Ghazi and Waleed Iraqi Journal of Science, 2021, Vol. 62, No. 5, pp: 1686-1694
DOI:10.24996/ijs.2021.62.5.31

ISSN: 0067-2904

Impacts of Porous Medium on Unsteady Helical Flows of Generalized Oldroyd-B Fluid with Two Infinite Coaxial Circular Cylinders

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Received: 15/11/2020 Accepted:31/3/2021

Abstract

This article deals with the influence of porous media on helical flows of generalized Oldroyd-B between two infinite coaxial circular cylinders.The fractional derivative is modeled for this problem and studied by using finite Hankel and Laplace transforms .The velocity fields are found by using the fundamentals of the series form in terms of Mittag-Leffler equation. The research focused on permeability parameters,fractional parameters(β) , relaxation (λ_1), retardation (λ_2), kinematic viscosity(υ), magnetic parameter(M),and time (t), which affected the velocity fields u and w.The influences of the various flow parameters of the problem on these distributions are debated and proved graphically by figures .

Key words :unsteady flows **,**circular cylinders , porous medium **,** Oldroyd-B fluid**.**

تأثير الوسط المسامي على التدفق الحلزوني غير المستقر لمائع من نوع اولدرويد–بي المعمم مع اسطوانتين دائريتين متحدتين لا متناهيتين

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الخلاصة

هذا البحث يتناول تأثير الوسائط المسامية على التدفقات اللولبيةلمائع من النمط اولرويد–بي المعمم بين أسطوانتين دائرية متحدتين . تمت صياغة المشتق الكسري لهذه المسألة ودراسته باستخدام تحويلات هنكلولابلاس مع ايجاد حقول السرعة بشكل سلسلة باستخدام دالةميتاغ–ليفلر. ركز البحث على المعلمات مثل (النفاذية z، المعلمات الكسرية(α، β)، الاسترخاءλ_1 ، التخلفλ_2 ، اللزوجة الحركية(υ) المعلمة المغناطيسية M والوقت t)والتيتؤثر على مجال السرعة u وw .تم مناقشة تأثير معلمات التدفق المختلفة ويثبت بيانيا من خلال الرسوم .

Introduction

The term“porous”is used for any material that has perforations on its outer side,with the possibility of penetration. The concept of porous media has many applications in various fields, including applied sciences, engineering, geological sciences, biology, etc. Therefore, it has been the focus of attention of researchers and scientists because of its significant impact on human life. For example, Al kanhal et al.[1] studiedthe thermal management of a nanofluid within a porous media with radiation heat and magnetic force.They found that the Nusselt number increases when radiation effect and Raleigh number increase, whereas temperature increases when the radiation and Raleigh parameters decrease. The article contains a digital analysis of heat transfer within a T-shaped cavity, such that the lower surface is hot and the upper surface is cold. The effects of extensive governing factors were examined

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to determine the evolution of nanofluid flow and heat transfer within the cavity.Sheikholeset al. [**2**] concluded that, to increase energy consumption, the permeability must be increased.In addition, the part of entropy depended on the active parameters (Hartmann number , Rayleigh number, and Darcy number **.**Yanget al.[3] studied the algorithm ofMarkov prior-based block-matching for super dimension of porous media.Ali et al.[4] noticed that both the Brownian motion and thermophoresis influence the concentration gradients of nanoparticles in the porous and free fluid layers. Mhammad [5] focused on the effect of the inclined magnetic field on the peristalsis flow.The long wavelength approximation method was used to solve the problem , while some important physical concepts that have a direct relationship werediscussed and analyzed. Sinha*et al*.[6] found that a back flow happened near the center line for the channel and it can be stopped by a strong external magnetic field. Mubashir*et al*.[7] constructed a model to analyze the flow of compressed , non-conducting, Newtonian fluids between two circular plates within a porous medium channel.They found a fourth-order nonlinear differential equation using similarity transformation. Mhammad and Abdulhadi [8]studied the effects of couple stress fluid on the peristaltic flow .Saleh and Abdulhadi [9] discussed the impactsof couple stress and porous medium on transient magneto peristaltic flow under the action of heat transfer.They established equations that described the model. The effects of several variables on the equation were analyzed.Shafiq*et al*.[10-11] discussed the Magneto hydrodynamic(MHD) between two permeable discs with heat transfer to a third degree liquid. A homotopy analysis method was used to simplify the equation.They tabulated both theskin friction coefficient and the Nusselt number to analyze the effects of dimensionless parameters .Breugem [12]found the expression for the effective viscosity through channel-type with porous medium .Tan et al[13] studiedthe influence of viscosity on the unsteady flow in porous media. Dengke [14] investigated the unidirectional flows of a visco-elastic fluid with the fractional Maxwell model of helical flows and a generalized Oldroyd-B fluid, with fractional calculus between two infinite coaxial circular cylinders . By selecting the probability-dependent image mass in the spectrum dynamics(SD) reconstruction method, the results obtained through the proposed algorithm were confirmed as an accurate and successful application to the fixed porous medium.

