belonging to this class, are then derived. Also, several special improving results for the associated classes involving the subordination are presented.

Keywords: Univalent functions, subordination, Quasi-subordination, Fekete-Szegö coefficient.

تطبيق شبه التابعية المرتبطة بدوال نوع ساكاغوتشي المعممة

نهاد حميد شهاب* , عبدالرحمن سلمان جمعه
قسم الرياضيات , كلية التربية للعلوم الصرفة , جامعة الانبار, الانبار , العراق

الخلاصة

في هذه المقالة , يتم تقديم فئة جديدة من الدوال التحليلية التي يتم تعريفها من خلال شروط شبه التابعية . ثم يتم اشتقاق تقديرات لمعاملات عدم المساواة فيكيتي سزيجو للدوال التي تنتمي الى هذه الفئة . كذلك، يتم عرض العديد من النتائج الخاصة المحسنة للفئات المرتبطة التي تتضمن التابعية

1.Introduction

Let $\mathcal{F}$ symbolizes the collection of normalized functions satisfying the condition $f(0)=f'(0)-1=0$ and given by Taylor expansion

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n , \quad (1)$$

which are analytic in the unit disk

$$\mathbb{D}=\{z\in\mathbb{C}, such\ that\ |z|<1\}, \quad (2)$$

where $\mathbb{C}$ is a complex plane.

Furthermore , let $\mathcal{A}$ symbolizes the class of all functions in $\mathcal{F}$ which are univalent in unit disk $\mathbb{D}$. Let w(z) be an analytic function in unit disk $\mathbb{D}$ with all coefficients are real and $|w(z)|\le 1$, such that

$$w(z)=w_0+w_1 z+w_2 z^2+........ \quad . \quad (3)$$

*Email: nih19u2012@uoanbar.edu.iq

Also, let Φ be a univalent and analytic function with positive real part in unit disk $\mathbb{D}$, with $\Phi(0)=1$, $\Phi'(0)>0$, which maps the unit disk onto a zone starlike with respect to 1 and symmetric with respect to the real axis . The Taylor's expansion with all coefficients is real and can be written in the form

$$\Phi(z)=1+C_1 z+C_2 z^2+........., \quad , \tag{4}$$

such that $C_1>0$.
Let $\mathcal{P}$ be the class of functions and written in the following form

$$\mathcal{P}(z)=1+\sum_{n=1}^{\infty}\mathcal{P}_n z^n \quad . \tag{5}$$

For any two analytic $f(z)$ and $g(z)$ functions in unit disk $\mathbb{D}$, we say that $f(z)$ is subordinate to $g(z)$, written as

$$f(z)\prec g(z) \ , \quad (z\in\mathbb{D}\ ), \tag{6}$$

if there exists $\hbar(z)$ being a Schwarz function and analytic in unit disk $\mathbb{D}$ with

$$\hbar(0)=0 \ \ and \ \ |\hbar(z)|<1\ , \ \ (z\in\mathbb{D})\ , \tag{7}$$

such that

$$f(z)=g\big(\hbar(z)\big), \qquad (z\in\mathbb{D}) \ \ . \tag{8}$$

Furthermore, if $g(z)$ is univalent in $\mathbb{D}$, then (see[1]):
$f(z)\prec g(z)\Leftrightarrow f(0)=g(0) and\ f(\mathbb{D})\subset g(\mathbb{D})$.
Robertson introduced the concept of quasi-subordination , in 1970 [2]. Moreover, if $f(z)$ and $g(z)$ are two analytic functions, we say that $f(z)$ is quasi-subordination to $g(z)$ in $\mathbb{D}$, which can be written in the form

$$f(z)\prec_q g(z) \qquad (z\in\mathbb{D})\ , \tag{9}$$

if there exist $\omega(z)$ and $\hbar(z)$ being analytic functions with $|\omega(z)|\le 1$, $\hbar(0)=0$, and $|\hbar(z)|<1$, such that

$$f(z)=\omega(z)g\big(\hbar(z)\big) \qquad (z\in\mathbb{D}) \ \ . \tag{10}$$

Note that , when $\omega(z)=1$, then $f(z)=g(\hbar(z))$ (see [3,4]), so that

$$f(z)\prec g(z) \ \text{ in } \mathbb{D} \ .$$

Furthermore, if $(z)=z$, then $f(z)=\omega(z)g(z)$ and, in this case, $f(z)$ is majorized by $g(z)$, written as

