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Effects of the Changes in the Neutron Number of Isotonic Nuclei on the Tow-Component Partial Level Density Formula Corrected for Pairing in Pre-Equilibrium Reactions

A. D. Salloum[1], J. F. Mohammad [2*], H. A. Al-Jabbar[1]
[1]*Physics Department, College of science for women, University of Baghdad, Iraq*
[2]*Physics Department, College of education for pure Sciences, University of Anbar, Iraq*

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Abstract

Ericson's formula describes the partial level density (PLD) of pre-equilibrium reactions. PLD with pairing correction can be calculated by modifying Ericson's formula using four methods, namely, pairing, improved pairing, exact Pauli and back shift energy corrections. The variations in the PLD values of each of the four formulas of strontium (^{88}Sr), Yttrium (^{89}Y) and Zirconium (^{90}Zr) isotones have been calculated. Results showed that the PLD values that uses pairing and improved pairing corrections do not vary for different isotones. However, a small change in PLD values was observed when exact Pauli correction and back shift energycorrection were utilised. The change in the PLD values using back shift energy correction was bigger than the values obtained using exact Pauli correction. Therefore, the use of back shift formula is recommended in nuclear cross-section calculations because it results in noticeable changes with any small increase in mass number.

Keywords: Exciton model, Pre-equilibrium reactions, Level density.

تأثير التغييرات في عدد النيوترون للنوى الايزوتونية على صيغة الكثافة الجزئية المكونة من عنصرين المصححة لتفاعلات الاقتران ما قبل التوازن

علي داود سلوم[1]، جمال فاضل محمد*[2]، هدى عبد الجبار [1]
[1]جامعة بغداد– كلية العلوم للبنات– قسم الفيزياء
[2]جامعة الانبار – كلية التربية للعلوم الصرفة– قسم الفيزياء

الخلاصة

كثافة الحالات الجزئية لتفاعلات ما قبل التوازن توصف بصيغة إريكسون وتصحيحاتها. يمكن حساب كثافة الحالات الجزئية مع تصحيح الاقتران باستخدام أربع طرق هي: الازدواج ، الازدواج المحسن، باولي التام الازاحة العكسية للطاقة. تمت دراسة التغيرات التي تطرأ على كل واحدة من الصيغ الاربعة للنظائر ^{88}Sr ، ^{89}Y ، ^{90}Zr. اظهرت النتائج ان قيم كثافة الحالات الجزئية لتصحيحي الزخم وتحسين الزخم لا تتغير باختلاف النواة، وفي حالة تصحيح باولي هناك تغير بسيط للقيم، اما في تصحيح طاقة التحول الخلفي فيكون التغير كبير مقارنة بتصحيح باولي، لذلك نوصي باستخدام تصحيح طاقة الازاحة العكسية في حسابات مساحة المقطع العرضي النووي لانها تعطي زيادة ملحوظة عند زيادة قليلة بالعدد الكتلي.

*Email: esp.jamalf.mohamad@uoanbar.edu.iq

1. Introduction

Griffin proposed the pre-equilibrium nuclear emission in 1966 to interpret the cross section of particle in the continuum region that cannot be interpreted theoretically neither by the compound nucleus cross section nor by the direct reaction cross section. Cross section depends basically on level density, which represents the number of energy levels per unit energy. In his work, Griffin used a level density formula used Ericson's formula for one-component because it considered protons and neutrons as distinguishable particles [1,2,3]. This formula represents the level density of excitation of some nucleons in the nucleus. Notably, not all nucleons in the nucleus are excited; therefore, it is called partial level density (PLD) [4]. Many corrections were added to the one-component Ericson's formula like; two-component Ericson's formula (which considers the protons and neutrons as distinguishable particles), William's formula, spin correction formula, linear momentum correction formula, surface correction formula and pairing corrections formula which include four corrections

