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A Stochastic Differential Equations Model for the Spread of Coronavirus COVID-19): The Case of Iraq)

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Abstract

In this paper, we model the spread of coronavirus (COVID -19) by introducing stochasticity into the deterministic differential equation susceptible -infected-recovered (SIR model). The stochastic SIR dynamics are expressed using Itô's formula. We then prove that this stochastic SIR has a unique global positive solution I(t).The main aim of this article is to study the spread of coronavirus COVID-19 in Iraq from 13/8/2020 to 13/9/2020. Our results provide a new insight into this issue, showing that the introduction of stochastic noise into the deterministic model for the spread of COVID-19 can cause the disease to die out, in scenarios where deterministic models predict disease persistence. These results were also clearly illustrated by Computer simulation.

Keywords: Mathematical modeling of COVID-19,basic reproduction number, pandemic ,stochastic SIR model, computer simulation.

نموذج المعادلات التفاضلية التصادفية لانتشار فيروس كورونا (COVID-19) حالة العراق

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الخلاصة

في هذا البحث, قمنا بنمذجة انتشار فيروس كورونا (COVID-19) بواسطة ادخال العشوائية في نموذج المعادلات التفاضلية الحتمية لنظام SIR.يتم التعبير عن ديناميكيات النظام العشوائية باستخدام صيغة ايتو . تم اثبات ان هذا النظام العشوائي لديه حل موجب عالمي وحيد I(t) .الهدف الرئيسي من هذا البحث هو دراسة انتشار فيروس كورونا في العراق للفترة من 13\8\2020 الى 13\9\2020 ووجدنا نتائج جديدة, تقدم نتائجنا نظرة ثاقبة جديدة حول انتشار فيروس كورونا في العراق, واظهرت النتائج أن ادخال الضوضاء العشوائية في نموذج المعادلات التفاضلية الحتمية لانتشار فيروس كورونا يمكن ان يتسبب في موت المرض ,في السيناريوهات التي تتنبأ فيها نماذج المعادلات التفاضلية الحتمية باستمرار المرض .وتتضح هذه النتائج أيضا من خلال المحاكاة الحاسوبية .

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1- INTRODUCTION

Comprehension and prediction of the novel COVID-19 has become very important owing to the huge global health burden. Until 18/9/2020, almost 30356725 persons became newly infected with COVID-19, while about 950625 died since the identification of the first cases in Wuhan City, China, in December 2019 [1, 2]. The global data indicate that the number of people infected with coronavirus continues to rise even though effective prevention strategies exist. No country of the world has been spared from coronavirus. The pandemic remains extremely dynamic, increasing and changing characters as the virus exploits new opportunities used for transmission [1] . Actually, coronavirus is infiltrating almost every aspect of life , damaging global economy, and altering both man- made and natural environments .The pandemic varies in impact within regions; some countries are more affected than others and within countries there are usually wide variations in infection levels between different provinces [1-8]. The large amount of work conducted on modeling the spread of COVID-19 has been largely restricted to ordinary differential equations [9-13]. These models do not take into account the inherent randomness that is associated with the spread of COVID-19. In this manuscript, we propose examining the effects of the introduction of environmental noise into such a system. Thus, we adopt the SIR model assumption for the spread of COVID-19 [7], as follows.

$$\begin{cases} \frac{dS(t)}{dt} = -aS(t)I(t), \\ \frac{dI(t)}{dt} = aS(t)I(t) - bI(t), \\ \frac{dR_m(t)}{dt} = bI(t), \end{cases} \tag{1.1}$$

And the introduction of environmental noise into the system (1.1). Hence, we propose a system of stochastic differential equations for modeling the spread of coronavirus . The rest of this article is structured as follows. Section 2 introduces the mathematical definition of the Stochastic Differential Equations (SDEs), including the stochastic process, Brownian motion, Itô's integral, and the theorem about Ito's formula. Section 3 describes the SDEs SIR Model for the spread of COVID-19. Also, a table of all the parameters used during our work with the basic reproduction number for the stochastic model is presented. In section 4, we prove the existence of the unique nonnegative solution [14,15]. In section 5, we consider the conditions required for COVID-19 to die out, i.e., for the disease to become extinct. The main results are presented in section 6. Finally, Section 7 is devoted to the conclusion part (e.g., see[7,8,16 - 19]).

2- Basic Concept of the Stochastic Differential Equations

In this section, mathematical definitions of the SDEs are described. Additionally, we explain some theorems that we use in this work.

Definition (2.1)

The stochastic process W(t) is defined as a family of random variables X(t, ω) of two variables t $\in T$ and $\omega \in \Omega$ on a common probability space (Ω, A, P).

