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Jordan Generalized (μ,ρ)-Reverse Derivation from Γ-Semirings Sinto ΓS-Modules

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Abstract

In this study, we introduce and study the concepts of generalized (μ,ρ)-reverse derivation, Jordan generalized (μ,ρ)-reverse derivation, and Jordan generalized triple (μ,ρ)-reverse derivation from Γ-semiring S into ΓS-module X. The most important findings of this paper are as follows:
If S is Γ-semiring and X is ΓS-module, then every Jordan generalized (μ,ρ)- reverse derivations from S into X associated with Jordan (μ,ρ)-reverse derivation d from S into X is (μ,ρ)-reverse derivation from S into X.

Keywords: generalized (μ,ρ)-reverse derivation, Jordan generalized (μ,ρ)-reverse derivation, Jordan generalized triple (μ,ρ)-reverse derivation, Γ-semiring

تعميمات جوردان للمشتقات العكسيه من النمط – (μ,ρ) من شبه الحلقه S الى المقاسات X من النمط –S Γ

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الخلاصه

في هذه الدراسه قدمنا المفاهيم التاليه : تعميمات المشتقات العكسيه من النمط–(μ,ρ) وتعميمات جوردان للمشتقات العكسية من النمط –(μ,ρ) وتعميمات جوردان الثلاثيه للمشتقات العكسيه من النمط –(μ,ρ) اهم نتائج هذا البحث كمايلي: اذا كان S شبه حلقه من النمط –Γ و X مقاس من النمط –ΓS فان كل تعميم جوردان للمشتقات العكسيه من النمط –(μ,ρ) من S الى X المرتبط بالمشتقة جوردان العكسية من النمط – (μ,ρ) من S الى X هي مشتقه عكسيه من نمط –(μ,ρ) من S الى X.

1. Introduction

The concept of generalized (μ,ρ)- reverse derivation from Γ-semiring S into ΓS-module X is one of most important topics in non-commutative algebra. The definition of Γ-ring was presented by Nobusawa in 1964 [1] and generalized by Barnes in 1966 [2]. Sen and Saha in 1986 [3] and Saha [4] in 1989 presented the concept of Γ-semiring as a generalization of Γ-ring. The definition of Γ-semiring was introduced in [3].

The definition of prime Γ-semiring and semi-prime Γ-semiring was introduced in [5]. The definition of 2-torsion free Γ-semiring was introduced in [6]. These definitions and identity properties of multiplication of inverse elements, were introduced in [6].The definitions of additive , identity, inverse abelian elements were introduce in [5]

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Let S be a Γ-semiring and X be an additive abelian group, X is called a left ΓS- module if there exists a mapping S×Γ×X →X (sending (a,α,x) into aαx where a∈S, α ∈Γ, and x∈X) satisfying the following:

for all a,a_1,a_2∈S, α,β∈Γand x,x_1,x_2 ∈X:

i) $(a_1+a_2)\alpha x= a_1\alpha x+a_2\alpha x$

ii) a(α+β)x=aαx+aβ x

iii) $a\alpha(x_1+x_2)=a\alpha x_1+a\alpha x_2$

iv) $(a_1\alpha a_2)\beta x= a_1\alpha(a_2\beta x)$

X is called a right ΓS- module if there exists a mapping X×Γ×S →X. Also, X is called ΓS-module if its both left and right ΓS-module. The definitions of left (respectively right) prime, semipeime, and 2-torsion free ΓS- modules were introduce in [7].

Paul and Halder [7] defined the left derivation and Jordan left derivation of Γ-ring M onto ΓM-module X and studied the relations between them. Salih [8] defined the derivation and Jordan derivation from Γ-ring M into ΓM-module, along with their generalization [9]. Mahmood, Nayef, and Salih [10] presented the concepts of generalized higher derivations and Jordan generalized higher derivations on ΓM-modules and studied the relations between them. For more information see [11,12].