which are: pairing correction, improved pairing, exact Pauli and back shift energy corrections [5,6]. Shafeq and Salloum, in 2013 [7], compared the one- and two-component formulae for Ericson's, William's and pairing corrections. Small differences were observed between the one- and two-component formula results. They also formulated and compared the composite formula, which was a new formula that included all corrections, with other corrections. They found that the comprehensive formula was equivalent to William's formula, which included the correction due to Pauli principle. In 2015, Ahmed [8] investigated the behaviour of the PLD formula corrected for Pauli exclusion principle, symmetric, spin, surface effect and pairing correction using single particle level density formula dependent on energy, and the results were compared with the experimental data from OSLO and the theoretical data from Hauser-Feshbach formula. The calculation were conducted using isotopes ^{114}Sn and^{116}Sn of energies up to 80 MeV. At low energies, a good agreement was observed between the results. At high energies, the deformation effect causes a mismatch amongst the results. Moreover, the results differ greatly from the experimental data. Therefore, new formulae must be formulated on the basis of the non-equidistant spacing model. Selman and Jasim, in 2016, [9] calculated the reaction inside the core of their different main sequence stars; Sun, Sirius and Vega. They used the PLD from the exciton model, Ericson's formula, William's formula and pairing formula; the calculation was performed for ^{4}H, ^{12}C, ^{14}N and ^{16}O; and the state density of ^{54}Fe was included for comparison. The results showed that any change in the exciton configuration will result in a remarkable change in the PLD values. The change in the PLD value with energy is not linear and declines at high energies. In this paper, the changes in the PLD values from each of the four formulae for the isotones ^{88}Sr, ^{89}Y and ^{90}Zr were studied.

2. Theory Part

Pairing correction refers to a correction that considers energy lost from excitation energy due to the coupling amongst nucleons. Therefore, the energy that is distributed among the nucleons is less than the incident energy because some of the incident energy was used to dissociate coupling nucleus. Pairing effect was added to Ericson's formula, which represents the crude formula in the one- and two-component pairing corrected formulae.

Now, considering the two-component pairing corrected formulae from the four methods.

1. Pairing correction: In this method, the two-component PLD formula is $\omega_2(n, E)$, where $P_2(\Delta)$ is the pairing energy given by [5,10]

$$\omega_2(n,E) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu} \left(E - P_2(\Delta) - B_{p_\pi,h_\pi,p_\nu h_\nu}\right)^{n-1}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} \, \Theta\left(E - P_2(\Delta) - B_{p_\pi,h_\pi,p_\nu h_\nu}\right) \quad (1)$$

where p_π, h_π, p_ν *and* h_ν are the proton particle, proton hole, neutron particle and neutron hole, respectively. The proton exciton number is $n_\pi = p_\pi + h_\pi$, the neutron exciton number is $n_\pi = p_\nu + h_\nu$, and the total exciton number $n = n_\pi + n_\nu$. The quantities $g_\pi^{n_\pi}$ *and* $g_\nu^{n_\nu}$

represent the single particle level density for protons and neutrons, respectively and are expressed as:

$$g_\pi = \frac{Z}{A}\ g \ \text{ and } \ g_\pi = \frac{Z}{A}\ g \tag{2}$$

where g is the single particle level density in the case of the one-component equidistant spacing exciton ESM:

$$g = \frac{A}{d} \tag{3}$$

where A is the mass number, and d is the distance between the spacing. $B_{p_\pi,h_\pi,p_\nu h_\nu}$ is the modified Pauli blocking factor:

$$B_{p_\pi,h_\pi,p_\nu h_\nu} = A_{p_\pi,h_\pi,p_\nu h_\nu}\sqrt{1+\left(\frac{2g\Delta}{n}\right)^2} \tag{4}$$

$A_{p_\pi,h_\pi,p_\nu h_\nu}$ is Pauli blocking factor, which is:

$$A_{p_\pi,h_\pi,p_\nu h_\nu} = \frac{p_\pi(p_\pi+1)+h_\pi(h_\pi-3)}{4g_\pi} + \frac{p_\nu(p_\nu+1)+h_\nu(h_\nu-3)}{4g_\nu} \tag{5}$$

where Δ is the energy gap of the excited state that is obtained from the curve fitting, and $P_2(\Delta)$ is the two-component pairing energy.