$$f(z)\ll g(z) \text{ in } .$$

In this case, $f(z)\prec_q g(z)\Longrightarrow f(z)=\omega(z)g(z)\Longrightarrow f(z)\ll g(z). z\in\mathbb{D}$.
Therefore, quasi-subordination is a generalization of subordination and also of majorization . [5, 6, 7].
Sakaguchi [8] introduced the class starlike $\mathcal{S}^*$ functions with respect to symmetric points in unit disk $\mathbb{D}$, for $f\in\mathcal{A}$ satisfying $\mathrm{Re}\left(\frac{zf'(z)}{(f(z)-f(-z))}\right)>0, (z\in\mathbb{D})$. Similarly, Wang et al. in [9] introduced the class convex functions $C_{\mathcal{S}}$ with respect to symmetric points in unit disk $\mathbb{D}$, for $f\in\mathcal{A}$ satifying $\mathrm{Re}\left(\frac{zf''(z)}{(f'(z)-f'(-z))}\right)>0, (z\in\mathbb{D})$, (see, for details, [10]).
In mathematics, the Fekete-Szego is an inequality for the coefficients of univalent analytic functions found by Fekete-Szego in 1933 [11], for $0\le\lambda\le 1$, and then for the Fekete-Szego functional $|a^3-\lambda a_2^2|$ for normalized univalent functions given by (1) [12,13,14,15,16,17,18].
The aim of the present paper is to introduce a new class of univalent functions by applying the generalized Salagean operator [19, 20] .
We define the following differential operator

$$G^0 f(z)= \ f(z)$$

$$G^1 f(z) = (1-\kappa) f(z) + \kappa z\, f'(z), \kappa \geq 0$$
$$G^n f(z) = G_\kappa (G^{n-1} f(z) \ . \tag{11}$$

If f is given by (1) , then from (11), we see that

$$G^n f(z) = z + \sum_{n=2}^{\infty} [1+(n-1)\kappa]^m a_n z^n \quad , \tag{12}$$

where $m \in N_0 = \{0,1,2,3,4, \ldots\ldots\}$ and $\kappa \geq 0$.

2. Preliminary Results

We use a special sigmoid function, which is a differentiable, bounded, and real function that is defined for all real input values and has a non-negative derivative at each point. We can write this sigmoid function as

$$\delta(z) = \frac{1}{1+e^{-z}} \tag{13}$$

The sigmoid function is salutary and has very important properties (see [21]) of which, a sigmoid function is monotonic and has a first derivative which is bell shaped. It outputs real numbers between zero and one and since it is one-one, then it never loses information .

Lemma 1.[22] . Let $\hbar(z)$ be the Schwarz function given by

$$\hbar(z) = \hbar_1 z + \hbar_2 z^2 + \cdots \qquad , \quad z \in \mathbb{D} \tag{14}$$

then

$$|\hbar_1| \leq 1 \ , \ \ |\hbar_2 - \mu \hbar_1^2| \leq 1 + (|\mu| - 1)|\hbar_1|^2 \leq \max\{1, |\mu|\}, \tag{15}$$

where $\mu \in \mathbb{C}$.

Lemma 2.[23]. We symbolize $\mathcal{S}$ to a sigmoid function and

$$H(z) = 2\mathcal{S}(z) = \frac{2}{1+e^{-z}} = 1 + \sum_{m=1}^{\infty} \frac{(-1)^m}{2^m} \left(\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} z^n \right)^m , \tag{16}$$

then $H(z) \in \mathcal{P}$, $|z| < 1$, where $H(z)$ is a modified sigmoid function .

Lemma3.[23]. Let

$$H_{\boldsymbol{n,m}}(z) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^m}{2^m} \left(\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \right)^m \tag{17}$$

.Then $|H_{n,m}| < 2$.

3. Main Result

Definition1. A function $f \in \mathcal{A}$ given by (1) is said to be in class $\mathcal{G}_q^\beta(H, 2u, v)$ if the next quasi-subordination holds :

$$\left[\left(G^n f(z)\right)' \left(\frac{(2u-v)z}{G^n f(2uz) - G^n f(vz)} \right)^\beta \right] - 1 \prec_q H(z) - 1 \quad , z \in \mathbb{D} \tag{18}$$

where $u, v \in \mathbb{C}$, with $u \neq v$, $|v| \leq 1$ and $\beta \geq 0$.

