



Splitting the e-Abacus Diagram in the Partition Theory

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Abstract

In the partition theory, there is more than one form of representation of dedication, most notably the e -Abacus diagram, which gives an accurate and specific description. In the year 2019, Mahmood and Mahmood presented the idea of merging more than two plans, and then the following question was raised: Is the process of separating any somewhat large diagram into smaller schemes possible? The general formula to split e -abacus diagram into two or more equal or unequal parts was achieved in this study now.

Keywords: Partition Theory, e -Abacus diagram, β -number, The beads.

تقسيم مخطط المعداد- e في نظرية التجزئة

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الخلاصة

في نظرية التجزئة هناك أكثر من صيغة لتمثيل التجزئة ومن أبرزها e -Abacus diagram والتي تعطي لنا وصفاً دقيقاً ومحدد. في العام 2019 قدم كل من Mahmood and Mahmood فكرة دمج أكثر من مخطط ثم تم طرح السؤال التالي:
هل ان عملية الفصل لاي مخطط كبير تؤدي الى مخطط أصغر ممكنة؟
وتم التوصل الى إيجاد الصيغة العامة لتقسيم مخطط المعداد e الى جزئين او أكثر متساويين او غير متساويين.

Introduction

Despite all studies and researches that have been presented on the subject of partition theory, the subject of large partition diagrams makes it difficult to conduct studies about coding letters and other topics, in addition to the time and great effort made to fulfil that. Therefore, a method was employed that assists the study of these large diagrams, related to partition theory, after splitting the diagram into a number of small diagrams, determining the corresponding diagrams for each new case. According to specific laws and rules and depending on the value of e , we can split e into $e_1, e_2, e_3, \dots \geq 2$, and it will be quite natural that these parts will be equal or unequal if the value of e chosen is an even number. On the other hand, if e value is odd, then the resulting splits will be unequal. All the processes of splitting are based on the rules and laws proved in this research.

Let n be an integer positive number. A sequence λ of non-negative integers, such that $[\lambda] = \sum_{j=1}^r \lambda_j = n$, for $j \geq 1$ and $\lambda_j > \lambda_{j+1}$, then the sequence $\lambda = (\lambda_1, \dots, \lambda_r)$ is called a partition [1]. The partitions theory was discovered by Andrew [2, 3].

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The partition theory is considered as the cornerstone of algebra. James [4] defined $\beta_i = \lambda_i + b - i, 1 \leq i \leq b$, which are called beta numbers, for λ and b as a non-negative integer. The $e -$ Abacus has vertical runners, labelled as $0, 1, 2, \dots, e - 1$ from left to right from top to bottom.

0	1	2	$e - 1$
e	$e + 1$	$e + 2$	$2e - 1$
$2e$	$2e + 1$	$3e - 1$

Mahmood [5] defined Abacus James diagram and β - numbers as the values of $b = r, r + 1 \dots, r + (e - 1)$, which are called the "guides", where r is the number of the parts of the partition λ of n , and the b guide represents the "Main diagram" or "Guide diagram".

All additions in this area motivated the researchers to create several ideas in the partition theory, including the direct application of this topic. One of the most vital applications are what Ilango and Marudh discussed [6], in which they depended on the idea of the nodes and voids when using the sonar to obtain the best images and then treating the patient as quickly as possible. Also, Andrews [7] applied his idea on the Tile domino surfaces, which he regarded as the most important application in the composition of Aztec diamonds. The subject of the main diagram that has been expanded by Mahmood and others [8] presented new additions that resulted in more important topics.

Also, new additions to the topic of partition theory and β -numbers [9] led to the emergence of the idea of coding the Syriac letters in 2017 [10,11]. In 2018 and 2019, both Mahmood and Mahmood [12,13] presented the idea of coding English letters and, where adding these letters according to the rule was remarkably useful. In fact, the idea of coding is common for many researches and other different topics [14]. As for the current topic, i.e. the partition theory, which has been expanded widely, it has become difficult to study large $e -$ Abacus diagrams and β - numbers [15,16]. Hence, it was decided to split large $e -$ Abacus diagrams into several smaller tables and then to find out the corresponding partition for each split to facilitate future studies.