In this paper we present the concepts of (μ,ρ)-reverse derivation, Jordan (μ,ρ)-reverse derivation, and Jordan triple (μ,ρ)-reverse derivation from Γ-semiring S into ΓS-module X. We prove that every Jordan (μ,ρ)- reverse derivation from Γ-semiring S, with additive inverse identity into ΓS-module X where σ,τ are automorphisms on S, is (μ,ρ)-reverse derivation from S into X. In addition, we introduce the concepts of generalized (μ,ρ)-reverse derivation, Jordan generalized (μ,ρ)-reverse derivation, and Jordan generalized triple (μ,ρ)-reverse derivation from Γ-semiring S into ΓS-module X and we study the relations among them. In this paper, μ and ρ are automorphisms on S.

2. (μ,ρ)-Reverse Derivations from Γ-Semirings into ΓS-Modules

Definitions 2.1: Let S be Γ-semiring and X be ΓS-module. An additive map δ from S into X is called (μ,ρ)- reverse derivation if an only if, for all a,b∈S, α∈Γ

$\delta(a\alpha b) = \delta(b)\alpha\mu(a)+\rho(b)\alpha\delta(a)$.

δ is called Jordan (μ,ρ)-reverse derivation from S into X if, for all a,b ∈S, α∈Γ:

$\delta(a\alpha a) = \delta(a)\alpha\mu(a) + \rho(a)\alpha\delta(a)$.

δ is called Jordan triple (μ,ρ)-reverse derivation from S into X if, for all a,b∈S, α,β∈Γ:

$\delta(a\alpha b\beta a) = \delta(a)\beta\mu(b)\alpha\mu(a) + \mu(a)\beta\delta(b)\alpha\rho(a) + \rho(a)\beta\rho(b)\alpha\delta(a)$.

Example 2.2 : Let S=$\left\{\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} : a \in Z\right\}$, Γ=$\left\{\begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix} : n \in Z\right\}$, S is Γ-semiring, and X=$\left\{\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in Z\right\}$, then X is ΓS-module.

We define μ,ρ:S→S by

$\mu\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}=\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, $\rho\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$, then μ and ρ are automorphisms.

Now, we define an additive mapping δ: S→X by $\delta\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} =\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$.

Then, δ is (μ,ρ)-reverse derivation on S into X.

Example 2.3: Let R be a ring, A=$\left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} : a,b \in R\right\}$, Γ=$\left\{\begin{pmatrix} n & 0 \\ 0 & m \end{pmatrix} : n,m \in Z\right\}$, and Y=$\left\{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a,b,c \in R\right\}$, then A is Γ- semiring, Y is ΓS-module, and d is (μ,ρ)-reverse derivation of A into Y. Let $S = A \times A$,Γ_1=Γ×Γ and X= Y×Y, then S is Γ_1- semiring and X is Γ_1S-module. We use the usual addition and multiplication on matrices of S and X. We define Δ as additive mappings from S into X, such that $\Delta(B , C) = (\delta(B), \delta(C))$ for all B,C∈ Y. Then, Δ is (μ,ρ)-reverse derivation from S into X.

Lemma 2.4: Let δ be Jordan (μ,ρ)-reverse derivation from Γ-semiring S into ΓS-module X, then for all a,b∈S:

$\delta(a\alpha b+b\alpha a) = \delta(b)\alpha\mu(a) + \rho(b)\alpha\delta(a) + \delta(a)\alpha\mu(b) + \rho(a)\alpha\delta(b)$.

Proof:
$$\begin{aligned}\delta((a+b)\alpha(a+b)) &= \delta(a+b)\alpha\mu(a+b)+ \rho(a+b)\alpha\delta(a+b)\\ &= (\delta(a)+\delta(b))\alpha(\mu(a)+\mu(b))+ (\rho(a)+\rho(b))\alpha(\delta(a)+\delta(b))\\ &= \delta(a)\alpha\mu(a)+\delta(a)\alpha\mu(b)+\delta(b)\alpha\mu(a)+\delta(b)\alpha\mu(b) + \rho(a)\alpha\delta(a)+\\ &\quad \rho(a)\alpha\delta(b) + \rho(b)\alpha\delta(a)+\rho(b)\alpha\delta(b) \qquad \ldots(1)\end{aligned}$$