Definition (2.2)

In the stochastic process W(t), t $\in [0, \infty]$ is said to be a Brownian motion or Wiener process if the following conditions are satisfied:

1. P(W(0)=0)=1.
2. For $0 < t_0 < t_1 < \cdots < t_n$, the increments $W(t_1) - W(t_0), W(t_n) - W(t_n - 1)$, are independent.
3. For arbitrary (t) and (h $>$ 0), $W(t + h) - W(t)$ has a Gaussian distribution with a mean value of zero and variance h. The Wiener process has the properties that E(w(t))=0 and Var $(W(t) - W(s)) = t - s$, for all $0 \le s \le t$. Thus, they have stationary increments.

Definition (2.3)

The stochastic differential equations (SDEs) take the form.

$$dX(t) = f(X(t), t)dt + g(X(t), t)dW(t)\,, \quad X(t_0) = X_0,\ t \in [t_0, T]\ , T > 0, \tag{2.3}$$

where $f(X(t), t)$ is the drift coefficients function and $g(X(t), t)$ is defined as a diffusion coefficient function. The solution to SDEs in equation (2.3) takes the following form in the integral formula.

$$X(t) = X_0 + \int_{t_0}^{t} f(X(s), s)ds + \int_{t_0}^{t} g(X(s), s)dW(s), \quad t \in [t_0, T], \tag{2.4}$$

where the first integral on the right side of equation (2.4) is Riemann integral, and the second is stochastic integral.

Theorem (2.1) (Itô's formula)
Suppose that X_t has SDE:
$$dX_t = f(X_t, t)dt + g(X_t, t)dw_t, \quad (2.5)$$
for $f, g \in C^{1,2}(J \times R, R)$, assume that $F: J \times R \to R$ is continuous and has $\frac{\partial F}{\partial t}$,$\frac{\partial F}{\partial X_t}$ and $\frac{\partial^2 F}{\partial X_t^2}$ that exist and are a continuous set $F = F(X_t, t)$, then F has the stochastic differential
$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial X_t}dX_t + \frac{1}{2}\frac{\partial^2 F}{\partial X_t^2}g^2 dt,$$
$$dF(X_t, t) = \left[\frac{\partial F}{\partial t} + \frac{\partial F}{\partial X_t}f + \frac{1}{2}\frac{\partial^2 F}{\partial X_t^2}g^2\right]dt + \frac{\partial F}{\partial X_t}gdW_t\,, \quad (2.6)$$
The last equation (2.6), is called Ito's formula or Ito's chain rule. Equation (2.5) is sufficiently general to represent an m- dimensional d-wiener process system. In the equation, $W_t = (W_t^1, W_t^2, \dots, W_t^d)^T$ is an ad-dimensional vector consists of d independent Wiener processes and g(X_t, t) is an $m \times d$matrix. If we labeled the columns of $g(X_t, t\,)$ to be as $g_1(X_t, t), g_2(X_t, t), \dots, g_d(X_t, t)$; then, the m-dimensional d-wiener process system is written as, $dX_t = f(X_t, t)dt + \sum_{j=1}^{d} g_j(X_t, t)dw_t^j$. Here, the component-by-component of the Ito's formula can be K= 1,2,…, m.

$$dF_k(X_t, t) = \left[\frac{\partial F_k}{\partial t} + \sum_{i=1}^{m} f_i \frac{\partial F_k}{\partial X} + \frac{1}{2}\sum_{i,j=1}^{m} g_{i1}g_{j1}\frac{\partial^2 F_k}{\partial X_i \partial X_j}\right]dt + \sum_{i=1}^{d}\sum_{i=1}^{m} g_{i1}\frac{\partial F_k}{\partial X_i}\,dW_t, \quad (2.7)$$

Definition (2.4)
We can define the stochastic integral or Itô's integral as follows; let g(t) be a stochastic function having a continuous derivative in the region $[\alpha, \beta]$, and let W(t), t≥ 0 denote a standard wiener process the Ito's integral:

$$\int_{\propto}^{\beta} g(t)dW(t) \equiv \lim_{n\to\infty}\sum_{i=1}^{n} g(t_{i-1})[W(\,t_i) - W(t_{i-1})\,], \quad (2.8)$$

where $\propto = t_0 < t_1 < \cdots < t_n = \beta$ is an apparition of the region $[\alpha, \beta]$. By applying the integration by parts, we found that:
$$\int_{\propto}^{\beta} g(t)dW(t) = g(\beta)W(\beta) - g(\propto)W(\propto) - \int_{\propto}^{\beta} W(t)dg(t), \quad (2.9)$$

3. The stochastic Differential Equations SIR Model for the spread of COVID-19

In this paper, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ to be a complete probability space with a filtration$\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the normal condition (i.e.it is growing and right continuous whereas $\mathcal{F}_0$ covers all $\mathbb{P}$ -null sets), and we let W(t) be a scalar Wiener process or Brownian motion defined on the probability space. We use $a \wedge$ b to denote min(a ,b) and $a \vee$ b to denote max(a ,b). The growth of COVID-19 infections in each region is modeled by the SDEs SIR model which is given as follows.

