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On the Soft Stability of Soft Picard and Soft Mann Iteration Processes

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Abstract.

In this paper, we introduce new concepts that relates to soft space based on work that was previously presented by researchers in this regard. First we give the definition of Soft Contraction Operator and some examples. After that we introduce the concepts of soft Picard iteration and soft Mann iteration processes. We also give some examples to illustrate them.

Many concepts in normed spaces have been generalized in soft normed spaces. One of the important concepts is the concept of stability of soft iteration in soft normed spaces. We discuss this concept by giving some lemmas that are used to prove some theorems about stability of soft iteration processes (with soft contraction operator) with soft Picard iteration procedure as well as soft Mann iteration procedure.

Keywords: soft Picard iteration processes

عن الاستقرارية الناعمة لعمليات بيكارد ومان التكرارية الناعمة

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الخلاصة :

في هذا البحث تم تقديم مفاهيم جديدة تتعلق بالفضاء الناعم بناءً على ما تم قدمه الباحثون سابقًا في هذا الحقل من الدراسة. أولاً ، تماعطاء تعريف دالة الانكماش الناعم و بعض الأمثلة بعد ذلك قدمنا مفاهيم العمليات التكرارية الناعمة لـ (بيكارد) والعمليات التكرارية الناعمة لـ (مان) مع بعض الأمثلة.

تم تعميم العديد من مفاهيم الفضاءات المعيارية الى مفاهيم في الفضاءات المعيارية اللينة. أحد هذه المفاهيم المهمة هو مفهوم استقرار التكرار الناعم في الفضاءات المعيارية اللينة. قدمنا هذا المفهوم وأعطينا بعض اللمسات التي تم استخدامها لإثبات بعض النظريات حول الاستقرار مثل استقرار عمليات التكرار الناعمة (مع دالة الانكماش الناعم) على التوالي مع عمليات التكرار الناعم لـ (بيكارد) وعمليات التكرار الناعمة لـ (مان).

1. INTRODUCTION

In the year 1999, Molodtsov [1]initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. He has shown several applications of this theory to solve many practical problems in economics, engineering, social science, medical science, etc. Research works in soft set theory and its applications in various fields have been rapidly progressed since 2002 until now. Jafari, S. , Sadati, S. and Yaghobi, A.[2] introduced new Results on Soft Banach Algebra. Recently Das , S. and Samanta , S in [3,4] introduced a notation of soft real sets, soft real numbers, soft complex sets, and soft complex numbers as well as some of

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their basic properties have been investigated. Some applications of soft real sets and soft real numbers have been presented in real life problems. Two different notions of 'soft metric' are presented in [5,6] and some properties of soft metric spaces are studied in both cases. In [7,8], the authors introduced the notion of soft linear operators in soft normed linear spaces, and soft complex sets and soft complex numbers.

The aim of this paper is to introduce new concepts that relates to soft spaces like the soft fixed point concept, soft Picard iteration and soft Mann iteration. We also introduce the concepts of stability of soft iteration processes and discuss the stability of these kinds of soft iterations processes with soft contraction operator.

2. PRELIMINARIES

Definition 2.1 [1] Assume that τ is a universe set and W is a non-empty set of parameters. Let $\wp(\tau)$ be symbolize the power set of τ.. A pair (H, W) is called a soft set over τ, where H is a mapping given by $H: W \to \wp(\tau)$. In other words, for $w \in \mathrm{W}$, $\mathrm{H}(w)$ can be represented as the set of w- approximate elements of the soft set (H, W).

Definition 2.2 [9] A soft set (H, W) over τ is said to be an absolute soft set symbolized via $\tilde{\tau}$ if for every $\lambda \in \mathrm{W}, \mathrm{H}(\lambda) = \tau$.

Definition 2.3 [9] A soft set (H, W) over τ is said to be a null soft set symbolized by $\widetilde{\Phi}$ if for every $\lambda \in \mathrm{W}, \mathrm{H}(\lambda) = \phi$.

Definition 2.4 [2] Let τ be a non-empty set of elements and $\mathrm{W} \neq \emptyset$ is a set of parameter. ρ: $\mathrm{W} \to \tau$ is called a soft element of τ. ρ is belongs to a soft set B of τ, which is symbolized by $\rho \tilde{\in} \mathrm{B}$, if $\rho(\lambda) \in \mathrm{B}(\lambda)$ for every $\lambda \in \mathrm{W}$. Thus for a soft set B of τ with respect to the index set W, we have $\mathrm{B}(\lambda) = \{\rho(\lambda), \rho \tilde{\in} \mathrm{B}\}\ \lambda \in \mathrm{W}$.

It is well-known that each singleton soft set "a soft set $(\mathrm{H}, \mathrm{W})$for which $\mathrm{H}(\lambda)$ is a singleton set, for every $\lambda \in \mathrm{W}$" can be recognized with a soft element by just identifying the singleton set with the element that it contains for all $\lambda \in \mathrm{W}$.

Definition 2.5 [7] Let $\mathfrak{B}(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$, and B is parameters set. Then, a mapping H: $\mathrm{W} \to \mathfrak{B}(\mathbb{R})$ is called a soft real set and symbolized with (H, W). If $(\mathrm{H}, \mathrm{W})$is specifically a singleton soft set, then when identifying (H, W) with the corresponding soft element, it will call a soft real number.

The collection of each soft real numbers is symbolized by $\mathbb{R}(\mathrm{W})$ while the collection of non-negative is only symbolized by $\mathbb{R}(W)^*$.

Definition 2.6 [8] Let $(\mathrm{H}, \mathrm{W}), (\mathrm{J}, \mathrm{W}) \tilde{\in} \mathbb{C}(\mathrm{W})$, then the sum, difference, product and division are defined as following:

$(\mathrm{H} + \mathrm{J})(\mathrm{n}) = \mathrm{u} + \mathrm{v}, \mathrm{u} \in \mathrm{H}(\mathrm{n})$ and $\mathrm{v} \in \mathrm{J}(\mathrm{n})$ for all $\mathrm{n} \in \mathrm{W}$

$(\mathrm{H} - \mathrm{J})(\mathrm{n}) = \mathrm{u} - \mathrm{v}, \mathrm{u} \in \mathrm{H}(\mathrm{n})$ and $\mathrm{v} \in \mathrm{J}(\mathrm{n})$ for all $\mathrm{n} \in \mathrm{W}$

