DOI: 10.24996/ijs.2021.62.7.29

ISSN: 0067-2904

Stability Analysis and Assortment of Exact Traveling Wave Solutions for the (2+1)-Dimensional Boiti-Leon-Pempinelli System

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Received: 9/11/2020 Accepted: 27/2/2021

Abstract
In this research, the Boiti–Leon–Pempinelli (BLP) system was used to understand the physical meaning of exact and solitary traveling wave solutions. To establish modern exact results, considered. In addition, the results obtained were compared with those obtained by using other existing methods, such as the standard hyperbolic tanh function method, and the stability analysis for the results was discussed.

Keywords: Hyperbolic tanh function; Solitary traveling wave solutions; Nonlinear Boiti–Leon–Pempinelli (BLP) system.

1 Introduction

Recently**,** exact solutions of nonlinear differential equations are very important because many complex universal phenomena are related to physics, chemistry, geometry, and many other fields. The set of these exact solutions represents real characteristics of these phenomena and describes them on a large scale. Based on this fact, there have been serious attempts to evolve new ways for constructing the accurate traveling wave results of nonlinear partial differential equations **[1 - 9]**. The main purpose of this research is to implement the modern extension of the hyperbolic tanh function method with the aim of debriefing new accurate traveling wave results using nonlinear BLP system.
This research illustrates the essence of using the modern extension of hyperbolic tanh function method for finding traveling wave solutions of nonlinear differential equations. The mathematical formulation is provided in Section 2 of this paper while applications of the method are explicitly explained and provided in section 3. In section 4, comparisons with other existing methods are made, and the stability analyses for the solutions are discussed in section 5, followed by concluding remarks in section 6.

2 Mathematical Formulation

Consider a nonlinear differential equation in two independent variables x and t in the form:

$$G\,(s, s_t\,, s_x\,, s_{xt}, s_{xx}, s_{xxx}\,...) = 0\,, \tag{1}$$

where G is a polynomial for unknown function $s = s(x\,,t)$ and the integrability of the BLP system [**10,11**] is:

$$s_{ty} = (s^2 - s_x)_{xy} + 2v_{xxx}\,,$$
$$v_t = v_{xx} + 2sv_x\,. \tag{2}$$

The process of finding a traveling wave solution for the modern extension of the hyperbolic tanh function method is explained in the following three steps:
Phase 1: To obtain the solutions of equation (1), the following variables are used

$$s(x,t) = s(\xi), \qquad \xi = x + y - ct\,, \tag{3}$$

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where c is a constant. Convert equation (1) to ordinary differential equation (ODE) as follows:

$$g(s, cs', s', cs'', s'', s''' \dots) = 0 \,, \tag{4}$$

where c is a constant and $s(x,t) = s(\xi)$.

Phase 2: Integrate equation (4) and assume the solution of ODE as:

$$s(\xi) = B_0 + \sum_{i=1}^{N}(B_i W^i + Y_i W^{-i}) + W^{-2}\left(b_0 + \sum_{i=1}^{N}\left(b_i W^i + y_i W^{-i}\right)\right) \,, \tag{5}$$

where N is a positive integer, B_0, B_i, Y_i, b_0, b_i and y_i are constants, $i = (1,2,3, \dots N)$, and W is given by:

$$W = \tanh(\xi), \quad and \quad W^{-1} = coth(\xi). \tag{6}$$

W satisfies (i) two differential equations;

$$\frac{dW}{d\xi} = (1 - W^2), \;\; also \;\; \frac{d}{d\xi} = (1 - W^2)\frac{d}{dW} \quad , \tag{7}$$

and, (ii) the Riccati equation (Reid [**12**]);

$$W'(\xi) = k_0 + k_1 W(\xi) + k_2 W^2(\xi) \,. \tag{8}$$

Equation (8) has specific solutions for $k_1 = 0$, given by Wang *et al*. **[13]**

