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Application of Two Rowed Weyl Module in the Case of Partition (7,6) and Skew- Partition (7,6)/(1,0)

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Abstract
The aim of this work is to study the application of Weyl module resolution in the case of two rows, which will be specified in the partition (7, 6) and skew- partition (7,6)/(1,0) by using the homological Weyl (i.e. the contracting homotopy and place polarization).

Keywords: Divided power algebra, resolution of Weyl module, place polarization, mapping Cone.

تطبيق مقاس وايل لصفين في حالة التجزئة (7,6) و (7,6)/(1,0)

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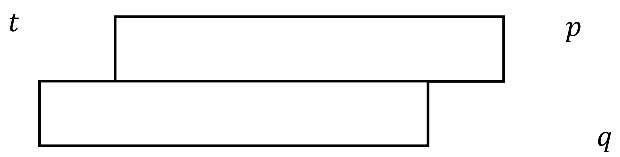
الخلاصة
الغرض من هذا البحث هو دراسة تطبيق تحلل مقاس وايل في حالة الصفين والتي ستكون محددة بالتجزئة (7 ,6) وشبه التجزئة (7,6)/(1,0) وذلك باستخدام طرق همولوجية(أي التوافق الهوموتوبي ودالة المكان).

1.Introduction

Let R be a commutative ring with identity , F be a free R-module and D_bF be the divided power of degree b.

Consider the figure below which is associated to the resolution of two-rowed Weyl module $K_{\lambda/\mu}F$ = Im($d'_{\lambda/\mu}$) where $d'_{\lambda/\mu}$ is the Weyl map that is described in [1], as follows:

λ/μ=



(1)

We have:

$$\sum D_{p+k} \otimes D_{q-k} \xrightarrow{\square} D_p \otimes D_q \xrightarrow{d'_{\lambda/\mu}} K_{\lambda/\mu} \to 0 \quad (2)$$

And by using letter-place algebra, the maps will be explained now as follows:

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$$\begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(p+k)} \\ 2^{(q-k)} \end{matrix}\right) \xrightarrow{\partial_{21}^{(k)}} \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(p)}2^{(k)} \\ 2^{(q-k)} \end{matrix}\right) \longrightarrow \sum\nolimits_{w} \begin{pmatrix} w_{(1)} \\ w'w_{(2)} \end{pmatrix}\!\!\left|\begin{matrix} (t+1)'(t+2)'\ldots(p+t)' \\ 1'2'3'\ldots q' \end{matrix}\right) \quad (3)$$

where

$$w \otimes w' \in D_{p+k} \otimes D_{q-k} \quad , \qquad \Box = \sum\nolimits_{k=t+1}^{q} \partial_{21}^{(k)}$$

and

$$d'_{\lambda/\mu} = \partial_{q'2} \ldots \partial_{1'2} \partial_{(p+t)'1} \ldots \partial_{(t+1)'1}$$

is the composition of place polarization, from positive places {1,2} to negative places {1', 2', … , (p+t)'}.

And, as shown in [2], $\Box$ is deliver a component $x \otimes y$ of $D_{p+k} \otimes D_{q-k}$ to $\sum x_p \otimes x'_k y$ where $\sum x_p \otimes x'_k$ is the element of the diagonal of x in $D_p \otimes D_k$.

Let Z_{21} be the free generator of divided power algebra $D(Z_{21})$ in one generator, then the divided power component $Z_{21}^{(k)}$ of degree k of the free generator Z_{21} acts on $D_{p+k} \otimes D_{q-k}$ by place polarization of degree k from place 1 to place 2.

Particularly, the 'graded' algebra 'with identity' $A = D(Z_{21})$ acts on the graded module $M = \sum D_{p+k} \otimes D_{q-k} = \sum M_{q-k}$, where the degree of the 2nd factor dictates the grading, see [3, 4, 5].

Therefore, M is a graded left A-module, where, for $w = Z_{21}^{(k)} \in A$ and $v \in D_{\beta_1} \otimes D_{\beta_2}$, by definition, we have:

$$w(v) = Z_{21}^{(k)}(v) = \partial_{21}^{(k)}(v) . \quad (4)$$

And if we have (t^+), which is the graded strand of degree q

$$M_{\bullet} : 0 \to M_{q-t} \xrightarrow{\partial_s} \ldots \to M_l \xrightarrow{\partial_s} \ldots M_1 \xrightarrow{\partial_s} M_0 \quad (5)$$

of the normalized bar complex, Bar(M, A; S, •), and S = {x}.