$(\mathrm{HJ})(\mathrm{n}) = \mathrm{uv}, \mathrm{u} \in \mathrm{H}(\mathrm{n})$ and $\mathrm{v} \in \mathrm{J}(\mathrm{n})$ for all $\mathrm{n} \in \mathrm{W}$

$(\mathrm{H} / \mathrm{J})(\mathrm{n}) = \mathrm{u}/\mathrm{v}, \mathrm{u} \in \mathrm{H}(\mathrm{n})$ and $\mathrm{v} \in \mathrm{J}(\mathrm{n})$ provided $\mathrm{J}(\mathrm{n}) \neq \emptyset$ for all $\mathrm{n} \in \mathrm{W}$

Let τ be a non-empty set and $\tilde{\tau}$ is the absolute soft set i.e., $\tau(\lambda) = \tau)$, for each $\lambda \in \mathrm{W}$. Suppose $\mathrm{S}(\tilde{\tau})$ be the collection of all soft sets (H, W) over τ for which $\mathrm{H}(\lambda) \neq \phi$, for all $\lambda \in \mathrm{W}$ together with the null soft set $\widetilde{\Phi}$.

For any non-null soft set $(\mathrm{H}, \mathrm{W}) \tilde{\in} \mathrm{S}(\tilde{\tau})$ the collection of all soft elements of (H, W) will be denoted by $\mathrm{SE}(\mathrm{H}, \mathrm{W})$ for a collection $\mathfrak{B}$ of soft elements of $\tilde{\tau}$, $\mathrm{SS}(\mathfrak{B})$ is defined as a soft set which is generated by $\mathfrak{B}$.

Definition 2.7 [7] Let $\mathcal{V}$ be a vector space over a field $\mathcal{K}$and W is a set of parameters. Assume that $\mathcal{G}$ is a soft set over $(\mathcal{V}, \mathrm{W})$. If for all $\lambda \in \mathrm{W}$, $\mathcal{G}(\lambda)$ is a subspace of $\mathcal{V}$, then $\mathcal{G}$ is called a soft vector space of $\mathcal{V}$ over $\mathcal{K}$.

Definition 2.8 [10] Suppose that $\mathcal{H}$ is a soft vector space of $\mathcal{V}$ over $\mathcal{K}$. Let $\mathcal{G}$: W → $\wp$ ($\mathcal{V}$) be a soft set over ($\mathcal{V}$, W) If $\mathcal{G}$ (λ) is a vector subspace of $\mathcal{V}$ over $\mathcal{K}$for each λ ∈ W, and $\mathcal{H}(\lambda) \supseteq \mathcal{G}(\lambda)$, then $\mathcal{G}$ is called a soft vector subspace of $\mathcal{H}$.

Definition 2.9 [7] Suppose that $\mathcal{G}$ is a soft vector space of $\mathcal{V}$ over $\mathcal{K}$. A soft element of $\mathcal{G}$ is called a soft vector of $\mathcal{G}$, and a soft element of the soft set ($\mathcal{K}$, W) (In a similar way) is said to be a soft scalar, $\mathcal{K}$ is the scalar field.

Definition 2.10 [7] Let $\tilde{x}$, $\tilde{y}$ be soft vectors of $\mathcal{G}$ and $\tilde{k}$ is a soft scalar. Then the operation addition $\tilde{x} + \tilde{y}$ is defined by: $(\tilde{x} + \tilde{y})(\lambda) = \tilde{x}(\lambda) + \tilde{y}(\lambda)$, and the operation multiplication $\tilde{k}.\tilde{x}$ is defined by: $\tilde{k}.\tilde{x}(\lambda) = \tilde{k}(\lambda).\tilde{x}(\lambda)$ for all λ ∈ W. Obviously, $\tilde{x} + \tilde{y}$ and $\tilde{k}.\tilde{x}$ are soft vectors of $\mathcal{G}$.

Definition 2.11 [7] Let $\tilde{\tau}$ be the absolute soft vector space i.e., $\tilde{\tau}(\lambda) = \tau$, for all $\lambda \in W$. A mapping $\|.\|$: SE($\tilde{\tau}$) → $R(W)^*$is called a soft norm on the soft vector space $\tilde{\tau}$ if $\|.\|$ satisfies the following:

(1). $\|.\| \tilde{\geq} \bar{0}$ for every $\tilde{x} \tilde{\in} \tilde{\tau}$.

(2). $\|\tilde{x}\| = \bar{0}$ if and only if $\tilde{x} = \Theta$.

(3). $\|\tilde{\alpha}.\tilde{x}\| = |\tilde{\alpha}|\,\|\tilde{x}\|$ for each $\tilde{x} \tilde{\in} \tilde{X}$ as well as for each soft scalar $\tilde{\alpha}$.

(4).With any $\tilde{x}, \tilde{y} \tilde{\in} \tilde{\tau}$, $\|\tilde{x} + \tilde{y}\| \tilde{\leq} \|\tilde{x}\| + \|\tilde{y}\|$

The soft vector space $\tilde{\tau}$ with a soft norm $\|.\|$on $\tilde{\tau}$ is called a soft normed linear space and it is symbolized with ($\tilde{\tau}$,$\|.\|$, B) or ($\tilde{\tau}$, $\|.\|$). The exceeding conditions are called soft norm axioms.

Suppose a soft norm $\|.\|$ has the following feature: The set $\{\|\tilde{x}\|(\lambda) : \tilde{x}(\lambda) = \xi\}$ is a one element set for $\xi \in$ X and $\lambda \in W$. Then with any , the function $\|.\|_\lambda: \tau \to R^+$ is defined by $\|\xi\|_\lambda = \|\tilde{x}\|(\lambda)$, with any $\xi \in \tau, \lambda \in W$, and $\tilde{x} \tilde{\in} \tilde{\tau}$ such that $\tilde{x}(\lambda) = \xi$ can be considered as a norm on τ. This property named the axiom $N5$.

Definition 2.12 [7] A sequence of soft elements $\{\widetilde{x_n}\}$ in a soft normed space ($\tilde{\tau}$,$\|.\|$,W) is called soft convergent sequence if $\|\widetilde{x_n} - \tilde{x}\| \to \bar{0}$ as n → ∞, we say the sequence converges to a soft element $\tilde{x}$. In other words for each $\tilde{\epsilon} \tilde{\geq} \bar{0}$, there exists N∈ ℕ , N = N ($\tilde{\epsilon}$) and $\bar{0} \tilde{\leq} \|\widetilde{x_n} - \tilde{x}\| \tilde{\leq} \tilde{\epsilon}$ whenever n > N.

