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Zariski Topology of Intuitionistic Fuzzy d-filter

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Abstract:
In this paper we discuss the Zariski topology of intuitionistic fuzzy d-filter in d-algebra, with some topological properties on the spectrum of intuitionistic fuzzy d-filter in d-algebra X which have algebraic features such as connectedness. We find that this topology is a strongly connected, and T_0 space. We also define the invariant map on intuitionistic fuzzy prime d-filter with a homomorphism map.

Keywords: d-algebra, intuitionistic fuzzy set, fuzzy set, general topology, Zariski topology.

الحدسي الضبابي*d*تبولوجي زارسكي على فلتر

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مديرية تربية كربلاء، وزارة التربية، العراق

الخلاصة:
في هذا البحث تم تقديم توبولوجيا زارسكي على الفلتر d الحدسي الضبابي في الجبر d، وكذلك درسنا العلاقة بين الخصائص الطبولوجية لطوبولوجيا زارسكي المتكون من هذا الطيف ذي السمات الجبرية للطيف كالترابط ان هذا التبولوجي يحقق الترابط القوي وبديهية T0. وكذلك عرفنا التطبيق اللامتغير على الفلتر d الحدسي الضبابي الأولي في الجبر d ، مع تطبيق متشاكل على هذا الفضاء .

1. Introduction

A class of abstract algebras (BCK-algebra) is introduced by Imai and Iseki in 1966,then after Neggers and Kim introduced a valuable generalisation as a d-algebra in 1999 [1]. Atanassov generalised the fuzzy set idea which is introduced by Zadeh in 1965 to the notion of intuitionistic fuzzy set in 1986 [2]. In [3], A. K. Hasan in 2014 considered the idea of a d-filter in d-algebra with other notations such as semi d-ideal, and fuzzy semi d-ideal of d-algebra. In [4], H. K. Abdullah and A. K. Hasan studied the spectrum of intuitionistic fuzzy semi d-ideal. The concept of intuitionistic fuzzy d-algebra is introduced by Jun et al. in 2006 [5].

In 2020 the notation of intuitionistic fuzzy d-filter of d-algebra and intuitionistic fuzzy prime d-filter are discussed by Hasan in [6] ,and he also gave as a generalization of his work in [3]. Algebraic geometry is a branch of mathematics that deals with solving many algebraic equations and structures using advanced techniques to attain applications based on algebraic geometry. In this paper we discuss the Zariski topology of intuitionistic fuzzy d-filter in d-algebra to study these theories, and we also develop a tool based on algebraic structures and

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topology. This paper also examines the topological properties of Zariski topology on the spectrum of d-algebra X, which have algebraic features such as connectedness as well as we find this topology is a strongly connected and satisfying the T_0 space. We also define the invariant map on intuitionistic fuzzy prime d-filter with a homomorphism map.

2. Background

This section contains the essential concepts of the d-filter in d-algebra and intuitionistic fuzzy notions in d-filter and prime d-filter.

Definition (2.1): [1] A d-algebra is every non-empty set $\mathcal{M}$ with a binary operation $*$ and a constant 0 which satisfies that:
$\hbar * \hbar = 0$,
$0 * \hbar = 0$, and
If $\hbar * g = g * \hbar = 0$, then $\hbar = g \ \forall\, \hbar, g \in \mathcal{M}$.
We will write $\hbar g$ instead of $\hbar * g$. It is said to be commutative if $\hbar(\hbar g) = g(g\hbar)$ for all $\hbar, g \in \mathcal{M}$, and $g(g\hbar)$ is denoted by $(\hbar \wedge g)$.
Henceforth $\mathcal{M}$ denotes a d-algebra.

Definition (2.2): [1] A bounded d-algebra $\mathcal{M}$ has an element $e \in \mathcal{M}$ such that $\hbar \le e$ for all $\hbar \in \mathcal{M}$, that means $\hbar e = 0, \forall\, \hbar \in \mathcal{M}$. In bounded d-algebra we denote $e\hbar$ by $\hbar^*$ for every $\hbar \in \mathcal{M}$.

Definition (2.3): [3] A d-algebra $\mathcal{M}$ is called $d^s -$algebra if it satisfies the following conditions:
(S_1) $\hbar 0 = \hbar$
(S_2) $(\hbar g)\lambda = (\hbar\lambda)g.\ \forall\, \hbar, g, \lambda \in \mathcal{M}$.

Definition (2.4): [3] A non-empty subset $\aleph$ of a bounded d-algebra $\mathcal{M}$ is called a d-filter of $\mathcal{M}$ if
(F1) $e \in \aleph$,
(F2) $(\hbar^* g^*)^* \in \aleph$, and $g \in \aleph$ imply $\hbar \in \aleph$, $\forall\, \hbar, g \in \mathcal{M}$.

Definition (2.5): [3] A proper d-filter $\aleph$ of d-algebra $\mathcal{M}$ is said to be prime if $\hbar \wedge g \in \aleph$, for any $\hbar, g \in \mathcal{M}$ implies that either $\hbar \in \aleph$ or $g \in \aleph$.