2. Improved pairing correction: this was supposed by Calbach. The two-component PLD formula is given by[5,10]

$$\omega_2(n,E) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu}[E-A_k(p_\pi,h_\pi,p_\nu h_\nu)]^{n-1}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!}\ \Theta(E-A_k(p_\pi,h_\pi,p_\nu h_\nu)) \tag{6}$$

where $A_k(p_\pi,h_\pi,p_\nu h_\nu)$ is the improved pairing correction and is used for the calculation of protons and neutrons from the relation

$$A_k(p,h) = E_{th} - \frac{p(p+1)+h(h+1)}{4g} \tag{7}$$

$$E_{th}(p,h) = \frac{p_m^2}{g} \tag{8}$$

p_m is the maximum value of particle number.

3. Exact Pauli correction: Some of the excitation energy is lost in overcoming the binding and Fermi energies; therefore, they are added to the PLD formula [5,10,11]:

$$\omega_2(n,E) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} = \sum\sum\sum\sum\ (-1)^{i_\pi+j_\pi+i_\nu+j_\nu}\ C_{p_\pi}^i C_{h_\pi}^j C_{p_\nu}^i C_{h_\pi}^j C_{h_\nu}^j (E-E_{th}-i_\pi B_\pi - j_\pi F_\pi - i_\nu B_\nu - j_\nu F_\nu)\ \Theta(E-E_{th}-i_\pi B_\pi - j_\pi F_\pi - i_\nu B_\nu - j_\nu F_\nu) \tag{9}$$

where $C_{p_\pi}^i C_{h_\pi}^j C_{p_\nu}^i C_{h_\pi}^j C_{h_\nu}^j$ are the binomial coefficients for proton and neutron:

$C_p^i = \frac{p!}{i!(p-i)!}$, $C_h^i = \frac{h!}{j!(h-j)!}$

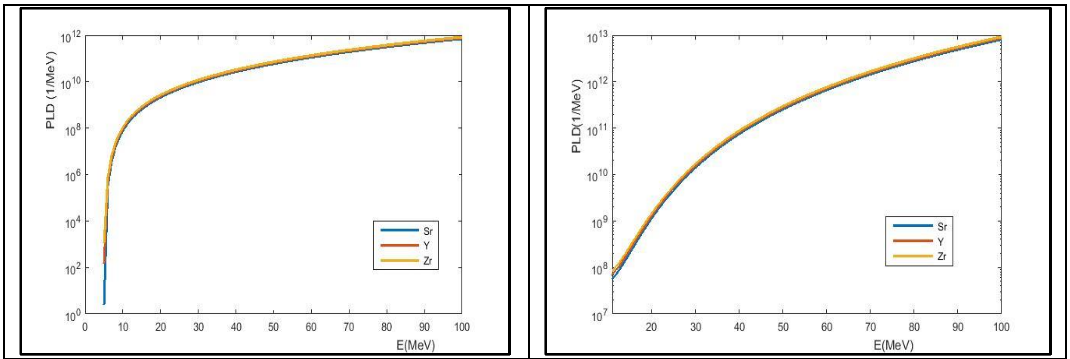
$B_\pi, F_\pi, B_\nu, F_\nu$ are the binding and Fermi energies for protons and neutrons. The binding energy value is B = 8 MeV, and the Fermi energy value is F = 38 MeV.

4. Back shift energy correction: Some of the excitation energy acts as a kinetic energy between the interacting nucleons that are represented by S, and the two-component PLD formula becomes [5,10,12]:

$$\omega_2(n,E) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!}\sum_{i_\pi=0}^{p_\pi}\sum_{j_\pi=0}^{h_\pi}\sum_{i_\nu=0}^{p_\nu}\sum_{j_\nu=0}^{h_\nu}(-1)^{i_\pi+j_\pi+i_\nu+j_\nu} C_{p_\pi}^i C_{h_\pi}^j C_{p_\nu}^i C_{h_\pi}^j C_{h_\nu}^j (E_{th} - i_\pi B_\pi - j_\pi F_\pi - i_\nu B_\nu - j_\nu F_\nu - S)\ \Theta\ (E_{th} - i_\pi B_\pi - j_\pi F_\pi - i_\nu B_\nu - j_\nu F_\nu - S) \tag{10}$$