Definition 2.13 [7] A sequence $\{\widetilde{x_n}\}$ of soft elements in a soft normed space ($\tilde{\tau}$, $\|.\|$,W) is said to be a soft Cauchy sequence in $\tilde{\tau}$ if corresponding to each $\tilde{\epsilon} \tilde{>} \bar{0}$, there exist m > N such that $\|\widetilde{x_i} - \widetilde{x_j}\| \tilde{\leq} \tilde{\epsilon}$, for all i,j ≥ m .

Definition 2.14 [7] Let $(\tilde{\tau}, \|.\|, W)$ be a soft normed space, if every soft Cauchy sequence in $\tilde{\tau}$ converges to a soft element of $\tilde{\tau}$, then $\tilde{\tau}$ is said to be complete. The complete soft normed space is said to be a soft Banach Space.

Definition 2.15[10] A series $\sum_{k=1}^{\infty} \widetilde{x_k}$ of soft elements called soft convergent, if the partial sum of the series $\widetilde{S_n} = \sum_{k=1}^{n} \widetilde{x_k}$ is soft convergent.

Let $\tilde{\tau}, \tilde{\omega}$ be the corresponding absolute soft normed spaces i.e., $\tilde{\tau}(w) = \tau$, $\tilde{\omega}(w) = \omega$ for all $w \in$ W. We use the notation $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ to represent soft vectors of a soft vector space.

Definition 2.16[7] Suppose $\mathcal{M}$: SE($\tilde{\tau}$) → SE($\tilde{\omega}$) is an operator. $\mathcal{M}$ is called soft linear if

($\mathcal{L}$1). $\mathcal{M}$ is additive, i.e., $\mathcal{M}(\widetilde{x_1} + \widetilde{x_2}) = \mathcal{M}(\widetilde{x_1}) + \mathcal{M}(\widetilde{x_2})$ for all soft vector $\widetilde{x_1}, \tilde{x}_2 \tilde{\in} \tilde{\tau}$.

($\mathcal{L}$2). $\mathcal{M}$ is homogeneous, i.e., for all soft scalar $\tilde{k}$, $\mathcal{M}(\tilde{k}.\tilde{x}) = \tilde{k}\,\mathcal{M}(\tilde{x})$, for all soft vector $\tilde{x} \tilde{\in} \tilde{\tau}$

The properties ($\mathcal{L}$1) and ($\mathcal{L}$2) can be put in a combined form $\mathcal{M}(\widetilde{k_1}.\widetilde{x_1} + \widetilde{k_2}.\widetilde{x_2}) = \widetilde{k_1}\,\mathcal{M}(\widetilde{x_1}) + \widetilde{k_2}\,\mathcal{M}(\widetilde{x_2})$ for every soft vectors $\widetilde{x_1}, \widetilde{x_2} \tilde{\in} \tilde{\tau}$ and every soft scalars $\widetilde{k_1}$, $\widetilde{k_2}$.

Definition 2.17[7] The operator $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\omega})$ is said to be continuous at $\widetilde{x_0} \tilde{\in} \tilde{\tau}$ if for every sequence $\{\widetilde{x_n}\}$ of soft elements of $\tilde{\tau}$ with $\widetilde{x_n} \to \widetilde{x_0}$ as n → ∞, we have $\mathcal{M}(\widetilde{x_n}) \to \mathcal{M}(\widetilde{x_0})$ as n → ∞ . If $\mathcal{M}$ is continuous at every soft element of $\tilde{\tau}$, then $\mathcal{M}$ is called a continuous operator.

Theorem 2.18[7] Let $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\omega})$ be a soft linear operator, where $\tilde{\tau}, \tilde{\omega}$ are soft normed linear spaces: If $\mathcal{M}$ is continuous at some soft element $\widetilde{x_0} \tilde{\in} \tilde{\tau}$ then $\mathcal{M}$ is continuous at every soft element of $\tilde{\tau}$.

3. ***Soft Contraction Operator, soft Picard and soft Mann iteration processes***

Definition 3.1:

Let $\tilde{\tau}$ be a soft normed space. A soft operator $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$is called a soft contraction operator if there exists a soft real number $\tilde{\alpha}$ such that $\overline{0} \tilde{\leq} \tilde{\alpha} \tilde{<} \overline{1}$, and for every $\tilde{x}, \tilde{y} \tilde{\in} \tilde{\tau}$ we have: $\|\mathcal{M}\tilde{x} - \mathcal{M}\tilde{y}\| \tilde{\leq} \tilde{\alpha}\|\tilde{x} - \tilde{y}\|$.

Example 3.2

Let $\tilde{\tau}$ be a soft vector space where $\tau = \mathcal{R}^n$ and W={1, 2, ..., n},and let $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ be a soft operator on $\tilde{\tau}$ such that $\mathcal{M}(\tilde{x}) = \overline{0.5}\tilde{x} - \overline{1}$
for all $\tilde{x}, \tilde{y} \tilde{\in} \tilde{\tau}$ and $\overline{0.5}$ is a soft real number for which $\overline{0.5}(\lambda) = 0.5 \; for \; all \; \lambda \in W$.

we have $\|\mathcal{M}\tilde{x} - \mathcal{M}\tilde{y}\| = \|\overline{0.5}\tilde{x} - \overline{1} - \overline{0.5}\tilde{y} + \overline{1}\|$

$$= \|\overline{0.5}\tilde{x} - \overline{0.5}\tilde{y}\| = \overline{0.5}\|\tilde{x} - \tilde{y}\|$$

So, we have $\|\mathcal{M}\tilde{x} - \mathcal{M}\tilde{y}\| \tilde{\leq} \overline{0.6}\|\tilde{x} - \tilde{y}\|$ for all $\tilde{x}, \tilde{y} \tilde{\in} \tilde{\tau}$. That is $\mathcal{M}$ is soft contraction.

Proposition 3.3

Every soft contraction operator is soft continuous operator.

Proof: Let $\tilde{x} \tilde{\in} \tilde{\tau}$ be arbitrary soft element. For any $\tilde{\epsilon} \tilde{>} \overline{0}$, let $\|\tilde{x} - \tilde{y}\| \tilde{<} \tilde{\delta}$. Choose $\tilde{\delta} \tilde{<} \tilde{\in}$. Since $\mathcal{M}$ is soft contraction, then $\|\mathcal{M}\tilde{x} - \mathcal{M}\tilde{y}\| \tilde{\leq} \tilde{\alpha}\|\tilde{x} - \tilde{y}\| \tilde{<} \tilde{\alpha}\tilde{\delta} \tilde{<} \tilde{\in}$. Hence $\mathcal{M}$ is soft continuous.