By definition, $M_{\bullet}$ is the following complex:

$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \ldots x Z_{21}^{(k_l)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_l}$$

$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \ldots x Z_{21}^{(k_l-1)} x D_{P+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_{l-1}}$$

$$\ldots \xrightarrow{d_1} \sum\nolimits_{k_i \geq 0} Z_{21}^{(t+k)} x D_{p+t+|k|} \otimes D_{q-t-k} \xrightarrow{d_0} D_p \otimes D_q \quad (6)$$

where $|k| = \sum k_i$ and d_l is the boundary operator ∂_x . Notice that (6) illustrates a left complex ($\partial_x^2 = 0$) over the Weyl module in terms of bar complex and letter-place algebra. Furthermore, in (6), when the separator x disappears between a $Z_{ab}^{(t)}$ and the components in the tensor product of divided powers, this means that $\partial_{ab}^{(t)}$ is applied to the tensor product (see [1] and [6]).

The authors in [4] and [5] exhibited the terms and the exactness of the Weyl module resolution in the case of partition (8,7) and skew-shape (8,6)/(2,1) . In this work, we locate the terms and the exactness of the Weyl module Resolution in the cases of partition(7,6) and skew- partition (7,6)/(1,0).

2. Application of Weyl Module Resolution in the Case of Partition (7,6).

2.1 The Terms of Weyl Module Resolution in the Case of Partition (7, 6).

In this section, we find the term for the resolution of Weyl module in the case of the Partition (7,6).

$M_0 = D_7 \otimes D_6$

$M_1 = Z_{21} x D_8 \otimes D_5 \oplus Z_{21}^{(2)} x D_9 \otimes D_4 \oplus Z_{21}^{(3)} x D_{10} \otimes D_3 \oplus Z_{21}^{(4)} x D_{11} \otimes D_2$
$\quad \oplus Z_{21}^{(5)} x D_{12} \otimes D_1 \oplus Z_{21}^{(6)} x D_{13} \otimes D_0$

$M_2 = Z_{21} x Z_{21} x D_9 \otimes D_4 \oplus Z_{21}^{(2)} x Z_{21} x D_{10} \otimes D_3 \oplus Z_{21} x Z_{21}^{(2)} x D_{10} \otimes D_3$
$\quad \oplus Z_{21}^{(3)} x Z_{21} x D_{11} \otimes D_2 \oplus Z_{21} x Z_{21}^{(3)} x D_{11} \otimes D_2 \oplus Z_{21}^{(2)} x Z_{21}^{(2)} x D_{11} \otimes D_2$
$\quad \oplus Z_{21}^{(4)} x Z_{21} x D_{12} \otimes D_1 \oplus Z_{21} x Z_{21}^{(4)} x D_{12} \otimes D_1 \oplus Z_{21}^{(2)} x Z_{21}^{(3)} x D_{12} \otimes D_1$

$$\oplus\, Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{12}\otimes D_1 \;\oplus\; Z_{21}^{(5)}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}\varkappa Z_{21}^{(5)}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}^{(2)}\varkappa Z_{21}^{(4)}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa D_{13}\otimes D_0$$

$$M_3 = Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10}\otimes D_3 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11}\otimes D_2 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{11}\otimes D_2$$
$$\oplus\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{11}\otimes D_2 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12}\otimes D_1 \oplus Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{12}\otimes D_1$$
$$\oplus\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{12}\otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{12}\otimes D_1 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_{12}\otimes D_1$$
$$\oplus\, Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{12}\otimes D_1 \oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \oplus Z_{21}\varkappa Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(4)}\varkappa D_{13}\otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{13}\otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa D_{13}\otimes D_0$$

$$M_4 = Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11}\otimes D_2 \;\oplus\; Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12}\otimes D_1$$
$$\oplus\, Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12}\otimes D_1 \;\oplus\; Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{12}\otimes D_1$$
$$\oplus\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{12}\otimes D_1 \;\oplus\; Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0$$
$$\oplus\, Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21} x D_{13}\otimes D_0$$
$$\oplus\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{13}\otimes D_0$$