Definition (2.6): [2] An intuitionistic fuzzy set (IFS) $\aleph$ in a set $\mathcal{M}$ is an object having the form $\aleph = \{< d, \alpha_\aleph(d), \beta_\aleph(d) >: d \in \mathcal{M}\}$, such that $\alpha_\aleph: \mathcal{M} \to (0,1)$ denotes the membership degree and $\beta_\aleph: \mathcal{M} \to (0,1)$ denotes the non-membership degree for all $d \in \mathcal{M}$ to set $\aleph$, and $0 \le \alpha_\aleph(d) + \beta_\aleph(d) \le 1$, for all $d \in \mathcal{M}$.
In this paper, we will use the notation $\aleph = \{< \alpha_\aleph, \beta_\aleph >\}$.

Definition (2.7): [7] Let $\mathcal{M}, \mathcal{R}$ be d-algebra with a homomorphism mapping $f: \mathcal{M} \to \mathcal{R}$, and let $\mathcal{L}$ be IFS in $\mathcal{M}$. We define IFS $f(\mathcal{L})$ in $\mathcal{R}$, as follows
$f(\mathcal{L})_{(\mathcal{R})} = < \alpha_{f(\mathcal{L})}(g), \beta_{f(\mathcal{L})}(g) >$, where

$$\alpha_{f(\mathcal{L})}(g) = \begin{cases} \sup \alpha_{\mathcal{L}}(\hbar)\ , \hbar \in \mathcal{M}, f(\hbar) = g & \text{if } f^{-1}(g) \ne \emptyset \\ 0 & \text{Otherwise} \end{cases}, \text{ and}$$

$$\beta_{f(\mathcal{L})}(g) = \begin{cases} \inf \beta_{\mathcal{L}}(\hbar) \quad \hbar \in \mathcal{M}, f(\hbar) = g & \text{if } f^{-1}(g) \ne \emptyset \\ 0 & \text{Otherwise} \end{cases},$$

$\forall\, g \in \mathcal{R}$.

Definition (2.8): [6] An intuitionistic fuzzy d-filter of $\mathcal{M}$ ('$IFd - filter$') is an IFS $\aleph = < \alpha_\aleph, \beta_\aleph >$ in $\mathcal{M}$ that satisfies:
$(IFdF_1)$ $\alpha_\aleph(e) \ge \alpha_\aleph(\hbar)$, $\beta_\aleph(e) \le \beta_\aleph(\hbar)$, and
$(IFdF_2)$ $\alpha_\aleph(\hbar) \ge \min\{\alpha_\aleph((\hbar^* g^*)^*), \alpha_\aleph(g)\}$, $\beta_\aleph(\hbar) \le \max\{\beta_\aleph((\hbar^* g^*)^*), \beta_\aleph(g)\}$,
for all $\hbar, g \in \mathcal{M}$.

Definition (2.9): [6] An $IFd - filter\ is$ called prime ('$IFPd - filter$') in $\mathcal{M}$ it is an $IFd - filter$ $\aleph = < \alpha_\aleph, \beta_\aleph >$ of $\mathcal{M}$ that satisfies:
$(IFPdF_1)$ $\alpha_\aleph(\hbar \wedge g) \le \max\{\alpha_\aleph(\hbar), \alpha_\aleph(g)\}$,and

