On the Characteristic Properties for Subclasses of the Higher Derivative of Analytic Functions

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Abstract

In the present paper, we introduce two subclasses, $S^*C(\rho,\sigma,\gamma,s,d)$ and $TS^*C(\rho,\sigma,\gamma,s,d)$, of analytic functions . Coefficients bounds for these subclasses are calculated.
The main purpose of this article is to originate characteristic properties of the functions in the above subclasses.

Keywords Multivalent function, Analytic function, Starlike function, Convex function.

حول الخصائص المميزة لاصناف جزئية من المشتقات العليا للدوال التحليلية

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1.Introduction

Let $\mathcal{H}(s)$ denotes the class of analytic functions $f(z)$ in the open unit disk $U = \{z \in C: |z| < 1\}$, of the form

$$f(z) = z^s + \sum_{r=s+1}^{\infty} a_n z^r \qquad \ldots\ldots\ldots(1)$$

Moreover, we will denote by $S(s)$ the family of all functions in $\mathcal{H}(s)$ which are univalent in U.
Let $\tau(s)$ denotes the subclass of all functions $f(z)$ involving non positive coefficients, in $\mathcal{H}(s)$ of the form

$$f(z) = z^s - \sum_{r=s+1}^{\infty} a_n z^r \qquad \ldots\ldots\ldots(2)$$

Many authors [1-7] studied classes like these and described many properties for analytic functions.

Prajapat [8] presented the subclasses $R_a^{\square}(r,\rho)$, $V_a^{\square}(r,\rho)$, $T_a^{\square}(r,\sigma,\rho)$ and $T_a^{\square}(r,\sigma,\rho)$ of the class of analytic functions $f(z)$ in the open unit disk U $\mathcal{H}(1)$ and a few incorporation connections were set up for the above subclasses. Likewise, Prajapat [9] presented an intriguing subclass $\chi_t(\gamma)$ of analytic and close to convex functions in U.

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Also, Mustafa [10] presented the subclass $K(\rho,\sigma)$, $\rho,\sigma \in [0,1)$, which is the speculation of the near curved capacities, called near arched, regarding a starlike capacity g(z) of order $\rho(\rho \in [0,1))$ and type $\sigma(\sigma \in [0,1))$ of analytic functions .

In [10,11], Mustafa discovered adequate conditions for the boundaries of the standardized Wright capacities to be in the class $K(\rho,\sigma)$.

As of late, Panigrahi and Murugusundaramoorthy [12] presented another subclass which is denoted by $M_{\lambda,\delta}^{k,t}(\rho)$. They found sharp gauges for the qualification of the coefficients of the limits having a spot with this class.

   As it will in general be seen from the recently referenced assessments, a segment of the noteworthy and all around investigated subclasses of S are the classes $S^*(\rho)$ and $C(\rho)$ defined as follows.

Definition 1.1. [13-15]. The class of starlike functions $S^*(\rho)$ of order ρ ($\rho \in [0,1)$) and the class of convex functions $C(\rho)$ of order ρ ($\rho \in [0,1)$) are defined, respectively, by

$$S^*(\rho) = \left\{ f \in \mathcal{H}(1): Re\left(\frac{zf'(z)}{f(z)}\right) > \rho\, , z \in U\right\}, \rho \in [0,1)$$

and

$$C(\rho) = \left\{ f \in \mathcal{H}(1): Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \rho\, , z \in U\right\}, \rho \in [0,1)$$

Moreover, we will denote

$\tau S^*(\rho,\sigma) = S^*(\rho,\sigma) \cap \tau$ and $\tau C(\rho,\sigma) = C(\rho,\sigma) \cap \tau$, where $\tau(1) = \tau$.