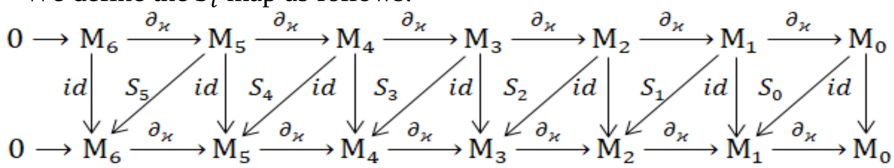
$$M_5 = Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12}\otimes D_1 \;\oplus\; Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0$$
$$\oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0$$
$$\oplus Z_{21}\varkappa\, Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0 \;\oplus\; Z_{21}\varkappa\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13}\otimes D_0$$
$$\oplus Z_{21}\varkappa\, Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13}\otimes D_0$$

$$M_6 = Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0$$

2.2 The Exactness of Weyl Resolution in the Case of Partition (7,6)

This section explains the building of contracting homotopies $\{S_i\}$, where i=1,2,…,5.

We define the S_i map as follows:



$S_0: M_0 \longrightarrow M_1$

$$S_0\left(\begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(7)}2^{(k)} \\ 2^{(6-k)} \end{matrix}\right.\right) = \begin{cases} Z_{21}^{(k)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(7+k)} \\ 2^{(6-k)} \end{matrix}\right. & ;if\ k>0 \\ 0 & ;if\ k\le 0 \end{cases}$$

$S_1: M_1 \longrightarrow M_2$

$$S_1\left(Z_{21}^{(k)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(7+k)}2^{(m)} \\ 2^{(6-k-m)} \end{matrix}\right.\right) = \begin{cases} Z_{21}^{(k)}\varkappa Z_{21}^{(m)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(7+k+m)} \\ 2^{(6-k-m)} \end{matrix}\right. & ;if\ m=1,2,3,4,5 \\ 0 & ;if\ m=0 \end{cases}$$

$S_2: M_2 \longrightarrow M_3$

$$S_2\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(7+|m|)}2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right.\right) =$$

$$\begin{cases} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(m)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right. & ;if\ m=1,2,3,4 \\ 0 & ;if\ m=0 \end{cases} \quad ;\text{ where } |k| = k_1+k_2$$

$S_3: M_3 \longrightarrow M_4$

$$S_3\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(7+|k|)}2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right.\right)$$

$$= \begin{cases} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right) & ; if\ m = 1,2,3 \\ 0 & ; if\ m = 0 \end{cases};$$

where $|k| = k_1 + k_2 + k_3$

$S_4 : M_4 \longrightarrow M_5$

$$S_4\left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right)\right)$$

$$= \begin{cases} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right) & ; if\ m = 1,2 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1 + k_2 + k_3 + k_4$

$S_5 : M_5 \longrightarrow M_6$

$$S_5\left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right)\right)$$

$$= \begin{cases} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right) & ; if\ m = 1 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1 + k_2 + k_3 + k_4 + k_5$

$$S_0 \partial_\varkappa \left(Z_{21}^{(k)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k)} 2^{(m)} \\ 2^{(6-k-m)} \end{matrix}\right)\right) = S_0 \partial_{21}^{(k)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k)} 2^{(m)} \\ 2^{(6-k-m)} \end{matrix}\right)$$

$$= \binom{k+m}{m} Z_{21}^{(k+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(6-k-m)} \end{matrix}\right)$$

and

$$\partial_\varkappa S_1 \left(Z_{21}^{(k)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k)} 2^{(m)} \\ 2^{(6-k-m)} \end{matrix}\right)\right) = \partial_\varkappa \left(Z_{21}^{(k)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(6-k-m)} \end{matrix}\right)\right)$$

$$= -\binom{k+m}{m} Z_{21}^{(k+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(6-k-m)} \end{matrix}\right) + Z_{21}^{(k)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k)} 2^{(m)} \\ 2^{(6-k-m)} \end{matrix}\right)$$

$$= Z_{21}^{(k)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k)} 2^{(m)} \\ 2^{(6-k-m)} \end{matrix}\right)$$

It is clear that $S_0 \partial_\varkappa + \partial_\varkappa S_1 = id_{M_1}$.