$(IFPdF_2)$ $\beta_{\aleph}(\hbar \wedge g) \geq \min\{\beta_{\aleph}(\hbar), \beta_{\aleph}(g)\}$ for all $\hbar, g \in \mathcal{M}$.
Theorem (2.10): [**6**] Let $f: \mathcal{M} \to \mathcal{F}$ be a d-homomorphism and $\aleph$ an $IFPd-filter$ of $\mathcal{F}$, then $f^{-1}(\aleph)$ is an $IFPd-filter$ of $\mathcal{M}$ (Hasan, 2020, p. 368).
Theorem (2.11): [**6**] Let f be an epimorphism of d-algebra from $\mathcal{M}$ to $\mathcal{F}$, and let $\aleph = <\alpha_{\aleph}, \beta_{\aleph}>$ be an IFS in d-algebra $\mathcal{F}$. If $f^{-1}(\aleph) = <\alpha_{f^{-1}(\aleph)}, \beta_{f^{-1}(\aleph)}>$ is an $IFPd-filter$ in $\mathcal{M}$, then $\aleph = <\alpha_{\aleph}, \beta_{\aleph}>$ is an $IFPd-filter$ of $\mathcal{F}$.
Definition (2.12): [**8**] Let $\omega, \vartheta \in (0,1)$ such that $\omega + \vartheta \leq 1$ is an intuitionistic fuzzy point and $\hbar_{(\omega,\vartheta)}$ is defined to be an IFS in $\mathcal{M}$, which is defined by $\hbar_{(\omega,\vartheta)}(g) = \begin{cases}(\omega,\vartheta) & \text{if } g = \hbar \\ (0,1) & \text{if } g \neq \hbar\end{cases}$ for all g in $\mathcal{M}$, and $\hbar_{(\omega,\vartheta)} \in \aleph$ if and only if $\alpha \leq \omega_{\aleph}(\hbar)$ and $\beta \geq \vartheta_{\aleph}(\hbar)$.
Definition (2.13): **[7]** IFS $\tilde{0}$ and $\tilde{1}$ in $\mathcal{M}$ are defined as $\tilde{0} = \{\langle \hbar, 0,1\rangle, \hbar \in \mathcal{M}\}$ and $\tilde{1} = \{\langle \hbar, 1,0\rangle, \hbar \in \mathcal{M}\}$, where 1 and 0 are maps constantly sending the elements of $\mathcal{M}$ to 1 and 0, respectively.
Definition (2.14): **[9]** If $\aleph\ and\ \mathcal{H}\ are\ IFS$, and they are defined as follows: $\aleph.\mathcal{H} = \{<\hbar, \alpha_{\aleph.\mathcal{H}}(\hbar), \beta_{\aleph.\mathcal{H}}(\hbar)>: \hbar \in \mathcal{M}\} = <\alpha_{\aleph}.\alpha_{\mathcal{H}}, \beta_{\aleph}.\beta_{\mathcal{H}}>$.
Definition (2.15): **[10]** Let $\aleph\ be\ IFS$ of $\mathcal{M}$, set $\wedge(\aleph) = \{(\omega_0, \vartheta_0), (\omega_1, \vartheta_1), \ldots, (\omega_n, \vartheta_n)\}$, where $(\omega_i, \vartheta_i) \in [0,1]$ such that $\omega_i + \vartheta_i \leq 1$, for all $i = 1,2, \ldots, n$
Notation (2.16): Let $\aleph\ be\ IFd-filter$ of $\mathcal{M}$. A level cut set is defined as $\aleph_* = \{\hbar \in \mathcal{M}, \alpha_{\aleph}(\hbar) = \alpha_{\aleph}(0), \beta_{\aleph}(\hbar) = \beta_{\aleph}(0)\}$
Definition (2.17): A non-constant $IFd-filter$ $\aleph$ of $\mathcal{M}$ is called intuitionistic fuzzy maximal d-filter $(IFMd-filter)$, if for any $IFd-filter\ \mathcal{H}$ of $\mathcal{M}$ if $\aleph \subseteq \mathcal{H}$, then either $\mathcal{H}_* = \aleph_*$ or $\mathcal{H}_* = \mathcal{M}$.
It is easy to proof the next theorem.
Theorem (2.18): Let $\aleph\ be\ IFPd-filter(IFMd-filter)$ of $\mathcal{M}$, then $\aleph_*$ is a prime (maximal) d-filter of $\mathcal{M}$

3. Zariski Topology on Intuitionistic Fuzzy d-filter

In this section we discuss Zariski topology from the intuitionistic d-filter spectrum and examine some of its properties.

Notation (3.1):
(i) $\chi = P\ \{P\ is\ an\ IFPd-filter\ of\ \mathcal{M}\}$.
(ii) $V(\aleph) = \{P \in \chi, \aleph \subseteq P, where\ \aleph\ is\ an\ IFd-filter\ of\ \mathcal{M}\}$.
(iii) $\chi(\aleph) = \chi \setminus V(\aleph)$, the complement of $V(\aleph)$ in $\mathcal{M}$, $\chi(\aleph) = \{P \in \chi, \aleph \not\subseteq P\}$.
Proposition (3.2): Let $\aleph$ and $\mathcal{H}$ be $IFd-filter$. If $\aleph \subseteq \mathcal{H}$, then $V(\mathcal{H}) \subseteq V(\aleph)$.
Proof: Let $P \in V(\mathcal{H})$ so we have $\mathcal{H} \subseteq P$. Then, $\aleph \subseteq \mathcal{H} \subseteq P$, and $P \in V(\aleph)$.
Proposition (3.3): If the smallest $IFPd-filter\ P$ contains $\aleph$, then $V(\aleph) = V(P)$.
Proof: We hold that $V(P) \subseteq V(\aleph)$ by proposition (3.2). Therefore, let $S \in V(\aleph)$ where $\aleph \subseteq S$. However, P is the smallest $IFPd-filter$ containing $\aleph$ $P \subseteq S$, so $S \in V(\aleph)$. Thus, $V(\aleph) = V(P)$.
Proposition (3.4): If $\aleph$ be an $IFd-filter$ then $V(\langle\aleph\rangle) = V(\aleph)$.
Proof: Let $S \in V(\aleph)$ so that $\aleph \subseteq S$, and this gives $\langle\aleph\rangle \subseteq S$ and $S \in V(\langle\aleph\rangle)$.
Conversely, let $S \in V(\langle\aleph\rangle)$ and $\langle\aleph\rangle \subseteq S$,since $\aleph \subseteq \langle\aleph\rangle \subseteq S$, we get $S \in V(\aleph)$. Therefore,$V(\langle\aleph\rangle) = V(\aleph)$.
Proposition (3.5): If $\aleph$ and $\mathcal{H}$ are two $IFd-filters$, then $V(\aleph \cup \mathcal{H}) \subseteq V(\aleph) \cup V(\mathcal{H})$.
Proof: Since $\aleph \subseteq \aleph \cup \mathcal{H}$ and $\mathcal{H} \subseteq \aleph \cup \mathcal{H}$, $V(\aleph \cup \mathcal{H}) \subseteq V(\aleph)$ and $V(\aleph \cup \mathcal{H}) \subseteq V(\mathcal{H})$. Thus, $V(\aleph \cup \mathcal{H}) \subseteq V(\aleph) \cup V(\mathcal{H})$.