$$\begin{cases} dS(t) = -aS(t)I(t)dt \quad, S(0) = S_0\,, \\ dI(t) = \Big(aS(t)I(t) - (\rho + \gamma)I(t)\Big)dt + \sigma S(t)I(t)dW_t \quad, I(0) = I_0, \\ dR_m(t) = (\rho + \gamma)I(t)dt - \sigma S(t)I(t)dW_t \;, \end{cases} \quad (3.1)$$
$$S(0) = N_{total} - R_m(0) - I(0), \quad (3.2)$$

Table 1-Model States and Model parameters

Parameter	Description
S(t)	the whole susceptible population at time t
I(t)	the number of active infections at time t
$R_m(t)$	the whole number of deaths and recoveries at time t
t	the daily –time parameter
dS(t),dI(t)and d$R_m(t)$	the change in the states at time t
dW_t	the increment in Weiner process which models the randomness in the evolution
a	disease transmission coefficient
ρ	per capita death rate
γ	the rate at which infected individuals become cured
σ	a parameter used to model the stochastic or randomness in the evolution, which will cause local deviation from the typical (exponential) trends
N_{total}	the populations of the regions I_0 and S_0 are the initial number of infections and susceptible individuals, respectively
R_e	The basic reproduction number for the deterministic model
R_e^S	The basic reproduction number for the stochastic model

Let us now consider the second equation of (3.1). To establish the basic reproduction number for the stochastic model

$$dI(t) = \Big(aS(t)I(t) - (\rho+\gamma)I(t)\Big)dt + \sigma S(t)I(t)dW_t \,, I(0) = I_0 \,, \tag{3.3}$$

we rewrite equation (3.3) as follows

$\frac{dI(t)}{I(t)} = \big(aS(t) - (\rho+\gamma)\big)dt + \sigma S(t)dW_t,$ then we see that the term $\ln I(t)$ should appear in the solution of (3.3).In this case, we set $F(I(t),t) = \ln I(t)$ and when we apply Itô's formula (2.6), we get

$$dF(I(t),t) = \left[0 + \Big(aS(t) - \big(\rho+\gamma\big)\Big)I(t)\left(\frac{1}{I(t)}\right) - 0.5\sigma^2 S(t)^2 I(t)^2\left(\frac{1}{I(t)^2}\right)\right]dt + \sigma S(t)I(t)\left(\frac{1}{I(t)}\right)dW_t,$$

$$\text{,So } d\ln I(t) = \left[\Big(aS(t) - \big(\rho+\gamma\big)\Big) - 0.5\sigma^2 S(t)^2\right]\mathrm{dt} + \sigma\mathrm{S(t)dW_t}$$

The integral for both sides gives

$$\ln I(t) - \ln I(0) = \left[\Big(aS(t) - \big(\rho+\gamma\big)\Big) - 0.5\sigma^2 S(t)^2\right]t + \sigma S(t)(W_t - W_0), \text{for } W_0 = 0,$$

So the solution of equation (3.3) is

$I(t) = I_0 \exp\left[aS(t) - \big(\rho+\gamma\big) - 0.5\sigma^2 S(t)^2\right]t + \sigma S(t)W_t$, so

$$R_e^S = \frac{as(t)}{(\rho+\gamma)} - \frac{\sigma^2 S(t)^2}{2(\rho+\gamma)} = R_e - \frac{\sigma^2 S(t)^2}{2(\rho+\gamma)}, \tag{3.4}$$

where $S(t)$ represents the whole susceptible population at time t. It can be expressed by N which, through this paper, will be always changing with the time t. Hence, the basic reproduction number for the stochastic model can be expressed as follows,

$$R_e^S = \frac{aN}{(\rho+\gamma)} - \frac{\sigma^2 N^2}{2(\rho+\gamma)} = R_e - \frac{\sigma^2 N^2}{2(\rho+\gamma)},$$

4.Existence of Unique Nonnegative Solution

Before we begin to investigate the dynamical behavior of the SDE SIR model for COVID-19 (3.3), it is important to prove that this module does not only has a single global solution, but also that the solution will remain within (0,N) when it starts from there . The current general existence and uniqueness theorem on SDEs (see e.g. [15]) does not apply to this special stochastic differential equation. To assure these properties, let us take the following theorem.

Theorem 4.1

For every given initial value $(0) = I_0 \in (0,N)$, the SDE SIR COVID-19 model (3.3) has a unique global nonnegative solution $I(t) \in (0,N)$ for all $T \geq 0$ with a probability that equals one, i.e.