$$S_1 \partial_\varkappa \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right)\right)$$

$$= S_1 \left(-\binom{|k|}{k_2} Z_{21}^{|k|} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right) + Z_{21}^{(k_1)} \varkappa\, \partial_{21}^{(k_2)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right)\right)$$

$$= -\binom{|k|}{k_2} Z_{21}^{|k|} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right) +$$

$$\binom{k_2+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right),$$

and

$$\partial_\varkappa S_2 \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right)\right) = \partial_\varkappa \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(6-k-m)} \end{matrix}\right)\right)$$

$$= \binom{|k|}{k_2} Z_{21}^{|k|} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right) -$$

$$\binom{k_2+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix}\right) + Z_{21}^{(k_1)} \varkappa\, Z_{21}^{(k_2)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right),$$

where $|k| = k_1 + k_2$.

It is clear that $S_1 \partial_\varkappa + \partial_\varkappa S_2 = id_{M_2}$.

$$S_2 \partial_\varkappa \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix}\right)\right)$$

$$=S_2\left(\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)-\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)+\right.$$
$$\left.Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)\right)$$
$$=\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_3+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right),$$

and

$$\partial_\varkappa S_3\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)\right)$$
$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_3+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\partial_{21}^{(m)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)$$
$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_3+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right),$$

where $|k| = k_1+k_2+k_3$.
It is clear that $S_2\partial_\varkappa+\partial_\varkappa S_3 = id_{M_3}$.

$$S_3\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)\right)$$
$$=S_3\left(-\binom{k_1+k_2}{k}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)+\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\right)$$
$$\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)\binom{k_3+k_4}{k_4}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)$$
$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_3+k_4}{k_4}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_4+m}{m}Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right),$$

and

$$\partial_\varkappa S_4\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(6-|k|-m)}\end{matrix}\right)\right)$$
$$=\partial_\varkappa\left(Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)\right)$$

$= \binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_4+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa \partial_{21}^{(m)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right)$

$= \binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_4+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right),$

where $|k| = k_1+k_2 + k_3+k_4$.

It is clear that $S_3\partial_\varkappa + \partial_\varkappa S_4 = id_{M_4}$.

$S_4\partial_\varkappa \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) \right)$

$= S_4 \Bigg(\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_4+k_5}{k_5} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4+k_5)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} Z_{21}^{(k_4)} \varkappa \partial_{21}^{(k_5)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) \Bigg)$

$= \binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$
$\binom{k_4+k_5}{k_5} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4+k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$= \binom{k_4+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5+m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right),$

and

$\partial_\varkappa S_5 \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) \right)$

$= \partial_\varkappa \left(Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) \right)$

$= -\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) +$
$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{matrix} \right) -$

$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix} +$

$\binom{k_4+k_5}{k_5} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4+k_5)} \varkappa Z_{21}^{(m)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix} -$

$\binom{k_4+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5+m)} \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix} +$

$Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \partial_{21}^{(m)} \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix}$

$= -\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix} +$

$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2+k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix} -$

$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3+k_4)} \varkappa Z_{21}^{(k_5)} \varkappa Z_{21}^{(m)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix} +$

$\binom{k_4+k_5}{k_5} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_{4+k_5})} \varkappa Z_{21}^{(m)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|+m)} \\ 2^{(6-|k|-m)} \end{pmatrix} -$

$\binom{k_5+m}{m} Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5+m)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{pmatrix} +$

$Z_{21}^{(k_1)} \varkappa Z_{21}^{(k_2)} \varkappa Z_{21}^{(k_3)} \varkappa Z_{21}^{(k_4)} \varkappa Z_{21}^{(k_5)} \varkappa \begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} 2^{(m)} \\ 2^{(6-|k|-m)} \end{pmatrix},$

where $|k| = k_1+k_2+k_3+k_4+k_5$.

It is clear that $S_4\partial_\varkappa + \partial_\varkappa S_5 = id_{M_5}$.

From the above, we get that $\{S_0, S_1, S_2, S_3, S_4, S_5\}$ is a contracting homotopy [7], which means that the complex

$0 \to M_6 \to M_5 \to M_4 \to M_3 \to M_2 \to M_1 \to M_0$ is exact.

3. Application of Weyl Module Resolution in the Case of the skew- Partition (7, 6)/(1, 0).

3.1 The Terms of Weyl Module Resolution

The resolution of Weyl Module associated to this case has the following terms.