From the above Definition, we note that $f \in \mathcal{G}_q^\beta(H, 2u, v)$ if and only if there exists $\hbar(z)$ being an analytic function with $|\hbar(z)| \leq 1$, such that

$$\frac{\left[\left(G^n f(z)\right)' ((2u-v)z / G^n f(2uz) - G^n f(vz)))^\beta \right] - 1}{\hbar(z)} \prec (H(z) - 1). \tag{19}$$

If, as in condition (19), $\hbar(z) = 1$, then the class $\mathcal{G}_q^\beta(H, 2u, v)$ is symbolized as $\mathcal{G}^\beta(H, 2u, v)$, satisfying the condition

$$\left(G^n f(z)\right)' \left(\frac{(2u-v)z}{G^n f(2uz) - G^n f(vz)} \right)^\beta \prec H(z) \ , \quad z \in \mathbb{D} \ . \tag{20}$$

Note that

$\left(\frac{(2u-v)z}{G^n f(2uz)-G^n f(vz)}\right)^\beta = [1-\beta(2u+v)a_2 z+\beta\left[\frac{\beta+1}{2}(2u+v)^2 a_2^2-(4u^2+2uv+\right.$
$\left.v^2)a_3\right] z^2+\ldots$ (21)

Theorem1. Let $f \in \mathcal{A}$ of the form (1) be a function in the class $\mathcal{G}_q^\beta(H,2u,v)$. Then

$$|a_2| \leq \frac{1}{2[1+\kappa]^m[2-\beta(2u+v)} \quad (22)$$

and for some $\gamma \in \mathbb{C}$,

$$|a_3-\mu a_2^2| \leq \frac{1}{2[1+2\kappa]^m|3-\beta(4+2v+v^2)|}$$
$$.max\left\{1,\left|\frac{1}{4}\left(\frac{2\gamma[1+2\kappa]^m(3-\beta(4u^2+2uv+v^2))}{[1+\kappa]^{2m}(2-\beta(2u+v))^2}-\frac{\beta(1+(2-(2u+v))/(2-\beta(2u+v)))(2u+v)}{2-\beta(2u+v)}\right)\right|\right\} \quad (23)$$

Proof

Let $f(z)=z+\sum_{n=2}^{\infty} a_n z^n$ be a function in class $\mathcal{G}_q^\beta(H,2u,v)$, then we get

$$f(z)=z+a_2z^2+a_3z^3+\cdots$$
$$f(2uz)=2uz+4a_2u^2z^2+8a_3u^3z^3+\cdots$$
$$f(vz)=vz+a_2v^2z^2+a_3v^3z^3+\cdots$$

Let $f \in \mathcal{G}_q^\beta(H,2u,v)$. From Definition1 we can write

$$\left[\left(G^n f(z)\right)'\left(\frac{(2u-v)z}{G^n f(2uz)-G^n f(vz)}\right)^\beta\right]-1 \prec_q (w(z)(H(\Bbbk(z)-1\ ), \quad (24)$$

A modified sigmoid function H(z) is given as below

$$H(z)=1+\frac{1}{2}z-\frac{1}{24}z^3+\frac{1}{240}z^5-\cdots \quad (25)$$

By combining (3), (14), and (25), we obtain

$$(w(z)\left(H(\Bbbk(z))-1\right)=\frac{1}{2}(w_0\Bbbk z+(w_0\Bbbk_2+w_1\Bbbk_1)z^2)+\cdots \quad (26)$$

Now, using the series expansion $\left(G^n f(z)\right)'$ from (1) and the expansion given by (21), we get

$\left[\left(G^n f(z)\right)'\left(\frac{(2u-v)z}{G^n f(2uz)-G^n f(vz)}\right)^\beta\right]-1 = \quad [[1+\kappa]^m[2-\beta(2u+v)]a_2z+\left[[1+2\kappa]^m[3-\right.$
$\left.\beta(4u^2+2uv+v^2)]a_3-\beta[1+\kappa]^{2m}(2u+v)\left(2-\frac{(\beta+1)(2u+v)}{2}\right)a_2^2\right]z^2$
(27)

From the expansions (24) and (27) , on equating the coefficients of z and z^2 in (24), we get

$[1+\kappa]^m[2-\beta(2u+v)]a_2=\frac{1}{2}w_0\Bbbk_1$, (28)

$[1+2\kappa]^m[3-\beta(4u^2+2uv+v^2)]a_3-\beta[1+\kappa]^{2m}(2u+v)\left(2-\frac{(\beta+1)(2u+v)}{2}\right)a_2^2=$
$\frac{1}{2}(w_0\Bbbk_2+w_1\Bbbk_1)$ (29)

Now, from (28), we get

$$a_2=\frac{w_0\Bbbk_1}{2[1+\kappa]^m[2-\beta(2u+v)]}\ . \quad (30)$$

From (29), it follows that

$[1+2\kappa]^m[3-\beta(4u^2+2uv+v^2)]a_3=\frac{\beta[4-(1+\beta)(2u+v)](2u+v)}{8[2-\beta(2u+v)]^2}w_0^2\Bbbk_1^2+\frac{1}{2}(w_0\Bbbk_2+w_1\Bbbk_1)$
.