On the other hand,
$\delta((a+b)\alpha(a+b)) = \delta(a\alpha a+a\alpha b+b\alpha a+b\alpha b)$
$= \delta(a\alpha a) + \delta(b\alpha b) + \delta(a\alpha b+b\alpha a)$
$= \delta(a)\alpha\mu(a) + \rho(a)\alpha\delta(a) + \delta(b)\alpha\mu(b) + \rho(b)\alpha\delta(b) + \delta(a\alpha b+b\alpha a)$ …(2)
By comparing (1) and (2), we have
$\delta(a\alpha b+b\alpha a) = \delta(b)\alpha\mu(a)+ \rho(b)\alpha\delta(a)+ \delta(a)\alpha\mu(b)+ \rho(a)\alpha\delta(b)$.

Definition 2.5: If δ is Jordan (μ,ρ)- reverse derivation from additive inverse Γ-semiring S into ΓS-module X, then we define ψ by
$\psi(a,b)_\alpha = \delta(a\alpha b)- \delta(b)\alpha\mu(a)-\rho(b)\alpha\delta(a)$, for all a,b∈S, α∈Γ.

Example 2.6:
Let S be a Γ-semiring, X be a ΓS-module, and a∈S, such that aΓa=(0) and xΓaΓx=0, for all x∈S. But xΓaΓy=0, for some x,y∈S, such that x≠y. Also, let δ be a mapping of S into X defined by:
$\delta(x)=x\alpha a+a\alpha x$, for all x∈S, α∈Γ.
It is clear that Γ is a Jordan (μ,ρ)- reverse derivation on S into X, which satisfies ψ.

In the following lemma, we present the properties of $\psi(a,b)_\alpha$.

Lemma 2.7: If S is additive inverse Γ-semiring , X is ΓS-module, and δ is Jordan (μ,ρ)-reverse derivation from S into X, then for all a,b,c ∈S, α∈Γ:
i) $\psi(a,b)_\alpha= -\psi(b,a)_\alpha$
ii) $\psi(a+c,b)_\alpha= \psi(a,b)_\alpha+ \psi(c,b)_\alpha$
iii) $\psi(a,b+c)_\alpha= \psi(a,b)_\alpha + \psi(a,c)_\alpha$.

Proof:
i) $\delta(a\alpha b+b\alpha a) = \delta(b)\alpha\mu(a)+ \rho(b)\alpha\delta(a)+ \delta(a)\alpha\mu(b)+\rho(a)\alpha\delta(b)$
$\delta(a\alpha b)+\delta(b\alpha a)= \delta(b)\alpha\mu(a)+ \rho(b)\alpha\delta(a)+ \delta(a)\alpha\mu(b)+ \rho(a)\alpha\delta(b)$
$\delta(a\alpha b)- \delta(b)\alpha\mu(a)- \rho(b)\alpha\delta(a)= -\delta(b\alpha a)+ \delta(a)\alpha\mu(b)+ \rho(a)\alpha\delta(b)$
$\psi(a,b)_\alpha= -\psi(b,a)_\alpha$
ii) $\psi(a+c,b)_\alpha= \delta((a+c)\alpha b)- \delta(b)\alpha\mu(a + c)-\rho(b)\alpha\delta(a+c)$
$= \delta(a\alpha b + c\alpha b) -\delta(b)\alpha(\mu(a)+\mu(c)) - \rho(b)\alpha(\delta(a)+\delta(c))$
$= \delta(a\alpha b)+\delta(c\alpha b) -\delta(b)\alpha\mu(a) - \delta(b)\alpha\mu(c)- \rho(b)\alpha\delta(a) - \rho(b)\alpha\delta(c)$
$= \delta(a\alpha b) -\delta(b)\alpha\mu(a) - \rho(b)\alpha\delta(a) + \delta(c\alpha b) -\delta(b)\alpha\mu(c) - \rho(b)\alpha\delta(c)$
$= \psi(a,b)_\alpha+ \psi(c,b)_\alpha$
iii) $\psi(a,b+c)_\alpha= \delta(a\alpha(b+c))- \delta(b+c)\alpha\mu(a)-\rho(b+c)\alpha\delta(a)$
$= \delta(a\alpha b+a\alpha c) - (\delta(b)+\delta(c))\alpha\mu(a) - (\rho(b)+\rho(c))\alpha\delta(a)$
$= \delta(a\alpha b)+\delta(a\alpha c) -\delta(b)\alpha\mu(a)- \delta(c)\alpha\mu(a) -\rho(b)\alpha\delta(a)-\rho(c)\alpha\delta(a)$
$= \delta(a\alpha b) -\delta(b)\alpha\mu(a) -\rho(b)\alpha\delta(a)+ \delta(a\alpha c) - \delta(c)\alpha\mu(a) -\rho(c)\alpha\delta(a)$
$= \psi(a,b)_\alpha + \psi(a,c)_\alpha$.