$M_0 = D_6 \otimes D_6$

$M_1 = Z_{21}^{(2)}\varkappa D_8 \otimes D_4 \oplus Z_{21}^{(3)}\varkappa D_9 \otimes D_3 \oplus Z_{21}^{(4)}\varkappa D_{10} \otimes D_2$
$\oplus Z_{21}^{(5)}\varkappa D_{11} \otimes D_1 \oplus Z_{21}^{(6)}\varkappa D_{12} \otimes D_0$

$M_2 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_9 \otimes D_3 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{10} \otimes D_2 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_2$
$\oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{11} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa D_{11} \otimes D_1$
$\oplus Z_{21}^{(5)}\varkappa Z_{21}\varkappa D_{12} \otimes D_0 \oplus Z_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(4)}\varkappa D_{12} \otimes D_0$
$\oplus Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa D_{12} \otimes D_0$

$M_3 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_2 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_1$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{11} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_1$
$\oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{12} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{12} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{12} \otimes D_0$

$M_4 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{12} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_0$

$M_5 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_0$

3.2 The Exactness of Weyl Resolution in case

This section explains the building of contracting homotopies $\{S_i\}$, where i=1,2,3,4 in the case of the skew-partition(7,6)/(1,0)

We have the following homotopies:

$S_0: D_6 \longrightarrow D_6 \longrightarrow \sum_{k>0} Z_{21}^{(k+1)}\varkappa\, D_{6+k} \otimes D_{6-k}$

$$S_0\left(\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6)}2^{(k)} \\ 2^{(6-k)} \end{matrix}\right)\right) = \begin{cases} Z_{21}^{(k)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)} \\ 2^{(6-k)} \end{matrix}\right) & ; if\ k = 1,2,3,4,5,6 \\ 0 & ; if\ k = 0 \end{cases}$$

$S_1: \sum_{k>0} Z_{21}^{(k+1)}\varkappa\, \mathcal{D}_{7+k} \otimes \mathcal{D}_{5-k} \longrightarrow Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa\, \mathcal{D}_{7+k} \otimes \mathcal{D}_{5-k}$ such that:

$$S_1\left(Z_{21}^{(k+1)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k)}2^{(m)} \\ 2^{(5-k-m)} \end{matrix}\right)\right) = \begin{cases} Z_{21}^{(k+1)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(5-k-m)} \end{matrix}\right) & ; if\ m = 1,2,3,4 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1 + k_2$

$S_2: \sum_{k_i>0} Z_{21}^{(k_1+1)}\varkappa\, Z_{21}^{(k_2)}\varkappa\, \mathcal{D}_{7+|k|} \otimes \mathcal{D}_{5-|k|} \longrightarrow Z_{21}^{(k_1+1)}\varkappa\, Z_{21}^{(k_2)}\varkappa\, Z_{21}^{(k_3)}\varkappa\, \mathcal{D}_{7+|k|} \otimes \mathcal{D}_{5-|k|}$
such that:

$$S_2\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)}2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)\right) =$$

$$\begin{cases} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) & ; if\ m = 1,2,3 \\ 0 & ; if\ m = 0 \end{cases};$$

where $|k| = k_1 + k_2$

$S_3: \sum_{k_i>0} Z_{21}^{(k_1+1)}\varkappa\ Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\ \mathcal{D}_{7+|k|} \otimes \mathcal{D}_{5-|k|} \longrightarrow Z_{21}^{(k_1+1)}\varkappa\ Z_{21}^{(k_2)}\varkappa\ Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\ \mathcal{D}_{7+|k|} \otimes \mathcal{D}_{5-|k|}$

$$S_3\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)}2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)\right)$$

$$= \begin{cases} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) & ; if\ m = 1,2 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1 + k_2 + k_3$