In this study, a special model was built to represent the problem and study the variables on which the equation depends. Through a quick look at the changes that have been represented by graphs, we can conclude that when increasing each of thevariables of the fractional parameters(β) , relaxation (λ_1), retardation (λ_2),kinematic viscosity(υ), andto the magnetic parameter(M), therate of speed increases, that is, the proportionality isdirect. As the variables of fractional parameters(α),time (t), and permeability (z) decrease, the speed increases, i.e. the proportion is inverse .

Mathematical analysis

Consideran unsteady helical flow of generalized Oldroyd-B fluid with porous media between two infinite coaxial cylinders, which are located at $r = R_1$ and $r = R_2 (R_1 < R_2)$and have thecoordinates $(r\,,\theta, z)$, where the helical velocity is

$$\boldsymbol{V} = r\, v\,(r,t).\, e_\theta + w.\,(r,t).\, e_z \quad , \tag{1}$$

where $e_\theta$and e_z are the unit vectors in the θ and z- directions .

For our two dimensional problem, the governing equations of motion are

$$\rho \frac{\partial w}{\partial t} = \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) S_{rz} - \sigma\beta_0^2 w - \frac{\mu\, Q}{k} w \ , \tag{2}$$

$$\rho r \frac{\partial \mathrm{v}}{\partial t} = \left(\frac{\partial}{\partial r} + \frac{2}{r}\right) S_{r\theta} - \ \sigma\beta_0^2 r\mathrm{v} - \frac{\mu\, Q}{k} r v \quad . \tag{3}$$

such that ρ represents the density of the fluid, σ refersto electric conductivity,and $B = [0, \beta_0, 0]$is the total magnetic field ,where

$S_{rz}(r,t)$ and $S_{r\theta}(r,t)$are defined by [15]:

$$(1 + \lambda_1^\alpha D_t^\alpha) S_{r\theta} = \mu\left(1 + \lambda_2^\beta D_t^\beta\right)\left(r\frac{\partial \mathrm{v}}{\partial r}\right) \ , \tag{4}$$

$$(1 \ + \lambda_1^\alpha D_t^\alpha) S_{rz} = \mu\left(1 + \lambda_2^\beta D_t^\beta\right)\left(\frac{\partial w}{\partial r}\right) \ . \tag{5}$$

$$D_t^\alpha[y(t)] = \frac{1}{\Gamma\,(n-a)}\int_0^t \frac{f^{(n)}\,(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \, ,$$

By the elimination of$S_{rz}$and $S_{r\theta}$among Eqs.(2)and (4) and Eqs. (3)and (5), respectively, then multiplying the result by

$(1+\lambda_1^\alpha D_t^\alpha)$,we have:

$$\rho(1+\lambda_1^\alpha D_t^\alpha)\frac{\partial w}{\partial t} = \mu\left(1+\lambda_2^\beta D_t^\beta\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\left(\sigma\beta_0^2+\frac{\mu Q}{k}\right)(1+\lambda_1^\alpha D_t^\alpha)w \quad , \tag{6}$$

$$\rho\, r(1+\lambda_1^\alpha D_t^\alpha)\frac{\partial \mathrm{v}}{\partial t} = \mu\left(1+\lambda_2^\beta D_t^\beta\right)\left(r\frac{\partial^2 \mathrm{v}}{\partial r^2}+\frac{3}{r}r\frac{\partial \mathrm{v}}{\partial r}\right)-r\left(\sigma\beta_0^2+\frac{\mu Q}{k}\right)(1+D_1^\alpha D_t^\alpha)\mathrm{v} \quad . \tag{7}$$

By dividing Eq. (6)by ρ and Eq. (7)by ρr, we get :

$$(1+\lambda_1^\alpha D_t^\alpha)\frac{\partial w}{\partial t} = \upsilon\left(1+\lambda_2^\beta D_t^\beta\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\rho\boldsymbol{k}}\right)(1+\lambda_1^\alpha D_t^\alpha)w \quad , \tag{8}$$

$$(1+\lambda_1^\alpha D_t^\alpha)\frac{\partial \mathrm{v}}{\partial t} = \upsilon\left(1+\lambda_2^\beta D_t^\beta\right)\left(\frac{\partial^2 \mathrm{v}}{\partial r^2}+\frac{3}{r}\frac{\partial \mathrm{v}}{\partial r}\right)-(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\rho\boldsymbol{k}})(1+\lambda_1^\alpha D_t^\alpha)\mathrm{v}, \tag{9}$$

where $\upsilon=\frac{\mu}{\rho}$ is the kinematic viscosity of the fluid.

The corresponding boundary conditions for the present problem are defined by:

$$w(R_1,t)=U_1\,,\quad w(R_2,t)=U_2\,,\ \ t>0 \tag{10}$$

and$\mathrm{v}(R_1,t)=\Omega_1\,,\quad \mathrm{v}(R_2,t)=\Omega_2\,,\ \ t>0\,.$ (11)

The initial conditions are expressed by:

$$\mathrm{v}(r,0)=w(r,0)=0 \qquad , \tag{12}$$

$$\partial_t w(r,0)=\partial_t \mathrm{v}(r,0)=0 \quad . \tag{13}$$

where $R_1<r<R_2$.