Therefore,the generalization classes $S^*(\rho)$ and $C(\rho)$ are denoted, respectively, as $S^*(\rho,\sigma)$ and $C(\rho,\sigma)$ and defined by

$$S^*(\rho,\sigma) = \left\{ f \in \mathcal{H}(1): Re\left(\frac{zf'(z)}{\sigma zf'(z) + (1-\sigma)f(z)}\right) > \rho\, , z \in U\right\}, \rho, \sigma \in [0,1)$$

and

$$C(\rho,\sigma) = \left\{ f \in \mathcal{H}(1): Re\left(\frac{f'(z) + zf''(z)}{f'(z) + \sigma zf''(z)}\right) > \rho\, , z \in U\right\}, \rho, \sigma \in [0,1)$$

The classes $\tau S^*(\rho,\sigma)$ and $\tau C(\rho,\sigma)$ were studied by Altıntaş and Owa [15] . Likewise, certain conditions for hypergeometric works and summed up Bessel capacities for these classes were emphasized by Altintas and Owa [16] , Moustafa [17] , and Porwal and Dixit [18].

Therefore, based on the above mentioned studies, we define a generalized unification of the function classes $S^*(\rho,\sigma)$ and $C(\rho,\sigma)$, as follows.

Definition 1.2. A function $f \in \mathcal{H}(s)$ given by (1) is said to be in the subclass $S^*C(\rho,\sigma,\gamma,s,d)(\rho,\sigma \in [0,1), \gamma \in [0,1])$ on the off chance that the accompanying condition is fulfilled.

$$Re\left\{\frac{z^{d+1}f^{d+1}(z) + \gamma z^{d+2}f^{d+2}(z)}{\gamma z^{d+1}(f^{d+1}(z) + \sigma zf^{d+2}(z)) + (1-\gamma)(\sigma zf^{d+1}(z) + (1-\sigma)z^d f^d(z))}\right\} > \rho \quad z \in U$$

Also, we will use

$\tau(s) S^*C(\rho,\sigma,\gamma,s,d) = S^*C(\rho,\sigma,\gamma,s,d) \cap \tau(s)$.

We note the following:

   $S^*C(\rho,\sigma, 0,1,0) = S^*(\rho,\sigma)$ studied by Mustafa [11];

$S^*C(\rho,\sigma,1,1,0) = C(\rho,\sigma)$ studied by Mustafa [11];

$S^*C(\rho, 0, 0,1,0) = S^*(\rho)$ studied by Siverman [19];

$S^*C(\rho, 0,1,1,0) = C(\rho)$ studied by Siverman [19];

$\tau S^*C(\rho,\sigma, 0,1,0) = \tau S^*(\rho,\sigma)$ studied by Altintas , Altintas and Irmak , altintas and Ozkan and Porwal [20-24];

$\tau S^*C(\rho,\sigma,1,1,0) = \tau C(\rho,\sigma)$ studied by Altintas and Owa[16];

$\tau\,S^{*}C\,(\alpha\,,0,0,1,0)=\tau\,S^{*}(\rho)$ studied by Siverman [19];

$\tau\,S^{*}C\,(\alpha\,,0,1,1,0)=\tau\,C(\rho\,)$ studied by Siverman [19].

The main purpose of this paper is to satisfy characteristic properties of the subclasses $S^{*}C(\rho,\sigma,\gamma,s,d\,)$ and $\tau\,(p)S^{*}C(\rho,\sigma,\gamma,s,d\,)$ ($\rho,\sigma\in[\,0,1)$, $\gamma\in[0,1]$) of analytic functions U. Hence, coefficient bounds are also given for the functions belonging to the above subclasses.

A sufficient condition for functions in $S^{*}C(\rho,\sigma,\gamma,s,d\,)$, $\rho,\sigma\in[0,1)$, $\gamma\in[0,1]$ is given by the following theorem.

Theorem 2.1. Let $f\in\mathcal{H}(s)$. Then, the capacity $f(z)$ has a place with the subclass $S^{*}C(\rho,\sigma,\gamma,s,d)$ where ($\rho,\sigma\in[\,0,1),\gamma\in[0,1]$) on the off chance that the accompanying condition is fulfilled:

$$\sum_{r=s+1}^{\infty}\frac{r!}{(r-d)!}\{(r-d)(\gamma(1-\rho\sigma)(n-d-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}\,|a_r|$$

$$\leq (s-d)\big(1+\sigma(s-d-1)\big)(\gamma(2-\rho)-1)$$
$$+(1-\gamma)(2-\rho)\big(\sigma(s-d)+(1-\sigma)\big)\dots\dots(3)$$