$\mathbb{P}\{I(t) \in (0,N) \text{ for all } t \geq 0\} = 1.$

Proof

Concerning equation (3.3) as an SDE on R, we see that its coefficients are locally Lipchitz continuous. It is well-known that for every given initial value $S_0 \in (0, N)$ there is a unique maximal local solution I(t) on $t \in [0, \tau_e]$, where τ_e is the explosion time. Let $k_0 > 0$ be sufficiently large for $\frac{1}{k_0} < I_0 < N - \left(\frac{1}{k_0}\right)$, then for each integer$k \geq k_0$, we define the stopping time as follows

$$\tau_k = \inf\left\{t \in [0, \tau_e]: I(t) \notin \left({}^{1}/_{k} \ , N - \left({}^{1}/_{k}\right)\right)\right\},$$

Everywhere during this paper we set $\inf(\emptyset) = \infty$, where $\emptyset =$ the empty set . Clearly, τ_k is growing as $k \to \infty$, then we set $\tau_\infty = \lim_{k\to\infty} \tau_k$, whence $\tau_\infty \leq \tau_e\ a.s$. If we can show that $\tau_\infty = \infty$ a.s., and $I(t) \in (0, N)$ a.s., for all $t \geq 0$. In other words , to complete the proof, all we need to show is that $\tau_\infty = \infty$ a.s. If this statement is false, then there is a pair of constant > 0 , and $\epsilon \in (0,1)$, such that $\mathbb{P}\{\tau_\infty \leq T\} > \epsilon$. Therefore, there is an integer $k_1 \geq k_0$ such that

$$\mathbb{P}\{\tau_k \leq T\} \geq \epsilon \quad \text{for all } k \geq k_1, \tag{4.1}$$

We define a function $V: (0, N) \to R_+$ by

$$V(x) = \frac{1}{x} + \frac{1}{N - x},$$

Via the Itô's formula we have , for any $t \in [0, T]$ and $k \geq k_1$,

$$\mathbb{E}V\big(I(t\Lambda\tau_k)\big) = V(I_0) + \mathbb{E}\int_0^{t\Lambda\tau_k} LV(I(s))d_s, \tag{4.2}$$

where $LV: (0, N) \to \mathbb{R}$ is defined by

$$LV(x) = x\left(-\frac{1}{x^2} + \frac{1}{(N - x)^2}\right)[aN - \rho - \gamma - ax]$$

$$+\sigma^2 x^2 (N - x)^2 \left(\frac{1}{x^3} + \frac{1}{(N - x)^3}\right), \tag{4.3}$$

It is easy to show that

$$LV(x) \leq \frac{\rho + \gamma}{x} + \frac{aN}{N - x} + \sigma^2 N^2 \left(\frac{1}{x} + \frac{1}{N - x}\right) \leq CV(x), \tag{4.4}$$

where $= (\rho + \gamma) \vee (\text{aN}) + \sigma^2 N^2$. By substituting this into (4.2), we get

$\mathbb{E}V\big(I(t\Lambda\tau_k)\big) \leq V(I_0) + \mathbb{E}\int_0^{t\Lambda\tau_k} CV\big(I(s)\big)d_s \leq V(I_0) + C\int_0^t \mathbb{E}V\big(I(s\Lambda\tau_k)\big)\,ds.$

The Gronwall inequality produces that

$$\mathbb{E}V\big(I(T\Lambda\tau_k)\big) \leq V(I_0)e^{CT}. \tag{4.5}$$

we set $\Omega_k = \{\tau_k \leq T\}$ for $k \geq k_1$ and, by (4.1), we get $\mathbb{P}(\Omega_k) \geq \epsilon$.

Note that for each $\omega \in \Omega_k$, $I(\tau_k, \omega)$ equals either $\frac{1}{k}$ or $N - \left(\frac{1}{k}\right)$, hence $V\big(I(\tau_k, \omega)\big) \geq k$.

It then follows from (4.5) that $V(I_0)e^{CT} \geq \mathbb{E}[I_{\Omega_k}\big(\omega\big)V(I(\tau_k, \omega))] \geq k\mathbb{P}(\Omega_k) \geq \epsilon k$.

Letting $k \to \infty$ leads to the contradiction $\infty > V(I_0)e^{CT} = \infty$, so we must therefore have $\tau_\infty = \infty$ a.s., whence the proof is complete.

5. Extinction

In the study of the dynamical behavior of population systems, it is important for us to consider the conditions required in order for the COVID-19 to die out , in other words, when the disease will become extinct.

Theorem 5.1. If the basic reproduction number for the stochastic model is

$$R_e^S = \frac{aN}{(\rho+\gamma)} - \frac{\sigma^2 N^2}{2(\rho+\gamma)} = R_e - \frac{\sigma^2 N^2}{2(\rho+\gamma)} < 1 \text{ and } \sigma^2 \leq \frac{a}{N}, \tag{5.1}$$

then, for any given initial value $I(0) = I_0 \in (0, N)$, the solution of SDE (3.3) follows

$$\lim_{t\to\infty} \sup \frac{1}{t}\log\big(I(t)\big) \leq aN - \rho - \gamma - 0.5\sigma^2 N^2 < 0\ a.s., \tag{5.2}$$

That is, I(t) tends to zero exponentially almost surely. In additional words, the disease (COVID-19) dies out with a probability of one .