$$\begin{aligned}
&k_0 = \frac{1}{2}\,, k_2 = -\frac{1}{2}\,, \;\; W_1 = \tanh\frac{\xi}{2}\,, \;\; \coth\frac{\xi}{2} \quad ,\\
&k_0 = \frac{1}{2}\,, k_2 = \frac{1}{2}\,, \;\; W_2 = \tan\frac{\xi}{2}\,, - \cot\frac{\xi}{2}, \tan\xi \, \pm \cot\xi \quad ,\\
&k_0 = 1\,, k_2 = -1\,, \;\; W_4 = \tanh\xi, \;\; \coth\xi\,,\\
&k_0 = 1\,, k_2 = 1\,, \;\; W_5 = \tan\xi\,, - \cot\xi\,,\\
&k_0 = 1\,, k_2 = -4\,, \;\; W_6 = \frac{1}{2}\tanh 2\xi, \;\; \frac{1}{2}\coth 2\xi \,,\\
&k_0 = 1, k_2 = 4, \;\; W_7 = \frac{1}{2}\tan 2\xi, \;\; \frac{1}{2}\cot 2\xi.
\end{aligned} \tag{9}$$

The solutions for $W_1, W_2, \dots, W_7$ were obtained by solving equation (8), which is the Riccati equation. In case that $k_2 = 0$ and $k_0 = k_1 = 1$, equation (8) has the solution $W = e^{\xi} - 1$. Other values for W can be derived for other arbitrary values for k_0 and k_2.

Phase 3: Substitute equations (5) and (7) or (8) into the ODE, integrate, introduce an algebraic equation in powers of W. N is determined; the coefficients of each power of W are equated to zero in the algebraic equation. This results in a system of algebraic equations involving the $(B_0,\, B_i, Y_i, b_0, b_i, and\; y_i)$, $(i = 1,2, \dots, N)$. If the original equation contains some arbitrary constant coefficients, these will appear in the system of algebraic equations. Observations involving hyperbolic functions can lead to trigonometric functions as $\tanh\xi = i\tan\xi$.

3. Applications of the Method

In this section, the applications of the modern extension of the hyperbolic tanh function method are explicitly provided. The method is used to structure more traveling wave solutions to the nonlinear Boiti–Leon–Pempinelli system by means of two equations; equations (5) and (8).

3.1 The precise solution of the nonlinear Boiti-Leon-Pempinelli system with respect to Equation (5):

In order to solve equation (1) by the modern extension of the hyperbolic tanh function method, the wave variables $s(x,t) = s(\xi)$ and $\xi = x + y - ct$ in equation (3) are used.

Then, equation (1) becomes;

$$\begin{aligned}
&cs'' = (s^2)'' - s''' + 2s'''\\
&cv' = v'' + 2sv
\end{aligned} \tag{10}$$

Integrating the first equation in (10) twice with respect to ξ gives;

$$v' = \frac{1}{2}s' - \frac{s^2 + cs}{2} = 0 \tag{11}$$

Substituting equation (11) into second equation of equation (10) gives the nonlinear ODE;

$$s'' - 2s^3 - 3cs^2 - c^2 s = 0 \,. \tag{12}$$

By balancing between s'' and s^3, at N=1, equation (12) has the following essential solution:

$$s(\xi) = B_0 + B_1 W + Y_1 W^{-1} + W^{-2}(b_0 + b_1 W + y_1 W^{-1}) \tag{13}$$

Substituting equations (13) and (7) into equation (12), the left-hand side of the resulting equation is converted into polynomials in $W^{\boldsymbol{i}}$. Setting each coefficient of these polynomials to zero, a set of system of algebraic equations for $(B_0,\, B_i, Y_i, b_0, b_i, and\; y_i)$ is obtained. Solving this system of algebraic equations by any computer program (MATHEMATICA, MAPLE, MATLAB,… etc.), the following unknown parameters is obtained.