It is clear that the converse of proposition (3.5) is not true, since if $P \in V(\aleph) \cup V(\mathcal{H})$, $P \in V(\aleph)$ or $P \in V(\mathcal{H})$ and $\aleph \subseteq P$ or $\mathcal{H} \subseteq P$, which does not include that $\aleph \cup \mathcal{H} \subseteq P$.

Definition (3.6): The prime radical $rad(\aleph)$ of $IFPd-filter$ $\aleph$ is the intersection of all $IFd-filters$ of $\mathcal{M}$ containing $\aleph$. In case there is no $IFPd-filter$, we say that $rad(\aleph)=\tilde{1}$.

The proof of the next two propositions are easy so that it is omitted

Proposition (3.7): If $\aleph$ is an $IFd-filter$, then

i) $\aleph \subseteq rad(\aleph)$

ii) $rad\big(rad(\aleph)\big)=rad(\aleph)$.

iii) If $\aleph$ $IFPd-filter,$ then $rad(\aleph)=\aleph$.

iv) If $\aleph \subseteq \mathcal{H},$ then $rad(\aleph)\subseteq rad(\mathcal{H})$.

Proposition (3.8): For any $IFd-filters$ $\aleph$ and $\mathcal{H}$, the following statement will hold:

i) $V(\aleph)=V(rad(\aleph))$ and

ii) $V(\aleph)=V(\mathcal{H})$ if and only if $rad(\aleph)=rad(\mathcal{H})$.

Note that the topological notations and concepts such as the connected space and T_0 in the following theorems are taken from Munkres' work [**11**].

Theorem (3.9): Let $\mathfrak{I}=\{\chi(\aleph), \aleph \text{ } is \text{ } an \text{ } IFd-filter \text{ } in \text{ } \mathcal{M}\}$. Then $\mathfrak{I}$ is a topological space on χ.

Proof: We know that $V\big(\tilde{0}\big)=\mathcal{M}$ and $V\big(\tilde{1}\big)=\emptyset$. Therefore, $\chi\big(\tilde{0}\big)=\emptyset$, $\chi\big(\tilde{1}\big)=\mathcal{M}$ and $\emptyset, \mathcal{M}\in\mathfrak{I}$.

Now let $\aleph_1$ and $\aleph_2$ be any two $IFd-filters$, and let $\mathcal{H}\in V(\aleph_1)\cup V(\aleph_2)$. This means that $\aleph_1\subseteq\mathcal{H}$ or $\aleph_2\subseteq\mathcal{H}$ (by notation [3.1, ii]) and $\aleph_1\cap\aleph_2\subseteq\mathcal{H}$. Therefore, $\mathcal{H}\in V(\aleph_1\cap\aleph_2)$, and $\aleph_1\cap\aleph_2\subseteq\mathcal{H}$, so $\aleph_1.\aleph_2\subseteq\mathcal{H}$ (by definition [2.15]). Then, $\aleph_1\subseteq\mathcal{H}$ or $\aleph_2\subseteq\mathcal{H}$, thus $\mathcal{H}\in V(\aleph_1)\cup V(\aleph_2)$. Hence, $V(\aleph_1)\cup V(\aleph_2)=V(\aleph_1\cap\aleph_2)$, and that means (by notation [3.1, iii]), that $\chi(\aleph_1)\cap\chi(\aleph_2)=\chi(\aleph_1\cap\aleph_2)$. $\mathfrak{I}$ is closed under the finite intersection.

Finally, if we have $\{\aleph_i, i\in\Lambda\}$ as a family of $IFd-filters$ of $\mathcal{M},$ we can validate that $\cup\{V(\aleph_i), i\in\Lambda\}=V(\langle\cup\{\aleph_i, i\in\Lambda\}\rangle)$, that means that $\bigcup_{i\in\Lambda}\chi(\aleph_i)=\chi(\langle\bigcup_{i\in\Lambda}\aleph_i\rangle)$. Hence $\mathfrak{I}$ is closed with arbitrary unions. Therefore $\mathfrak{I}$ is a topological space.