The result is sharp for the functions

$f_r(z)$
$=z$

$$+\frac{(s-d)\big(1+\sigma(s-d-1)\big)(\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)\big(\sigma(s-d)+(1-\sigma)\big)z^r}{\sum_{r=2}^{\infty}\dfrac{r!}{(r-d)!}\{(r-d)(\gamma(1-\rho\sigma)(r-d-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}}\quad(4)$$

$z\in U, r=2,3,\dots$

Proof . Let the inequality (3) holds true and $|z|=1$, then we have

$$\left|\frac{z^{d+1}f^{d+1}(z)+\gamma z^{d+2}f^{d+2}(z)}{\gamma z^{d+1}\big(f^{d+1}(z)+\sigma zf^{d+2}(z)\big)+(1-\gamma)\big(\sigma z^{d+1}f^{d+1}(z)+(1-\sigma)z^{d}f^{d}(z)\big)}-1\right|\leq 1-\rho$$

It suffices to show that

$|z^{d+1}f^{d+1}(z)+\gamma z^{d+2}f^{d+2}(z)-\gamma z^{d+1}\big(f^{d+1}(z)+\sigma zf^{d+2}(z)\big)-(1-\gamma)\sigma z^{d+1}f^{d+1}(z)+$

$\big((1-\sigma)z^{d}f^{d}(z)\big)|-(1-\rho)\quad\big|\gamma z^{d+1}\big(f^{d+1}(z)+\sigma zf^{d+2}(z)\big)+(1-\gamma)\big(\sigma z^{d+1}f^{d+1}(z)+(1-\sigma)z^{d}f^{d}(z)\big)\big|$

$=\big|\frac{s!}{(s-d)!}\big\{\big((s-d)(1+\gamma(s-d-1))-\gamma(s-d)\big(1+\sigma(s-d-1)\big)-(1-\gamma)\big(\sigma(s-d)+$

$(1-\sigma))\big)\big\}z^s+$

$$\sum_{r=s+1}^{\infty}\frac{r!}{(r-d)!}\Big((r-d)\big(1+\gamma(r-d-1)\big)-\gamma(r-d)\big(1+\sigma(r-d-1)\big)-(1$$
$$-\gamma)\big(\sigma(r-d)+(1-\sigma)\big)\Big)a_r z^r\big|$$

$-(1-\rho)\left|\frac{s!}{(s-d)!}\big\{\gamma(s-d)\big(\big(1+\sigma(s-d-1)\big)\big)+(1-\gamma)(\sigma(s-d)+(1-\sigma))\big\}z^s\right.$

$$\left.+\sum_{r=s+1}^{\infty}\frac{r!}{(r-d)!}\Big(\gamma(r-d)(1+\sigma(r-d-1))+(1-\gamma)(\sigma(r-d)+(1-\sigma))\Big)a_r\right|$$

$\leq\sum_{r=s+1}^{\infty}\frac{r!}{(r-d)!}\{(r-d)(\gamma(1-\alpha\sigma)(r-d-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}-$

$(s-d)\big(1+\sigma(s-d-1)\big)(\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)\big(\sigma(s-d)+(1-\sigma)\big)\leq 0$,

by hypothesis.

By setting s=1 and d=0 in Theorem 2.1, the accompanying results are obtained by Mustafa [11].

Corollary 2.1. The capacity f (z) characterized by (1) has a place with the subclass $S^{*}(\rho$ $,\sigma,\gamma,1,0\,)$,($\rho,\sigma\in[0,1)$, $\gamma\in[0,1]$) on the off chance that the accompanying condition is fulfilled:

$$\sum_{r=2}^{\infty}\{(r)(\gamma(1-\rho\sigma)(r-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}|a_r|$$
$$\leq (\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)$$

By setting $\gamma = 0$ and $\gamma = 1$, we can easily get the following corollaries.