It is quite natural that the process of splitting is not merely the division into two or more equal or different parts, because if we choose e as an odd number, then we cannot divide the diagram equally at all, which is what we will present in this research.

2. The Proposed Method

2.1 Splitting e-Abacus

There are large $e -$ Abacus diagrams with big partitions, which are hard to study or to perform mathematical operations on them. Thus, it is better to split these $e -$ Abacus diagrams such that each part may represent a partition and can be easily read from right to left. The basic rule of this partition is $e \geq 4$. The reason is that the smallest e that can be chosen is 2, and therefore, any diagram to be divided must be at least $e = 4$.

2.2. Splitting e-Abacus in Case e=4

If $e = 4$, $e -$ Abacus diagram should be split into $e_1 = e_2 = 2$, as in the Table 1, where the partition can be read according to the following:

Table 1- Splitting e-Abacus diagram if $e = 4$ into $e_1 = e_2 = 2$

$e = 4$					$e_1 = 2$			$e_2 = 2$	
Run1	Run2	Run3	Run4	\Rightarrow	Run1	Run2	\Rightarrow	Run1	Run2
0	1	2	3		0	1		0	1
4	5	6	7		2	3		2	3
8	9	10	11		4	5		4	5
12	13	14	15		6	7		6	7
.
.

Let OP_ℓ^* be the space that precedes the node in the original partition, which consists of four columns. Assume that b^{**}_ℓ is the number of beads in the line ℓ for all columns, except for the column ℓ itself. An example for that is the following.

2.3 Example

If $\lambda = (10, 8^3, 5^2, 4, 3^4, 2^2, 1)$ is a partition, the β –number of this partition where $b = 14$ such that $b \geq 14$ are $(23, 20, 19, 18, 14, 13, 11, 9, 8, 7, 6, 4, 3, 1)$, and $e = 4$, then we can represent it on the e –Abacus diagram in the form of a bead, as mentioned by James [4]. The following Table 2 shows this case.

Table 2- Representation of β -numbers on the e –Abacus diagram

$e = 4$			
Run1	Run2	Run3	Run4
1	•	2	•
•	3	•	•
•	•	4	•
5	•	•	6
7	8	•	•
•	9	10	•
$(10, 8^3, 5^2, 4, 3^4, 2^2, 1)$			

$e_1 = 2$	
Run1	Run2
–	•
•	–
•	•
–	•
–	–
•	–
$(5, 3, 2^2, 1^2)$	

$e_2 = 2$	
Run1	Run2
–	•
•	•
–	•
•	–
•	•
–	•
$(4, 3^2, 2^2, 1^3)$	

The partition of each split can be calculated in the case where $e = 4$, as shown in Table 3.

Table 3- Calculation of the partition of split (1) and split (2) in the case where $e = 4$