Lemma 2.8: : If S is Γ-semiring with additive invertible identity and X is ΓS-module, then δ is Jordan (μ,ρ)-reverse derivation from S into X iff $\psi(a,b)_\alpha=0$, for all a,b∈S, α∈Γ.
Proof: By Lemma 2.4, we get
$\delta(a\alpha b+b\alpha a) = \delta(b)\alpha\mu(a) + \rho(b)\alpha\delta(a) + \delta(a)\alpha\mu(b) + \rho(a)\alpha\delta(b)$ …(1)
Now, we have the following:
Since δ is additive mapping, then we have
$\delta(a\alpha b+b\alpha a) = \delta(a\alpha b)+ \delta(b\alpha a)$
$= \delta(a\alpha b) + \delta(a)\alpha\mu(b) +\rho(a)\alpha\delta(b)$ …(2)
By comparing (1) and (2), we get
$\delta(a\alpha b) – \delta(b)\alpha\mu(a) + \rho(b)\alpha\delta(a) =0$
$\psi(a,b)_\alpha=0$
Converse: Obvious.

Theorem 2.9: Every Jordan (μ,ρ)- reverse derivation from Γ-semiring S with additive inverse identity into ΓS-module X is (μ,ρ)-reverse derivation from S into X.
Proof: Let δ be Jordan (μ,ρ)- reverse derivation from Γ-semiring S into ΓS-module.
Then, by Lemma 2.8, we get
$\psi(a,b)_\alpha=0$
Now, by Lemma 2.8, we get

δ is (μ,ρ)-reverse derivations from S into X.

Proposition 2.10: Every Jordan (μ,ρ)-reverse derivation from Γ-semiring S into 2-torsion free ΓS-module is Jordan triple (μ,ρ)-reverse derivation from S into M.

Proof: Since δ is Jordan (μ,ρ)-reverse derivation from S into X,
Then, by replacing aβb+bβa by b in lemma 2.4, we get

δ(aα(aβb+bβa) +(aβb+bβa)αa) = δ((aαa)βb+ (aαb)βa +(aαb)βa+ (bαa)βa)
= δ(b)βμ(a)αμ(a) + μ(b)βδ(a)αρ(a) + μ(b)βρ(a)αδ(a)+ δ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)+ δ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)+δ(a)βμ(b)αμ(a) + μ(b)βδ(a)αρ(a) + ρ(a)βρ(a)αδ(b) …(1)

On the other hand,

δ(aα(aβb+bβa) +(aβb+bβa)αa)= δ(aαaβb+ aαbβa+ aαbβa+bαaβa)
= δ(b)βμ(a)αμ(a) + μ(b)βδ(a)αρ(a) + μ(b)βρ(a)αδ(a)+
δ(2aαbβa)+ δ(a)βμ(b)αμ(a) +μ(b)βδ(a)αρ(a) + ρ(a)βρ(a)αδ(b) …(2)

By comparing (1) and (2), we get

2δ (aαbβa) = δ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)+ δ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)

Since X is 2-torsion free, then we get

δ(aαbβa)= δ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)

Thus, δ is Jordan triple (μ,ρ)-reverse derivation from S into X.