To find the velocity field, we use the function

$$\mathrm{v}(r,t)=\frac{u(r,t)}{r} \tag{14}$$

We substitute Eq. (14) into Eq.(9), with both of the initial and boundary conditions. and multiply the result by r, then we have:

$$(1+\lambda_1^\alpha D_t^\alpha)\frac{\partial \mathrm{u}}{\partial t} = \upsilon\left(1+\lambda_2^\beta D_t^\beta\right)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)-\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}}\right)(1+\lambda_1^\alpha D_t^\alpha)u\,, \tag{15}$$

where

$$u(R_1,t)=R_1\Omega_1\,,\quad u(R_2,t)=R_2\Omega_2\,,\ \ t>0 \tag{16}$$

$$u(r,0)=\partial_t \mathrm{u}(r,0)=0\,,\quad R_1<r<R_2 \tag{17}$$

To find the analytical solution for the problems in Eq.(8) and Eq.(15), we use the initial conditions (12),(13) and(17).At first, we find thefractional derivatives for the Laplace transform with respect to t, as follows:

$$s(1+\lambda_1^\alpha s^\alpha)\overline{w} = \upsilon\left(1+\lambda_2^\beta s^\beta\right)\left(\frac{\partial^2\overline{w}}{\partial r^2}+\frac{1}{r}\frac{\partial\overline{w}}{\partial r}\right)-\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k}\right)(1+\lambda_1^\alpha s^\alpha)\overline{w}\,, \tag{18}$$

$$\overline{w}(R_1,s)=\frac{U_1}{s}\,,\quad \overline{w}(R_2,s)=\frac{U_2}{s} \tag{19}$$

$$s(1+\lambda_1^\alpha s^\alpha)\bar{u} = \upsilon\left(1+\lambda_2^\beta s^\beta\right)\left(\frac{\partial^2\bar{u}}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{u}}{\partial r}-\frac{\bar{u}}{r^2}\right)-(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})(1+\lambda_1^\alpha s^\alpha)\bar{u} \tag{20}$$

$$\bar{u}(R_1,s)=\frac{R_1\Omega_1}{s}\,,\quad \bar{u}(R_2,s)=\frac{R_2\Omega_2}{s}\ . \tag{21}$$

The finite Hankel transform with respect to r[15] is defined as:

$$\overline{\overline{w}} = \int_{R_1}^{R_2} r\,.\,\overline{w}(r,s)\,\psi_1(s_{1n}r)\,dr \tag{22}$$

$$\bar{\bar{u}} = \int_{R_1}^{R_2} r\,\bar{u}(r,s)\psi_2(s_{2n}r)dr \tag{23}$$

And the inverse of Hankel transform becomes :

$$\overline{w}(r,s)=\frac{\pi^2}{2}\sum_{n=1}^{\infty}\frac{s_{1n}^2 J_0^2(s_{1n}R_1)\overline{\overline{w}}(r,s)\psi_1(s_{1n}r)}{J_0^2(s_{1n}R_1)-J_0^2(s_{1n}R_2)} \tag{24}$$

$$\bar{u}(r,s) = \frac{\pi^2}{2}\sum_{n=1}^{\infty}\frac{s_{2n}^2 J_1^2(s_{2n}R_1)\bar{\bar{u}}(r,s)\psi_2(s_{2n}r)}{J_1^2(s_{2n}R_1)-J_1^2(s_{2n}R_2)} \tag{25}$$

where $s_{1n}$and s_{2n} are the positive roots of $\psi_1(s_{1n}R_1) = 0$ and$\psi_2(s_{2n}R_1) = 0$, respectively.

$\psi_1(s_{1n}r) = Y_0(s_{1n}R_2)J_0(s_{1n}r) - J_0(s_{1n}R_2)Y_0(s_{1n}r)$,

$\psi_2(s_{2n}r) = Y_1(s_{2n}R_2)J_1(s_{2n}r) - J_1(s_{2n}R_2)Y_1(s_{2n}r)$

whereY_i and J_i are the Bessel functions of the first and second orders zero and one , where (i=0,1), respectively.

We substitute the finite Hankel transform to Eqs.(24)and (25), then wehave:

$$\bar{\bar{w}} = \frac{2\upsilon\left(1+\lambda_2^\beta s^\beta\right)[U_2 J_0(s_{1n}R_1) - U_1 J_0(s_{1n}R_2)]}{\pi s\ J_0(s_{1n}R_1)\left[s(1+\lambda_1^\alpha s^\alpha) + (\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})(1+\lambda_1^\alpha s^\alpha) + s_{1n}^2\,\upsilon\left(1+\lambda_2^\beta s^\beta\right)\right]} \tag{26}$$

$$\bar{\bar{u}} = \frac{2\upsilon\left(1+\lambda_2^\beta s^\beta\right)[R_2\Omega_2 J_1(s_{2n}R_1) - R_1\Omega_1 J_1(s_{2n}R_2)]}{\pi s\ J_1(s_{2n}R_1)\left[s(1+\lambda_1^\alpha s^\alpha) + (\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})(1+\lambda_1^\alpha s^\alpha) + s_{2n}^2\,\upsilon\left(1+\lambda_2^\beta s^\beta\right)\right]} \tag{27}$$