State 1

$B_0 = 1, B_1 = 0, c = -2, b_0 = 0, b_1 = -Y_1 + 1, y_1 = 0$
State 2
$B_0 = 1, B_1 = 0, c = -2, b_0 = 0, b_1 = -Y_1 - 1, y_1 = 0$
State 3
$B_0 = -1, B_1 = 0, c = 2, b_0 = 0, b_1 = -Y_1 + 1, y_1 = 0$
State 4
$B_0 = -1, B_1 = 0, c = 2, b_0 = 0, b_1 = -Y_1 - 1, y_1 = 0$
State 5
$B_0 = 2, B_1 = 1, c = -4, b_0 = 0, b_1 = -Y_1 + 1, y_1 = 0$
State 6
$B_0 = 1, B_1 = 1, c = -2, b_0 = 0, b_1 = -Y_1, y_1 = 0$
State 7
$B_0 = -1, B_1 = 1, c = 2, b_0 = 0, b_1 = -Y_1, y_1 = 0$
State 8
$B_0 = -2, B_1 = 1, c = 4, b_0 = 0, b_1 = -Y_1 + 1, y_1 = 0$
State 9
$B_0 = \pm 2i, B_1 = 1, c = -2\sqrt{2}i, b_0 = 0, b_1 = -Y_1 - 1, y_1 = 0$
State 10
$B_0 = 2, B_1 = -1, c = -4, b_0 = 0, b_1 = -Y_1 - 1, y_1 = 0$
State 11
$B_0 = 1, B_1 = -1, c = -2, b_0 = 0, b_1 = -Y_1, y_1 = 0$
State 12
$B_0 = -1, B_1 = -1, c = 2, b_0 = 0, b_1 = -Y_1, y_1 = 0$
State 13
$B_0 = -2, B_1 = -1, c = 4, b_0 = 0, b_1 = -Y_1 - 1, y_1 = 0$
State 14
$B_0 = \pm 2i, B_1 = 1, c = -2\sqrt{2}i, b_0 = 0, b_1 = -Y_1 - 1, y_1 = 0$
By replacing the above cases, i.e. States 1–14, into equation (13), the abundant traveling wave results of equation (1) are obtained. Then, the hyperbolic function results are:

$$s_1(x,y,t) = 1 + \coth(x + y - 2t) \quad (14)$$
$$s_2(x,y,t) = 1 - \coth(x + y - 2t) \quad (15)$$
$$s_3(x,y,t) = -1 + \coth(x + y + 2t) \quad (16)$$
$$s_4(x,y,t) = -1 - \coth(x + y + 2t) \quad (17)$$
$$s_5(x,y,t) = 2 + \tanh(x + y - 4t) + \coth(x + y - 4t) \quad (18)$$
$$s_6(x,y,t) = 1 + \tanh(x + y - 2t) \quad (19)$$
$$s_7(x,y,t) = -1 + \tanh(x + y + 2t) \quad (20)$$
$$s_8(x,y,t) = -2 + \tanh(x + y + 4t) + \coth(x + y + 4t) \quad (21)$$
$$s_9(x,y,t) = \pm 2i + \tanh\left(x + y - 2\sqrt{2}it\right) - \coth(x + y - 2\sqrt{2}it) \quad (22)$$
$$s_{10}(x,y,t) = 2 - \tanh(x + y - 4t) - \coth(x + y - 4t) \quad (23)$$
$$s_{11}(x,y,t) = 1 - \tanh(x + y - 2t) \quad (24)$$
$$s_{12}(x,y,t) = -1 - \tanh(x + y + 2t) \quad (25)$$
$$s_{13}(x,y,t) = -2 - \tanh(x + y + 4t) - \coth(x + y + 4t) \quad (26)$$
$$s_{14}(x,y,t) = \pm 2i - \tanh\left(x + y - 2\sqrt{2}it\right) - \coth(x + y - 2\sqrt{2}it) \quad (27)$$

3.2 The precise solution of the Nonlinear Boiti-Leon-Pempinelli system with respect to equation (8)

Here, substituting Equations (13) and (8) into Equation (12) and summing up each term with similar power of W^i, the left side of Equation (13) is transmuted into a polynomial in W^i. If the coefficients of all powers of W^i are equated to zero, a group of algebraic nonlinear equations for $B_0, B_1, Y_1, b_0, b_1, y_1$ and c are obtained. Solving the algebraic equations using MAPLE, the following results are obtained;