Therefore,

$$a_3 = \frac{1}{2[1+2\kappa]^m[3-\beta(4u^2+2uv+v^2)]}\Big[w_1 \mathscr{k}_1 + w_0\Big(\mathscr{k}_2 \frac{\beta(1+(2-(2u+v))/(2-\beta(2u+v)))(2u+v)}{4[2-\beta(2u+v)]}\Big)\mathscr{k}_1^2\Big]. \qquad (31)$$

For some $\gamma \in \mathbb{C}$, from (30) and (31), we obtain

$$a_3 - \mu a_2^2 = \frac{1}{2[1+2\kappa]^m[3-\beta(4u^2+2uv+v^2)]}\Big[w_0\mathscr{k}_2 + w_1\mathscr{k}_1 - \frac{1}{4}\Big(\frac{2\gamma[1+2\kappa]^m(3-\beta(4u^2+2uv+v^2))}{[1+\kappa]^{2m}[2-\beta(2u+v)]^2}\Big) - \Big(\frac{\beta(1+(2-(2u+v))/(2-\beta(2u+v)))(2u+v)}{4[2-\beta(2u+v)]}\Big) w_0^2\mathscr{k}_2^2\Big] \qquad . \qquad (32)$$

We have that $w(z)$ given by (3) is bounded and analytic in unit disk $\mathbb{D}$, therefore, on using [15] (page 172) , we have for some $\mathcal{Y}$ $(|\mathcal{Y}| \le 1)$:

$$|w_0| \le 1 \quad and \quad w_1 = (1-w_0^2)\mathcal{Y}\ . \qquad (33)$$

By putting the value of w_1 from (32) into (33) , we get

$$a_3 - \mu a_2^2 = \frac{1}{2[1+2\kappa]^m[3-\beta(4u^2+2uv+v^2)]}\Big[w_0\mathscr{k}_2 + \mathcal{Y}\mathscr{k}_1 - \Big[\frac{1}{4}\Big(\frac{2\gamma[1+2\kappa]^m(3-\beta(4u^2+2uv+v^2))}{[1+\kappa]^{2m}(2-\beta(2u+v))^2}\Big) - \Big(\frac{\beta(1+(2-(2u+v))/(2-\beta(2u+v)))(2u+v)}{4[2-\beta(2u+v)]}\Big)\mathscr{k}_1^2 + \mathcal{Y}\mathscr{k}_1\Big]w_0^2\Big] \qquad (34)$$

Now , if $w_0 = 0$ in (34) , we get

$$|a_3 - \mu a_2^2| \le \frac{1}{2[1+2\kappa]^m|3-\beta(4u^2+2uv+v^2)|}\ . \qquad (35)$$

Otherwise , if $w_0 \ne 0$ in (34), we define a function

$$\Gamma(w_0) = w_0\mathscr{k}_2 + \mathcal{Y}\mathscr{k}_1 - \Big[\Big[\frac{1}{4}\Big(\frac{2\gamma[1+2\kappa]^m(3-\beta(4u^2+2uv+v^2))}{[1+\kappa]^{2m}(2-\beta(2u+v))^2}\Big) - \Big(\frac{\beta(1+(2-(2u+v))/(2-\beta(2u+v)))(2u+v)}{[2-\beta(2u+v)]}\Big)\mathscr{k}_1^2 + \mathcal{Y}\mathscr{k}_1\Big]w_0^2\Big] \qquad . \qquad (36)$$