By substituting Eqs.(26) and (27) into Eqs. (24) and (25),respectively, we have

$$\bar{w}(r,s) = \pi\sum_{n=1}^{\infty}\frac{J_0(s_{1n}R_1)\psi_1(s_{1n}r)[U_2 J_0(s_{1n}R_1) - U_1 J_0(s_{1n}R_2)]}{J_0^2(s_{1n}R_1) - J_0^2(s_{1n}R_2)} \times \bar{A}_1(s_{1n},s) \tag{28}$$

where$\bar{A}_1(s_{1n},s) = \frac{s_{1n}^2\,\upsilon\left(1+\lambda_2^\beta s^\beta\right)}{s\left[s(1+\lambda_1^\alpha s^\alpha)+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})(1+\lambda_1^\alpha s^\alpha)+s_{1n}^2\,\upsilon\left(1+\lambda_2^\beta s^\beta\right)\right]}$ (29)

and

$$\bar{u}(r,s) = \pi\sum_{n=1}^{\infty}\frac{J_1(s_{2n}R_1)\psi_2(s_{2n}r)[R_2\Omega_2 J_1(s_{2n}R_1) - R_1\Omega_1 J_1(s_{2n}R_2)]}{J_1^2(s_{2n}R_1) - J_1^2(s_{2n}R_2)} \times \bar{A}_1(\ s_{2n},s) \tag{30}$$

where$\bar{A}_1(s_{2n},s) = \frac{s_{2n}^2\,\upsilon\left(1+\lambda_2^\beta s^\beta\right)}{s\left[s(1+\lambda_1^\alpha s^\alpha)+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})(1+\lambda_1^\alpha s^\alpha)+s_{2n}^2\,\upsilon\left(1+\lambda_2^\beta s^\beta\right)\right]}$ (31)

We rewrite Eqs.(29) and (31) in a series form :

$$\bar{A}_1(s_{1n},s) = \frac{1}{s} - \left(s(1+\lambda_1^\alpha s^\alpha) + (\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})(1+\lambda_1^\alpha s^\alpha)\right)\sum_{m=0}^{\infty}(-1)^m\sum_{a+b+c+d=m}^{a,b,c,d\geq 0}(-1)^a$$
$$\frac{m!}{a!b!c!d!}\frac{\left(\frac{\sigma\beta_0^2}{\rho}++\frac{\mu Q}{k}\right)^b(s_{1n}^2\,\upsilon)^{c+d}\left(\lambda_2^\beta\right)^d s^\delta}{(\lambda_1^\alpha)^{m-b+1}} \times \frac{1}{\left(s^{\alpha+1}+\lambda_1^{-\alpha}+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})\lambda_1^{-\alpha}\right)^{m+1}} \tag{32}$$

.

where$\delta = m - 1 + (\alpha-1)b + (\ \beta-1)d - c - a$,

and

$$\bar{A}_1(s_{2n},s) = \frac{1}{s} - \left(s(1+\lambda_1^\alpha s^\alpha) + (\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})(1+\lambda_1^\alpha s^\alpha)\right)\times\sum_{m=0}^{\infty}(-1)^m\sum_{a+b+c+d=m}^{a,b,c,d\geq 0}(-1)^a$$
$$\frac{m!}{a!\,b!\,c!\,d!}\frac{\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}}\right)^{\boldsymbol{b}}(s_{2n}^2\,\upsilon)^{c+d}\left(\lambda_2^\beta\right)^d s^\delta}{(\lambda_1^\alpha)^{m-b+1}} \times \frac{1}{\left(s^{\alpha+1}+\lambda_1^{-\alpha}+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})\lambda_1^{-\alpha}\right)^{m+1}} \tag{33}$$

By applying the inverse of Laplace transform to Eqs.(32) and (33) and using the property of Mittag-Leffler function [15]

, we have:

$$L^{-1}\left\{\frac{n!\,s^{\mu-v}}{(s^{\mu}\mp c)^{n+1}}\right\}=t^{\mu n+v-1}E_{\mu,v}^{(n)}(\pm ct^{\mu})\ ,\left(Re(s)>|c|^{1/\mu}\right)$$

Hence, the results become :

$$A_1(s_{1n},t)=1-\sum_{m=0}^{\infty}(-1)^m\sum_{a+b+c+d=m}^{a,b,c,d\geq 0}\frac{(-1)^a}{a!\,b!\,c!\,d!}\frac{\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k}\right)^b(s_{1n}^2\,\upsilon)^{c+d}\left(\lambda_2^{\beta}\right)^d}{(\lambda_1^{\alpha})^{m-b+1}}$$
$$\times\Bigg((\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})t^{(\alpha+1)m+(\alpha-\delta)}E_{\alpha+1,((\alpha+1)-\delta)}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k}\right)\right)t^{\alpha+1}\right)$$
$$+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})\,\lambda_1^{\alpha}\,t^{(\alpha+1)m-\delta}E_{\alpha+1,(1-\delta)}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})\right)t^{\alpha+1}\right)$$
$$+t^{(\alpha+1)m+(\alpha-\delta-1)}E_{\alpha+1,(\alpha-\delta)}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})\right)t^{\alpha+1}\right)$$
$$+\ \lambda_1^{\alpha}\,t^{(\alpha+1)m-\delta-1}E_{\alpha+1\,,-\delta}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{k})\right)t^{\alpha+1}\right)\Bigg) \qquad (34)$$