Definition 3.4

Let $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ where $\tilde{\tau}$ is a soft normed space. A soft element $\tilde{x}$ called soft fixed element if, $\mathcal{M}(\tilde{x}) = \tilde{x}$.

Theorem 3.5

Let $\tilde{\tau}$ be a soft Banach space and$\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$. If $\mathcal{M}$ is a soft contraction operator, then there exists a unique soft element $\tilde{x} \tilde{\in} \tilde{\tau}$ such that $\mathcal{M}(\tilde{x}) = \tilde{x}$.

Proof: Let $\widetilde{x_0}$ be any soft element in $\tilde{\tau}$. We set $\widetilde{x_1} = \mathcal{M}(\widetilde{x_0})$, $\widetilde{x_2} = \mathcal{M}(\widetilde{x_1})$, ... , $\tilde{x}_{n+1} = \mathcal{M}(\widetilde{x_n})$.

$$\|\tilde{x}_{n+1} - \widetilde{x_n}\| = \|\mathcal{M}\widetilde{x_n} - \mathcal{M}\tilde{x}_{n-1}\| \tilde{\leq} \tilde{\alpha}\|\widetilde{x_n} - \tilde{x}_{n-1}\|$$
$$= \tilde{\alpha}\|\mathcal{M}\tilde{x}_{n-1} - \mathcal{M}\tilde{x}_{n-2}\|$$
$$\tilde{\leq} \tilde{\alpha}^2\|\tilde{x}_{n-1} - \tilde{x}_{n-2}\| \ldots\ldots \tilde{\leq} \tilde{\alpha}^n\|\widetilde{x_1} - \widetilde{x_0}\|.$$

Therefore, we have $\|\tilde{x}_{n+1} - \tilde{x}_n\| \tilde{\leq} \tilde{\alpha}^n\|\widetilde{x_1} - \widetilde{x_0}\|$.

Now, for $n > m$ we have:

$$\|\tilde{x}_n - \tilde{x}_m\| \tilde{\leq} \|\widetilde{x_n} - \tilde{x}_{n-1}\| + \|\tilde{x}_{n-1} - \tilde{x}_{n-2}\| + \cdots + \|\tilde{x}_{m+1} - \tilde{x}_m\|$$
$$\tilde{\leq} (\tilde{\alpha}^{n-1} + \tilde{\alpha}^{n-2} + \cdots + \tilde{\alpha}^m)\|\widetilde{x_1} - \widetilde{x_0}\|$$
$$\tilde{\leq} \frac{\tilde{\alpha}^m}{\overline{1} - \tilde{\alpha}} \|\widetilde{x_1} - \widetilde{x_0}\|$$

(Since $\frac{\tilde{\alpha}^m}{\overline{1} - \tilde{\alpha}} = (\tilde{\alpha}^{n-1} + \tilde{\alpha}^{n-2} + \cdots + \tilde{\alpha}^m) + \frac{\tilde{\alpha}^m}{\overline{1} - \tilde{\alpha}}$, $then \; \tilde{\alpha}^{n-1} + \tilde{\alpha}^{n-2} + \cdots + \tilde{\alpha}^m \tilde{\leq} \frac{\tilde{\alpha}^m}{\overline{1} - \tilde{\alpha}}$)

When n,m $\longrightarrow \infty$, $\|\widetilde{x_n} - \widetilde{x_m}\| \longrightarrow \overline{0}$. This implies that $\{\widetilde{x_n}\}$ is a soft Cauchy sequence. Since $\tilde{\tau}$ is complete so that there is a soft element $\tilde{x} \tilde{\in} \tilde{\tau}$ such that $\widetilde{x_0} \to \tilde{x} \; as \; n \to \infty$. Therefore,

$\|\mathcal{M}\tilde{x} - \tilde{x}\| \tilde{\leq} \|\mathcal{M}\widetilde{x_n} - \mathcal{M}\tilde{x}\| + \|\mathcal{M}\widetilde{x_n} - \tilde{x}\| \tilde{\leq} \tilde{\alpha}\|\widetilde{x_n} - \tilde{x}\| + \|\tilde{x}_{n+1} - \tilde{x}\|$.

We obtain that $\|\mathcal{M}\tilde{x} - \tilde{x}\| \to \overline{0} \; as \; n \to \infty$ (i.e., $\mathcal{M}\tilde{x} = \tilde{x}$).

If $\tilde{y}$ is another soft fixed element of $\mathcal{M}$, then:

$$\|\tilde{x} - \tilde{y}\| = \|\mathcal{M}\tilde{x} - \mathcal{M}\tilde{y}\| \tilde{\leq} \tilde{\alpha}\|\tilde{x} - \tilde{y}\|.$$

This implies that $\|\tilde{x} - \tilde{y}\| = \overline{0}$ (since $\tilde{\alpha} \tilde{<} \overline{1}$) and $\tilde{x} = \tilde{y}$. Hence, the soft fixed element of $\mathcal{M}$ is unique.

The iteration procedure using in the last theorem called (**soft Picard iteration procedure)**.

Definition 3.6 (soft *Mann iteration*)
Let $\tilde{\tau}$ be a soft normed space and $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ is a soft operator on. Let $\{\widetilde{\alpha_n}\}$ be a sequence of soft real number belong to $\mathbb{R}(W)^*$ such that $\bar{0} \leq \tilde{\alpha}_n < \bar{1}$ $for\ all\ n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} \tilde{\alpha}_n$ is diverge.
Define a soft sequence $\{\widetilde{x_n}\}$ in $\tilde{\tau}$ by $\tilde{x}_0 \tilde{\in} \tilde{\tau}$ and $\tilde{x}_{n+1} = M(\widetilde{x_n}, \alpha_n, \mathcal{M})\ n \in \mathbb{N}$
Where $M(\widetilde{x_n}, \alpha_n, \mathcal{M}) = (1 - \tilde{\alpha}_n)\widetilde{x_n} + \tilde{\alpha}_n \mathcal{M}\widetilde{x_n}$.
The sequence $\{\widetilde{x_n}\}$ is called the soft Mann iteration.