The equation (36) is polynomial in w_0 and hence analytic in $|w_0| \le 1$. The maximum $|\Gamma(w_0)|$ occurs at $w_0 = e^{i\theta}$, $(0 \le \theta \le 2\pi)$. Thus

$$\max_{0\le\theta\le2\pi}|\Gamma(e^{i\theta})| = |\Gamma(1)|,$$

$$|a_3 - \mu a_2^2| \le \frac{1}{2[1+2\kappa]^m|3-\beta(4u^2+2uv+v^2)|}\Big|\mathscr{k}_2 - \frac{1}{4}\Big(\frac{2\gamma([1+2\kappa]^m)(3-\beta(4u^2+2uv+v^2))}{[1+\kappa]^{2m}(2-\beta(2u+v))^2}\Big) - \Big(\frac{\beta(1+(2-(2u+v))/(2-\beta(2u+v)))(2u+v)}{2-\beta(2u+v)}\Big)\mathscr{k}_1^2\Big|$$

Therefore , by using Lemma1 , we get

$$|a_3 - \mu a_2^2| \le \frac{1}{2[1+2\kappa]^m|3-\beta(4+2v+v^2)|} .max\Big\{1, \Big|\frac{1}{4}\Big(\frac{2\gamma[1+2\kappa]^m(3-\beta(4u^2+2uv+v^2))}{[1+\kappa]^{2m}(2-\beta(2u+v))^2} - \frac{\beta(1+(2-(2u+v))/(2-\beta(2u+v)))(2u+v)}{2-\beta(2u+v)}\Big)\Big|\Big\}\ .$$

In case $u = 1$, we have the following:

Corollary1. Let $f \in \mathcal{A}$ of the form (1) be a function in the class $\mathcal{G}_q^{\beta}(H,2,v)$. Then

$$|a_2| \le \frac{1}{2[1+\kappa]^m[2-\beta(2+v)}$$

and for any complex number $\gamma \in \mathbb{C}$,

$$|a_3 - \mu a_2^2| \le \frac{1}{2[1+2\kappa]^m|3-\beta(4+2v+v^2)|} .max\Big\{1, \Big|\frac{1}{4}\Big(\frac{2\gamma[1+2\kappa]^m(3-\beta(4+2v+v^2))}{[1+\kappa]^{2m}(2-\beta(2+v))^2} - \frac{\beta(1+(2-(2+v))/(2-\beta(2+v)))(2+v)}{2-\beta(2+v)}\Big)\Big|\Big\}\ .$$

The result is sharp .

In Corollary1, in case $v=-2$, we obtain the next Corollary.

Corollary2. Let $f \in \mathcal{A}$ of the form (1) be a function in the class $\mathcal{G}_q^{\beta}(H,2,-2)$. Then

$$|a_2| \leq \frac{1}{4[1+\kappa]^m}$$

and for any complex number $\gamma \in \mathbb{C}$,

$$|a_3-\mu a_2^2| \leq \frac{1}{[1+2\kappa]^m|3-4\beta|} \;.max\left\{1,\left|\frac{1}{4}\left(\frac{2\gamma[1+2\kappa]^m(3-4\beta)}{4([1+\kappa]^{2m}}\right)\right|\right\} \;.$$

The result is sharp .

In case $\beta=1$ in Corollary2, we get the following:

Corollary3. Let $f \in \mathcal{A}$ of the form (1) be a function in the class $\mathcal{G}_q(H,2,-2)$. Then

$$|a_2| \leq \frac{1}{4[1+\kappa]^m}$$

and for any complex number $\gamma \in \mathbb{C}$,

$$|a_3-\mu a_2^2| \leq \frac{1}{[1+2\kappa]^m} \;.max\left\{1,\left|\left(\frac{3\gamma[1+2\kappa]^m}{8[1+\kappa]^m}\right)\right|\right\}.$$

The result is sharp .

In case $\beta=0$ in Corollary2, we deduce the following:

Corollary4: Let $f \in \mathcal{A}$ of the form (1) be a function in the class $\mathcal{G}_q(H,2,-2)$. Then

$$|a_2| \leq \frac{1}{4[1+\kappa]^m}$$

$$|a_3-\mu a_2^2| \leq \frac{1}{3[1+2\kappa]^m} \;.max\left\{1,\left|\left(\frac{3\gamma[1+2\kappa]^m}{8([1+\kappa]^m}\right)\right|\right\} \;.$$

The result is sharp.

References

[1] Miller, S. S., & Mocanu, P. T. *Differential subordinations: theory and applications. CRC Press*, 2000.

[2] Robertson, M. S. "Quasi-subordination and coefficient conjectures". *Bulletin of the American Mathematical Society*, vol. 76, no. 1, pp. 1-9, 1970.

[3] Juma, A. R. S., & Saloomi, M. H. "Coefficient Estimates for Subclasses of Regular Functions". *Iraqi Journal of Science*, pp. 1710-1716, 2018.