3. Generalized (μ,ρ)-Reverse Derivations from Γ-Semirings S into ΓS-module X

Definitions 3.1: Let S be Γ-semiring and X be ΓS-module. Then, an additive map ξ from S into X is called generalized (μ,ρ)- reverse derivation from S into X, associated with (μ,ρ)-reverse derivation δ from S into X, if and only if, for all a,b∈S, α∈Γ:

ξ(aαb) = ξ(b)α$\mu(a)$+ρ(b)αδ(a).

ξ is called Jordan generalized (μ,ρ)- reverse derivation from S into X, associated with Jordan (μ,ρ)-reverse derivation δ from S into X, if for all a,b ∈S, α∈Γ:

ξ(aαa) = ξ(a)αμ(a) + ρ(a)αδ(a).

ξ is called Jordan generalized triple (μ,ρ)-reverse derivation from S into X, associated with Jordan triple (μ,ρ)-reverse derivation δ from S into X, if for all a,b∈S, α, β∈Γ:

ξ(aαbβa) = ξ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a).

Example3.2: Let X be ΓS-semiring, as in example 2.2, and δ be (μ,ρ)-reverse derivation, as in example 2.2. We define μ,ρ:S→S by

$\mu\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$, $\rho\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$.

Then, μ and ρ are automorphisms.

Let ξ be an additive mapping on S into X defined by

$\xi\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$.

Then, ξ is generalized (μ,ρ)-reverse derivation on S into X.

Example 3.3: Let R be a ring, A , Γ, and Y are defined as in example 2.3, ξ be generalized (μ,ρ)-reverse derivation from A into Y associated with (μ,ρ)-reverse derivation δ from A into Y, and S, Γ_1, and X are as in example 2.3. We define ζ as an additive mapping from S into X such that $\zeta\ (a\ ,b) = \big(\xi(a), \xi(b)\big)$. Then, ζ is generalized (μ,ρ)-reverse derivation associated with (μ,ρ)-reverse derivation Δ from S into X, defined as in example 2.3.

Lemma 3.4: Let ξ be Jordan generalized (μ,ρ)-reverse derivation from Γ-semiring S into ΓS-module X, then for all a,b∈S:

ξ(aαb+bαa) = ξ(b)αμ(a) + ρ(b)αδ(a) + ξ(a)αμ(b) + ρ(a)αδ(b).

Proof: ξ((a+b)α(a+b)) = ξ(a+b)αμ(a+b)+ ρ(a+b)αδ(a+b)
= (ξ(a)+ξ(b))α(μ(a)+μ(b))+ (ρ(a)+ρ(b))α(δ(a)+δ(b))
= ξ(a)αμ(a)+ξ(a)αμ(b)+ξ(b)αμ(a)+ξ(b)αμ(b) + ρ(a)αδ(a)+
ρ(a)αδ(b) + ρ(b)αδ(a)+ρ(b)αδ(b) …(1)

On the other hand,

ξ((a+b)α(a+b)) = ξ(aαa+aαb+bαa+bαb)
= ξ(aαa) + ξ(bαb) +ξ(aαb+bαa)

= ξ(a)αμ(a) + ρ(a)αδ(a) + ξ(b)αμ(b) + ρ(b)αδ(b) + ξ(aαb+bαa) …(2)

By comparing (1) and (2), we have

ξ(aαb+bαa) = ξ(b)αμ(a)+ ρ(b)αδ(a)+ ξ(a)αμ(b)+ ρ(a)αδ(b)

Definition 3.5: If ξ is Jordan generalized (μ,ρ)- reverse derivation from additive inverse Γ-semiring S into ΓS-module, then we define φ by

$\varphi(a,b)_\alpha$ = ξ(aαb)- ξ(b)α$\mu(a)$-ρ(b)αδ(a), for all a,b∈S, α∈Γ.

Example 3.6:

Let S be a Γ-semiring, X be a ΓS-module, and a∈S, such that aΓa=(0) and xΓaΓx=0, for all x∈S, but xΓaΓy=0, for some x,y∈S, such that x≠y. Also, let δ be a mapping of S into X defined by:

δ(x)=xαa+aαx, for all x∈S, α∈Γ.

Let ξ be a mapping of S into X defined by:

$\xi(x) = x\alpha a \qquad for\ all\ x \in S, and\ \alpha \in \Gamma$

It is clear that ξ is a Jordan generalized (μ,ρ)- reverse derivation on S into X, which satisfies φ.