Family 1: If $k_0 = 1, k_1 = 1,$ and $k_2 = 0$ are set into the group of algebraic nonlinear equations and the resulting system have been solved, then

Case 1
$B_0 = 0\,, B_1 = 0, Y_1 = Y_1, c = 1\,, b_0 = 0, b_1 = 1 - Y_1, y_1 = 0$

Case 2
$B_0 = 0\,, B_1 = 0, Y_1 = Y_1, c = -1\,, b_0 = 0, b_1 = -1 - Y_1,\; y_1 = 0$
Case 3
$B_0 = 1\,, B_1 = 0, Y_1 = Y_1, c = -1\,, b_0 = 0, b_1 = 1 - Y_1, y_1 = 0$
Case 4
$B_0 = -1\,, B_1 = 0, Y_1 = Y_1, c = 1\,, b_0 = 0, b_1 = -1 - Y_1,\; y_1 = 0$
Substituting these values $W = e^{\xi} - 1$ in equation (13) gives

$$s_1(x,y,t) = \frac{1}{e^{(x+y+t)}} \tag{28}$$

$$s_2(x,y,t) = \frac{-1}{e^{(x+y-t)}} \tag{29}$$

$$s_3(x,y,t) = 1 + \frac{1}{e^{(x+y-t)}} \tag{30}$$

$$s_4(x,y,t) = -1 - \frac{1}{e^{(x+y+t)}} \tag{31}$$

Family 2: If $k_0 = \frac{1}{2}\,, k_1 = 0$ and $k_2 = -\frac{1}{2}$ are set into the group of algebraic nonlinear equations and the resulting system have been solved, then
Case 1
$B_0 = \frac{1}{2}\,, B_1 = 0, Y_1 = Y_1, c = -1\,, b_0 = 0, b_1 = \frac{1}{2} - Y_1, y_1 = 0$
Case 2
$B_0 = \frac{1}{2}\,, B_1 = 0, Y_1 = Y_1, c = -1\,, b_0 = 0, b_1 = -\frac{1}{2} - Y_1, y_1 = 0$
Case 3
$B_0 = -\frac{1}{2}\,, B_1 = 0, Y_1 = Y_1, c = 1\,, b_0 = 0, b_1 = \frac{1}{2} - Y_1, y_1 = 0$
Case 4
$B_0 = -\frac{1}{2}\,, B_1 = 0, Y_1 = Y_1, c = 1\,, b_0 = 0, b_1 = -\frac{1}{2} - Y_1, y_1 = 0$
Case 5
$B_0 = 1\,, B_1 = \frac{1}{2}, Y_1 = Y_1, c = -2\,, b_0 = 0, b_1 = \frac{1}{2} - Y_1, y_1 = 0$
Case 6
$B_0 = -1\,, B_1 = \frac{1}{2}, Y_1 = Y_1, c = 2\,, b_0 = 0, b_1 = \frac{1}{2} - Y_1, y_1 = 0$
Case 7
$B_0 = \frac{1}{2}\,, B_1 = \frac{1}{2}, Y_1 = Y_1, c = -1\,, b_0 = 0, b_1 = -Y_1\,, y_1 = 0$
Case 8
$B_0 = -\frac{1}{2}\,, B_1 = \frac{1}{2}, Y_1 = Y_1, c = 1\,, b_0 = 0, b_1 = -Y_1\,, y_1 = 0$
Case 9
$B_0 = \mp\frac{1}{2}\sqrt{2}i, B_1 = \frac{1}{2}, Y_1 = Y_1, c = \mp\sqrt{2}\,i, b_0 = 0,$
$b_1 = -\frac{1}{2} - Y_1\,, y_1 = 0$
Case 10
$B_0 = 1, B_1 = -\frac{1}{2}, Y_1 = Y_1, c = -2\,, b_0 = 0, b_1 = -\frac{1}{2} - Y_1\,, y_1 = 0$
Case 11
$B_0 = -1, B_1 = -\frac{1}{2}, Y_1 = Y_1, c = 2\,, b_0 = 0, b_1 = -\frac{1}{2} - Y_1\,, y_1 = 0$
Case 12