Remark (3.10): The topological space $(\mathcal{M},\mathfrak{I})$ that defined in theorem (3.9) is named the Zariski topology of the intuitionistic fuzzy prime d-filter of d-algebra and is signified by $IFPd-filter-Spec(\mathcal{M})$ or simply χ.

Theorem (3.11): The sub-family $\{\chi\big(\varkappa_{(\omega,\vartheta)}\big), \varkappa\in\mathcal{M}$ and $\omega,\vartheta\in(0,1]$, such that $\omega+\vartheta\leq 1\}$ of χ, is a base for $\mathfrak{I}$.

Proof: Let $\chi(\aleph)\in\mathfrak{I}$, and $\mathcal{H}\in\chi(\aleph)$. Then, $\alpha_{\mathcal{H}}(\varkappa)<\alpha_{\aleph}(\varkappa)$ and $\beta_{\mathcal{H}}(\varkappa)>\beta_{\aleph}(\varkappa)$ for some $\varkappa\in\mathcal{M}$. Let $\alpha_{\aleph}(\varkappa)=\omega$ and $\beta_{\aleph}(\varkappa)=\vartheta$, then $\varkappa_{(\omega,\vartheta)}\nsubseteq\aleph$ and $\aleph\in\chi\big(\varkappa_{(\omega,\vartheta)}\big)$.

Now $V(\aleph)\subseteq V\big(\varkappa_{(\omega,\vartheta)}\big)$, because if $P\in V(\aleph)$, then $\alpha_P(\varkappa)\geq\alpha_{\aleph}(\varkappa)=\omega=\alpha_{x_{(\omega,\vartheta)}}(x)$, and $\beta_P(x)\leq\beta_F(x)=v=\beta_{x_{(\omega,\vartheta)}}(x)$. Therefore, $x_{(\omega,\vartheta)}\subseteq P$ and $P\in V\big(\varkappa_{(\omega,\vartheta)}\big)$. Hence, $\chi\big(\varkappa_{(\omega,\vartheta)}\big)\subseteq\chi(\aleph)$ and $\mathcal{H}\in\chi\big(\varkappa_{(\omega,\vartheta)}\big)\subseteq\chi(\aleph)$. The proof is completed.

Theorem (3.12): The Zariski topology of $\mathcal{M}$ is disconnected if and only if there are two $-filters$ $\aleph,\mathcal{H}$ such that $\mathrm{rad}(\aleph\cup\mathcal{H})=\mathrm{rad}(1)$ and $\mathrm{rad}(\aleph\cap\mathcal{H})=\mathrm{rad}(0)$.

Proof: Assuming $\mathfrak{I}$ is a disconnected space, we have two $IFd-filters$ $\aleph,\mathcal{H}$ in $\mathcal{M}$ such that $\chi(\aleph)\neq\emptyset$, $\chi(\mathcal{H})\neq\emptyset$, $\chi(\aleph)\cap\chi(\mathcal{H})=\emptyset$ and $\chi(\aleph)\cup\chi(\mathcal{H})=\mathrm{Spec}(\mathcal{M})$. This means that $\chi(\aleph)\cap\chi(\mathcal{H})=\chi\big(\tilde{0}\big)$ and $\chi(\aleph)\cup\chi(\mathcal{H})=\chi\big(\tilde{1}\big)$. Consequently, we get $\chi(\aleph\cap\mathcal{H})=\chi\big(\tilde{0}\big)$ and $\chi(\aleph\cup\mathcal{H})=\chi\big(\tilde{1}\big)$. Accordingly, and by proposition (3.10, ii) $\mathrm{rad}(\aleph\cap\mathcal{H})=\mathrm{rad}\big(\tilde{0}\big)$ and $\mathrm{rad}(\aleph\cup\mathcal{H})=\mathrm{rad}\big(\tilde{1}\big)$. The proof is completed the converse way.

A strongly connected subset F of a topological space on $\mathcal{M}$ can be defined as follow: For any open subset U and V of $\mathcal{M}$; if $\mathrm{F}\subseteq U\bigcup V$, then $\mathrm{F}\subseteq U$ or $\mathrm{F}\subseteq V$ [**12**]. It is clear that any strongly connected is connected.

Theorem (3.13): Any open set of Zariski topology $\mathfrak{I}$ is strongly connected.

Proof: Let $\mathfrak{H}$ be a collection of an $IFPd-filter$ of $\mathfrak{J}$ and let $\aleph, \mathcal{H}$ be an $IFd-filter$ in $\mathcal{M}$. Since $\mathfrak{H} \subseteq \chi(\aleph) \cup \chi(\mathcal{H}) \subseteq \chi(\aleph \cup \mathcal{H})$, by proposition (3.5) it follows that $\mathfrak{H} \subseteq \chi(\aleph)$ or $\mathfrak{H} \subseteq \chi(\mathcal{H})$, and this completes the proof.