Theorem 3.7
Let $\tilde{\tau}$ be a soft Banach space and $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ is a soft continuous operator on $\tilde{\tau}$. If the soft Mann iteration $\{\widetilde{x_n}\}$ defined in previous definition converges strongly to a soft element $\tilde{p} \tilde{\in} \tilde{\tau}$, then $\tilde{p}$ is a soft fixed element of $\mathcal{M}$.

Proof: since $\{\widetilde{x_n}\}$ converges to $\tilde{p}$, so $\lim_{n\to\infty} \widetilde{x_n} = \tilde{p}$. We want to prove that $\mathcal{M}\tilde{p} = \tilde{p}$.
Suppose not, that is $\mathcal{M}\tilde{p} \neq \tilde{p}$, $i.e.$, $\|\mathcal{M}\tilde{p} - \tilde{p}\| \tilde{>} \bar{0}$.
We set $\tilde{\epsilon}_n = \widetilde{x_n} - \mathcal{M}\widetilde{x_n} - (\tilde{p} - \mathcal{M}\tilde{p})$.
Because $\lim_{n\to\infty} \widetilde{x_n} = \tilde{p}$ and $\mathcal{M}$ is soft continuous, we obtain that:
$\lim_{n\to\infty} \tilde{\epsilon}_n = lim_{n\to\infty} (\widetilde{x_n} - \mathcal{M}\widetilde{x_n} - (\tilde{p} - \mathcal{M}\tilde{p})) = \bar{0}$. So, $\|\tilde{\epsilon}_n\| \to \bar{0}$.
Now, since $\|\mathcal{M}\tilde{p} - \tilde{p}\| \tilde{>} \bar{0}$, there exists $k \in \mathbb{N}$ $such\ that$ $\|\tilde{\epsilon}_n\| \tilde{<} \|\mathcal{M}\tilde{p} - \tilde{p}\| / \bar{3}$.
For every Cauchy sequence in $\tilde{\tau}$, $\|\widetilde{x_n} - \widetilde{x_m}\| \tilde{<} \|\mathcal{M}\tilde{p} - \tilde{p}\|/\bar{3}$ $for\ all\ n, m \geq k$.
Let H be any positive integer such that $\sum_{i=k}^{k+H} \alpha_i \geq 1$.
We have: $\tilde{x}_{i+1} = (1 - \tilde{\alpha}_n)\tilde{x}_i + \tilde{\alpha}_i \mathcal{M}\tilde{x}_i$

$\tilde{x}_{i+1} - \tilde{x}_i = \tilde{\alpha}_i(\mathcal{M}\tilde{x}_i - \tilde{x}_i)$

Therefore,
$$\|\tilde{x}_k + \tilde{x}_{k+H+1}\| = \left\|\sum_{i=k}^{k+H} (\tilde{x}_i - \tilde{x}_{i+1}\right\|$$
$$= \left\|\sum_{i=k}^{k+H} \tilde{\alpha}_i(\tilde{p} - \mathcal{M}\tilde{p} + \tilde{\epsilon}_i)\right\|$$
$$\tilde{\geq} \left\|\sum_{i=k}^{k+H} \tilde{\alpha}_i(\tilde{p} - \mathcal{M}\tilde{p})\right\| - \left\|\sum_{i=k}^{k+H} \alpha_i\, \tilde{\epsilon}_n\right\|$$
$$\tilde{\geq} \sum_{i=k}^{k+H} \tilde{\alpha}_i[\|\mathcal{M}\tilde{p} - \tilde{p}\| - \|\mathcal{M}\tilde{p} - \tilde{p}\|/\bar{3}]$$
$$\tilde{\geq} \frac{\bar{2}\|\mathcal{M}\tilde{p} - \tilde{p}\|}{\bar{3}}$$
But $\|\tilde{x}_k + \tilde{x}_{k+H+1}\| \tilde{<} \|\mathcal{M}\tilde{p} - \tilde{p}\| / \bar{3}$, which is contradiction.
So, $\mathcal{M}\tilde{p} = \tilde{p}$. That is $\tilde{p}$ is a soft fixed element.

Example 3.8
Let $\tilde{\tau}$ be an absolute soft vector space where $\tau = \mathcal{R}^3$ and W = {1, 2, 3}, and let $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ be a soft operator on $\tilde{\tau}$ such that $\mathcal{M}(\tilde{x}) = \overline{1 - \tilde{x}}$
It is clear that $\mathcal{M}$ is continuous. We choose $\tilde{\alpha}_n = \overline{(\frac{1}{n})}$, $n \in \mathbb{N}$ and $\bar{0} \leq \tilde{\alpha}_n < \bar{1}$
Let $\widetilde{x_1} = \{(1, (1,1,1)), (2, (2,2,2)), (3, (3,3,3))\}$.
We have
$$\tilde{x}_{n+1} = (\bar{1} - \tilde{\alpha}_n)\widetilde{x_n} + \tilde{\alpha}_n \mathcal{M}\widetilde{x_n} \quad for\ all\ n \in \mathbb{N}$$
$$= \left(\bar{1} - \overline{(\frac{1}{n})}\right)\widetilde{x_n} + \overline{(\frac{1}{n})}(\bar{1} - \tilde{x})$$
$$= \left(\bar{1} - \overline{(\frac{2}{n})}\right)\widetilde{x_n} + \overline{(\frac{1}{n})}$$

n =				
1	1	2	3	$\widetilde{x_1}$={(1,(1,1,1)),(2,(2,2,2)),(3,(3,3,3))}
2	0	-1	-2	$\widetilde{x_2}$={(1,(0,0,0)),(2,(-1,-1,-1)),(3,(-2,-2,-2))}
3	0.5	0.5	0.5	$\widetilde{x_3}$={(1,(0.5,0.5,0.5)),(2,(0.5,0.5,0.5)),(3,(0.5,0.5,0.5))}
4	0.5	0.5	0.5	$\widetilde{x_4}$={(1,(0.5,0.5,0.5)),(2,(0.5,0.5,0.5)),(3,(0.5,0.5,0.5))}
5	0.5	0.5	0.5	$\widetilde{x_5}$={(1,(0.5,0.5,0.5)),(2,(0.5,0.5,0.5)),(3,(0.5,0.5,0.5))}

It is clear that $\widetilde{x_n} \to \tilde{x}$ $where$ $\tilde{x} = \{(1, (0.5,0.5,0.5)), (2, (0.5,0.5,0.5)), (3, (0.5,0.5,0.5))\}$

So by (theorem.3.7), $\tilde{x}$ is a soft fixed element of $\mathcal{M}$.
Let $\widetilde{x_1} = \{(1,(-1,0,1)),(2,(1,2,0)),(3,(3-2,31,-1))\}$.