$B_0 = \frac{1}{2}, B_1 = -\frac{1}{2}, Y_1 = Y_1, c = -1, b_0 = 0, b_1 = -Y_1, y_1 = 0$

Case 13

$B_0 = -\frac{1}{2}, B_1 = -\frac{1}{2}, Y_1 = Y_1, c = 1, b_0 = 0, b_1 = -Y_1, y_1 = 0$

Case 14

$B_0 = \pm\frac{1}{2}\sqrt{2}i, B_1 = -\frac{1}{2}, Y_1 = Y_1, c = \pm\sqrt{2}\,i, b_0 = 0,$

$b_1 = \frac{1}{2} - Y_1, y_1 = 0$

Solutions of s_1 to s_{14} in the second family after substituting $W = e^{\xi} - 1$ in equation (13) are:

$$s_1(x,y,t) = \frac{1}{2} + \frac{1}{2}\coth\frac{\xi}{2} \quad (32)$$
$$s_2(x,y,t) = \frac{1}{2} - \frac{1}{2}\coth\frac{\xi}{2} \quad (33)$$
$$s_3(x,y,t) = -\frac{1}{2} + \frac{1}{2}\coth\frac{\xi}{2} \quad (34)$$
$$s_4(x,y,t) = -\frac{1}{2} - \frac{1}{2}\coth\frac{\xi}{2} \quad (35)$$
$$s_5(x,y,t) = 1 + \frac{1}{2}\tanh\frac{\xi}{2} + \frac{1}{2}\coth\frac{\xi}{2} \quad (36)$$
$$s_6(x,y,t) = -1 + \frac{1}{2}\tanh\frac{\xi}{2} + \frac{1}{2}coth\frac{\xi}{2} \quad (37)$$
$$s_7(x,y,t) = \frac{1}{2} + \frac{1}{2}\coth\frac{\xi}{2} \quad (38)$$
$$s_8(x,y,t) = -\frac{1}{2} + \frac{1}{2}coth\frac{\xi}{2} \quad (39)$$
$$s_9(x,y,t) = \pm\frac{1}{2}\sqrt{2}i + \frac{1}{2}\tanh\frac{\xi}{2} + \frac{1}{2}\coth\frac{\xi}{2} \quad (40)$$
$$s_{10}(x,y,t) = 1 - \frac{1}{2}\tanh\frac{\xi}{2} - \frac{1}{2}\coth\frac{\xi}{2} \quad (41)$$
$$s_{11}(x,y,t) = -1 - \frac{1}{2}tanh\frac{\xi}{2} - \frac{1}{2}coth\frac{\xi}{2} \quad (42)$$
$$s_{12}(x,y,t) = \frac{1}{2} - \frac{1}{2}tanh\frac{\xi}{2} \quad (43)$$
$$s_{13}(x,y,t) = -\frac{1}{2} - \frac{1}{2}tanh\frac{\xi}{2} \quad (44)$$
$$s_{14}(x,y,t) = \pm\frac{1}{2}\sqrt{2}i - \frac{1}{2}\tanh\frac{\xi}{2} + \frac{1}{2}\coth\frac{\xi}{2} \quad (45)$$

4. Comparison with other methods

In this section, the results from equations (5) and (8) are compared with the standard hyperbolic tanh function method from equation (14) in Wazwaz *et al*. [11] and presented in Table 1. Furthermore, the solutions of modern extension of the hyperbolic tanh function method with respect to equation (5) are more diverse and extensive, compared to the one in equation (8).