3. Result and Discussion

In this section, the behaviour of each formula, mentioned in Section 2, for isotonic nuclei is discussed. The equations were programmed using MATLAB 2015. Figure 1 shows the behaviour of the PLD formula, Eq.(1), for pairing correction.The PLD curves increase rapidly up to 10 MeV. Subsequently, their change with energy decreases because the PLD value arrives at a maximum value at 10 MeV. Then, the change in PLD declines. No difference was observed amongst the PLD values of isotonic nuclei 88Sr, 89Y and 90Zr, indicating that the PLD value given by Eq.(1) does not change with the change of the neutron number. The unchange in PLD values my be interpreted in that Eq. (1) is not affected by the change in mass number

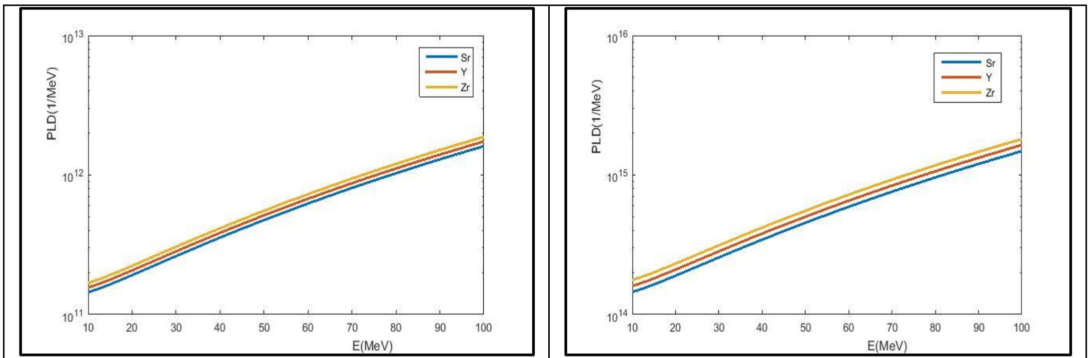


(a)	(b)

Figure 1-(a) the PLD results calculated by Pairing formula for the isotones $^{88}_{38}$Sr, $^{89}_{39}$y and $^{90}_{40}$Zr, (b) the PLD results calculated by improved pairing for the isotones $^{88}_{38}$Sr, $^{89}_{39}$y and $^{90}_{40}$Zr.

Figure 1 (a) compares the PLD results calculated from Eq.(6) for improved pairing correction for the different isotones. The PLD value decreased as the energy increased up to 5 MeV because $A_k(p_\pi, h_\pi, p_\nu h_\nu$ value is greater than 5 MeV after the increase in PLD with energy. Furthermore, the increase in PLD accelerated up to 30 MeV and the decelerated after this value. No difference was observed among the PLD results of different isotones that were calculated using Eq. (6), this can be interpreted that Eq.(6) values is not affected by the change in mass number.

Figure 2 (a) shows the results of PLD formula for the isotones from exact pauli, Eq. (9). The PLD decreased up to 5 MeV then increased linearly with energy. That is, the PLD value changes in the same amount for all energy values, and the increase varies Figures. 1 and 2. Moreover, the PLD values increased slightly with the isotones mass number. This may be interpreted that the increase in mass number make the excited nucleons more and this will lead to increase in excited states



(a)	(b)

Figure 2-(a) the PLD results calculated by exact pauli for the isotones $^{88}_{38}$Sr, $^{89}_{39}$y and $^{90}_{40}$Zr, (b) the PLD results calculated by back shift energy for the isotones $^{88}_{38}$Sr, $^{89}_{39}$y and $^{90}_{40}$Zr.

Finally, Figure2 (b) illustrates the behaviour of the PLD formula from back shift, Eq. (10) for the isotones. The PLD values decreased up to 5 MeV and then begin to increase after that value. Their change with energy is linear. The changes in the PLD values with the neutron numbers are clearer than that in Figure2 (b), this can be interpreteped as in Figure2 (a) the increase in nucleons causes increase in excited states and this affected the PLD values.

4. Conclusions

The behaviour of the PLD values, given by Eqs. (1) and (6), changed with energy while, those given by Eqs. (9) and (10) changed linearly . The PLD values from Eqs. (1) and (6) were the same but have different neutron numbers. In Eq.(9), PLD showed minimal changes for different neutron numbers. In Eq. (10), the PLD value changed linearly with energy, and the change in the PLD value was more evident than that from Eq.(9). Therefore, the use of Eq. (10) for describing the PLD value is recommended because it provides more realistic results than the other equations.

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