Proof: By apply Itô's formula, we have

$$\log(I(t)) = \log(I_0) + \int_0^t f(I(s)ds + \int_0^t \sigma\big(N - I(s)\big)dW_s, \tag{5.3}$$

where $f:\mathbb{R}\to\mathbb{R}$ is well-defined by

$$f(x) = aN - \rho - \sigma - ax - 0.5\sigma^2(N-x)^2. \quad (5.4)$$

Though, under condition (5.1), we have

$$f(I(s)) = aN - \rho - \gamma - 0.5\sigma^2N^2 - (a-\sigma^2N)I(s) - 0.5\sigma^2 I^2(s)$$
$$\le aN - \rho - \gamma - 0.5\sigma^2N^2,$$

For $(s) \in (0, N)$. It now follows from (5.3) that

$$\log(I(t)) \le \log(I_0) + (aN - \rho - \gamma - 0.5\sigma^2N^2)t + \int_0^t \sigma(N - I(s))dW_s, \quad (5.5)$$

This indicates that

$$\lim_{t\to\infty}\sup\frac{1}{t}\log(I(t)) \le aN - \rho - \gamma - 0.5\sigma^2N^2 + \lim_{t\to\infty}\sup\frac{1}{t}\int_0^t \sigma(N-I(s))dW_s, \quad (5.6)$$

But, by the large number theorem for martingales (see e.g. [15]), we have

$$\lim_{t\to\infty}\sup\frac{1}{t}\int_0^t \sigma(N-I(s))dW_s = 0\ , a.s.$$

We as a result get the desired assertion (5.2) from (5.6). It is useful to note that in the deterministic SIR model (1.1), *I(t)* tends to 0 if and only if $R_e \le 1$, while in the SDE SIR model (3.1), *I(t)* tends to 0 if $R_e^s = R_e - \frac{\sigma^2N^2}{2(\rho+\gamma)} < 1$ and $\sigma^2 \le \frac{a}{N}$, in other words, $I(t)$ tends to zero exponentialy

Theorem 5.2. If the basic reproduction number for the stochastic model is

$$R_e^S = \frac{aN}{(\rho+\gamma)} - \frac{\sigma^2N^2}{2(\rho+\gamma)} = R_e - \frac{\sigma^2N^2}{2(\rho+\gamma)} > 1, \quad (5.7)$$

Then for every given initial value $I(0) = I_0 \in (0, N)$, the solution of the SDE SIR model (3.3) follows

$$\lim_{t\to\infty}\sup I(t) \ge \xi \quad a.s. \quad (5.8)$$

and

$$\lim_{t\to\infty}\inf I(t) \le \xi \quad a.s\ , \quad (5.9)$$

where

$$\xi = \frac{1}{\sigma^2}\left(\sqrt{a^2 - 2\sigma^2(\rho+\gamma)} - (a - \sigma^2N)\right), \quad (5.10)$$

which is the unique root in (0,N) of

$$aN - \rho - \gamma - a\xi - 0.5\sigma^2(N-\xi)^2 = 0. \quad (5.11)$$

Namely, $I(t)$ will rise to ξ or above the level ξ infinitely, often with the probability of one.

Proof. Recall the definition (5.4) of function $f:\mathbb{R}\to\mathbb{R}$. By condition (5.7), it is easy to see that equation $f(x) = 0$ has a positive root and a negative root. The positive one is

$$\frac{1}{\sigma^2}\left(\sqrt{(a-\sigma^2N)^2 + 2\sigma^2(aN - \rho - \gamma - 0.5\sigma^2N^2)} - (a-\sigma^2N)\right)$$
$$= \frac{1}{\sigma^2}\left(\sqrt{a^2 - 2\sigma^2(\rho+\gamma)} - (a-\sigma^2N)\right) = \xi.$$

Noting that

$$f(0) = aN - \rho - \gamma - 0.5\sigma^2N^2 > 0 \text{ and } f(N) = -\rho - \gamma < 0,$$

we see that $\xi \in (0, N)$ and

$$f(x) > 0 \text{ is strictly increasing on } x \in (0, 0 \vee \hat{x}), \quad (5.12)$$

$$f(x) > 0 \text{ is strictly decreasing on } x \in (0 \vee \hat{x}, \xi), \quad (5.13)$$

While

$$f(x) < 0 \text{ is strictly decreasing on } x \in (\xi, N), \quad (5.14)$$

We now begin to prove assertion (5.8). If it is not true, then there is a sufficiently small $\epsilon \in (0,1)$ such that

$$\mathbb{P}(\Omega_1) > \epsilon, \quad (5.15)$$

where $\Omega_1 = \{\limsup_{t\to\infty} I(t) \le \xi - 2\epsilon\}$. Hence , for every $\omega \in \Omega_1$, there is $T = T(\omega) > 0$, such that

$$I(t,\omega) \le \xi - \epsilon \text{ whenever } t \ge T(\omega). \quad (5.16)$$