Table 1- Comparison of our solutions due to Boiti–Leon–Pempinelli system with standard hyperbolic tanh function method

Standard hyperbolic tanh function method due to Wazwaz *et al*. [11]	**The solution of modern extension of the hyperbolic tanh function method with respect to equation (5)**	**The solution of modern extension of the hyperbolic tanh function method depending with respect to equation (8)**
If $c = -1.1$, then equation (14) in **[11]** becomes: $s_1(\xi) = \frac{1}{2} \mp \frac{1}{2}\tanh(\mp\xi)$ $s_1(\xi) = -\frac{1}{2} \pm \frac{1}{2}\tanh(\xi)$	If $c = -2.2$, then equations (19) and (24) become: $s_6(\xi) = 1 + \tanh(\xi)$ $s_{11}(\xi) = 1 - \tanh(\xi)$	If $c = -1.1$, then equations (43) and (44) become: $s_{12}(\xi) = \frac{1}{2} - \frac{1}{2}\tanh(\xi)$ $s_{13}(\xi) = -\frac{1}{2} - \frac{1}{2}\tanh(\xi)$
If $c = -1.1$, then equation (14) in **[11]** becomes: $s_2(\xi) = \frac{1}{2} \mp \frac{1}{2}\coth(\xi)$	If $c = -2.2$, then equations (14) and (15) become: $s_1(\xi) = 1 + \coth(\xi)$ $s_2(\xi) = 1 - \coth(\xi)$	If $c = -1.1$, then equations (32) and (35) become: $s_{12}(\xi) = \frac{1}{2} - \frac{1}{2}\coth(\xi)$

$s_2(\xi) = -\frac{1}{2} \pm \frac{1}{2}\coth(\xi)$		$s_{13}(\xi) = -\frac{1}{2} - \frac{1}{2}\coth(\xi)$
If $c = -2.2$, then equation (14) in **[11]** becomes: $s_3(\xi) = 1 \pm \frac{1}{2}\tanh(\xi) \mp \frac{1}{2}\coth(\xi)$ $s_3(\xi) = -1 \pm \frac{1}{2}\tanh(\xi) \pm \frac{1}{2}\coth(\xi)$	If $c = -4.4$, then equations (23) and (26) become: $s_{10}(\xi) = 2 - \tanh(\xi) - \coth(\xi)$ $s_{13}(\xi) = -2 - \tanh(\xi) - \coth(\xi)$	If $c = -2.2$, then equations (41) and (42) become: $s_{10}(\xi) = 1 - \frac{1}{2}\tanh(\xi) - \frac{1}{2}\coth(\xi)$ $s_{11}(\xi) = -1 - \frac{1}{2}\tanh(\xi) - \frac{1}{2}\coth(\xi)$

From Table-1, it is noted that the solutions obtained by equation (8) are more diverse and extensive, compared to the solutions obtained by equation (5).

5. Stability

Equation (1) is a Hamiltonian system given by **[14 -16].** Consider the expression $M = \frac{1}{2}\int_{-\infty}^{\infty} s^2(\xi)d\xi$, where M is the momentum and $s(\xi)$ is the electric field potential. The sufficient condition for soliton stability is $\frac{\partial M}{\partial c} > 0$, where c is the frequency. The sufficient condition for instability is $\frac{\partial M}{\partial c} < 0$. Moreover, ξ is the soliton of velocity. A stability analysis was also performed in Ali *et al*., [17]

The plots for the solitary wave solutions and stability contour of solitary waves at different intervals are given in Figures 1 to 3.

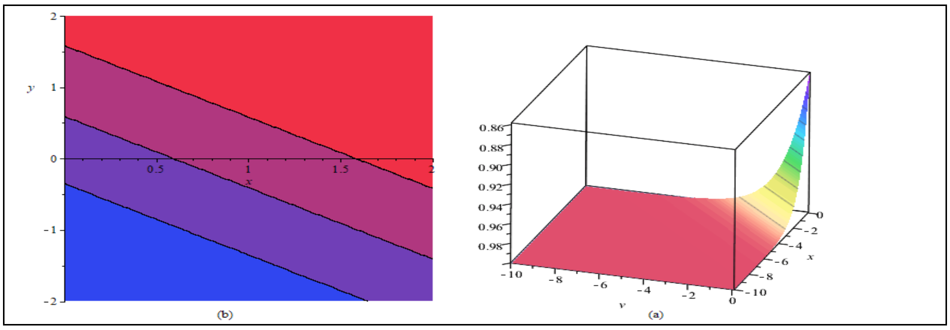

Figure 1- (a) Solitary waves solutions of Equation (43), (b) stability contour of solitary waves in interval [0,2].