and

$$A_1(s_{2n},t)=1-\sum_{m=0}^{\infty}(-1)^m\sum_{a+b+c+d=m}^{a,b,c,d\geq 0}\frac{(-1)^a}{a!\,b!\,c!\,d!}\frac{\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}}\right)^{\boldsymbol{b}}(s_{1n}^2\,\upsilon)^{c+d}\left(\lambda_2^{\beta}\right)^d}{(\lambda_1^{\alpha})^{m-b+1}}$$
$$\times\Bigg((\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})t^{(\alpha+1)m+(\alpha-\delta)}E_{\alpha+1,((\alpha+1)-\delta)}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+\left(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}}\right)\right)t^{\alpha+1}\right)$$
$$+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})\,\lambda_1^{\alpha}\,t^{(\alpha+1)m-\delta}E_{\alpha+1,(1-\delta)}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})\right)t^{\alpha+1}\right)$$
$$+t^{(\alpha+1)m+(\alpha-\delta-1)}E_{\alpha+1,(\alpha-\delta)}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})\right)t^{\alpha+1}\right)$$
$$+\ \lambda_1^{\alpha}\,t^{(\alpha+1)m-\delta-1}E_{\alpha+1\,,-\delta}^{(m)}\left(-\frac{1}{\lambda_1^{\alpha}}\left(1+(\frac{\sigma\beta_0^2}{\rho}+\frac{\mu Q}{\boldsymbol{k}})\right)t^{\alpha+1}\right)\Bigg), \qquad (35)$$

By substituting Eqs.(34) and (35) into Eqs. (28) and (30), respectively, the inverse finite Hankel transform on W(r) and U(r) becomes:

$$w(r,t)=W(r)-\pi\sum_{n=1}^{\infty}\frac{J_0(s_{1n}R_1)\psi_1(s_{1n}r)[U_2J_0(s_{1n}R_1)-U_1J_0(s_{1n}R_2)]}{J_0^2(s_{1n}R_1)-J_0^2(s_{1n}R_2)}$$
$$\times G_1(s_{1n},t) \qquad (36)$$

where$W(r)=\left[U_2+\left(\frac{In\left(\frac{r}{R_2}\right)}{In\left(\frac{R_2}{R_1}\right)}\right)*(U_2-U_1)\right]$

and

$$u(r,t)=U(r)-\pi\sum_{n=1}^{\infty}\frac{J_1(s_{2n}R_1)\psi_2(s_{2n}r)[R_2\Omega_2J_1(s_{2n}R_1)-R_1\Omega_1J_1(s_{2n}R_2)]}{J_1^2(s_{2n}R_1)-J_1^2(s_{2n}R_2)}$$
$$\times G_1(s_{2n},t) \qquad (37)$$

where $U(r)=\left[r\Omega_2+\left(\frac{R_1^2(r^2-R_2^2)}{r(R_2^2-R_1^2)}\right)*(\Omega_2-\Omega_1)\right]$.

Results and Discussion

In this section, the results are discussed throught the graphical iillustrations for different physical quantities.

Figs.[1-10]show thevelocity field w for different values,whereas Figs.[11-20]show the velocity field u for different values.By looking at the graphs, we notice that both the velocity field values of u and w change directly with the change of each of the tested parameters(β , λ_1 , λ_2, (υ), and M, i.e. the increases in these variables will increase the values of velocity fields of u and w, and the opposite is true . Also, the values of velocity fileds of u and w are inversely changing with the change of each of the parameters of α, t, and z, i.e. the increase in these variables will decrease the values of the velocity fileds of u and w, and the oppositeistrue.

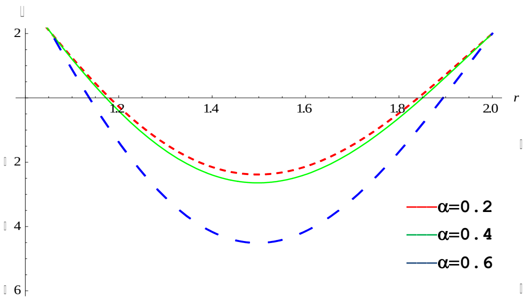

Figure1-The velocity **w** for different values of**α** at **λ_1=15, λ_2=8 ,v=0.165, M=0.1, τ=2, β=0.6, K_1=3.31114, Q=.2,Z=2, μ=.2**