Row	Split(1)= e_1	Row	Split(1)= e_2
1	$op_1^* = \boxed{1}$	1	$op_1^* = 2, e_1 = 2, b_1^{**} = 1$ $op_1^* - (e_1 - b_1^{**}) = 2 - (2 - 1) = \boxed{1}$
2	$op_2^* = 2, e_2 = 2, b_{1,2}^{**} = 1$ $op_2^* - (e_2 - b_{1,2}^{**}) = 2 - (2 - 1) = \boxed{1}$	2	$op_2^* = 3, 2e_1 = 4, b_{1,2}^{**} = 2$ $(3 - (4 - 2))^2 = \boxed{1^2}$
3	$op_3^* = 3, 2e_2 = 4, b_{1,2}^{**} = 3$ $(3 - (4 - 3))^2 = \boxed{2^2}$	3	$op_3^* = 4, 3e_1 = 6, b_{1,2,3}^{**} = 4$ $4 - (6 - 4) = \boxed{2}$
4	$op_4^* = 5, 3e_2 = 6, b_{1,2,3}^{**} = 4$ $5 - (6 - 4) = \boxed{3}$	4	$op_4^* = 5, 4e_1 = 8, b_{1,2,3,4}^{**} = 5$ $5 - (8 - 5) = \boxed{2}$
5	There are no beads	5	$op_5^* = 8, 5e_1 = 10, b_\ell^{**} = 5$ $(8 - (10 - 5))^2 = \boxed{3^2}$, where $\ell = 1, 2, 3, 4, 5$
6	$op_6^* = 8, 5e_2 = 10, b_\ell^{**} = 7$ $8 - (10 - 7) = \boxed{5}$, where $\ell = 1, \dots, 5$	6	$op_6^* = 10, 6e_1 = 12, b_\ell^{**} = 6$ $10 - (12 - 6) = \boxed{4}$, where $\ell = 1, \dots, 6$

2.4. General Method for Splitting e-Abacus Diagram Where $e_1 = e_2 = 2$

The general rule for finding the partition is drawing the e –Abacus diagram, as shown in Table 4.

Table 4- The general rule to find the partition of each split in the case where $e = 4$

Row	$e_1 = 2$	$e_2 = 2$
1	$op_1^* = 1$	$op_1^* = (e_1 - b_{\ell}^{**})$
2	$op_2^* - (e_2 - \sum_{\ell=1} b_{\ell}^{**})$ Where $\ell = 1$	$op_2^* - (e_1 - \sum_{\ell=1}^2 b_{\ell}^{**})$ Where $\ell = 1,2$
3	$op_3^* - (2e_2 - \sum_{\ell=1}^2 b_{\ell}^{**})$ Where $\ell = 1,2$ ⋮ ⋮	$op_3^* - (3e_1 - \sum_{\ell=1}^3 b_{\ell}^{**})$ Where $\ell = 1,2,3$ ⋮ ⋮
$\ell + 1$	$op_{\ell+1}^* - (\ell e_2 - \sum_{\ell=1} b_{\ell}^{**})$	$op_{\ell+1}^* - ((\ell + 1)e_1 - \sum_{\ell=1} b_{\ell}^{**})$

The suggestion can be clarified when $e = 4$, as shown in Table 5.

Table 5- Clarification of the general formula when $e = 4$

	Split(1) : $e_1 = 2$	Split(2) : $e_2 = 2$
1	$op_1^* = 1$	$op_1^* = 0$ if there is bead $op_1^* = 1$ if it is a vacuum
2	op_2^* minus (positions of (1) row of split(2) minus all the beads in 1 row of split (2))	op_2^* minus (positions of 1,2 row of split 1 minus all the beads in 1,2 row of split (1))
3	p_3^* minus (positions of 1,2 row of split 2 minus all the beads in 1,2 row of split (2)) ⋮ It stops when the last void is reached before last bead	⋮ ⋮ It stops when the last void is reached before last bead

2.5. Splitting e-Abacus Diagrams in Case $e = 5$

If $e = 5$, then $e - Abacus$ diagram is split into $e_1 = 2, e_2 = 3$, or the opposite, that we get two partitions, which can be represented on the $e - Abacus$ diagram, as shown in Table 6.

Table 6- Splitting $e - Abacus$ diagram if $e = 5$

$e = 5$				
Run1	Run2	Run3	Run4	Run5
0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮

⇒

$e_1 = 2$	
Run1	Run2
0	1
2	3
4	5
6	7
8	9
⋮	⋮
⋮	⋮

$e_2 = 3$		
Run1	Run2	Run3
0	1	2
3	4	5
6	7	8
9	10	11
⋮	⋮	⋮
⋮	⋮	⋮

The $\beta -$ numbers, if $e = 5$ as in the previous example, can be represented as (23,12,18,14,13,11,9,8,7,6,4,3,1).