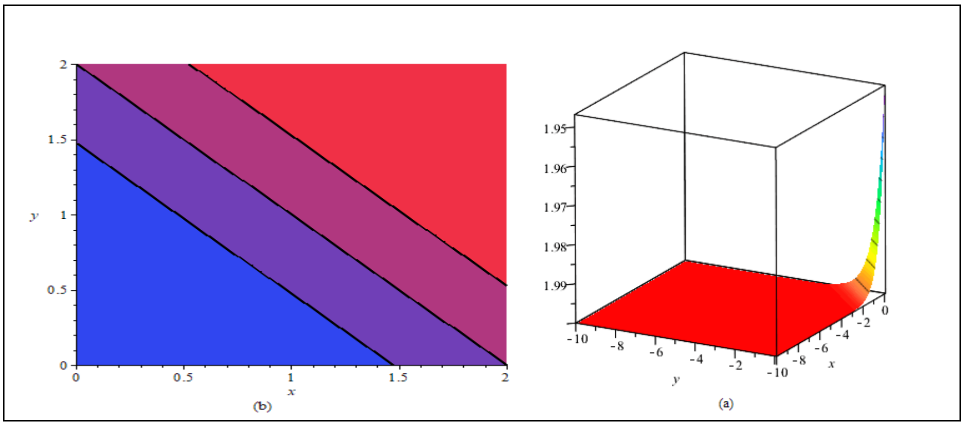

Figure 2-(a) Solitary waves solutions of Equation (24), (b) stability contour of solitary waves in interval [0,2].

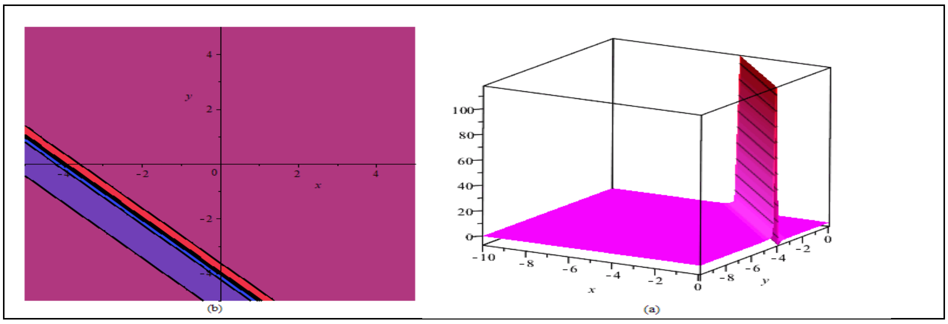

Figure 3- (a) Solitary waves solutions of Equation (26), (b) stability contour of solitary waves in interval [-5,5].

Based to the conditions of stability, Equations (43) and (24) are stable in the interval [0, 2], while Equation (26) is unstable in the interval [-5, 5].

6.0 Conclusions

In this treatise, the solitary wave of the nonlinear Boiti–Leon–Pempinelli system by means of the modern extension of the hyperbolic tanh function method was considered. Auxiliary equations were used to establish modern exact results. The results obtained assist in estimating the multiplex physical phenomena and have decisive significance in different life applications. Subsequently, the solutions were supported and established by making comparisons with the standard hyperbolic tanh function method. We conclude that the solutions in this research were diversified and simple. It is also shown that some special cases from the solutions are in agreement with the other solutions in Seadawy *et al.*, **[16]**. Moreover, the current method is beneficial, efficacious, and extremely straightforward in exploring accurate solutions. Lastly, the stability of these solutions and the waves were analyzed by making graphs of the exact solutions using 3D plots.

Funding Information

The authors did not receive any funding for this research.

Conflict of Interest

The authors declare that there is no conflict of interest.

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