In the following lemma we present the properties of $\varphi(a,b)_\alpha$.

Lemma 3.7: If S is additive inverse Γ-semiring, X is ΓS-module, and ξ is Jordan generalized (μ,ρ)-reverse derivations from S into X, then for all a,b,c ∈S, α∈Γ:

i) $\varphi(a,b)_\alpha = -\varphi(b,a)_\alpha$

ii) $\varphi(a+c,b)_\alpha = \varphi(a,b)_\alpha + \varphi(c,b)_\alpha$

iii) $\varphi(a,b+c)_\alpha = \varphi(a,b)_\alpha + \varphi(a,c)_\alpha$.

Proof:

i) ξ(aαb+bαa) = ξ(b)αμ(a)+ ρ(b)αδ(a)+ ξ(a)αμ(b)+ ρ(a)αδ(b)

ξ(aαb)+f(bαa)= ξ(b)αμ(a)+ ρ(b)αδ(a)+ ξ(a)αμ(b)+ ρ(a)αδ(b)

ξ(aαb)- ξ(b)αμ(a)- ρ(b)αδ(a)= -ξ(bαa)+ ξ(a)αμ(b)+ ρ(a)αδ(b)

$\varphi(a,b)_\alpha = -\varphi(b,a)_\alpha$

ii) $\varphi(a+c,b)_\alpha$= ξ((a+c)αb)- ξ(b)α$\mu(a+c)$-ρ(b)αδ(a+c)

= φ(aαb + cαb) –ξ(b)α(μ(a)+μ(c)) - ρ(b)α(δ(a)+δ(c))

= ξ(aαb)+ξ(cαb) –ξ(b)αμ(a) –ξ(b)αμ(c)- ρ(b)αδ(a) - ρ(b)αδ(c)

= ξ(aαb) –ξ(b)ασ(a) - ρ(b)αδ(a) + ξ(cαb) –ξ(b)αμ(c) - μ(b)αδ(c)

= $\varphi(a,b)_\alpha + \varphi(c,b)_\alpha$

iii) $\varphi(a,b+c)_\alpha$= φ(aα(b+c))- φ(b+c)α$\mu(a)$-τ(b+c)αδ(a)

= ξ(aαb+aαc) – (ξ(b)+ξ(c))αμ(a) - (ρ(b)+ρ(c))αδ(a)

= ξ(aαb)+ξ(aαc) –ξ(b)αμ(a)- ξ(c)αμ(a) –ρ(b)αδ(a)-ρ(c)αδ(a)

= ξ(aαb) –ξ(b)αμ(a) –ρ(b)αδ(a)+ ξ(aαc) - ξ(c)αμ(a) -ρ(c)αδ(a)

= $\varphi(a,b)_\alpha + \varphi(a,c)_\alpha$

Lemma 3.8: If S is Γ-semiring with additive invertible identity and X is ΓS-module, then ξ is Jordan generalized (μ,ρ)-reverse derivation from S into X iff $\varphi(a,b)_\alpha=0$, for all a,b∈S, α∈Γ.

Proof: By Lemma 3.4, we get

ξ(aαb+bαa) = ξ(b)αμ(a) + ρ(b)αδ(a) + ξ(a)αμ(b) + ρ(a)αδ(b) …(1)

Now, we have

Since ξ is additive mapping, we have

ξ(aαb+bαa) = ξ(aαb)+ ξ(bαa)

= ξ(aαb) + ξ(a)αμ(b) +ρ(a)αδ(b) …(2)

By comparing (1) and (2), we get

ξ(aαb) – ξ(b)αμ(a) + ρ(b)αδ(a) =0

$\varphi(a,b)_\alpha=0$.

Converse: Obvious.

Theorem 3.9: If S is Γ- semiring and X is ΓS-module, then every Jordan generalized (μ,ρ)- reverse derivations from S into X is (μ,ρ)-reverse derivation from S into X.