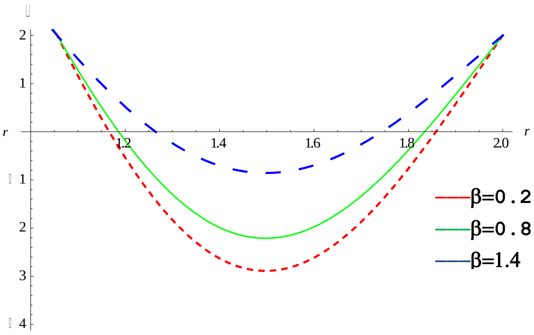

Figure 2-The velocity **w** for different values of**β** at **λ_1=15, λ_2=8, v=0.165, M=0.1, τ=2, α=0.4, K_1=3.31114,**

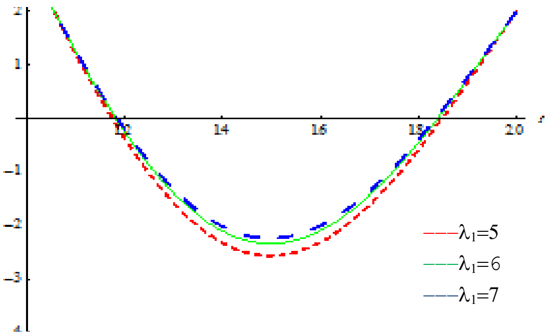

Figure3- the velocity w for different value ofλ_1 at λ_2=8,v=0.165,M=0.1,α=0.4, τ=2 β=0.6, K1=3.31114,Q=.2,Z=2,μ=.2

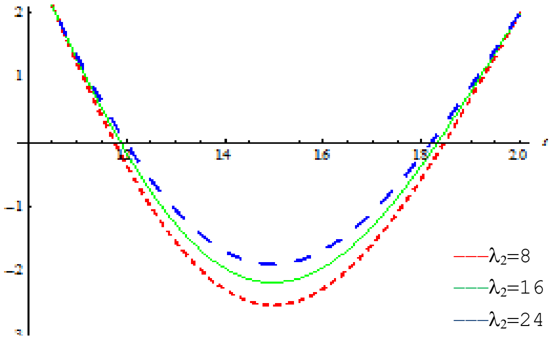

Figure 4- the velocity w for different value of λ_2 λ_1=15,v=0.165,M=0.1,α=0.4, τ=2 β=0.6, K_1=3.31114,Q=.2, Z=2,μ=.2

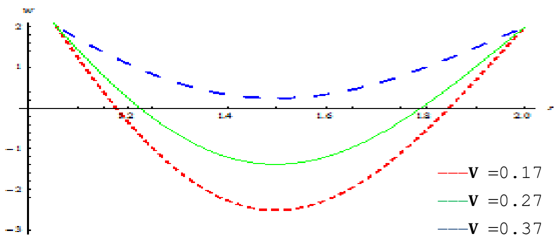

Figure 5- the velocity w for different value of**V** λ_1=15,λ_2=8,M=0.1,α=0.4, τ=2 β=0.6, K_1=3.31114,Q=.2,Z=2,μ=.2

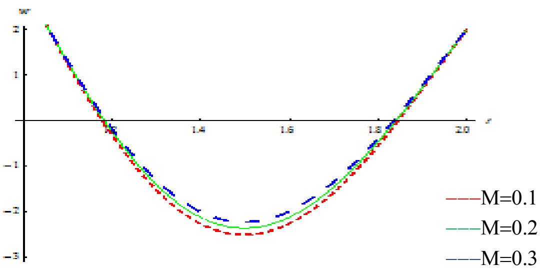

Figure 6- the velocity w for different value of M λ_1=15,λ_2=8, V=0.165, α=0.4, τ=2 β=0.6, K_1=3.31114,Q=.2, Z=2,μ=.2

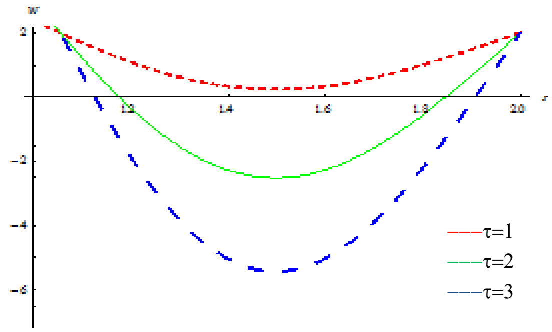

Figure 7- the velocity w for different value of τ
λ_1=15,λ_2=8,v=0.165,M=0.1,α=0.4 β=0.6,
K_1=3.31114,Q=.2,Z=2,μ=.2

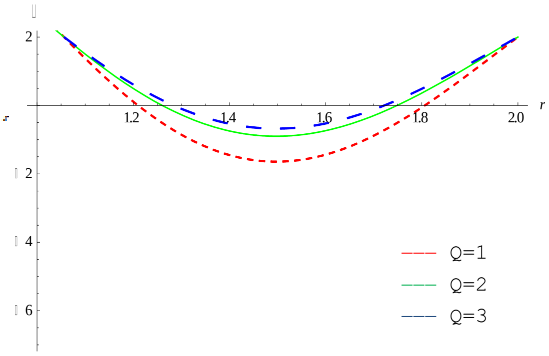

Figure 8- the velocity w for different value of Q
λ_1=15,λ_2=8,v=0.165,M=0.1,α=0.4 β=0.6,
K1=3.31114.Z=2.μ=.2. τ=2