n=										
1	-1	0	1	1	2	0	-2	1	-1	$\widetilde{x_1}$={(1,(-1,0,1)),(2,(1,2,0)),(3,(-2,1,-1))}
2	2	1	0	0	-1	1	3	0	2	$\widetilde{x_2}$={(1,(2,1,0)),(2,(0,-1,1)),(3,(3,0,2))}
3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	$\widetilde{x_3}$={(1,(0.5,0.5,0.5)),(2,(0.5,0.5,0.5)),(3,(0.5, 0.5,0.5))}
4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	$\widetilde{x_4}$={(1,(0.5,0.5,0.5)),(2,(0.5,0.5,0.5)),(3,(0.5, 0.5,0.5))}
5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	$\widetilde{x_5}$={(1,(0.5,0.5,0.5)),(2,(0.5,0.5,0.5)),(3,(0.5, 0.5,0.5))}

It is clear that $\widetilde{x_n} \to \tilde{x} \quad where\ \tilde{x} = \{(1,(0.5,0.5,0.5)),(2,(0.5,0.5,0.5)),(3,(0.5,0.5,0.5))\}$
So by (theorem 3.7), $\tilde{x}$ is a soft fixed element of $\mathcal{M}$.

Example 3.9

Let $\tilde{\tau}$ be an absolute soft vector space where τ = $\mathcal{R}^3$ and W = {1, 2, 3}, and let $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ be a soft operator on $\tilde{\tau}$ such that $\mathcal{M}(\tilde{x}) = \bar{2}\tilde{x}$

It is clear that $\mathcal{M}$ is continuous. We choose $\tilde{\alpha}_n = \overline{(\frac{1}{n})}\ , n \in \mathbb{N}\ and\ \bar{0} \leq \tilde{\alpha}_n < \bar{1}$

Let $\widetilde{x_1} = \{(1,(1,1,1)),(2,(2,2,2)),(3,(3,3,3))\}$.

We have $\tilde{x}_{n+1} = (\bar{1} - \tilde{\alpha}_n)\widetilde{x_n} + \tilde{\alpha}_n \mathcal{M}\widetilde{x_n} \quad for\ all\ n \in \mathbb{N}$

$$= \left(\bar{1} - \overline{\frac{1}{n}}\right)\widetilde{x_n} + \overline{\frac{1}{n}}(\bar{2}\tilde{x})$$

$$= \overline{\left(\frac{n+1}{n}\right)}\widetilde{x_n}$$

n=					
1	1	2	3		$\widetilde{x_1}$={(1,(1,1,1)),(2,(2,2,2)),(3,(3,3,3))}
2	2	4	6		$\widetilde{x_2}$={(1,(2,2,2)),(2,(4,4,4)),(3,(6,6,6))}
3	3	6	9		$\widetilde{x_3}$={(1,(3,3,3)),(2,(6,6,6)),(3,(9,9,9))}
4	4	8	12		$\widetilde{x_4}$={(1,(4,4,4)),(2,(8,8,8)),(3,(12,12,12))}
5	5	10	15		$\widetilde{x_5}$={(1,(5,5,5)),(2,(10,10,10)),(3,(15,15,15))}
6	6	12	18		$\widetilde{x_6}$={(1,(6,6,6)),(2,(12,12,12)),(3,(18,18,18))}
7	7	14	21		$\widetilde{x_7}$={(1,(7,7,7)),(2,(14,14,14)),(3,(21,21,21))}
8	8	16	24		$\widetilde{x_8}$={(1,(8,8,8)),(2,(16,16,16)),(3,(24,24,24))}
9	9	18	27		$\widetilde{x_9}$={(1,(9,9,9)),(2,(18,18,18)),(3,(27,27,27))}
10	10	20	30		$\tilde{x}_{10}$={(1,(10,10,10)),(2,(20,20,20)),(3,(30,30,30))}

Although that $\mathcal{M}$ is soft continuous operator, the soft Mann iteration not converges to a soft element in $\tilde{\tau}$.

Proposition 3.10

Let $\tilde{\tau}$ be a soft normed space and $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ is a soft operator on $\tilde{\tau}$. $\tilde{p}$ is a fixed element of $\mathcal{M}$ such that $\|\mathcal{M}\tilde{x} - \tilde{p}\| \,\tilde{\leqq}\, \|\tilde{x} - \tilde{p}\|\ for\ all\ \tilde{x} \,\tilde{\in}\, \tilde{\tau}$, then for the soft Mann iteration $\tilde{x}_{n+1} = (\bar{1} - \tilde{\alpha}_n)\widetilde{x_n} + \tilde{\alpha}_n \mathcal{M}\widetilde{x_n}$ the $\lim_{n\to\infty}\|\widetilde{x_n} - \tilde{p}\|\ exists\ for\ all\ n \in \mathbb{N}$.

Proof: Because $\|\tilde{x}_{n+1} - \tilde{p}\| = \|(\bar{1} - \tilde{\alpha}_n)\widetilde{x_n} + \tilde{\alpha}_n \mathcal{M}\widetilde{x_n} \quad - \tilde{p}\|$

$$= \|(\bar{1} - \tilde{\alpha}_n)\widetilde{x_n} + \tilde{\alpha}_n \mathcal{M}\widetilde{x_n} \quad - (1 - \alpha_n)\tilde{p} - \alpha_n \tilde{p}\|$$

$$\tilde{\leqq} \|(\bar{1} - \tilde{\alpha}_n)[\widetilde{x_n} - \tilde{p}]\| + \|\tilde{\alpha}_n(\mathcal{M}\widetilde{x_n} - \tilde{p})\|$$

$$= (\bar{1} - \tilde{\alpha}_n)\|\widetilde{x_n} = \tilde{p}\| + \tilde{\alpha}_n\|\mathcal{M}\widetilde{x_n} - \tilde{p}\|$$

$$\tilde{\leq} (\bar{1} - \tilde{\alpha}_n)\|\widetilde{x_n} = \tilde{p}\| + \tilde{\alpha}_n\|\widetilde{x_n} - \tilde{p}\|$$
$$= \|\widetilde{x_n} = \tilde{p}\| \quad for\ all\ n \in \mathbb{N}$$