Proof: Let ξ be Jordan generalized (μ,ρ)- reverse derivation from Γ-semiring S into ΓS-module X, then by Lemma 3.8, we get

$\varphi(a,b)_\alpha=0$.

Now, by Lemma 3.8, we get

ξ is (μ,ρ)-reverse derivation from Γ-semiring S into X.

Proposition 3.10: Every Jordan generalized (μ,ρ)-reverse derivation from Γ-semiring S into 2-torsion free ΓS-module X is Jordan generalized triple (μ,ρ)-reverse derivation from S into X.

Proof: Since ξ is Jordan generalized (μ,ρ)-reverse derivation from S into X, then by replacing aβb+bβa by b in lemma 3.4, we get

ξ(aα(aβb+bβa) +(aβb+bβa)αa) = ξ((aαa)βb+ (aαb)βa +(aαb)βa+ (bαa)βa)
= ξ(b)βμ(a)αμ(a) + μ(b)βδ(a)αρ(a) + μ(b)βρ(a)αδ(a)+ ξ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)+ ξ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)+ξ(a)βμ(b)αμ(a) + μ(b)βδ(a)αρ(a) + ρ(a)βρ(a)αδ(b) …(1)

On the other hand,

ξ(aα(aβb+bβa) +(aβb+bβa)αa)= ξ(aαaβb+ aαbβa+ aαbβa+bαaβa)
= ξ(b)βμ(a)αμ(a) + μ(b)βδ(a)αρ(a) + μ(b)βρ(a)αδ(a)+
ξ(2aαbβa)+ ξ(a)βμ(b)αμ(a) +μ(b)βδ(a)αρ(a) + ρ(a)βρ(a)αδ(b) …(2)

By comparing (1) and (2), we get

2ξ(aαbβa) = ξ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)+ξ(a)βμ(b)αμ(a) + μ(a)βδ(b)αρ(a) + ρ(a)βρ(b)αδ(a)

Since X is 2-torsion free, we get

ξ(aαbβa)= ξ(a)βμ(b)αμ(a) + μ(a)βδ(b)αμ(a) + ρ(a)βρ(b)αδ(a)

Thus ξ is Jordan generalized triple (μ,ρ)-reverse derivation from S into X.

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References

1. Nobusawa, **N. 1964**. "On a generalization of the ring theory", *Osaka Journal of Math*., **1**:81-89.
2. W.E. Barnes, **1966**. "On The Γ-Rings of Nobusawa". *Pac. J. Math*., **18**(3): 411-422 .
3. N.K. Saha, **1989** "Gamma semiring" Ph.D. thesis, Univ. of Calcutta, India.
4. M.K. Sen and N.K. Saha, **1986** ."On Γ-semigroup I", *Bull. Cal. Math. Soc*. **78**:180–186.
5. M.M.K.Rao, **1997**. " Γ-Semiring II" *Southeast Asian Bulletin of Mathematics*, **21**: 281-87.
6. M. Chandramouleeswarn and V.Thiruveni , **2010**. "On derivation of semiring", *Advances in Algebra*, **3**(1): 123-131.
7. A.C. Paul and A.K. Halder, **2009**. "Jordan left derivations of two torsion free ΓM-modules" *Journal of Physical Sciences*, **13**: 13-19.
8. S. M. Salih, **2016**. "Jordan Generalized Derivation onΓM-Module" *Magistra J*. No. 98 Th. XXIX.
9. A.H. Mahmood and E.J. Harjan, **2016** " On dependent elements of reveres bi-multipliers" *Iraqi Journal of Science*, **57**(2A): 972-978.
10. A.H. Mahmood, M. S. Nayef and S.M. Salih, **2020** "Generalized Higher Derivations on ΓM-Modules" *Iraqi Journal of Science*, **Special Issue**: 35-44
11. S. M. Salih and N. N. Sulaiman, **2020**. " Jordan Triple Higher (**σ**,)-Homomorphisms on Prime Rings" *Iraqi Journal of Science*, **61**(10): 2671-2680.
12. S.M.Salih, **2013**. "Orthogonal Derivations and Orthogonal Generalized Derivations on ΓM-Modules" *Iraqi Journal of Science*, **54**(3): 658-665.