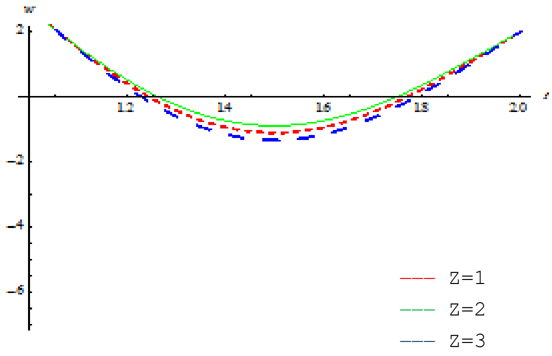

Figure 9- the velocity w for different value of Z
λ_1=15,λ_2=8,v=0.165,M=0.1,α=0.4 β=0.6,
K1=3.31114,Q=.2,μ=.2, τ=2

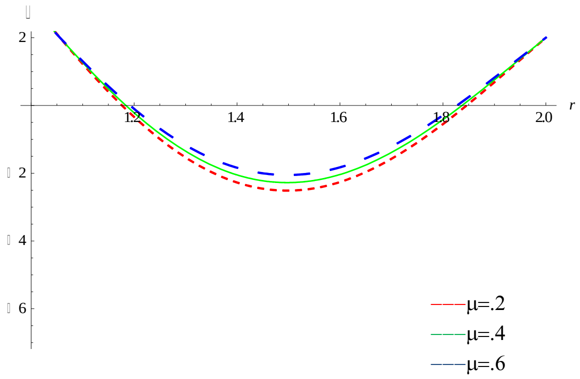

Figure 10- the velocity w for different value ofμ
λ1=15,λ2=8,v=0.165,M=0.1,α=0.4 β=0.6,
K1=3.31114,Q=.2,Z=2, τ=2

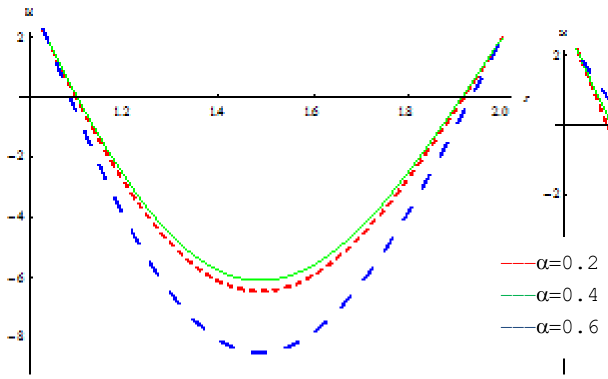

Figure 11- the velocity u for different value ofα
λ_1=15,λ_2=8,v=0.165,M=0.1, τ=2 β=0.6,
K_1=3.31114,Q=.2,μ=.2,Z=2

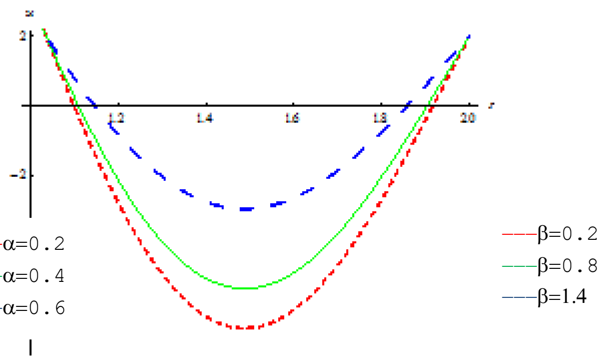

Figure 12-the velocity u for different value ofβ
λ_1=15,λ_2=8,v=0.165,M=0.1,α=0.4 ,
K_1=3.31114,Q=.2,μ=.2, τ=2, Z=2

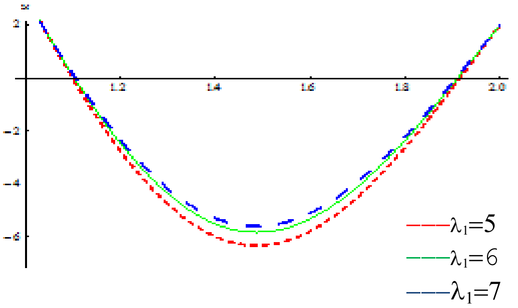

Figure 13- the velocity u for different va
λ_2=8,v=0.165,M=0.1,α=0.4,β=0.6 ,
K_1=3.31114,Q=.2,μ=.2, τ=2, Z=2

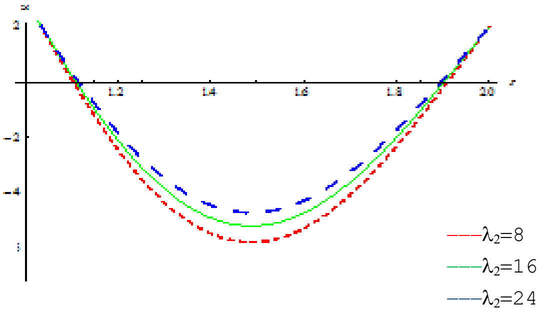

Figure 14 -the velocity u for different value ofλ_2
λ_1=15,v=0.165,M=0.1,α=0.4,β=0.6 ,
K_1=3.31114,Q=.2,μ=.2, τ=2, Z=2