Table 7-Representation of The β –numbers on the e-Abacus diagram if $e = 5$

Run1	Run2	Run3	Run4	Run5
1	•	2	•	•
3	•	•	•	•
4	•	5	•	•
6	7	8	•	•
•	9	10	•	–
(10, 8 ³ , 5 ² , 4, 3 ⁴ , 2 ² , 1)				

 \Rightarrow

Run1	Run2
–	•
–	•
–	•
–	–
•	–
(5, 3, 2, 1)	

Run1	Run2	Run3
–	•	•
•	•	•
–	•	•
–	•	•
–	•	–
(4, 3 ² , 2 ² , 1 ⁵)		

Table 8- The calculation of the partition of split (1) and split (2) in the case where $e=5$

Row	Split(1): $e_1 = 2$	Split(1): $e_2 = 3$
1	$op_1^* = \boxed{1}$	$op_1^* = 2, e_1 = 2, b_\ell^{**} = 1$ $(2 - (2 - 1))^2 = \boxed{1^2}$ where $\ell = 1$
2	$op_2^* = 3, e_2 = 3, b_\ell^{**} = 2$ where $\ell = 1; 3 - (3 - 2) = \boxed{2}$	$op_2^* = 3, 2 e_1 = 4, b_\ell^{**} = 2$ where $\ell = 1, 2; (3 - (4 - 2))^3 = \boxed{1^3}$
3	$op_3^* = 4, e_2 = 6, b_\ell^{**} = 5$ where $\ell = 1, 2; 4 - (6 - 5) = \boxed{3}$	$op_3^* = 5, 3e_1 = 6, b_\ell^{**} = 3$ where $\ell = 1, 2, 3; (5 - (6 - 3))^2 = \boxed{2^2}$
4	There are no beads	$op_4^* = 8, 4e_1 = 8, b_\ell^{**} = 3$ where $\ell = 1, 2, 3, 4; (8 - (8 - 3))^2 = \boxed{3^2}$
5	$op_5^* = 8, 4 e_2 = 12, b_\ell^{**} = 9$ where $\ell = 1, 2, 3, 4; 8 - (12 - 9) = \boxed{5}$	$op_5^* = 10, 5e_1 = 10, b_\ell^{**} = 4$ where $\ell = 1, \dots, 5; (10 - (10 - 4)) = \boxed{4}$

2.6. Splitting e –Abacus Diagram in the Case where $e = 6$.

If $e = 6$, then the e –Abacus diagram is split into $e_1 = e_2 = e_3 = 2, e_1 = 2, e_1 = 2, e_2 = 4, e_1 = e_2 = 3$, or $e_1 = 4, e_2 = 2$ and so on.

Now, we can select the splitting, which includes three splits, as shown in Table 9.

Table 9- Splitting e –Abacus diagram if $e = 6$

$e = 6$					
Run1	Run2	Run3	Run4	Run5	Run6
0	1	2	3	4	5
6	7	8	9	10	11
12	13	14	15	16	17
18	19	20	21	22	23
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•

 \Rightarrow

$e_1 = 2$	
Run1	Run2
0	1
2	3
4	5
6	7
•	•
•	•
•	•

$e_2 = 2$	
Run1	Run2
0	1
2	3
4	5
6	7
•	•
•	•
•	•

$e_3 = 2$	
Run1	Run2
0	1
2	3
4	5
6	7
•	•
•	•
•	•

The β -numbers for the previous example, when $e = 6$, can be represented as (23,20,19,18,14,13,11,9,8,7,6,4,3,1), as shown in Table 10.