It follows that $\lim_{n\to\infty}\|\widetilde{x_n} - \tilde{p}\|$ $exists.$

4. Stability of soft iteration processes

Definition 4.1

Let $\mathcal{M}: SE(\tilde{\tau}) \to SE(\tilde{\tau})$ be a soft operator on $\tilde{\tau}$, where $\tilde{\tau}$ is a soft normed space. Suppose that $FIX(\mathcal{M}) = \{\tilde{p} \tilde{\in} \tilde{X}, \mathcal{M}\tilde{p} = \tilde{p}\}$ is a set of soft fixed element of $\mathcal{M}$ with $\tilde{x}_0 \tilde{\in} \tilde{\tau}$ and $\{\tilde{x}_n\}$ be a soft sequence such that:

$\tilde{x}_n = f(\mathcal{M}, \tilde{x}_n)$ $, n = 0,1,2, \ldots$ ……. (1)

Where $\tilde{x}_0 \tilde{\in} \tilde{\tau}$, is the initial soft element and f is function that connects between $\mathcal{M}$ and $\tilde{x}_n.$ Assume that $\{\tilde{x}_n\}$ converges to a soft fixed element $\tilde{p}$. If, we take $\{\tilde{y}_n\}$ another soft sequence in $\tilde{\tau}$ and put $\tilde{\epsilon} = \|\tilde{y}_{n+1} - f(\mathcal{M}, \tilde{y}_n)\|$ $, n = 0,1,2,3, \ldots$, then the soft iteration procedure (1) is called soft $\mathcal{M}$-stable or soft stable with respect to $\mathcal{M}$ if and only if $\lim_{n\to\infty} \tilde{\epsilon} = \bar{0}$ implies $\lim_{n\to\infty} \tilde{y}_n = \tilde{p}$.

Lemma4.2

if $\tilde{\delta}$ is a soft real number such that $\bar{0} \tilde{\leq} \tilde{\delta} \tilde{<} \bar{1}$, and $\{\tilde{\epsilon}_n\}_{n=0}^{\infty}$ is a soft sequence of positive soft real number with $\lim_{n\to\infty} \tilde{\epsilon} = \bar{0}$, then for all $\{\tilde{u}_n\}_{n=0}^{\infty} \subset R(W)^*$satisfies:

$\tilde{u}_{n+1} \tilde{\leq} \tilde{\delta}\tilde{u}_n + \tilde{\epsilon}$ $, n = 0,1,2,3, \ldots\ldots$, we have $\lim_{n\to\infty} \tilde{u}_n = \bar{0}$.

Proof: if $\tilde{\delta} = \bar{0}$, the statement is true. Assume $\bar{0} \tilde{<} \tilde{\delta} \tilde{<} \bar{1}$, we can multiply both side of Inequality by $\frac{\bar{1}}{\tilde{\delta}^{k+1}} = \tilde{\delta}^{-k-1}$, we obtain that:

$$\tilde{u}_{k+1}\tilde{\delta}^{-k-1} \tilde{\leq} \tilde{\delta}^{-k}\tilde{u}_k + \tilde{\delta}^{-k-1}\tilde{\epsilon}_k \qquad for \quad k = 0,1,2, \ldots\ldots$$

By sum all inequalities for $k = 0,1,2,3, \ldots\ldots n$ and after simplify we obtained that:

$$\bar{0} \tilde{\leq} \tilde{u}_{n+1} \tilde{\leq} \tilde{\delta}^{n+1}\tilde{u}_0 + \textstyle\sum_{k=0}^{n} \tilde{\delta}^{n-k}\tilde{\epsilon}_k$$

Now, using lemma (1) in [11] we get;

$\lim_{n\to\infty}[\sum_{k=0}^{n} \tilde{\delta}^{n-k}\tilde{\epsilon}_k] = \bar{0}$. Therefore, $\lim_{n\to\infty} \tilde{u}_n = \bar{0}$.

Stability of soft iteration processes (with contraction operator)

Theorem 4.3 (stability of Picard iteration procedure)

Let $\tilde{\tau}$ $\mathcal{M}: SE(\tilde{X}) \to SE(\tilde{X})$ is a soft operator on τ, $where$ $\tilde{\tau}$ is $\widetilde{\text{a soft}}$ banach space that satisfies the condition:

$\|\mathcal{M}\tilde{x} - \mathcal{M}\tilde{y}\| \tilde{\leq} \tilde{k}\|\tilde{x} - \tilde{y}\|$ Where, $\bar{0} \tilde{\leq} \tilde{k} \tilde{<} \bar{1}$.

then the soft Picard iteration process where $\tilde{x}_0 \tilde{\in} \tilde{\tau}$ and $\tilde{x}_{n+1} = \mathcal{M}\tilde{x}_n$ $, n \geq 0$, is soft $\mathcal{M}$ -stable.

Proof: by soft contraction theorem, $\mathcal{M}$ has unique soft fixed point $\tilde{p}$. Consider $\{\tilde{y}_n\}_{n=0}^{\infty}$ be a soft sequence in $\tilde{\tau}$ such that $\tilde{y}_{n+1} = \mathcal{M}\tilde{y}_n$ and let $\tilde{\epsilon}_n = \|\tilde{y}_{n+1} - \mathcal{M}\tilde{y}_n\|$.

Suppose that $\lim_{n\to\infty} \tilde{\epsilon}_n = \bar{0}$ to prove that $\lim_{n\to\infty} \tilde{y}_n = \tilde{p}$.

$$\|\tilde{y}_{n+1} - \tilde{p}\| \tilde{\leq} \|\tilde{y}_{n+1} - \mathcal{M}\tilde{y}_n\| + \|\mathcal{M}\tilde{y}_n - \tilde{p}\|$$
$$= \|\mathcal{M}\tilde{y}_n - \mathcal{M}\tilde{p}\| + \tilde{\epsilon}_n$$
$$\tilde{\leq} \tilde{k}\|\tilde{y}_n - \tilde{p}\| + \tilde{\epsilon}_n$$

Since $\bar{0} \tilde{\leq} \tilde{k} \tilde{<} \bar{1}$ and by (Lemma4.2), we obtain that $\lim_{n\to\infty}\|\tilde{y}_n - \tilde{p}\| = \bar{0}$ that is $\lim_{n\to\infty} \tilde{y}_n = \tilde{p}$.

On the other hand, let $\lim_{n\to\infty} \tilde{y}_n = \tilde{p}$.

$$\tilde{\epsilon}_n = \|\tilde{y}_{n+1} - \mathcal{M}\tilde{y}_n\| \tilde{\leq} \|\tilde{y}_{n+1} - \tilde{p}\| + \|\tilde{p} - \mathcal{M}\tilde{y}_n\|$$
$$\tilde{\leq} \|\tilde{y}_{n+1} - \tilde{p}\| + \tilde{k}\|\tilde{y}_n - \tilde{p}\|$$

When $n \to \infty$, we have $\lim_{n\to\infty} \tilde{\epsilon}_n = \bar{0}$.