Obviously, we may choose ϵ to be so small (if it is essential to reduce it) that $f(0) > f(\xi - \epsilon)$. It therefore
follows from (5.12), (5.13), and (5.16) that

$$f\big(I(t,\omega)\big) \geq f(\xi - \epsilon) \text{ whenever } t \geq T(\omega). \quad (5.17)$$

Furthermore, by the large number theorem for martingales, there is a $\Omega_2 \in \Omega$ with $\mathbb{P}(\Omega_2) = 1$ such that for each $\omega \in \Omega_2$,

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t \sigma\big(N - I(s,\omega)\big)dW(s,\omega) = 0. \quad (5.18)$$

Now, by fixing any $\in \Omega_1 \cap \Omega_2$, it then follows from (5.3) and (5.17) that , for t $\geq T(\omega)$,

$$\log(I(t,\omega)) \geq \log(I_0) + \int_0^{T(\omega)} f(I(s,\omega))ds + f(\xi - \epsilon)\big(t - T(\omega)\big) + \int_0^t \sigma\big(N - I(s,\omega)\big)dW(s,\omega). \quad (5.19)$$

This produces $\liminf_{t\to\infty} \frac{1}{t}\log(I(t,\omega)) \geq f(\xi - \epsilon) > 0$,
whence $\lim_{t\to\infty} I(t,\omega) = \infty$.
Nonetheless, this contradicts (5.16). Hence, we necessarily have the desired assertion (5.8). Let us now prove the assertion (5.9). If it is not true, then there is a sufficiently small $\delta \in (0,1)$ such that

$$\mathbb{P}(\Omega_3) > \delta, \quad (5.20)$$

where $\Omega_3 = \{\liminf_{t\to\infty} I(t) \geq \xi + 2\delta\}$. Hence, for every $\omega \in \Omega_3$, there is a $\tau = \tau(\omega) > 0$ such that

$$I(t,\omega) \geq \xi + \delta \text{ whenever } t \geq \tau(\omega). \quad (5.21)$$

Now , we fix any $\omega \in \Omega_2 \cap \Omega_3$. It then follows from (5.3) and (5.14) that , for $t \geq \tau(\omega)$,

$$\log(I(t,\omega)) \leq \log(I_0) + \int_0^{\tau(\omega)} f(I(s,\omega))ds + f(\xi + \delta)\big(t - \tau(\omega)\big) + \int_0^t \sigma\big(N - I(s,\omega)\big)dW(s,\omega). \quad (5.22)$$

This, together with (5.18), yields

$$\limsup_{t\to\infty} \frac{1}{t}\log(I(t,\omega)) \leq f(\xi + \delta) < 0,$$

whence $\lim_{t\to\infty} I(t,\omega) = 0$.
But this contradicts (5.21). We therefore must have the desired assertion (5.9).

6. Main results

To comprehend the effects of introducing environmental stochasticity into the system (3.1) on the stability of the system and the extinction of the disease, we will take an example of the spread of coronavirus in Iraq. All data were obtained from the Iraqi Ministry of Health and from another report [1].

Table 2- The spread of Covid-19 in Iraq, finding the basic reproduction number for the deterministic model and the stochastic model when the population is 40397492 [1] and $\sigma = 0.0001$.

Date	New cases	Active cases	New deaths	New recoveries	R_e	R_e^s
13/8/2020	3841	592	53	2667	2.205882353	0.367648235
14/8/2020	4013	627	68	2921	2.007360321	0.334560053
15/8/2020	4293	590	76	2571	2.266717038	0.377786172
16/8/2020	4348	572	75	2674	2.182611859	0.363768643
17/8/2020	3202	605	94	3571	1.637107776	0.272851295

18/8/2020	4576	574	82	2895	2.015451797	0.335908632
19/8/2020	4093	582	85	2529	2.295332823	0.38255547
20/8/2020	3995	614	87	2831	2.o56202879	0.34270048
21/8/2020	4288	587	75	3246	1.806684734	0.301114122
22/8/2020	3965	637	70	2947	1.988730527	0.331455087
23/8/2020	3291	661	75	3016	1.941119379	0.323519896
24/8/2020	3644	538	91	3980	1.473839352	0.245639892
25/8/2020	3962	574	77	3372	1.739634677	0.289939113
26/8/2020	3837	595	72	3454	1.701644923	0.283607486
27/8/2020	3651	566	72	3794	1.551991723	0.258665287
28/8/2020	4177	582	74	3865	1.523229246	0.253871541
29/8/2020	3834	579	77	4146	1.4207909007	0.236798477
30/8/2020	3731	542	68	3860	1.527494908	0.254582484
31/8/2020	3757	579	83	3722	1.576872536	0.262812089
1/9/2020	3404	521	81	3871	1.518218623	0.253036436
2/9/2020	3946	527	78	3732	1.57480315	0.262467192
3/9/2020	4755	496	74	3552	1.65471594	0.275785989
4/9/2020	5026	495	84	3611	1.623815968	0.270635995
5/9/2020	4644	523	63	3891	1.737116387	0.472574151
6/9/2020	3651	507	90	3301	1.769389561	0.294898826
7/9/2020	4314	534	77	4299	1.371115174	0.228519195
8/9/2020	4894	564	68	3465	1.698273422	0.28304557
9/9/2020	4243	540	75	3669	1.602564103	0.267094017
10/9/2020	4597	542	82	3824	1.53609831	0.256016384
11/9/2020	4254	566	67	3579	1.645639057	0.274273177
12/9/2020	4106	564	60	3887	1.52014188	0.25335698
13/9/2020	3531	564	73	3422	1.716738197	0.286122032