[4] Juma, A. R. S., & Saloomi, M. H. "Quasi-subordination conditions on bi-univalent functions involving Hurwitz-Lerch zeta functions". *Science International* , vol. 30, no.1 , pp. 127-131, 2018.

[5] MacGregor, T. H. "Majorization by univalent functions". *Duke Mathematical Journal*, vol. 34, no. 1, pp. 95-102, 1967.

[6] Ren, F., Owa, S., & Fukui, S. "Some inequalities on quasi-subordinate functions". *Bulletin of the Australian Mathematical Society*, vol. 43, no. 2, pp. 317-324, 1991.

[7] Nehari, Z. *Conformal Mapping, McGraw-Hill, New York, NY,USA, 1st edition*, 1952.

[8] Sakaguchi, K. "On a certain univalent mapping". *Journal of the Mathematical Society of Japan*, vol. 11, no. 1, pp. 72-75, 1959.

[9] Wang, C.Y. &, Yuan, S.-M. "On certain subclasses of close-to-convex and quasi-convex functions with respect to k-symmetric points," *Journal of Mathematical Analysis and Applications*, vol. 322, no. 1, pp. 97–106, 2006.

[10] Olatunji, S. O., Dansu, E. J., & Abidemi, A. "On a sakaguchi type class of analytic functions associated with quasi-subordination in the space of modified sigmoid functions". *Electronic Journal of Mathematical Analysis and Applications*, vol. 5, no. 1, pp. 97-105, 2017.

[11] Fekete, M., & Szegö, G. "Eine Bemerkung über ungerade schlichte Funktionen". *Journal of the london mathematical society*, vol. 1, no. 2, pp. 85-89, 1933.

[12] Bucur, R., Andrei, L., & Breaz, D. "Coefficient bounds and Fekete-Szegö problem for a class of analytic functions defined by using a new differential operator". *Appl. Math. Sci*, vol. 9, pp. 1355-1368, 2015.

[13] Bulut, S. "Fekete–Szegö problem for subclasses of analytic functions defined by Komatu integral operator". *Arabian Journal of Mathematics*, vol. 2, no. 2, pp. 177-183, 2013.

[14] Goyal, S. P., & Kumar, R. "Fekete-Szegö problem for a class of complex order related to Salagean operator". *Bull. Math. Anal. Appl*, vol. 3, no. 4, pp. 240-246, 2011.

[15] Goyal, S. P., Vijaywargiya, P., & Goswami, P. "Sufficient Conditions for Sakaguchi Type Functions of Order β". *European Journal of Pure and Applied Mathematics*, vol. 4, no. 3, pp. 230-236, 2011.

[16] Owa, S., Sekine, T., & Yamakawa, R. "On Sakaguchi type functions". *Applied mathematics and computation*, vol. 187, no. 1, pp. 356-361, 2007.

[17] Panigrahi, T., & Raina, R. K. "Fekete-Szegö problems for generalized Sakaguchi type functions associated with quasi-subordination". *Studia Universitatis Babes-Bolyai, Mathematica*, vol. 63, no. 3, 2018.

[18] SAKAR, F. M., AYTAŞ, S., & GÜNEY, H. Ö. "On The Fekete-Szegö problem for generalized class M α, γ (β) defined by differential operator". *Süleyman Demirel Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, vol. 20, no. 3, pp. 456-459, 2016.

[19] Al-Oboudi, F. M. "On univalent functions defined by a generalized Sălăgean operator". *International Journal of Mathematics and Mathematical Sciences*, 2004.

[20] Goyal, S. P., & Goswami, P. "Certain coefficient inequalities for Sakaguchi type functions and applications to fractional derivative operator". *Acta Universitatis Apulensis. Mathematics-Informatics*, vol. 19, pp. 159-166, 2009.

[21] Han, J., & Moraga, C. "The influence of the sigmoid function parameters on the speed of backpropagation learning". In International Workshop on Artificial Neural Networks, (pp. 195-201). Springer, Berlin, Heidelberg, (1995, June)..

[22] Keogh, F. R., & Merkes, E. P. "A coefficient inequality for certain classes of analytic functions". *Proceedings of the American Mathematical Society*, vol. 20, no. 1, pp. 8-12, 1996.

[23] Fadipe-Joseph, O. A., Oladipo, A. T., & Ezeafulukwe, U. A. "Modified sigmoid function in univalent function theory". *Int. J. Math. Sci. Eng. Appl*, vol. 7, no. 7, pp. 313-317, 2013.