The converse of previous theorem need not to be true in general to see that if one takes any closed strongly connected set in this variety.

Theorem (3.14): The Zariski topology $\mathfrak{J}$ on $\mathcal{M}$ is a T_0.

Proof: For $\aleph, \mathcal{H} \in \chi$ and $\aleph \neq \mathcal{H}$, we have either $\aleph \nsubseteq \mathcal{H}$ or $\mathcal{H} \nsubseteq \aleph$. Let $\aleph \nsubseteq \mathcal{H}$ and $\mathcal{H} \notin \mathrm{V}(\aleph)$ but $\aleph \in \mathrm{V}(\aleph)$; then $\mathcal{H} \in \mathrm{X}(\aleph)$ and $\aleph \notin \mathrm{X}(\aleph)$. If $\mathcal{H} \nsubseteq \aleph$, we notice that $\aleph \in \mathrm{X}(\mathcal{H})$, but $\mathcal{H} \notin \mathrm{X}(\mathcal{H})$. $\mathrm{Spec}(\mathcal{M})$ is a T_0.

Theorem (3.15): In $\mathrm{Spec}(\mathcal{M})$, $\mathrm{V}(\aleph) = \{\overline{\aleph}\}$ for all $IFd-filters$ in $\mathcal{M}$.

Proof: We know that $V(\aleph)$ is a closed set containing $\aleph$, so $\{\overline{\aleph}\} \subseteq V(\aleph)$. Now let $\mathcal{H} \notin \{\overline{\aleph}\}$; now there is an open set X \ V(F) embracing $\mathcal{H}$ but not $\aleph$. Therefore, $F \nsubseteq \mathcal{H}$, but $F \subseteq \aleph$ and $\mathcal{H} \notin \mathrm{V}(\aleph)$. Thus, $V(\aleph) \subseteq \{\overline{\aleph}\}$.

Corollary (3.16): For any $IFd-filter\ \mathcal{H}$, $\mathcal{H} \in \{\overline{\aleph}\}$ if and only if $\aleph \subseteq \mathcal{H}$.

The proof comes directly from theorem (3.15).

Theorem (3.17): Let $\omega, \vartheta \in [0,1]$ such that $\omega + \vartheta \leq 1$ and let $Y = \{P \in \chi : \wedge(P) = \{(0,1), (\omega, \vartheta)\}\}$, then the subspace Y is T_1 if and only if every singleton element of Y is $IFMd-filter$ of $\mathcal{M}$.

Proof: Let $\{\aleph\}$ be a closed set, then $V(\aleph) \cap Y = \{\aleph\}$, by theorem (3.15). In order to show that $\aleph$ is $IFMd-filter$ of $\mathcal{M}$, we have to show that the d-filter $\aleph_* = \{\hbar \in \mathcal{M}, \alpha_\aleph(\hbar) = \alpha_\aleph(0), \beta_\aleph(\hbar) = \beta_\aleph(0)\}$ is a maximal, so we need to show that there is no prime d-filter of $\mathcal{M}$ probably containing $\aleph_*$.

Let J be some prime d-filter of $\mathcal{M}$ probably containing $\aleph_*$. Consider an $IFd-filter\ \mathcal{H}$ of $\mathcal{M}$ which is defined by $\alpha_\mathcal{H}(\hbar) = \begin{cases} 1 & if\ \hbar \in J \\ \omega & otherwisw \end{cases}$, $\beta_\mathcal{H}(\hbar) = \begin{cases} 0 & if\ \hbar \in J \\ \omega & otherwisw \end{cases}$.

Then $\mathcal{H} \in Y$, and $\aleph$ probably contained in $\mathcal{H}$. This contradicts the fact that $V(\aleph) \cap Y = \{\aleph\}$.

Conversely, let $\aleph$ be $IFMd-filter$ of $\mathcal{M}$, then $\aleph_* = \{\hbar \in \mathcal{M}, \alpha_\aleph(\hbar) = \alpha_\aleph(0), \beta_\aleph(\hbar) = \beta_\aleph(0)\}$ is a maximal. We claim that $V(\aleph) \cap Y = \{\aleph\}$. Clearly $\{\aleph\} \subseteq V(\aleph) \cap Y$. Next let $\mathcal{H} \in V(\aleph) \cap Y$, then $\aleph \subseteq \mathcal{H}$ and $\aleph_* \subseteq \mathcal{H}_*$. That means $\aleph_* = \mathcal{H}_*$, since $\aleph_*$ is a maximal d-filter. Hence $\aleph = \mathcal{H}$, since $\wedge(\aleph) = \wedge(\mathcal{H}) = \{(0,1), (\omega, \vartheta)\}$. Therefore, $(\aleph) \cap Y = \{\aleph\}$. Consequently, $\{\aleph\}$ is a closed subset of Y

Theorem (3.18): The Zariski topology $\mathfrak{J}$ on $\mathcal{M}$ is not T_2.