When calculating the average of parameters from 13/8/2020 to 13/9/2020 and substitute it into equation (3.3), the equation will be as follows

$$dI(t) = (2481.5625)I(t)dt + 100I(t)dW_t \quad, I(0) = 1 , \qquad (6.1)$$

Now by Itô's formula ,we have the solution as:

$I(t) = \exp[-2518.4375\, t + 100W_t]$, (6.2)

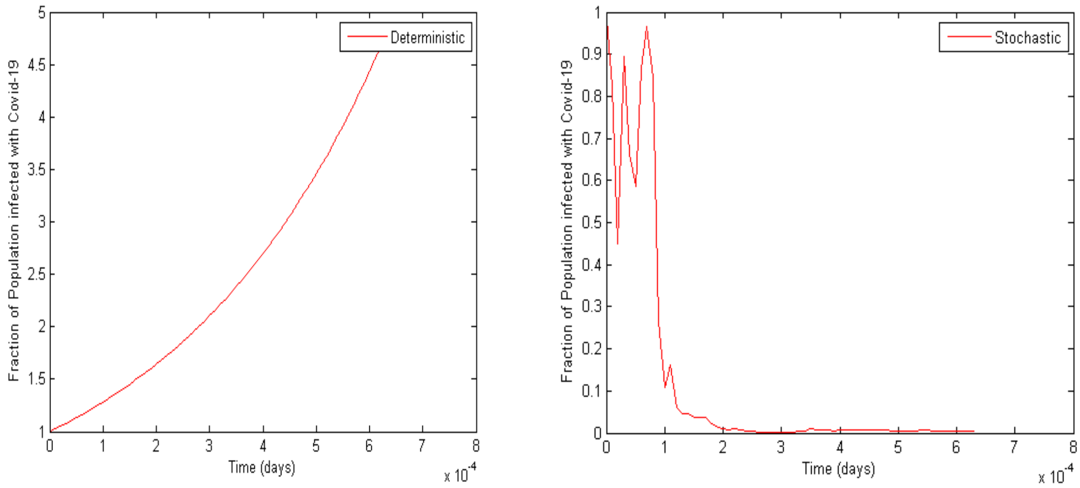
So, we can therefore conclude, by theorem (5.1), that for any initial value $I(0) = I_0 \in (0,1000000)$ the solution (6.2) obeys

$$\limsup_{t\to\infty} \frac{1}{t}\log(I(t)) \le -2518.4375\, .a.s,$$

This implies that I(t) will tend to zero exponentially with the probability of one.

But, on the other hand, for the corresponding deterministic SIR model (1.1) , the basic reproduction number for the deterministic model is $R_e > 1$.

The computer simulation in Figure-1, by using Euler Maruyama method (EM), supports these result clearly, illustrating the extinction of COVID-19 .



(b) (a)

Figure 1-Computer simulation of the path I(t) using the EM method with step size Δ= 0.00001, &I(0) = 1, equation (6.1); (a) Deterministic SIR model (b) Stochastic SIR model .

Table 3-The spread of COVID-19 in Iraq , finding the basic reproduction number for the deterministic model and the stochastic model when the population is 40397492 and $\sigma = 0.00001$.

Date	New cases	Active cases	New Deaths	New recoveries	R_e	R_e^s
13/8/2020	3841	592	53	2667	2.205882353	2.205882353
14/8/2020	4013	627	68	2921	2.007360321	1.990632318
15/8/2020	4293	590	76	2571	2.266717038	2.247827729
16/8/2020	4348	572	75	2674	2.182611859	2.164423427
17/8/2020	3202	605	94	3571	1.637107776	1.623465211
18/8/2020	4576	574	82	2895	2.015451797	1.998656365
19/8/2020	4093	582	85	2529	2.295332823	2.276205049
20/8/2020	3995	614	87	2831	2.o56202879	2.039067855
21/8/2020	4288	587	75	3246	1.806684734	1.791629028
22/8/2020	3965	637	70	2947	1.988730527	1.972157773
23/8/2020	3291	661	75	3016	1.941119379	1.924943384
24/8/2020	3644	538	91	3980	1.473839352	1.461557357
25/8/2020	3962	574	77	3372	1.739634677	1.725137721
26/8/2020	3837	595	72	3454	1.701644923	1.687464549
27/8/2020	3651	566	72	3794	1.551991723	1.539058459
28/8/2020	4177	582	74	3865	1.523229246	1.510535669
29/8/2020	3834	579	77	4146	1.4207909007	1.408950976
30/8/2020	3731	542	68	3860	1.527494908	1.514765784
31/8/2020	3757	579	83	3722	1.576872536	1.563731932