Corollary 2.2. Let f(z) characterized by (1) has a place with the class $S^*(\rho,\sigma)$ ($\rho,\sigma \in [0,1)$) if the following condition is

$$\sum_{r=2}^{\infty}\{r(1-\rho\sigma)-\rho(1-\sigma)\}|a_r| \leq (1-\rho)$$

Sharpness is fulfilled for the capacities

$$f_r(z) = z + \frac{(1-\rho)}{r(1-\rho\sigma)-\rho(1-\sigma)}z^r , z \in U, r = 2,3,\dots$$

Corollary 2.3. Let f(z) characterized by (1) has a place with the class $C(\rho,\sigma)$ ($\rho,\sigma \in [0,1)$) if the following condition

$$\sum_{r=2}^{\infty}\{r((1-(\rho\sigma)(r-1)-(\rho+1)\}|a_r| \leq (1-\rho)$$

Sharpness is fulfilled for the capacities

$$f_r(z) = z + \frac{(1-\rho)}{r((1-(\rho\sigma)(r-1)-\rho+1)}z^r , z \in U, r = 2,3,\dots$$

By taking $\sigma = 0$ in Corollaries 2.2 and 2.3, separately, we have the accompanying outcomes that are emphasized by Siverman [19].

Corollary 2.4. Let $f(z)$ characterized by (1) has a place with the class $S^*(\rho)$ ($\rho \in [0,1)$) on the off chance that the accompanying condition is fulfilled:

$$\sum_{r=2}^{\infty}(r-\rho)|a_r| \leq 1-\rho$$

Sharpness is fulfilled for the capacities

$$f_r(z) = z + \frac{(1-\rho)}{(r-\rho)}z^r \quad z \in U, r = 2,3,\dots$$

Corollary 2.5. Let f(z) characterized by (1) has a place with the class $C(\rho)$ ($\rho \in [0,1)$) on the off chance that the accompanying condition is fulfilled $\sum_{r=2}^{\infty} r(r-\rho)|a_r| \leq (1-\rho)$
Sharpness is fulfilled for the capacities

$$f_r(z) = z + \frac{(1-\rho)}{r(r-\rho)}z^r \quad z \in U, r = 2,3,\dots$$

Remark 2.1. Multiple results of the properties given by Corollaries 2.3 and 2.4 can be obtained for each of the classes studied by many researchers, especially including the various parameters.
For the function in the class $\tau(s)S^*C(\rho,\sigma,\gamma,s,d)$, the converse of Theorem 2.1 is also true.

Theorem 2.2. Let $f \in \tau(s)$. Then, the function $f(z)$ belongs to the class $S^*C(\rho,\sigma,\gamma,s,d)$, where ($\rho,\sigma \in [0,1), \gamma \in [0,1]$) if and only if

$$\sum_{r=s+1}^{\infty}\frac{r!}{(r-d)!}\{(r-d)(\gamma(1-\rho\sigma)(r-d-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}a_r$$
$$\leq (s-d)(1+\sigma(s-d-1))(\gamma(2-\rho)-1)$$
$$+ (1-\gamma)(2-\rho)(\sigma(s-d)+(1-\sigma))\dots\dots(3)$$

Sharpness is fulfilled for the capacities

$$f_r(z)$$
$$= z$$
$$- \frac{(s-d)(1+\sigma(s-d-1))(\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)(\sigma(s-d)+(1-\sigma))z^r}{\sum_{r=2}^{\infty}\frac{r!}{(r-d)!}\{(r-d)(\gamma(1-\rho\sigma)(r-d-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}} \quad (4)$$

$z \in U, r = 2,3, \dots$

Proof. Let $f \in S^*C(\rho, \sigma; \gamma)$, ρ, $\sigma \in [0,1)$, $\gamma \in [0,1]$. Then, according to Definition 1.2, we have

$$\text{Re}\left\{\frac{z^{d+1}f^{d+1}(z)+\gamma z^{d+2}f^{d+2}(z)}{\gamma z^{d+1}\left(f^{d+1}(z)+\sigma z f^{d+2}(z)\right)+(1-\gamma)\left(\sigma z^{d+1}f^{d+1}(z)+(1-\sigma)z^d f^d(z)\right)}\right\} > \rho, z \in U \qquad \dots (5)$$