Therefore, the Picard iteration procedure is $\mathcal{M}$-stable.

Theorem 4.4 : (stability of Mann iteration procedure)
Let $\mathcal{M}: SE(\tilde{X}) \rightarrow SE(\tilde{X})$ be a soft operator on $\tilde{\tau}$, where $\tilde{\tau}$ is a soft banach space that satisfies the condition: $\|\mathcal{M}\tilde{x} - \mathcal{M}\tilde{y}\| \tilde{\leq} \tilde{k}\|\tilde{x} - \tilde{y}\|$ where $\bar{0} \tilde{\leq} \tilde{k} \tilde{<} \bar{1}$.
then the soft Mann iteration process where $\tilde{x}_0 \tilde{\in} \tilde{\tau}$ and
$\tilde{x}_{n+1} = (\bar{1} - \tilde{\alpha}_n)\tilde{x}_n + \tilde{\alpha}_n\mathcal{M}\tilde{x}_n$, $\tilde{\alpha}_0 = 1$, $\bar{0} \tilde{\leq} \tilde{\alpha} \tilde{<} \bar{1}$ $for\ all\ n \geq 1$, is soft $\mathcal{M}$ -stable.
Proof: by soft contraction theorem, $\mathcal{M}$ has unique soft fixed point $\tilde{p}$. Consider $\{\tilde{y}_n\}_{n=0}^{\infty}$ be a soft sequence in $\tilde{\tau}$ such that $\tilde{y}_{n+1} = (\bar{1} - \tilde{\alpha}_n)\tilde{y}_n + \tilde{\alpha}_n\mathcal{M}\tilde{y}_n$ and let
$\tilde{\epsilon}_n = \|\tilde{y}_{n+1} - (\bar{1} - \tilde{\alpha}_n)\tilde{y}_n + \tilde{\alpha}_n\mathcal{M}\tilde{y}_n\|$.
Suppose that $\lim_{n\to\infty} \tilde{\epsilon}_n = \bar{0}$ to prove that $\lim_{n\to\infty} \tilde{y}_n = \tilde{p}$.

$$\begin{aligned}
\|\tilde{y}_{n+1} - \tilde{p}\| &\tilde{\leq} \|\tilde{y}_{n+1} - (\bar{1} - \tilde{\alpha}_n)\tilde{y}_n - \tilde{\alpha}_n\mathcal{M}\tilde{y}_n\| + \|(\bar{1} - \tilde{\alpha}_n)\tilde{y}_n + \tilde{\alpha}_n\mathcal{M}\tilde{y}_n - \tilde{p}\| \\
&= \|(\bar{1} - \tilde{\alpha}_n)\tilde{y}_n + \tilde{\alpha}_n\mathcal{M}\tilde{y}_n - ((\bar{1} - \tilde{\alpha}_n) + \tilde{\alpha}_n)\tilde{p}\| + \tilde{\epsilon}_n \\
&= \|(\bar{1} - \tilde{\alpha}_n)(\tilde{y}_n - \tilde{p}) + \tilde{\alpha}_n(\mathcal{M}\tilde{y}_n - \tilde{p})\| + \tilde{\epsilon}_n \\
&\tilde{\leq} (\bar{1} - \tilde{\alpha}_n)\|\tilde{y}_n - \tilde{p}\| + \tilde{\alpha}_n\|\mathcal{M}\tilde{y}_n - \mathcal{M}\tilde{p}\| + \tilde{\epsilon}_n \\
&\tilde{\leq} (\bar{1} - \tilde{\alpha}_n)\|\tilde{y}_n - \tilde{p}\| + \tilde{\alpha}_n\tilde{k}\|\tilde{y}_n - \tilde{p}\| + \tilde{\epsilon}_n \\
&= \left(\bar{1} - \tilde{\alpha}_n + \tilde{\alpha}_n\tilde{k}\right)\|\tilde{y}_n - \tilde{p}\| + \tilde{\epsilon}_n
\end{aligned}$$

Since $\left(\bar{1} - \tilde{\alpha}_n + \tilde{\alpha}_n\tilde{k}\right) \tilde{<} \bar{1}$ and by (lemma 4.2), we obtained that $\lim_{n\to\infty}\|\tilde{y}_n - \tilde{p}\| = \bar{0}$ that is $\lim_{n\to\infty} \tilde{y}_n = \tilde{p}$.
On the other hand, let $\lim_{n\to\infty} \tilde{y}_n = \tilde{p}$.

$$\begin{aligned}
\tilde{\epsilon}_n &= \|\tilde{y}_{n+1} - (\bar{1} - \tilde{\alpha}_n)\tilde{y}_n - \tilde{\alpha}_n\mathcal{M}\tilde{y}_n\| \\
&\tilde{\leq} \|\tilde{y}_{n+1} - \tilde{p}\| + \|\tilde{p} - (\bar{1} - \tilde{\alpha}_n)\tilde{y}_n - \tilde{\alpha}_n\mathcal{M}\tilde{y}_n\| \\
&\tilde{\leq} \|\tilde{y}_{n+1} - \tilde{p}\| + \|((\bar{1} - \tilde{\alpha}_n) + \tilde{\alpha}_n)\tilde{p} - (\bar{1} - \tilde{\alpha}_n)\tilde{y}_n - \tilde{\alpha}_n\mathcal{M}\tilde{y}_n\| \\
&\tilde{\leq} \|\tilde{y}_{n+1} - \tilde{p}\| + (\bar{1} - \tilde{\alpha}_n)\|\tilde{y}_n - \tilde{p}\| + \tilde{\alpha}_n\|\mathcal{M}\tilde{y}_n - \tilde{p}\| \\
&\tilde{\leq} \|\tilde{y}_{n+1} - \tilde{p}\| + (\bar{1} - \tilde{\alpha}_n)\|\tilde{y}_n - \tilde{p}\| + \tilde{\alpha}_n\tilde{k}\|\tilde{y}_n - \tilde{p}\|
\end{aligned}$$

When $n \to \infty$, the $\lim_{n\to\infty} \tilde{\epsilon}_n = \bar{0}$.
Therefore, the Mann iteration procedure is $\mathcal{M}$-stable.

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