Table 10- Representation of the β –numbers on the e –Abacus diagram if $e = 6$

$e = 6$					
Run1	Run2	Run3	Run4	Run5	Run6
1	•	2	•	•	3
•	•	•	•	4	•

 \Rightarrow

$e_1 = 2$	
Run1	Run2
–	•
•	•

$e_2 = 2$	
Run1	Run2
–	•
•	•

$e_3 = 2$	
Run1	Run2
•	–
–	•

5	•	•	6	7	8	-	•	•	-	-	-
•	•	•	9	10	•	•	•	•	-	-	•
(10, 8 ³ , 5 ² , 4, 3 ⁴ , 2 ² , 1)						(2 ³ , 1 ³)		(2, 1 ⁴)		(5, 2)	

We can calculate the partition of each split, as shown in Table 11.

Table 11- Calculating the partition of split (1), split (2), and split (3) when $e = 6$

Row	split(1): $e_1 = 2$	Split(2): $e_2 = 2$	split(3): $e_3 = 2$
1	$op_1^* = \boxed{1}$	$op_1^* = 2, e_1 = 2, b_{\ell}^{**} = 1$ Where $\ell = 1$ $2 - (2 - 1) = \boxed{1}$	$op_1^* = 2, e_1 + e_2 = 4$ $b_{\ell}^{**} = 2$ Where $\ell = 1$ $2 - (4 - 2) = \boxed{0}$
2	$op_2^* = 3, (e_1 + e_2) = 4,$ $b_{\ell}^{**} = 2$ Where $\ell = 1$ $(3 - (4 - 2))^2 = \boxed{1^2}$	$op_2^* = 3, (2 e_1 + e_3) = 6,$ $b_{\ell}^{**} = 4$ Where $\ell = 1$ $(3 - (6 - 4))^2 = \boxed{1^2}$	$op_2^* = 4, 2(e_1 + e_2) = 8,$ $b_{\ell}^{**} = 6$ Where $\ell = 1, 2$ $(4 - (8 - 6)) = \boxed{2}$
3	$op_3^* = 5, 2(e_2 + e_3) = 8,$ $b_{\ell}^{**} = 5$ Where $\ell = 1, 2$ $(5 - (8 - 5)) = \boxed{2}$	$op_3^* = 5, (3 e_1 + 2e_3) = 10,$ $b_{\ell}^{**} = 6$ Where $\ell = 1, 2, 3$ $(5 - (10 - 6)) = \boxed{1}$	There is no beads
4	$op_4^* = 8, 3(e_2 + e_3)$ $b_{\ell}^{**} = 6$ Where $\ell = 1, 2, 3$ $(8 - (12 - 6))^2 = \boxed{2^2}$	$op_4^* = 8, (4 e_1 + 3e_3) = 14$ $b_{\ell}^{**} = 8$ Where $\ell = \text{*****}$ $(8 - (14 - 9)) = \boxed{2}$	$op_4^* = 10, 4(e_1 + e_2) = 16$ $b_{\ell}^{**} = 11$ Where $\ell = 1, \dots, 4$ $(10 - (16 - 11)) = \boxed{5}$

2.7 A Suggested Method for Finding Partitions for e_1, e_2 and e_3

The general rule for finding the partition is drawing the $e -$ Abacus diagram, where $e_1 = e_2 = e_3 = 2$, as shown in Table 12.

Table 12- The general rule for finding the partition of each split in the case where $e = 6$