Also, we can easily show that the condition (5) holds true if

$$\text{Re}\left\{\frac{z^{d+1}f^{d+1}(z)+\gamma z^{d+2}f^{d+2}(z)}{\gamma z^{d+1}\left(f^{d+1}(z)+\sigma z f^{d+2}(z)\right)+(1-\gamma)\left(\sigma z^{d+1}f^{d+1}(z)+(1-\sigma)z^d f^d(z)\right)} - 1\right\} \le 1 - \rho \qquad \dots\dots (6)$$

$$\text{Re}$$
$$\left\{\frac{\frac{s!}{(s-d)!}\left\{\left((s-d)(1+\gamma(s-d-1))-\gamma(s-d)(1+\sigma(s-d-1))-(1-\gamma)(\sigma(s-d)+(1-\sigma))\right)\right\}z^s+\sum_{r=2}^{\infty}\frac{r!}{(r-d)!}\left((r-d)(1+\gamma(r-d-1))-\gamma(r-d)(1+\sigma(r-d-1))-(1-\gamma)(\sigma(r-d)+(1-\sigma))\right)a_r}{\frac{s!}{(s-d)!}\left\{\gamma(s-d)\left((1+\sigma(s-d-1))\right)+(1-\gamma)(\sigma(s-d)+(1-\sigma))\right\}z^s+\sum_{r=2}^{\infty}\frac{r!}{(r-d)!}\left(\gamma(r-d)(1+\sigma(r-d-1))+(1-\gamma)(\sigma(r-d)+(1-\sigma))\right)a_n}\right\} \le$$
$$1 - \rho$$

The value of z on the real axis so that $f(z)$ is real and letting $z \rightarrow 1^-$, we obtain

$$\sum_{r=s+1}^{\infty}\frac{r!}{(r-d)!}\{(r-d)(\gamma(1-s\sigma)(r-d-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}a_r$$
$$\le (s-d)\left(1+\sigma(s-d-1)\right)(\gamma(2-\rho)-1)$$
$$+ (1-\gamma)(2-\rho)\left(\sigma(s-d)+(1-\sigma)\right)$$

By setting $s=1$ and $d=0$ in Theorem 2.2, the accompanying culminations are acquired by Mustafa [11].

Corollary 2.6. Let f(z) characterized by (2) has a place with the class $S^*C(\rho, \sigma, \gamma, 1, 0)(\rho, \sigma \in [0,1))$ on the off chance that the accompanying condition is fulfilled

$$\sum_{r=2}^{\infty}\{(r)(\gamma(1-\rho\sigma)(r-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}a_r$$
$$\le (\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)$$

By setting $\gamma = 0$ and $\gamma = 1$, we can easily get the following corollaries.

Corollary 2.7. Let f(z) characterized by (2) has a place with the class $\tau S^*(\rho, \sigma)(\rho, \sigma \in [0,1))$ if the following condition is

$$\sum_{r=2}^{\infty}\{(r)(1-\rho\sigma)-\rho(1-\sigma)\}a_r \le (1-\rho)$$

Sharpness is fulfilled for the capacities

$$f_r(z) = z - \frac{(1-\rho)}{r(1-\rho\sigma)-\rho(1-\sigma)}z^r, z \in U, r = 2,3,\dots$$

Corollary 2.8. Let f(z) characterized by (2) has a place with the class $\tau C(\rho, \sigma)(\rho, \sigma \in [0,1))$ on the off chance that the accompanying condition is fulfilled

$$\sum_{r=2}^{\infty}\{r((1-\rho\sigma)(r-1)-\rho+1)\}a_r \le (1-\rho)$$

Sharpness is fulfilled for the capacities

$$f_r(z) = z - \frac{(1-\rho)}{r((1-\rho\sigma)(r-1)-\rho+1)}z^r, z \in U, r = 2,3,\dots$$

By taking $\sigma = 0$ in Corollaries 2.7 and 2.8, individually, we have the accompanying outcomes.

Corollary 2.9. Let f(z) characterized by (2) has a place with the class $\tau S^*(\rho)(\rho \in [0,1))$ on the off chance that the accompanying condition is fulfilled

$$\sum_{r=2}^{\infty}(r-\rho)a_r \le (1-\rho)$$

Sharpness is fulfilled for the capacities

$$f_r(z) = z - \frac{(1-\rho)}{(r-\rho)} \quad z \in U, r = 2,3,\dots$$

Corollary 2.10. Let $f(z)$ characterized by (2) has a place with the class $\tau C(\rho)$ ($\rho \in [0,1)$) on the off chance that the accompanying condition is fulfilled

$$\sum_{r=2}^{\infty} r(r-\rho)\, a_r \leq (1-\rho)$$

Sharpness is fulfilled for the capacities

$$f_r(z) = z - \frac{(1-\rho)}{r(r-\rho)} \quad z \in U, r = 2,3,\ldots$$

Theorem 2.3. Let $f(z)$ characterized by (2) has a place with the class $\tau(s)S^*C(\rho,\sigma,\gamma,s,d)$ ($\rho, \sigma \epsilon [0,1)$, $\gamma \in [0,1)$)