Proof: Let J be a prime d-filter of $\mathcal{M}$. Consider $that$ $\aleph$, and $\mathcal{H}$ are $IFPd-filters$ of $\mathcal{M}$, defined by

$\alpha_\aleph(\hbar) = \begin{cases} 1 & if\ \hbar \in J \\ 0.1 & otherwisw \end{cases}$, $\beta_\aleph(\hbar) = \begin{cases} 0 & if\ \hbar \in J \\ 0.2 & otherwisw \end{cases}$, and

$\alpha_\mathcal{H}(\hbar) = \begin{cases} 1 & if\ \hbar \in J \\ 0.3 & otherwisw \end{cases}$, $\beta_\mathcal{H}(\hbar) = \begin{cases} 1 & if\ \hbar \in J \\ 0.4 & otherwisw \end{cases}$.

Let $\chi(\varkappa_{(\omega,\vartheta)})$ and $\chi(y_{(\omega,\vartheta)})$ be any two basic open set in χ containing $\aleph$, and $\mathcal{H}$ respectively, where $\varkappa, y \in \mathcal{M}$ and $\omega, \vartheta \in (0,1]$ such that $\omega + \vartheta \leq 1$. Then $\varkappa_{(\omega,\vartheta)} \nsubseteq \aleph$ and $y_{(\omega,\vartheta)} \nsubseteq \mathcal{H}$, and so $\varkappa \notin \aleph_* = J$ and $y \notin \mathcal{H}_* = J$, since J is prime d-filter so $\varkappa y \notin J$. Then by theorem (3.15) and corollary (3.16) we have $(\varkappa_{(\omega,\vartheta)}) \cap \chi(y_{(\omega,\vartheta)}) = \chi((\varkappa y)_{(\omega,\vartheta)}) \neq \emptyset$. Hence χ is not Hausdorff.

Definition (3.19): Let S be any subset of d-algebra $\mathcal{M}$ and let $f: \mathcal{M} \to S$. An IFS $\aleph$ of $\mathcal{M}$ is called $f-invariant$ if $f(\hbar) = f(g)$, which implies $\alpha_\aleph(\hbar) = \alpha_\aleph(g)$ and $\beta_\aleph(\hbar) = \beta_\aleph(g)$, where $\hbar, g \in \mathcal{M}$.

If $\aleph$ is an $f-invariant\ IFS$ of $\mathcal{M}$ then $f^{-1}(f(\aleph)) = \aleph$.