Run	Split(1)	Split(2)	Split(3)
1	$op_1^* = 1$	$op_1^* = 0$ or 1	$op_1^* = 0$ or 1
2	$op_2^* - ((e_2 + e_3)) - \sum_{\ell=1} b_{\ell}^{**}$	$op_2^* - ((2e_1 + e_3) - \sum_{\ell=1} b_{\ell}^{**})$	$op_2^* - (2(e_1 + e_2) - \sum_{\ell=1} b_{\ell}^{**})$
3	$op_3^* - (2(e_2 + e_3) - \sum_{\ell=1} b_{\ell}^{**})$	$op_3^* - ((3e_1 + 2e_3) - \sum_{\ell=1} b_{\ell}^{**})$	$op_3^* - (3(e_1 + e_2) - \sum_{\ell=1} b_{\ell}^{**})$
4	$op_4^* - (3(e_2 + e_3) - \sum_{\ell=1} b_{\ell}^{**})$	$op_4^* - ((4e_1 + 3e_3) - \sum_{\ell=1} b_{\ell}^{**})$	$op_4^* - (4(e_1 + e_2) - \sum_{\ell=1} b_{\ell}^{**})$
⋮	⋮	⋮	⋮
$\ell + 1$	$op_{\ell+1}^* - (\ell(e_2 + e_3) - \sum_{\ell=1} b_{\ell}^{**})$	$op_{\ell+1}^* - (((\ell + 1)e_1 + \ell e_3) - \sum_{\ell=1} b_{\ell}^{**})$	$op_{\ell+1}^* - ((\ell + 1)(e_1 + e_2) - \sum_{\ell=1} b_{\ell}^{**})$

Note: If the splits are not equal, we apply the same previous rule.

2.8. The General Rule for Finding Partitions for e_1 and e_2, e_3, \dots, e_n

By 2.2-2.7, the proof of the general rule of splitting a large e-Abacus diagram is defined by Table 13 f.

Table 13-The general rule for finding the partition of each split in the case of large $e -$ Abacus diagrams.

Run	e_1	e_2	e_3	e_k
1	op_1^*	$op_1^* - \{e_1 - \sum b_{\ell}^{**}\}$	$op_1^* - \left\{ \sum_{t=1}^2 e_t - \sum b_{\ell}^{**} \right\}$	$op_1^* - \left\{ \sum_{t=1}^{k-1} e_t - \sum b_{\ell}^{**} \right\}$
2	$op_2^* - \left\{ \sum_{t=2}^k e_t - \sum b_{\ell}^{**} \right\}$	$op_2^* - \left\{ (2e_1 + \sum_{t=3}^k e_t) - \sum b_{\ell}^{**} \right\}$	$op_2^* - \left\{ (2(e_1 + e_2) + \sum_{t=4}^k e_t) - \sum b_{\ell}^{**} \right\}$	$op_2^* - \left\{ \sum_{t=1}^k 2e_t - \sum b_{\ell}^{**} \right\}$
3	$op_3^* - \left\{ \sum_{t=2}^k 2e_t - \sum_{\ell=1} b_{\ell}^{**} \right\}$	$op_3^* - \left\{ (3e_1 + \sum_{t=3}^k 2e_t) - \sum b_{\ell}^{**} \right\}$	$op_3^* - \left\{ (3(e_1 + e_2) + \sum_{t=4}^k 2e_t) - \sum b_{\ell}^{**} \right\}$	$op_3^* - \left\{ \sum_{t=1}^k 3e_t - \sum b_{\ell}^{**} \right\}$
	⋮	⋮	⋮	⋮
$\ell + 1$	$op_{\ell+1}^* - \left(\sum_{t=2}^k e_t - \sum_{\ell=1} b_{\ell}^{**} \right)$	$op_{\ell+1}^* - \left\{ \left((\ell + 1)e_1 + \sum_{t=3}^k \ell e_t \right) - \sum b_{\ell}^{**} \right\}$	$op_{\ell+1}^* - \left\{ (\ell + 1)(e_1 + e_2) + \ell \sum_{t=4}^k e_t - \sum b_{\ell}^{**} \right\}$	$op_{\ell+1}^* - \left\{ (\ell + 1) \sum_{t=1}^k e_t - \sum b_{\ell}^{**} \right\}$

Conclusions

Through the work performed in this research, a conclusion was reached in terms of finding the general formula to find the partition for each e-Abacus diagram, which results from splitting the large diagrams of any partition.

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