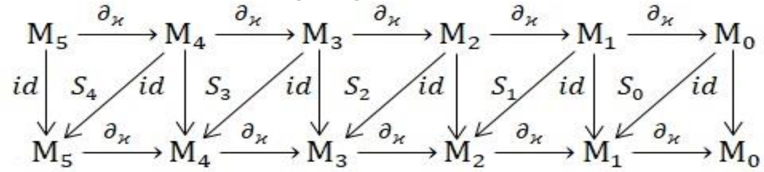
$S_4: \sum_{k_i>0} Z_{21}^{(k_1+1)}\varkappa\ Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\ Z_{21}^{(k_4)}\varkappa\, \mathcal{D}_{7+|k|} \otimes \mathcal{D}_{5-|k|} \longrightarrow Z_{21}^{(k_1+1)}\varkappa\ Z_{21}^{(k_2)}\varkappa\ Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\ Z_{21}^{(k_5)}\varkappa\, \mathcal{D}_{7+|k|} \otimes \mathcal{D}_{5-|k|}$

$$S_4\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)}2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)\right)$$

$$= \begin{cases} Z_{21}^{(k_1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{matrix}\right) & ; if\ m = 1 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1 + k_2 + k_3 + k_4$

So, we have the following diagram:



$$S_0\partial_\varkappa\left(Z_{21}^{(k+1)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k)}2^{(m)} \\ 2^{(5-k-m)} \end{matrix}\right)\right) = S_0\partial_{21}^{(k+1)}\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7)}2^{(k+m)} \\ 2^{(5-k-m)} \end{matrix}\right) =$$

$$= \binom{k+1+m}{m} Z_{21}^{(k+1+m)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(5-k-m)} \end{matrix}\right),$$

and

$$\partial_\varkappa S_1\left(Z_{21}^{(k+1)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k)}2^{(m)} \\ 2^{(5-k-m)} \end{matrix}\right)\right) = \partial_\varkappa\left(Z_{21}^{(k+1)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(5-k-m)} \end{matrix}\right)\right)$$

$$= -\binom{k+1+m}{m} Z_{21}^{(k+1+m)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(5-k-m)} \end{matrix}\right) + Z_{21}^{(k+1)}\varkappa\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k)}2^{(m)} \\ 2^{(5-k-m)} \end{matrix}\right),$$

It is clear that $S_0\partial_\varkappa + \partial_\varkappa S_1 = id_{M_1}$.

$S_1\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=S_1\left(-\binom{|k|+1}{k_2}Z_{21}^{|k|+1}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+\quad Z_{21}^{(k_1+1)}\varkappa\,\partial_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=-\binom{|k|+1}{k_2}Z_{21}^{|k|+1}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_2+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

and

$\partial_\varkappa S_2\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+k+m)}\\2^{(5-k-m)}\end{matrix}\right)\right)$

$=\binom{|k|+1}{k_2}Z_{21}^{|k|+1}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_2+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(6-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1+1)}\varkappa\,Z_{21}^{(k_2)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

where $|k|= k_1+k_2$.

It is clear that $S_1\partial_\varkappa+\partial_\varkappa S_2=id_{M_2}$.

$S_2\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=S_2\left(\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)-\right.$

$\left.\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+\quad Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_3+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right),$

and

$\partial_\varkappa S_3\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=-\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_3+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+m)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\2^{(5-|k|-m)}\end{matrix}\right)$

where $|k| = k_1+k_2+k_3$.

It is clear that $S_2\partial_\varkappa+\partial_\varkappa S_3=id_{M_3}$.

$S_3\partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)\right)$

$=S_3\left(-\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+\right.$

$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa)\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)-$

$\binom{k_3+k_4}{k_4}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\2^{(5-|k|-m)}\end{matrix}\right)+$

$$Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa\partial_{21}^{(k_4)}\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|)}2^{(m)} \\ 2^{(5-|k|-m)} \end{vmatrix}$$

$$= -\binom{k_1+1+k_2}{k_2} Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} +$$

$$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\ Z_{21}^{(m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} -$$

$$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} +$$

$$\binom{k_4+m}{m} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix}$$

and

$$\partial_\varkappa S_4\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|)}2^{(m)} \\ 2^{(5-|k|-m)} \end{vmatrix}\right)$$

$$= \partial_\varkappa\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix}\right)$$

$$= \binom{k_1+1+k_2}{k_2} Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} -$$

$$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} +$$

$$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} -$$

$$\binom{k_4+m}{m} Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+m)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix} +$$

$$Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(7+|k|+m)} \\ 2^{(5-|k|-m)} \end{vmatrix}$$

From the above, we have that $\{S_0, S_1, S_2, S_3, S_4\}$ is a contracting homotopy [7], which means that our complex is exact.

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