Then

$$\sum_{r=s+1}^{\infty}|a_r| \leq \frac{(s-d)\left(1+\sigma(s-d-1)\right)(\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)\left(\sigma(s-d)+(1-\sigma)\right)}{\frac{(s+1)!}{(s+1-d)!}\{(s+1-d)(\gamma(1-\rho\sigma)(s-d)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}} \qquad \ldots (9)$$

Proof.
Using Theorem 2.2, we write

$$\frac{(s+1)!}{(s+1-d)!}\{(s+1-d)(\gamma(1-\rho\sigma)(s-d)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}\sum_{r=2}^{\infty}|a_r|$$

$$\leq \sum_{r=2}^{\infty}\frac{r!}{(r-d)!}\{(r-d)(\gamma(1-\rho\sigma)(r-d-1)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}|a_r|$$

$$\leq (s-d)\left(1+\sigma(s-d-1)\right)(\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)\left(\sigma(s-d)+(1-\sigma)\right).$$

That is

$$\sum_{r=s+1}^{\infty}|a_r| \leq \frac{(s-d)\left(1+\sigma(s-d-1)\right)(\gamma(2-\rho)-1)+(1-\gamma)(2-\rho)\left(\sigma(s-d)+(1-\sigma)\right)}{\frac{(s+1)!}{(s+1-d)!}\{(s+1-d)(\gamma(1-\rho\sigma)(s-d)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}}$$

Thus, inequality (9) is provided.
By setting $s=1$ and $d=0$, the following corollaries are obtained by Mustafa [11].

Corollary 2.11. Let $f(z)$ characterized by (2) has a place with the class $\tau(s)S^*C(\rho, \sigma, \gamma, 1, 0)$ ($\rho, \sigma \epsilon [0,1)$, $\gamma \in [0,1)$), we get

$$\sum_{r=2}^{\infty}|a_r| \leq \frac{1-\rho}{\{2(\gamma(1-\rho\sigma)-\rho(\sigma(1-\gamma)+\gamma)+1)-\rho(1-\gamma)(1-\sigma)\}}$$

Also, by setting $\gamma = 0$ and $\gamma = 1$ in Theorem 2.3, we show the accompanying outcomes separately emphasized by Mustafa [11].

Corollary 2.12. Let $f(z)$ characterized by (2) has a place with the class $\tau(1)S^*(\rho, \sigma)(\rho, \sigma \in [0,1))$. Then,

$$\sum_{r=s+1}^{\infty}|a_r| \leq \frac{1-\rho}{2-(1+\sigma)\rho}$$

Corollary 2.13. Let $f(z)$ characterized by (2) has a place with the class $\tau(1)C(\rho, \sigma)(\rho, \sigma \in [0,1))$. Then,

$$\sum_{r=s+1}^{\infty}|a_r| \leq \frac{1-\rho}{2(2-(1+\sigma)\rho)}$$

Remark 2.4. Different aftereffects of the properties given by Corollaries 2.3 and 2.4 can be obtained for every one of the classes emohasized by numerous scientists, particularly including the different boundaries.
Numerous aftereffects of the coefficient gauges (given by Corollary 2.12 and Corollary 2.13) can in reality be found by altering the different boundaries included.
Moreover, by setting $\rho = \sigma = 0$ in Corollaries 2.12 and 2.13, we obtain interesting results for the classes τS^* and τC, respectively, obtained by Mustafa [11].

Corollary 2.14. Let $f(z)$ characterized by (2) has a place with the class τS^*. Then,

$$\sum_{r=2}^{\infty}|a_r| \leq \frac{1}{2}$$

Corollary 2.15. Let $f(z)$ characterized by (2) has a place with the class τC. Then,

$$\sum_{r=2}^{\infty}|a_r| \leq \frac{1}{4}$$

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