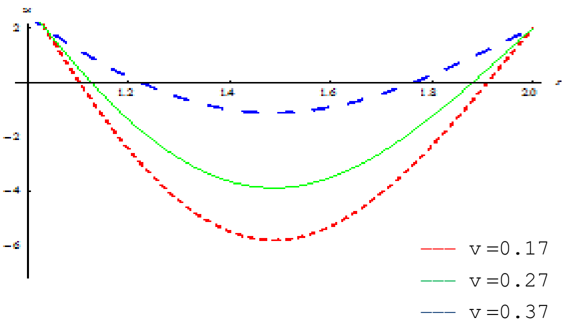

Figure 15- the velocity u for different value of v
λ_1=15 , λ_2=8,M=0.1,α=0.4,Z=2 β=0.6,
K1=3.31114,Θ=.2,μ=.2, τ=2

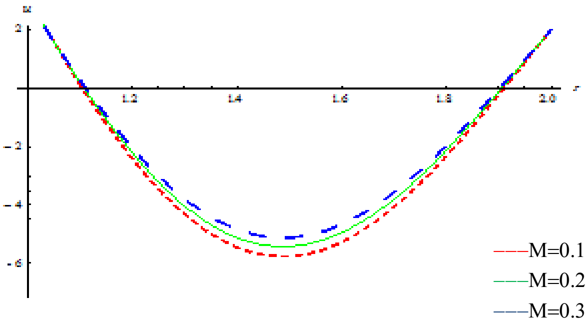

Figure 16 -the velocity u for different value of M
λ_1=15,λ_2=8,v =0.17,α=0.4,Z=2 β=0.6,
K_1=3. 31114,Q=.2,μ=.2, τ=2

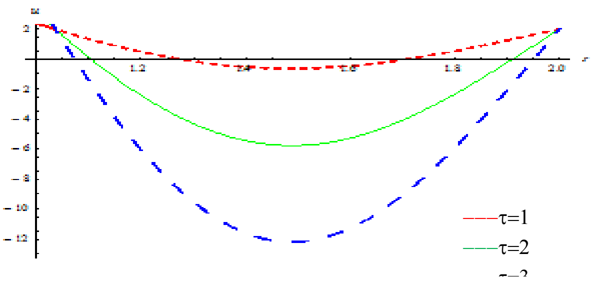

Figure 17- the velocity u for different value ofτ
λ_1=15,λ_2=8,M=0.1,v =0.17,α=0.4, τ=2
β=0.6,K_1=3.31114,Q=.2,μ=.2, Z=2

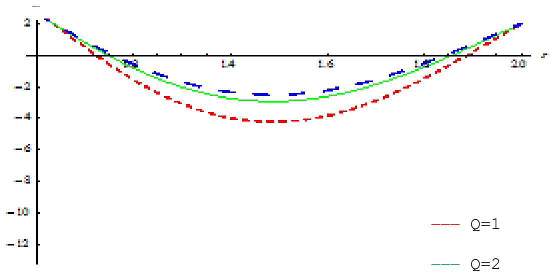

Figure 18- the velocity u for different value of Q
λ_1=15,λ_2=8,M=0.1,v =0.17,α=0.4, β=0.6,
K_1=3.31114,μ=.2, Z=2, τ=2

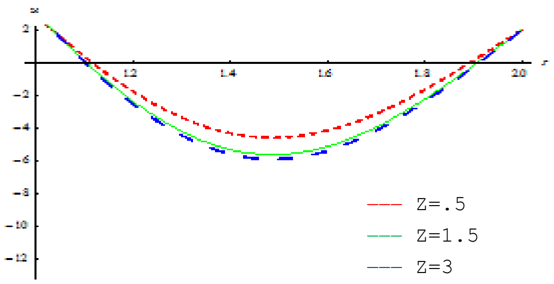

Figure 19- the velocity u for different value of Z
λ_1=15,λ_2=8,M=0.1,v =0.17,α=0.4,
τ=2 β=0.6,K_1=3.31114,Q=.2,μ=.2,

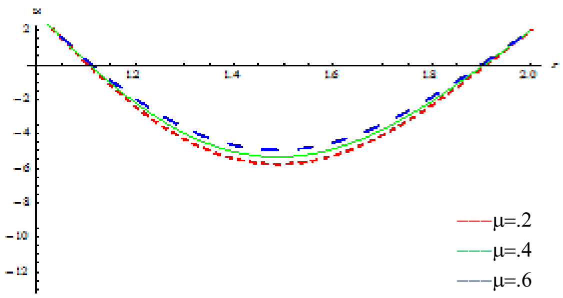

Figure 20- the velocity u for different value ofμ
λ_1=15,λ_2=8,M=0.1,v =0.17,α=0.4, τ=2
β=0.6,K_1=3.31114,Q=.2 , Z=2

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