Theorem (3.20): Let $f: \mathcal{M} \to \acute{\mathcal{M}}$ be homomorphism and let $\aleph$ be any $f-invariant$ $IFPd-filter$ of $\mathcal{M}$. Also let $\acute{\aleph}$ be any $f-invariant$ $IFPd-filter$ of $\acute{\mathcal{M}}$. Then $f(\aleph)$, and $f^{-1}(\acute{\aleph})$ are $IFPd-filter$ of $\acute{\mathcal{M}}$ and $\mathcal{M}$, respectively.
Proof: It comes directly from definition (3.19) and theorems (2.10) and (2.11).
Theorem (3.21): Let $f: \aleph \to \acute{\aleph}$ be a homomorphism, and let $\acute{\chi} = IFPd - filter - Spec(\acute{\aleph})$, such that $\chi^* = \{P \in \mathcal{M}, P\ is\ f-invariant\}$ $\acute{\chi}(\mathcal{H}) = \acute{\chi} \backslash V(\mathcal{H})$, where $\mathcal{H}$ is any $IFd-filter$ in $\acute{\mathcal{M}}$. Let g be a map from $\acute{\mathcal{M}}$ to $\mathcal{M}$ defined by $g(\acute{\aleph}) = f^{-1}(\acute{\aleph})$, such that $\acute{\aleph} \in \acute{\mathcal{M}}$. Then the following conditions hold:
i) g is continuous.
ii) g is open
iii) g is a homomorphism of $\acute{\chi}$ into χ^*.
Proof:
i) If $\acute{\aleph} \in \acute{\mathcal{M}}$, it follows that $f^{-1}(\acute{\aleph}) \in \mathcal{M}$. Also, $f^{-1}(\acute{F})$ is $f-invariant$, since for all $c, d \in \mathcal{M}$, if $f(c) = f(d)$, then $\alpha_{\acute{\aleph}}(f(c)) = \alpha_{\acute{\aleph}}(f(d))$ and $\beta_{\acute{\aleph}}(f(c)) = \beta_{\acute{\aleph}}(f(d))$. That implies $\alpha_{f^{-1}(\acute{\aleph})}(c) = \alpha_{f^{-1}(\acute{\aleph})}(d)$ and $\beta_{f^{-1}(\acute{\aleph})}(c) = \beta_{f^{-1}(\acute{\aleph})}(d)$. Hence, $g(\acute{\aleph}) = f^{-1}(\acute{\aleph}) \in \chi^*$.
Next, we must prove that $g^{-1}(\chi(\varkappa_{(\omega,\vartheta)}) \cap \chi^*) = \acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})$.
Let $\acute{\aleph} \in \acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})$ this is true if and only if $f(\varkappa)_{(\omega,\vartheta)} \not\subset \acute{\aleph}$, which means that $f(\varkappa_{(\omega,\vartheta)}) \not\subset \acute{\aleph}$, so we get $\varkappa_{(\omega,\vartheta)} \not\subset f^{-1}(\acute{\aleph}) = g(\acute{\aleph})$. Hence, $g(\acute{\aleph}) \in \chi(\varkappa_{(\omega,\vartheta)})$.
We also get $g(\acute{\aleph}) = f^{-1}(\acute{\aleph}) \in \chi^*$, which means that $g(\acute{\aleph}) \in \chi(\varkappa_{(\omega,\vartheta)}) \cap \chi^*$, so $\acute{\aleph} \in g^{-1}(\chi(\varkappa_{(\omega,\vartheta)}) \cap \chi^*)$. Hence, $g^{-1}(\chi(\varkappa_{(\omega,\vartheta)}) \cap \chi^*) = \acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})$, and g is continuous.
ii) Let $\acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})$ with $\varkappa \in \mathcal{M}$ and $\omega, \vartheta \in (0,1]$ such that $\omega + \vartheta \leq 1$ is any open set in $\acute{\chi}$. Let $\mathcal{H} \in \acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})$. Then, $\mathcal{H} = g(\acute{\aleph})$ for some $\acute{\aleph} \in \acute{\chi}$ such that $f(\varkappa)_{(\omega,\vartheta)} \not\subset \acute{\aleph}$. It is easy to see that $\mathcal{H}$ is $f-invariant$ as is shown in part (i).
Next, $g\left(\acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})\right) = \chi(\varkappa_{(\omega,\vartheta)}) \cap \chi^*$. Since $\aleph \in g\left(\acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})\right)$, we get $g^{-1}(\aleph) \in \acute{\chi}(f(\varkappa)_{(\omega,\vartheta)})$ and $f-invariant$ if and only if $f(\varkappa_{(\omega,\vartheta)}) = f(\varkappa)_{(\omega,\vartheta)} \not\subset g^{-1}(\aleph) = f(\aleph)$. Since $\aleph$ is $f-invariant$, we get $\aleph \in \chi(\varkappa_{(\omega,\vartheta)}) \cap \chi^*$. The image of every basic open set in $\acute{\chi}$ is open in χ^*, which means that g is open.
iii) We can prove that g is surjective and injective using the previous two points [(i) and (ii)]. Let $\acute{\aleph}, \acute{\mathcal{H}} \in \acute{\chi}$ and $g(\acute{\aleph}) = g(\acute{\mathcal{H}})$. Then we get $f^{-1}(\acute{\aleph}) = f^{-1}(\acute{\mathcal{H}})$. Therefore $f(f^{-1}(\acute{\aleph})) = f(f^{-1}(\acute{\mathcal{H}}))$, since f is one to one, the we get $\acute{\aleph} = \acute{\mathcal{H}}$, and g is one-one. Finally, let $\aleph \in \chi^*$. $\aleph$ be an $f-invariant\ IFPd-filter$ of $\mathcal{M}$, and by theorem (2.10), $f(\aleph)$ is an $IFPd-filter$ of $\acute{\mathcal{M}}$. Thus $g(f(\aleph)) = f^{-1}(f(\aleph)) = \aleph$. That means g is onto, and this completes the proof.

Conclusion

This paper demonstrates that the spectrum of intuitionistic fuzzy d-filters defines a Zariski topology $\mathfrak{I}$ on d-algebra $\mathcal{M}$ and $\chi(\varkappa_{(\omega,\vartheta)})$ forms a basis for this topology for any $\varkappa$ in $\mathcal{M}$. We also find that:

- $\mathfrak{I}$ is strongly connected
- $\mathfrak{I}$ can be disconnected if there are two $IFd-filters$ $\aleph$, $\mathcal{H}$ such that $\mathrm{rad}(\aleph \cup \mathcal{H}) = \mathrm{rad}(1)$ and $\mathrm{rad}(\aleph \cap \mathcal{H}) = \mathrm{rad}(0)$.
- $\mathfrak{I}$ is a T_0.
- The map g from $\acute{\mathcal{M}}$ to $\mathcal{M}$ is a homomorphism, such that: $g(\acute{\aleph}) = f^{-1}(\acute{\aleph})$ for any intuitionistic fuzzy d-filter $\acute{\aleph}$ in $\acute{M}$ with $f-invariant$.

It is believed that this topology can be T_1 if one defines the intuitionistic maximal d-filter and he can study the separation axioms on this notation. We hope that this work enhances the scope for further study in this field of Zariski topology and that it can impact upcoming research and discussions on other topological properties or work in other algebraic structures.

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