1/9/2020	3404	521	81	3871	1.518218623	1.505566801
2/9/2020	3946	527	78	3732	1.57480315	1.56167979
3/9/2020	4755	496	74	3552	1.65471594	1.64092664
4/9/2020	5026	495	84	3611	1.623815968	1.610284168
5/9/2020	4644	523	63	3891	1.737116387	1.724470965
6/9/2020	3651	507	90	3301	1.769389561	1.754644648
7/9/2020	4314	534	77	4299	1.371115174	1.359689214
8/9/2020	4894	564	68	3465	1.698273422	1.684121143
9/9/2020	4243	540	75	3669	1.602564103	1.589209402
10/9/2020	4597	542	82	3824	1.53609831	1.523297491
11/9/2020	4254	566	67	3579	1.645639057	1.631925398
12/9/2020	4106	564	60	3887	1.52014188	1.507473151
13/9/2020	3531	564	73	3422	1.716738197	1.702432045

When calculating the average of parameters from 13/8/2020 to 13/9/2020 and substitute it into equation (3.3), the equation will be, as follows

$$dI(t) = (2481.5625)I(t)dt + 10I(t)dW_t \quad , I(0) = 1 , \tag{6.3}$$

Now, by Itô's formula , the solution will be as follows :

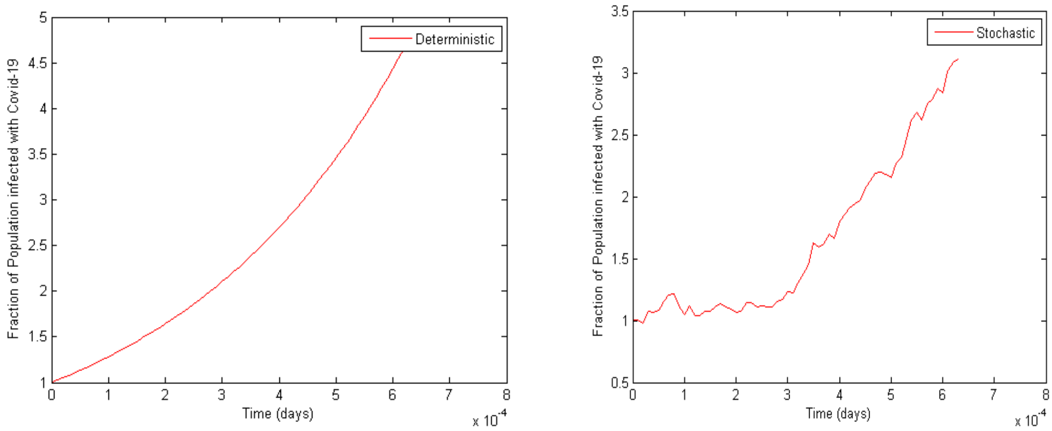
$$I(t) = \exp[2481.5625\, t + 10W_t] , \tag{6.4}$$

$$\xi = \frac{1}{\sigma^2}\left(\sqrt{a^2 - 2\sigma^2(\rho + \gamma)} - (a - \sigma^2 N)\right) = 382762.65,$$

Then by theorem (5.2), for every given initial value $I(0) = I_0 \in (0 , 1000000)$, the solution of equation (6.3) obeys

$$\lim_{t\to\infty} \inf I(t) \leq 382762.65 \quad \leq \limsup_{t\to\infty} I(t), \quad a.s$$

Which implies that I(t) will not tend to zero exponentially with the probability of one. The computer simulation in Figure-2 supports these results .



(a) (b)

Figure 2-Computer simulation of the path I(t) using the EM method with Δ= 0.00001 and I(0) = 1, equation (6.3); (a) Stochastic SIR model, (b) Deterministic SIR model.

7.Conclusions

In this article, we introduced environmental stochasticity into the deterministic SIR model [7]. We explored the properties for the resulting stochastic SIR model for the spread of COVID-19. This was achieved by first proving that there exists a unique positive solution $I(t)$ for any given initial value $I_0 \in (0, N)$. Furthermore, we constructed R_e^s and the condition required for the extinction and the persistence for our Solution $I(t)$. In general, if $R_e^s < 1$, the solution will almost surely extinct, as shown in theorem (5.1).The computer simulation shown in Figure-1b supports these result clearly, illustrating the extinction of Covid-19. If $R_e^s > 1$. This means that the solution will not tend to zero, as shown in theorem (5.2). The computer simulation shown in Figure-2a supports these result. Also, we conclude through this paper that we can control the stability of the system through environmental stochasticity (σ), if it is large, as shown in Table 2. We find that $R_e^s < 1$ and the system will be stable. Moreover, if σ is small, as in Table 3, we find that $R_e^s > 1$ which implies that the system is unstable.

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